Designing Two-Dimensional Spectrum Auctions for Mobile Secondary Users

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***Abstract*—Dynamic spectrum access by non-licensed users has emerged as a promising solution to address the bandwidth scarcity challenge. In a secondary spectrum market, primary users lease chunks of unused spectrum to secondary users. Auctions perform as one of the natural mechanisms for allocating the spectrum, generating an economic incentive for the licensed user to relinquish channels. Existing spectrum auction designs, while taking *externality* introduced by interference into account, fail to consider the potential *mobility* of secondary users, which leads to another dimension of externality: mobile communication motivates a secondary user to exclusively occupy a channel, *i.e.*, forbidding channel reuse in its mobility region. In this work, we design two expressive auctions for mobility support, by introducing two-dimensional bids that reflect a secondary user's willingness to pay for exclusive and non-exclusive channel usage, for the single-channel and multiple-channel scenarios, respectively. In the outcome of our 2D auctions, a channel is either monopolized or simultaneously reused without interference, whereas a secondary user can be mobile or is regulated to be static. We prove the existence of desirable equilibria in both auctions, where $\frac{1}{10}$ and $\frac{c}{7(1+c)}$ of optimal social welfare are guaranteed to be recoverable, respectively (c is the number of channels).**

***Index Terms*—Exclusive Bidding, Two-Dimensional Spectrum Auction, Mobile Secondary Users**

I. Introduction

Due to the paradigm shift to smartphones and related mobile Internet usage, wireless spectrum is now subject to heavy congestion. At the same time, studies from regulatory bodies such as the Federal Communications Commission (FCC) [1] in the United States have recognized that traditional fixed spectrum allocation schemes can be rather inefficient, in both temporal and spatial domains. An apparent remedy to the spectrum scarcity myth, introduced by the *status quo* static allocation, is to have a spectrum licensee resell its idling spectrum chunks (channels) to non-licenced users, with the support of a well designed *secondary spectrum market*.

In such a secondary spectrum market, auctions are a natural choice for the spectrum owner, or the *primary user* (PU), to relinquish its unused channels to *secondary users* (SUs) for monetary remuneration. A spectrum auction elicits bids for channel access from SUs, computes a channel allocation and the associated channel charges. An important goal in spectrum auction design is to maximize *social welfare* — the aggregated 'happiness' of everyone in the system. A 'good' auction tends to allocate channels to SUs who value them the most.

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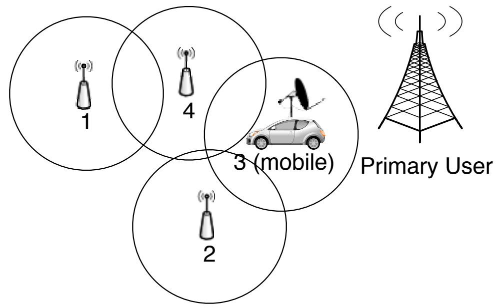


Fig. 1. An exclusivity-enabled secondary spectrum market with 4 SUs and 2 channels.

As one of the most efficient allocation mechanisms, spectrum auctions have attracted strong research attention in recent years. A salient feature of spectrum auctions, as compared to classic auctions from the field of economics, is the need to handle wireless interference among SUs properly. Most existing spectrum auctions in the literature [2]–[5] are designed to appropriately model the externality resulting from such interference, for better channel reuse and hence a larger social welfare. Unfortunately, to the best of our knowledge, all existing spectrum auction designs are based on the implicit assumption that SUs are static. Although the auction may be held periodically [5], and the location of an SU may vary from one round to another, generally all SUs are required to be static — at least within one round of the auction. In practice, a secondary network may indeed be heterogeneous. While some SUs are content with static communication, others prefer not to compromise their ability to communicate on the move — after all, being mobile has been one of the original driving forces behind wireless communication [6]. A highly mobile SU incurs potential interference with all other SUs within its mobility range. It would demand for exclusive usage of a channel.

In the economy of a secondary spectrum market, whether exclusivity is a desirable choice depends on the behaviour of all SUs, particularly, on their submitted bids. Intuitively, if an SU's bid is high enough to exclude all the other users from accessing a specific channel, it may own that channel with the guarantee that no one else will reuse it in the entire region. For example, in Fig. 1, there are 4 SUs (SU 1, 2, 3, 4) and two available channels (ch1, ch2) in the region. SUs whose transmission regions overlap interfere with one another. Imagine SU 3 has the demand to move within the entire region without communication conflict, and SU 3 submits a dominantly high bid leading itself to the exclusive use of channel 1. The other users, given their relatively uncompetitive bids, can still share (reuse) the other channel available. One possible scenario is that SU 1 and 2 share channel 2, while SU 4 is not allocated with a channel due to interference and its low bid.

To enable such exclusive channel access to support mobility, we adopt a two-dimensional bidding language, assuming SUs have two-dimensional valuations: one for exclusively accessing a channel, and another for reusing a channel. Let's denote a 2D bid as $(b, \bar{b})$. If an SU prefers either exclusive channel access or no access at all, it may submit a bid of zero for reuse ($\bar{b} = 0$). An SU without mobility demand may submit two identical values in its bid ($b = \bar{b}$). More generally, an SU may submit two non-zero values in its bid ($b \geq \bar{b} \geq 0$), a higher value for exclusive, mobile channel access and a lower value for non-exclusive, static channel access.

A traditional spectrum auction design essentially has the second bid $\bar{b}$ (for channel reuse) only, or can be viewed as a special case of our 2D auction where SUs are required to bid $b = \bar{b}$. The additional dimension of information in b helps our 2D spectrum auction decide whether channels are to be allocated exclusively or reused. For each channel, we have two possible allocations: S, where the channel is entitled to a single SU in the region, and M, where the channel is shared by multiple SUs. Designing a spectrum auction that accommodates such 2D bids and judiciously decides between the two possible channel allocation outcomes — for the goal of optimal social welfare — becomes a new, intriguing subject of study.

In this paper, we design two such auctions for auctioning a single channel and multiple channels, respectively, with provable constant approximation ratios in terms of the social welfare achieved. When there is only one channel to be sold by the PU in the region, we propose GR_{2D}, a 2D channel auction that refers to recent techniques due to Gopinathan *et al.* [4] for channel allocation and payment calculation in outcome M, with the classic Vickrey auction mechanism [7] to determine the pricing rules. We design the rule for deciding whether the outcome should be S or M, and tailor the pricing rule to elicit good social welfare. We prove that, in all "good" equilibria where losers bid at least their true valuations (a practical assumption that ensures losers are envy-free; otherwise they have the motivation to bid higher for a chance to win), the social welfare is guaranteed to be at least $\frac{1}{10}$ of the optimal. Furthermore, we prove that such good equilibria always exist.

Our second auction, VCG_{2D}, which adopts the technique proposed by Al-Ayyoub and Gupta [8] to reduce allocation complexity due to interference, is tailored for the multiple channel setting. We partition the network region into hexagon cells, and colour the cells. Each reusable channel is allocated to hexagons with the same colour, and SUs in co-coloured hexagons do not interfere with each other. The purpose of such network partitioning is to simplify the interference structure, preparing for the application of the canonical VCG mechanism [7], [9], [10] towards channel allocation under both outcome M (in each hexagon) and outcome S. The pricing rule, bearing the idea of VCG, is carefully designed to control the dependence between different channels, making sure that good equilibria of the auction exist. Our multi-channel 2D auction achieves similar properties as GR_{2D}, with a $\frac{c}{7(1+c)}$-optimal social welfare guarantee, where c is the number of channels.

In the remainder of this paper, we review related literature in Sec. II, present preliminary material in Sec. III, design and analyze our 2D spectrum auction for the single channel case in Sec. IV, and for the multiple channel case in Sec. V. Extensive simulation results are presented in Sec. VI. Sec. VII concludes the paper.

II. Related Work

Dynamic spectrum access has been envisioned as a solution to the spectrum sparsity problem. The cognitive approach relies on spectrum sensing technologies, so that secondary users are able to probe the holes of spectrum and ensure that their transmissions do not interfere with those of the primary user [11]. In the auction approach, the primary user periodically leases idling channels to secondary users for short time durations using spectrum auctions. This coordinated scheme does not assume that the primary user is even unaware of the existence of the secondary users as the uncoordinated one. The primary user obtains extensive control over spectrum from the spectrum regulator, and is able to resell channels in a secondary spectrum market.

The large volume of work on spectrum auction designs dates back to almost a decade ago. Huang *et al.* [12] propose two auction-based mechanisms for sharing spectrum, highlighting the unique challenge from wireless interference constraints. Another early solution is due to Buddhikot and Ryan [13], in which spectrum access is coordinated and controlled by a spectrum broker. VERITAS [14] is the first spectrum auction based on a monotonic channel allocation rule, and is thus truthful. Zhou and Zheng propose TRUST [15], a truthful double auction with multiple sellers (licensed users). To enable more flexible requirements including the usage of time and frequency, combinatorial auctions are considered recently [16]. Unfortunately, none of these auctions are expressive enough to enable exclusive channel access, which is critical for the support of mobile wireless users.

Not restricted to exclusivity, the issue of externality in auctions has been considered more generally in economics. Jehiel *et al.* [17] design multi-dimensional auctions where winners not only care about just winning, but also who else wins. While general and expressive enough, it does not take computational challenges into account. A number of works have introduced externalities into their designs for online advertisement [18]–[21]. However, they cannot be directly applied to spectrum auctions, as the externalities in spectrum auctions are far more complicated when wireless interference is considered.

Kash *et al.* [22] propose a spectrum auction enabling the sharing of a channel within an SU's interference range. By introducing a binary variable into the valuation of each SU to indicate the willingness to share, this auction primitively models exclusive/non-exclusive access of a channel for static SUs of heterogeneous types, with different transmission power and spectrum needs. However, their work is less expressive than ours, and more importantly, is still limited to static SUs. Deek *et al*. [23] design *Topaz*, an online spectrum auction that adopts three-dimensional bids — including channel valuation and the claimed arrival and departure time. Their goal is to discourage bidders from misreporting both their valuations and channel access time window. Our approach, as a two-dimensional auction, is quite different from *Topaz*, since the extra dimension(s) are from exclusive and mobility channel access instead of from the temporal domain.

III. Preliminaries

In this section, we present some background knowledge on auction design in Sec. III-A, and then precisely define our problem in Sec. III-B.

A. Auction Design

In an auction, agents compete over a set of items through a bidding system. Generally, it can be described as the following [24]:

1) A finite set $\mathcal{O}$ of *allowed outcomes*.
2) Each agent has a privately-known real value, called its valuation, which quantifies the bidder's benefit from the outcome.
3) Bidders are required to submit/declare their valuations in terms of bids. The bidders may lie about their valuations. Thus the bid of an agent may not be equal to its valuation.
4) An auction chooses an outcome $\mathbf{o}$ based on some criteria over the vector of declared bids.
5) In addition to determining an outcome, the auction also charges each bidder a certain amount of currency.
6) The utility of each bidder is the difference between its true valuation and its payment, based on the outcome.
7) The utility of the auctioneer is the revenue it gathers, *i.e*., the sum of the payments from all the bidders.

In this paper, we will focus on a natural and important goal of auction design, *social welfare maximization*. Social welfare is defined as the sum of all the winning agents' valuations, which can be viewed as the aggregated 'happiness' (utility) of everyone in the system, including the auction holder (note that the sum of the payments cancels the revenue of the auctioneer when computing the total social welfare). Adopting conventional assumptions in the literature, we assume that each agent is selfish and rational. A *selfish* agent is one that acts strategically to maximize its utility. An agent is said to be *rational* when it always prefers the outcome that brings itself a higher utility. Hence, an agent may lie about its valuation if doing so yields a higher utility.

An auction is said to be *truthful* when bidders' optimal behaviour is to report their true valuations, regardless of others' bids, *i.e*., declaring their valuation truthfully can maximize their utilities in such an auction setting. In our paper, considering truthful auctions is necessary because simplifying bidder strategies in one dimension (without changing outcome of the auction) is desirable.

The only general auction that aims at optimizing social welfare and guarantees truthfulness is due to Vickrey, Clarke, and Groves (VCG) [7], [9], [10]. Informally, the celebrated VCG auction finds the optimal outcome $\mathbf{o}$ that maximizes the social welfare, and charges each winning agent i an amount equal to the total "damage" that it causes to the other bidders, *i.e*., the difference between the social welfare of the others with and without i's participation [25]. However, it requires optimal channel allocation, whose computation is NP-complete [26], and hence real-time spectrum auctions rely on other heuristic algorithms.

B. Problem Definition

We denote the set of secondary users (SUs) in the network as $\mathcal{N}$, where $|\mathcal{N}| = n$. A primary user (PU) is referred to as an *auctioneer*, who owns the set of all the available channels, $\mathcal{C}$, and $|\mathcal{C}| = c$. The PU is responsible to run the auction in multiple rounds repeatedly, in which secondary users perform as *bidders* (or *agents*). Each bidder is interested in at most one channel in each round. An SU is a wireless node equipped with a single radio, capable of switching among channels. Denote by $d(i, j)$ the geographical distance between nodes i and j, and by r the homogeneous service range of all the nodes. We say i and j interfere if both are assigned the same channel, and $d(i, j) \leq 2r$. These interference constraints can be captured by a conflict graph G [4]. A channel allocation is feasible if no two agents interfere.

A bidder i's valuation can be represented as a pair $(v_i, \bar{v}_i)$, which is often private information known only to the bidder itself. v_i and $\bar{v}_i$ are the largest monetary values that bidder i is willing to pay. Here v_i is bidder i's valuation for exclusively winning a channel, allowing its mobile access to the channel in the entire network region without interference. $\bar{v}_i$ is i's valuation for jointly winning a channel with other users, *i.e*., it wins a channel under the condition that it (statically) shares the channel with other SUs. A bidder (weakly) prefers exclusivity, *i.e*., $v_i \geq \bar{v}_i$.

We consider the scenario in which all c channels are identical and orthogonal. However, channels may experience different levels of fading and path loss, for secondary users who are geographically distributed in the region. To make the problem tractable, we assume that all the channels are identical for the same user at a specific location, *i.e*., all the channels experience the same fading and path loss at the same location. By employing a learning framework, an SU's values at round t can be formed into functions as in (1) and (2) [27], where s_i^t is user i's status (on/off etc.), y_i^t is the channel status for all the channels viewed by i, x_i^t is the interference experienced by i, $\mathbf{o}_i^t$ is the outcome vector for all the channels for i and p_i^t is i's payment, all of which are statistics to learn for SU i at round t [27]. In this work, we will focus on a specific round of an auction, so the superscript t will be omitted. However, values can still be learned and computed from historical statistics. They are monetary worth of a channel, which is the maximum price a bidder is willing to pay.

An auction determines an item allocation to competitive bidders, based on their submitted bids. In our case, each bidder submits a two-dimensional bid denoted by $(b_i, \bar{b}_i)$, for

$$v_i^t = f(s_i^0, y_i^0, x_i^0, \mathbf{o}_i^0, p_i^0, \ldots, s_i^{t-1}, y_i^{t-1}, x_i^{t-1}, \mathbf{o}_i^{t-1}, p_i^{t-1}, s_i^t, y_i^t) \tag{1}$$

$$\bar{v}_i^t = \bar{f}(s_i^0, y_i^0, x_i^0, \mathbf{o}_i^0, p_i^0, \ldots, s_i^{t-1}, y_i^{t-1}, x_i^{t-1}, \mathbf{o}_i^{t-1}, p_i^{t-1}, s_i^t, y_i^t) \tag{2}$$

exclusive or non-exclusive access to a channel, respectively. Again, let S (for exclusive access) and M (for reuse) represent these two types of outcomes. Furthermore, the auction computes payments for winning bidders, and charges them with monetary amounts accordingly. We denote by p_i the payment of bidder i. Then the utility of i can be calculated as:

$$u_i = \begin{cases} v_i - p_i & \text{if } i \text{ obtains a channel and its outcome is S} \\ \bar{v}_i - p_i & \text{if } i \text{ obtains a channel and its outcome is M} \\ 0 & \text{if } i \text{ loses} \end{cases}$$

We refer to v_i and b_i as S-value and S-bid of agent i, and $\bar{v}_i$ and $\bar{b}_i$ as its M-value and M-bid, respectively. We employ such a quasi-linear utility assumption, which is almost universal in the auction theory literature.

Note that we employ such a utility function in an economic setting, where in some spectrum auction designs, v_i and $\bar{v}_i$ are viewed as users' utility. For example, f and $\bar{f}$ can be in logarithmic form, with respect to the channel quality [2]. A user is asked to submit this utility in terms of bid (it may lie) as a monetary value, the amount of which reflects an economic benefit from channel bandwidth or transmission rate if it wins.

In an equilibrium of the spectrum auction game, no SU can improve its utility by unilaterally changing its bid. We focus on the equilibria where losers bid at least their true valuations, which we refer to as "good" equilibria. A good equilibrium is more stable in practice due to its 'envy free' property, *i.e.*, a losing SU clearly has no motivation to bid higher for a chance to win a channel. A 'bad' equilibrium, on the other hand, is not stable, especially in our 2D auction setting, where it is possible for an SU to bid higher to change the allocation of a channel from S to M.

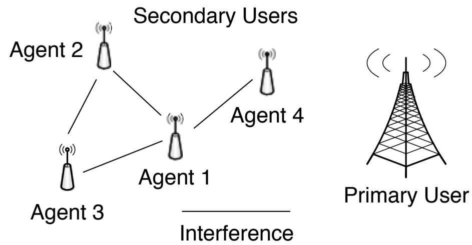


Fig. 2. A simple illustration for Gopinathan's auction.

IV. GR_{2D}: A Greedy Auction Enabling Exclusivity for The Single Channel Case

We start our 2D auction design for enabling exclusive channel access, and hence SU mobility, with the simpler case where a single channel is to be auctioned by the PU. We extend our studies to the case of multiple channels in Sec. V. We present the design of our auction, GR_{2D}, explain the rationale and technique behind its design, and analyze its properties. We prove that good equilibria always exist under GR_{2D}, under which a constant fraction of optimal social welfare can be achieved.

A. The design of GR_{2D}

Our design of GR_{2D} has been inspired by two previous works: the classic Vickrey auction [7] and a recent greedy spectrum auction due to Gopinathan *et al*. [4]. More specifically, we are inspired by and adapt ideas in the former for computing charges in outcome S, and the latter for outcome M. The Vickrey auction is a simple yet truthful mechanism suitable for allocating a single item to competing bidders. It selects a bidder with the highest bid, and charges it the second-highest bid. Outcome S in our 2D auction of a single channel resembles the setting of a Vickrey auction. The auction of Gopinathan *et al.*, on the other hand, is specifically designed for channel allocation in secondary spectrum markets, with spatial reuse of the channel in mind. It is a greedy and truthful auction that approximately optimizes social welfare. The auction greedily allocates the channel to agents, as long as the assignment is feasible, in non-increasing order of their bids. The payment calculation is based on *critical bids*: Setting some winning agent i's bid to 0 and run the greedy allocation algorithm based on the new vector of bids. The first agent j that makes it infeasible to allocate the channel to i is said to be the *critical bidder* of i, and the actual winning bidder i is charged by j's bid.

For example, in the single-channel secondary spectrum market shown in Fig. 2, we assume SU 1, SU 2, SU 3, and SU 4 bid 7, 8, 6 and 5, respectively. Then SU 2 and SU 4 will win the channel with payments 7 and 0 respectively, while SU 1 and SU 3 lose. Note here that SU 1 is SU 2's critical bidder and there is no critical bidder for SU 4.

The auction of Gopinathan *et al.* achieves 5-approximate social welfare [4]. We apply the same technique of greedy channel allocation coupled with charging critical bids, to ensure truthfulness for outcome M in our 2D spectrum auction.

We are now ready to describe in detail GR_{2D}, a greedy auction enabling exclusive bidding. We order agents in non-decreasing order of their M-bids, so that $\bar{b}_1 \geq \bar{b}_2 \geq \bar{b}_3 \geq \cdots \geq \bar{b}_n$. Indices max and $\max^2$, in our case, are used to represent the bidders with the highest S-value and second highest S-value, *i.e.*, $v_{\max} \geq v_i$ for every i, and $v_{\max^2} \geq v_i$ for every $i \neq \max$. We will abuse notations to use $b_{\max}$ and $b_{\max^2}$ to denote the bidders with the highest and second highest S-bids, respectively. We use a vector of binary variables $\mathbf{o} = \{o_1, o_2, \cdots, o_n\}$ to indicate whether an agent is granted channel access or not in outcome M, by following the greedy channel allocation rule. We denote by another vector of binary variables $\mathbf{o}' = \{o'_1, o'_2, \cdots, o'_n\}$ the assignment result of running the algorithm with $(\bar{\mathbf{b}}_{-\max}, 0)$, where $\bar{\mathbf{b}}_{-\max}$ is the vector of M-bids except $\max$. Note that $o_i = o'_i$ for $i < \max$ because the algorithm is greedy and the allocation result of agents with larger M-bids than agent $\max$ is not affected. To decide between outcome S and M, a criterion is defined as follows. Assume $\max = j$, *i.e.*, the agent with $b_{\max}$ has the j-th largest M-bid. Compare $b_{\max}$ with $\Gamma = \sum_{i=1}^{n} o_i \bar{b}_i + \sum_{i \in \mathcal{I}(j)} \bar{b}_i$, where $\mathcal{I}(j) = \{j+1, j+2, \cdots, j+5\}$. The term $\sum_{i \in \mathcal{I}(j)} \bar{b}_i$ can be viewed as the maximum possible "damage" introduced by agent j. Given the underlying valuations, we decide whether the outcome is efficient or not based on this criterion, *i.e.*, based on whether $v_{\max} > \sum_{i=1}^{n} o_i \bar{v}_i + \sum_{i \in \mathcal{I}(j)} \bar{v}_i$.

The auction GR_{2D} is summarized in Algorithm 1.

Algorithm 1 GR_{2D} — A greedy auction enabling exclusive bidding

Input: All two-dimensional bids $(b_i, \bar{b}_i)$, conflict graph G
Select the agent with $b_{\max}$, and let $j \Leftarrow \max$;
Run Gopinathan's allocation algorithm on $\bar{\mathbf{b}}$ and $(\bar{\mathbf{b}}_{-j}, 0)$, obtaining $\mathbf{o}$ and $\mathbf{o}'$ respectively;
if $b_{\max} \geq \Gamma$ **then**
  The outcome is S with a single winner j;
  $p_j \Leftarrow \max(b_{\max^2}, \sum_{i=1}^{j-1} o_i \bar{b}_i + \sum_{i=j+1}^{n} o'_i \bar{b}_i)$;
else
  The outcome is M by running Gopinathan's allocation algorithm on $\bar{\mathbf{b}}$;
  Let $c(\cdot)$ be the critical bidder of an agent;
  $p_j \Leftarrow \max(\bar{b}_{c(j)}, b_{\max^2} - \Gamma + \bar{b}_j)$, if j wins;
  $p_i \Leftarrow \max(\bar{b}_{c(i)}, b_{\max} - \Gamma + \bar{b}_i)$, for $i \neq j, i \notin \mathcal{I}(j)$;
  $p_i \Leftarrow \max(\bar{b}_{c(i)}, b_{\max} - \Gamma + 2\bar{b}_i)$, for $i \in \mathcal{I}(j)$;
end if

B. Analysis of GR_{2D}

In the remainder of this section, we will analyze the performance of GR_{2D}, by (a) proving the existence of good equilibria under this auction, and (b) proving its constant approximation property in terms of social welfare, under any good equilibrium.

We start by proving the following two lemmas, which will assist us to prove the performance of GR_{2D} in its good equilibria.

Lemma 1. *Assume agent* $\max$ *bids truthfully. If the outcome of the auction for a given vector of bids is S, then the winner* $\max$ *cannot benefit from any deviation that changes the outcome to M.*

Proof: We first assume $\max = j$, *i.e.*, the agent who has $b_{\max}$ has the j-th largest M-bid. By the assumption that bidder $\max$ bids truthfully, we have $b_{\max} = v_{\max}$ and $\bar{b}_j = \bar{v}_j$. We need to show that bidder $\max$ always prefers outcome S to outcome M. That is, the utility of $\max$ in outcome S is always no less than that in outcome M.

If bidder $\max$ loses in outcome M, then its utility in M is 0. In the computation of the price for outcome S, the second term is smaller than Γ, and $\max$ will have nonnegative utility in outcome S. In this case, it will not benefit from changing the outcome to M.

If bidder $\max$ wins in outcome M, then we need to compare its utility in the two outcomes, *i.e.*, we need to show

$$v_{\max} - \max(b_{\max^2}, \sum_{i=1}^{j-1} o_i \bar{b}_i + \sum_{i=j+1}^{n} o'_i \bar{b}_i) \geq \bar{v}_j - \max(\bar{b}_{c(j)}, b_{\max^2} - \Gamma + \bar{b}_j)$$

We then prove this lemma by considering two cases, based on whether the dominant term for the payment of $\max$ in the outcome is $b_{\max^2}$ or not.

(i). Assume the dominant term is not $b_{\max^2}$, *i.e.*, $\max(b_{\max^2}, \sum_{i=1}^{j-1} o_i \bar{b}_i + \sum_{i=j+1}^{n} o'_i \bar{b}_i) = \sum_{i=1}^{j-1} o_i \bar{b}_i + \sum_{i=j+1}^{n} o'_i \bar{b}_i$. Then, we have

$$\begin{aligned} v_{\max} - (\sum_{i=1}^{j-1} o_i \bar{b}_i + \sum_{i=j+1}^{n} o'_i \bar{b}_i) &\geq \Gamma - \sum_{i=1}^{j-1} o_i \bar{b}_i - \sum_{i=j+1}^{n} o'_i \bar{b}_i \\ &= \bar{v}_j - \sum_{i=j+1}^{n} (o_i - o'_i) \bar{b}_i \\ &\quad + \sum_{i \in \mathcal{I}(j)} \bar{b}_i \\ &\geq \bar{v}_j \end{aligned}$$

which implies

$$v_{\max} - \max(b_{\max^2}, \sum_{i=1}^{j-1} o_i \bar{b}_i + \sum_{i=j+1}^{n} o'_i \bar{b}_i) \geq \bar{v}_j - \max(\bar{b}_{c(j)}, b_{\max^2} - \Gamma + \bar{b}_j)$$

(ii). Assume $\max(b_{\max^2}, \sum_{i=1}^{j-1} o_i \bar{b}_i + \sum_{i=j+1}^{n} o'_i \bar{b}_i) = b_{\max^2}$, we have

$$v_{\max} - b_{\max^2} \geq \Gamma - b_{\max^2} = \bar{v}_j - (b_{\max^2} - \Gamma + \bar{b}_j)$$

which again implies

$$v_{\max} - \max(b_{\max^2}, \sum_{i=1}^{j-1} o_i \bar{b}_i + \sum_{i=j+1}^{n} o'_i \bar{b}_i) \geq \bar{v}_j - \max(\bar{b}_{c(j)}, b_{\max^2} - \Gamma + \bar{b}_j)$$

■

Lemma 2. *Overstating the M-value is weakly dominated in GR_{2D}. That is, for agent i who participates in the auction, any strategy that sometimes overstates the M-value is dominated by another strategy that never overstates the M-value.*

Proof: Fix the vector of all the bids except i's. First, assume the outcome is S. In this case, from Lemma 1, bidder $\max$ will not benefit from any deviations to outcome M, so it will not overstate its M-value. For bidder $i \neq \max$, if it changes the outcome to M by overstating its value, its payment will be larger than its true value because of the pricing policy. Next, assume the outcome is M. If bidder i chooses to overstate its M-value, either its critical bidder does not change or it wins with a negative utility. Therefore, it cannot benefit from overstating its value in this case either. ■

We are now ready to present the following result:

Theorem 1. *The social welfare in any good equilibrium of GR_{2D} is at least $\frac{1}{10}$ of the optimal.*

Proof: We prove this theorem in two cases: the efficient outcome is S, or M, based on the underlying valuations such that $v_{\max} > \sum_{i=1}^{n} o_i\bar{v}_i + \sum_{i\in\mathcal{I}(j)} \bar{v}_i$ or $v_{\max} <= \sum_{i=1}^{n} o_i\bar{v}_i + \sum_{i\in\mathcal{I}(j)} \bar{v}_i$, respectively.

Efficient outcome is S: Suppose the efficient outcome of our auction is S, *i.e.*, $v_{\max} > \sum_{i=1}^{n} o_i\bar{v}_i + \sum_{i\in\mathcal{I}(j)} \bar{v}_i$. We show that all bidders except $\max = j$ bid truthfully, and $\max$ bids $(v_{\max}, 0)$, is an equilibrium of the auction.

First, these bids lead to outcome S because

$$v_{\max} > \sum_{i=1}^{n} o_i\bar{v}_i + \sum_{i\in\mathcal{I}(j)} \bar{v}_i = \Gamma$$

Then we need to show that the outcome under this vector of bids is such that no individual bidder has an incentive to deviate from. If bidder i wishes to change the outcome to M by raising its current bid to $\bar{b}_i > \bar{v}_i$, and change itself to a winner, its utility will be negative: By the pricing rule we have $p_i \geq v_{\max} - \sum_{l=1,l\neq i}^{n} o_l\bar{b}_l$; since the initial outcome is S, we have $v_{\max} - \sum_{l=1,l\neq i}^{n} o_l\bar{b}_l > \bar{v}_i$. Therefore, $p_i > \bar{v}_i$, leading it to a negative utility. Now we need to show bidder $\max$ has no incentive to deviate, *i.e.*, it has no benefit from any deviation that changes the outcome to M. This is directly implied by Lemma 1. Hence, the vector of bids where every bidder bids their true value except that $\max$ bids $(v_{\max}, 0)$ is a good equilibrium of our auction.

Further, we have proved in Lemma 2 that bidding above one's M-value is a weakly dominated strategy in our auction. Suppose all bidders are playing undominated strategies, then no bidder is bidding higher than its M-value. Since the efficient outcome is S, if $\max$ bids truthfully and every other bidder bids less or equal to its M-value, the outcome will be S. We know from Lemma 1 that bidder $\max$ always prefers this outcome to outcome M. Therefore, bidding truthfully is always profitable for $\max$.

Note that inefficient equilibria with outcome M can occur when the efficient outcome is S. However, in all such equilibria, there must be some SU bidding above its true value, which is a weakly dominated strategy in GR$_{2D}$, by Lemma 2. Therefore, when the efficient outcome is S, the social welfare in its good equilibria would be $v_{\max} > \sum_{i=1}^{n} o_i\bar{v}_i$, which is at least $\frac{1}{5}$ optimal social welfare.

Efficient outcome is M: Then suppose the efficient outcome is M. If the actual outcome is M as well, the agents in equilibria would bid truthfully as in Gopinathan's [4], and the social welfare is at least $\frac{1}{5}$ of the optimal. If the outcome is S, then the winner would be $\max$ in equilibria; otherwise, $\max$ would be a loser and it would bid at least its true S-value, in which others are bidding at least their true values as well, leading to $\max$ a negative utility. Hence it would be a contradiction with the equilibrium assumption. Therefore, the social welfare is $v_{\max}$. Let $\max = j$. In such kind of equilibria, bidder $\max$ must lie about its M-value so that $\bar{b}_j < \bar{v}_j$ or raise its S-value to $b_{\max} > v_{\max}$, to make the auction outcome to S.

Since the losers bid at least their true values, we have

$$2v_{\max} \geq \bar{v}_j + \sum_{i=1}^{j-1} o_i\bar{v}_i + \sum_{i=j+1}^{n} o_i'\bar{v}_i \geq \sum_{i=1}^{n} o_i\bar{v}_i \quad (3)$$

Since $\sum_{i=1}^{n} o_i\bar{v}_i$ achieves $\frac{1}{5}$ optimal social welfare [4], the social welfare in any good equilibrium of GR$_{2D}$ is at least $\frac{1}{10}$ of the optimal. ∎

Note that employing truthful auctions for outcome S and M ensures that bidders have no incentives to lie if outcome does not change. Therefore, the challenge of our auction design is to eliminate the incentives of losers raising their bids so as to change between the channel allocation results, which is achieved by the design of our payment scheme.

Intuitively, the lower bound of social welfare tells how much we will lose at most if we enable exclusive bidding, since winning bidders may lie about their true values to change the underlying outcome. Our simulation results in Sec. VI show that GR$_{2D}$ can do much better than this lower bound on average.

V. VCG$_{2D}$: A VCG-Based 2D Auction for The Multiple Channel Case

A. GR$_{2D}$ Is Not Suitable for Auctioning Multiple Channels

When addressing the problem of designing auctions for multiple channels, GR$_{2D}$ is no longer applicable. As explained below, GR$_{2D}$ becomes highly complicated when there is more than one channel.

In the single channel case, if the agent with $b_{\max}$ loses in outcome S, it will either lose or win the channel in outcome M, leading to a simple payment calculation. When multiple channels are auctioned instead, we need to first run a specific allocation algorithm to obtain an initial result for outcome M for all the channels, which is represented by a $c \times n$ matrix $\mathbb{O} = \{\mathbf{o}(1), \mathbf{o}(2), \cdots, \mathbf{o}(c)\}$, where channels are indexed by $1, 2, \cdots, c$. For every channel $k \in \mathcal{C}$, $\mathbf{o}(k)$ is a vector of binary variables $\{o_i(k)\}_{i\in\mathcal{N}}$ indicating whether an agent i is allocated channel k. Afterwards, a mechanism needs to be designed to decide which channel will be exclusively used according to some agent's S-bid and the initial allocation result. When the allocation scheme of Gopinathan *et al.* is adopted for outcome M, an agent winning channel c exclusively may have incentives to deviate to outcome M, because it may win another different channel c', and by doing so, it may have a higher utility. That is, with a similar payment scheme design as in GR$_{2D}$, it is difficult to compare the utilities of an agent in outcome S and M winning two different channels, incurring a possibility that it prefers outcome M to S.

B. Network Partitioning for Interference Control

The design difficulty explained above motivated us to employ another auction mechanism for outcome M, which aims to reduce the complexity due to interference. Our approach here is based on the classic idea of network partitioning, which was also applied by Al-Ayyoub and Gupta [8] in their spectrum auction design that tries to maximize revenue. We next describe the partitioning process and its role in the auction design.

Given a network with SUs remaining at their initial locations and the conflict graph under the unit disk model, we compute a valid allocation with approximate social welfare as the following:

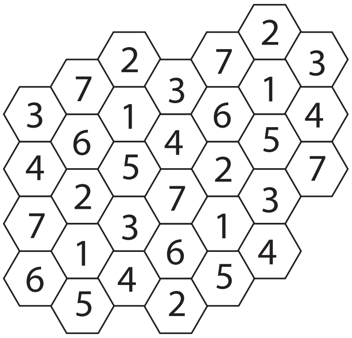


Fig. 3. Uniformly coloured hexagons, with 7 colours.

1) Divide the entire region into a set $\mathcal{H}$ of small regular hexagons, each with side length r.
2) Uniformly colour the hexagons with seven colours, as illustrated in Fig. 3.
3) Allocate channels to SUs in each hexagon *independently* according to the VCG mechanism.
4) For each colour, sum up the valuations of all the winners in hexagons of that colour.
5) Select the colour that has the maximum welfare, and allocate the channels to the winners accordingly.

By dividing the region into hexagons of side length r, we ensure that any two of SUs in the same hexagon interfere with each other, *i.e.*, the conflict graph in a hexagon is a complete graph. After uniformly colouring the hexagons with 7 colours as in Fig. 3, we further have the following property for SUs:

Property 1. *SUs in different co-coloured hexagons do not interfere with each other.*

This property can be obtained directly from the following observation: the distance between any two points in two co-coloured hexagons is at least $\sqrt{7}r > 2r$, as shown in Fig. 4.

A channel cannot be reused within the same hexagon, but can be reused across different hexagons of the same colour. Therefore, in each hexagon, we can apply the VCG mechanism to allocate channels, in which the welfare is optimized. Since SUs do not interfere with each other in different hexagons of the same colour, we can combine allocations of co-coloured hexagons to form a network-side allocation. As a result, we obtain 7 allocations, one for each colour [8]. Among these allocations, we select the one with the highest social welfare, which is guaranteed to have at least a $1/7$-factor of the optimal. We denote this algorithm as A.

C. Design of VCG$_{2D}$ for Multiple Channels

We now describe how to enable exclusivity for mobile SUs based on the auction described above. We first run the allocation process, obtaining matrix $\mathbb{O}$. For each channel $k \in \mathcal{C}$, let $\Lambda(k) = \sum_{i=1}^{n} o_i(k)\bar{b}_i$. Without loss of generality, assume $\Lambda(1) \geq \Lambda(2) \geq \cdots \geq \Lambda(c)$. Then we order agents according to their M-bids so that $\bar{b}_1 \geq \bar{b}_2 \geq \cdots \geq \bar{b}_n$, and order them by S-bids so that $b_{(1)} \geq b_{(2)} \geq \cdots \geq b_{(n)}$. Select c agents with $b_{(1)}, b_{(2)}, \cdots, b_{(c)}$, and compare their S-bids with $\frac{1}{c}\sum_{k=1}^{c}\Lambda(k) + \bar{b}_1$ in order. Once we have $b_{(l)} > \frac{1}{c}\sum_{k=1}^{c}\Lambda(k) + \bar{b}_1$, a randomly selected channel will be exclusively used by agent (l). Intuitively, in this criterion, social welfare is averaged over the number of channels to make the payment calculation independent of channels, which we will present later. Similar to GR$_{2D}$, we use the criterion to decide whether the outcome for a channel c is efficient or not, *i.e.*, whether $v_{(l)} > \frac{1}{c}\sum_{k=1}^{c}\Lambda'(k) + \bar{v}_1$, where $\Lambda'(k) = \sum_{i=1}^{n} o_i(k)\bar{v}_i$. After the comparisons and assignments for outcome S, the channels that can be reused will be assigned to SUs according to the allocation for outcome M.

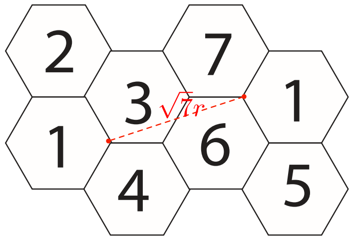


Fig. 4. Minimum distance between two points in co-coloured hexagons.

We then proceed to payment calculation. We assume that in outcome S, the set of winners are $\mathcal{W}_s$, and $|\mathcal{W}_s| = w_s$. For a winner $j = (l)$ and $l \leq c$, we assume it is located in hexagon $h_j \in \mathcal{H}$. We order agents in h_j according to their M-bids so that $\bar{b}_1^{h_j} \geq \bar{b}_2^{h_j} \cdots \geq \bar{b}_{n(h_j)}^{h_j}$, where $\bar{b}_i^{h_j}$ is the M-bids of agent i in hexagon h_j and $n(h_j)$ is the number of agents in h_j. For simplicity, we assume that $n(h_j) > c$, *i.e.*, the number of agents in each hexagon is larger than the number of channels. When we run the VCG mechanism in each hexagon, there are $c - w_s$ winners. If j wins a channel in outcome S, then we charge j either the sum of averaged total social welfare resulted from $\mathbb{O}$ (except j) and $\bar{b}_{c-w_s+2}^{h_j}$ (the payment according to VCG auction without j in h_j), or the (w_s+1)-th maximum S-bid, $b_{(w_s+1)}$, whichever is larger, *i.e.*,

$$p_j = \max(b_{(w_s+1)}, \frac{1}{c}\sum_{k=1}^{c}\Lambda(k) - \frac{1}{c}I(j)\bar{b}_j + I(j)\bar{b}_{c-w_s+2}^{h_j})$$

where $I(j) = \sum_{k=1}^{c} o_j(k)$, a binary variable indicating whether agent j is allocated a channel or not in initial outcome $\mathbb{O}$.

For other winners i in outcome M, which is located in hexagon h_i, we charge it with payment

$$p_i = \max(\bar{b}_{c-w_s+1}^{h_i}, b_{(w_s+1)} - \frac{1}{c}\sum_{k=1}^{c}\Lambda(k) + \frac{1}{c}\bar{b}_i)$$

We can now summarize VCG$_{2D}$ in Algorithm 2.

D. Analysis of VCG$_{2D}$ for Multiple Channels

Similar to the case of GR$_{2D}$, we show that good equilibria exist under VCG$_{2D}$, and analyze the social welfare achieved under such a good equilibrium.

Theorem 2. *The social welfare in any good equilibrium is at least $\frac{c}{7(1+c)}$ of the optimal, where c is the number of channels.*

Proof: Suppose the underlying valuations are such that the efficient outcomes for all the channels are S, *i.e.*, $v_{(1)} > \frac{1}{c}\sum_{k=1}^{c}\Lambda'(k) + \bar{v}_1$, $v_{(2)} > \frac{1}{c}\sum_{k=1}^{c}\Lambda'(k) + \bar{v}_1$, $\cdots$, $v_{(c)} > \frac{1}{c}\sum_{k=1}^{c}\Lambda'(k) + \bar{v}_1$. We show that bidder (1), (2), $\cdots$, (c) bid $(v_{(1)}, 0)$, $(v_{(2)}, 0)$, $\cdots$ $(v_{(c)}, 0)$ respectively and others bid truthfully, is an equilibrium of the auction.

Algorithm 2 VCG_{2D}: A VCG-based auction enabling exclusive bidding

```
Input: All the two-dimensional bids (b_i, \bar{b}_i), conflict graph G, a set of available channels C
Select the agents with b_(1), b_(2), ..., b_(c);
Let W_s <= ∅, w_s <= 0;
Run algorithm A with the M-bids of all the agents, obtaining Λ(k) for each channel k;
for each agent (l) with b_(1), b(2), ..., b_(c) do
    if b_(l) > Σ_{k=1}^{c} Λ(k) + \bar{b}_1 then
        W_s <= W_s ∪ {(l)}, w_s <= |W_s|;
    end if
end for
for each j = (l) ∈ W_s do
    j wins a randomly selected and unused channel k ∈ C exclusively;
    p_j <= max(b_(w_s+1), (1/c) Σ_{k=1}^{c} Λ(k) − (1/c) I(j) \bar{b}_j + I(j) \bar{b}^{h_j}_{c−w_s+2});
end for
For every other agent i, run algorithm A on their M-bids, with the remaining channels;
p_i <= max(\bar{b}^{h_i}_{c−w_s+1}, b_(w_s+1) − (1/c) Σ_{k=1}^{c} Λ(k) + (1/c) \bar{b}_i);
```

First of all, these bids lead the auction to outcome S for all the channels, because

$$v_{(l)} > \frac{1}{c}\sum_{k=1}^{c}\Lambda'(k) + \bar{v}_1, \quad \forall l = 1,\ 2,\ \cdots,\ c$$

Next we show that no individual bidder has incentive to deviate from the current auction result. If a loser i wishes to raise its S-bid to exclusively use a channel, its payment will be $p_i \geq v_{(w_s+1)}$, where $w_s = c$. Note that agent (w_s+1) is a winner before the change. Therefore we have $p_i \geq v_{(w_s+1)} \geq v_i$. If agent i wants to raise its M-bid to $\bar{b}_i > \bar{v}_i$, to change the outcome of a channel to M, its payment will be $p_i \geq v_{(w_s)} - \frac{1}{c}\sum_{k=1}^{c}\Lambda(k) + \frac{1}{c}\bar{b}_i$. However, the initial outcome of this channel is S, so we have $v_{(w_s)} - \frac{1}{c}\sum_{k=1}^{c}\Lambda(k) + \frac{1}{c}\bar{b}_i \geq \frac{1+c}{c}\bar{b}_i > \bar{v}_i$, leading i to a negative utility.

We then need to prove that winners have no incentive to deviate. For a winning SU $j = (l)$, there is no incentive for it to raise or reduce its S-bid while still winning a channel exclusively. We only need to prove that it has no incentive to raise its M-bid to change the outcome for some channel(s) to M. The proof is similar to that of Lemma 1, and is omitted here.

Since $\sum_{i=1}^{c} v_{(i)} > \sum_{k=1}^{c}\Lambda'(k)$, the social welfare in this case is at least $\frac{1}{7}$ of the optimal.

Suppose the efficient outcomes for some channels are S, and those of the others are M. All the winners in outcome S bid their S-values truthfully and their M-bids as 0, the other winners bid truthfully and the losers bid at least their true values, is a good equilibrium.

We assume that there are w_s winners in $\mathcal{W}_s$ in the auction if all the agents bid truthfully. Then apparently, these winners have the incentive to (potentially) increase their utility by reducing their M-bids to 0, thereby possibly reducing $\frac{1}{c}\sum_{k=1}^{c}\Lambda(k)$ and $b_{(w_s)}$. Note that this is dependent on $I(j),\ \forall j \in \mathcal{W}_s$. If so, we assume that the set of winners who can win a channel changes to $\mathcal{W}'_s$, and $w'_s = |\mathcal{W}'_s|$, where $w'_s \geq w_s$.

On the other hand, the winners in outcome M or the losers do not want to deviate from the current auction result. Consider a winning SU i who wins channel k in outcome M. If i raises its S-bid or lowers its M-bid to $b'_i > v_i$ or $\bar{b}'_i > \bar{v}_i$ respectively and wins a channel exclusively, its payment will be at least $v_{w'_s} \geq v_i$. If i lowers its S-bid or raises its M-bid, there is no benefit either. For a losing agent i, it bids at least its true value, and it cannot benefit by raising its S-bid or M-bid, which will not increase its utility due to the payment scheme.

Therefore, all the winners in outcome S bid their S-values truthfully and their M-bids as 0, the other winners bid truthfully and the losers bid at least their true values, is a good equilibrium.

The most inefficient case happens when the efficient outcomes for all the channels are M and the actual outcomes in the equilibrium are all S. In that case, for a winning agent $j \in \mathcal{W}_s$, we have

$$\begin{aligned} v_j &\geq \frac{1}{c}\sum_{k=1}^{c}\Lambda'(k) - \frac{1}{c}I(j)\bar{b}_j + I(j)\bar{b}^{h_j}_{c-w_s+2} \\ &\geq \frac{1}{c}\sum_{k=1}^{c}\Lambda'(k) - \frac{1}{c}\bar{v}_j \end{aligned}$$

Add $\frac{1}{c}v_j$ on both sides, we obtain

$$\frac{1+c}{c}v_j \geq \frac{1}{c}\sum_{k=1}^{c}\Lambda'(k) - \frac{1}{c}\bar{v}_j + \frac{1}{c}v_j \geq \frac{1}{c}\sum_{k=1}^{c}\Lambda'(k)$$

since $v_j \geq \bar{v}_j$.

We arrive at the following result by adding up for all the winning agents,

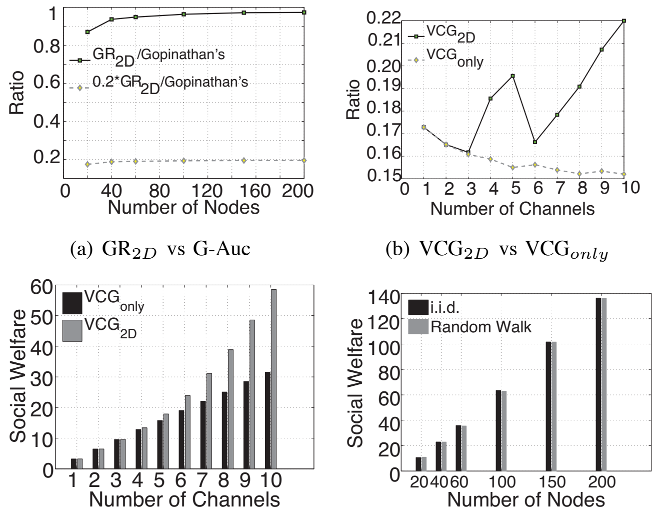
$$\frac{1+c}{c}\sum_{k=1}^{c}v_j \geq \sum_{k=1}^{c}\Lambda'(k)$$

Recall that $\sum_{k=1}^{c}\Lambda'(k)$ is the total social welfare achieved by simply partitioning and colouring the network, using the VCG mechanism for each hexagon, and selecting the colour with maximum social welfare, which achieves $\frac{1}{7}$ optimal social welfare. Therefore, the social welfare collected by our auction in any good equilibrium is at least $\frac{c}{7(1+c)}$ of the optimal. ■

We notice that, the approximation ratio for the multi-channel 2D auction improves as the number of channels grows. This is because our design of the payment scheme for outcome S is directly related to $\frac{1}{c}\bar{v}_j$ for a winner j, whose negative influence on the computation of performance bound is alleviated when c increases.

VI. Simulation Studies

We evaluate the performance of our auctions using MATLAB simulations. We randomly and uniformly distribute wireless nodes in a 1×1 geographical region. Two nodes interfere with one another if their Euclidean distance is smaller than 0.1. All M-bids follow a uniform distribution in $(0, 1)$. When setting S-bids, we need to make sure that outcomes



(a) GR_{2D} vs G-Auc

(b) VCG_{2D} vs VCG_{only}

(c) VCG_{2D} vs VCG_{only}

(d) Impact of mobility

Fig. 5. Performance evaluation of the 2D auctions.

change between S and M in different simulation instances. For simplicity, we generate S-bids from normal distributions with mean 1 and varying standard deviations for different settings. For those S-bids that are less than 1, we simply change them to 1, making sure that they are greater than corresponding M-bids. All the simulation results are averaged over 100 runs.

We first vary the number of nodes in the region, and draw S-bids from normal distributions with mean 1, standard deviation $0.25\times$*number of nodes*. We observe that the social welfare achieved by our auction under equilibrium is quite close to that of Gopinathan's, as shown in Fig. 5(a). It is validated that GR_{2D} is guaranteed to achieve $\frac{1}{10}$ optimal social welfare (by the grey dashed line in Fig. 5(a)). However, although the lower bound of Gopinathan's is $\frac{1}{5}$, in our simulation, it can actually achieve 0.8-0.9 optimal social welfare for all the cases in outcome M (we do not show here for want of space). Hence, as shown in Fig. 5(a), since on average GR_{2D} can achieve a ratio of more than 0.87 over Gopinathan's, the social welfare that it can collect on average is relatively high.

Fig. 5(b) and Fig. 5(c) show the performance of our second auction, VCG_{2D}, by varying the number of channels from 1 to 10 and drawing S-bids from normal distributions with mean 1, standard deviation $0.2\times$*number of channels*. We denote by VCG_{only} the method introduced in Sec. V-B, where network partitioning is performed and VCG auctions are conducted within every single hexagon, with M-bids. We can see that if the number of channels is less than 3, all the outcomes will be M for every channel (for almost all the simulation instances). However, when the number of channels is larger than 6, all the outcomes are S, and these may be in equilibria in the worst case regarding social welfare. We can observe from Fig. 5(b) that, when the number of channels increases to 6, the ratio of achieved social welfare by VCG_{2D} drops to 0.166 and then continues to increase. This is due to the fact that in some cases the bidders winning exclusive channels have the incentive to lie about their M values and may hurt the total social welfare that we may possibly achieve. However, VCG_{2D} can still outperform the one-dimensional auction in terms of social welfare in most cases on average, shown in Fig. 5(c).

Our analyses in this paper are independent of specific mobility models, because we assume that if a secondary user is allowed to move, it must have won a channel exclusively. That is, it can move in the region freely without any interference. In addition, we focus on a specific round of the auctions in our analyses, so new conflict graphs and bids are always obtained at the beginning of each round as inputs. These new inputs may change the outcomes of our auctions (*i.e.*, changes between S and M), but the analyses still hold. Nonetheless, it is conceivable that the movement of secondary users may affect the social welfare due to the change of the conflict graph. We investigate the role of mobility through simulations, by examining the random walk model. If a user wins a channel exclusively, it selects a direction from $\{\theta|\theta = 45^\circ \times N,\ N = 0, 1, 2, 3, ..., 7\}$ with the same probability and a step distance 0.1 for moving at this round (it stays on the edge of the 1×1 region if it "moves out" of the region). We found no significant difference from the i.i.d. model where mobile secondary users are redistributed in an i.i.d. way at the beginning of each auction round. Fig. 5(d) shows the result of simulating GR_{2D}. We omit the result of VCG_{2D} here, which is basically the same as Fig. 5(d). These results suggest the chances that a small number of moving users influence Γ or $\Lambda(k)$ are low, especially when there are a large population of secondary users in the region.

VII. Conclusion

Our main contribution in this work is to propose the first spectrum auctions that enable exclusive channel bidding, and therefore provide mobility support to secondary users. Our auctions take 2D bids by nature, and allocate a channel either exclusively or non-exclusively. We have combined recent techniques in spectrum auction design and interference control with classic tools from auction theory, and designed two auctions that work in the single channel and multi-channel scenarios, respectively. We have proved that good equilibria always exist in our auctions, and further proved that a constant fraction of optimal social welfare can always be achieved. A particular challenge in our proofs is the possibility for an SU to deviate from its current bid that changes a channel allocation status between exclusive and non-exclusive outcomes. As in multi-round auctions, rational bidders can learn from history and estimate a suitable bid amount for itself, so a good equilibrium can be reached. Investigating bidder behaviour in such auctions can be interesting future work.

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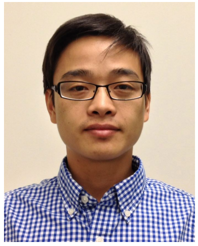

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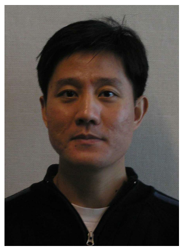

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