Ensuring Minimum Spectrum Requirement in Matching-Based Spectrum Allocation

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Abstract—To enable dynamic spectrum access, service providers with spare spectrum (sellers) trade with those who are in need of additional spectrum (buyers). In a spectrum market, the transaction result is essentially a matching between sellers and buyers. Though it is tempting to optimize the matching over certain utility functions, a stable matching is more desirable, since no participants have incentives to deviate from the matching result. Existing spectrum matching algorithms only consider the *maximum* number of channels a buyer can purchase, but ignore *minimum* spectrum requirement that is essential to support proper operation of wireless communications. In this paper, we present a new framework of spectrum matching with both maximum quota and minimum requirements. Different from conventional matching problems, the spectrum market poses distinctive challenges due to spectrum reusability. To tackle this problem, we design two novel algorithms that satisfy different stability criterion: Extended Deferred Acceptance (EDA) algorithm that is fair but wasteful and the Multistage Deferred Acceptance (MDA) algorithm that is non-wasteful but weakly fair. Both algorithms converge to an interference-free matching and guarantees the minimum spectrum requirement. The simulation results show that the two proposed algorithms can raise buyer happiness and the channel utilization.

Index Terms—Spectrum allocation, stable matching, minimum requirement

1 Introduction

Spectrum is an indispensable resource for wireless communication, yet it is also a limited resource that is strained for supporting the ever-increasing wireless traffic. To make the best use of available spectrum and to avoid underutilization due to static spectrum assignment, dynamic spectrum access has emerged to enable wireless service providers to buy or sell spare channels according to their demands [1].

Auction is a traditional way of dynamic spectrum redistribution in a spectrum market. The allocation process in a spectrum auction is essentially a matching that aims at maximizing social welfare or revenue. However, as opposed to optimizing the matching over certain utility functions, a stable matching is more desirable for a free spectrum market, due to two major reasons. First, buyers and sellers are selfish individuals, acting out of their own interests, which are not necessarily aligned with the system optimization. While the optimal matching may only be enforced, stable matching takes into account individual preferences of buyers and sellers, producing a matching result that no participants have incentives to deviate from. Second, optimization solvers are usually computationally hard, suffering from scalability problems, whereas the running time of stable matching algorithms is polynomial.

Stable matching for college admission problems was first studied by Gale and Shapley in their pioneering work [2], in which the *Deferred Acceptance* (DA) algorithm was proposed to match students to schools subject to maximum quotas of the schools. Since then, stable matching has been widely applied to resource allocation in computer science, such as virtual machine management in the cloud [3], user association in small cells [4], and spectrum sharing in device-to-device communication [5]. However, unlike traditional matching problems, spectrum matching features reusability: in wireless communications, due to signal attenuation, two transmission pairs who are distant enough will not interfere with each other, thus can reuse the same channel.[1] This indicates that a seller is allowed to sell her channel to multiple buyers as long as they do not interfere with each other, which poses special challenge for a stable matching.

Spectrum matching was first studied in [6], in which an adapted two-stage deferred acceptance algorithm was designed to accommodate spectrum reusability, and produces a stable matching result. However, in [6], it is only designated that every buyer has a maximum quota that cannot be exceeded, but in real-world scenarios, in order to support the proper operations of wireless communications, buyers also have a minimum spectrum requirement that should be

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1. We refer to the divisible units of spectrum as channels.

TABLE 1
Comparison of Spectrum Auction and the Proposed Matching Algorithms

	Revenue/social welfare optimization	Stable	Individual rational	Fair	Non-wasteful	Minimum requirement
Auction	✓	×	✓	×	×	×
ADA	×	✓	✓	✓	✓	×
EDA	×	Weakly	✓	✓	×	✓
MDA	×	Weakly	✓	Weakly	✓	✓

satisfied. Since the classic deferred acceptance algorithm cannot address the minimum requirement in matching, D. Fragiadakis et al. [7] proposed an extended deferred acceptance algorithm for the school admission problem where schools have maximum and minimum quotas. Nevertheless, as discussed before, the algorithm cannot be directly applied to spectrum matching due to the unique feature of spectrum reusability.

In this paper, we present the framework of spectrum matching, where buyers with a maximum quota and a minimum requirement intend to purchase channels, and sellers may sell the same channel to multiple non-interfering buyers (spectrum reuse). A matching is strongly stable if it is individual rational, fair and non-wasteful. Though it is tempting to design strongly stable spectrum matching algorithms, we prove it is impossible to satisfy the three criteria altogether. Therefore, we propose two spectrum matching algorithms that caters to different stability criterion. The Extended Deferred Acceptance (EDA) algorithm is fair but wasteful, while the Multistage Deferred Acceptance (MDA) algorithm is non-wasteful but unfair. We summarize the differences of spectrum auction, ADA, EDA and MDA algorithm in Table 1. The simulation results show that the EDA algorithm achieves higher buyer happiness due to its fairness, since a buyer is matched to more preferred channels, and the MDA algorithm achieves higher quota fulfillment due to its non-wastefulness, since a buyer's maximum quota is fully exploited to take in more channels.

2 Related Work

Auction-Based Spectrum Allocation. Dynamic spectrum access is a fundamental feature of cognitive radio networks [8]. The double auction is also an important spectrum allocation paradigm for dynamic spectrum access. In double auctions, a third-party auctioneer executes certain auction mechanisms to decide the spectrum allocation based on the bids of buyers and the asks of sellers, which is indeed a matching process. Such a matching is enforced by the auctioneer, whose objective is usually revenue maximization or truthfulness. While matching results with these properties are desirable, they may not be achieved in the absence of a centralized controller, i.e., the auctioneer, because selfish buyers and sellers have incentives to deviate from the matching result if they have better choices. A truthful spectrum double auction was first proposed in [9]. To deal with spectrum heterogeneity, truthful auction mechanisms were designed in [10], [11], [12]. Typical revenue maximization double auction mechanisms include [13], [14], [15], [16]. In [17], [18], online double auction mechanisms were proposed to address the dynamics in spectrum availability and demand. Furthermore, concerns for user privacy has given rise to privacy-preservation double spectrum auction mechanisms [19], [20].

Stable Matching. Stable matching has been extensively studied since 1962, when Gale and Shapley first analyzed the school admission problem [2] and proposed deferred acceptance algorithm to achieve a stable matching [21]. Variants of matching problems in economics have been examined in [22], [23], [24]. Stable matching has been widely used for resource allocation in computer science. Matching problems with minimum quotas were studied in [7], [24], [25], [26], [27]. However, these matching algorithms cannot be directly applied to spectrum matching, due to the unique feature of spectrum reusibility. In our previous work [28], we designed the EDA algorithm to reach a fair but wasteful spectrum matching results. In this paper, we propose the MDA algorithm to realize a non-wasteful and weakly fair spectrum matching results. Moreover, we analyze the computational complexity of all proposed algorithm and compare the performance of EDA and MDA algorithms through simulations.

3 System Model

Market Participants. In a free spectrum market, service providers with spare channels serve as sellers, and service providers who need additional channels serve as buyers. Assume that seller s owns m_s channels. Inspired by the idea in [10], we create m_s dummies for seller s, each of whom possesses one channel.[2] Let $\sum_s m_s = M$ denote the total number of all available channels, and $\mathcal{M} = \{1, 2, \ldots, M\}$ denote the set of these channels. We use the index interchangeably for a seller and her channel, e.g., seller s's channel is referred to as channel s. Therefore, we have M virtual sellers and every seller trades exactly one channel. In the remainder of this paper, we omit the term "virtual" without confusion. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ denote the set of buyers. To ensure the operation of her base station or terminal device, buyer c has a minimum spectrum requirement of l_c. Although buyer c is willing to acquire as many channels as possible, she has a maximum quota of h_c due to limitations such as the budget constraint.

Interference Relationship. The key feature of spectrum is reusability, which must conform to interference constraints. Interference relationship among buyers is usually characterized by interference graph, in which the nodes denote buyers, and two nodes share an edge if the two buyers interfere with each other. Different channels have different transmission ranges, resulting in a diversity of interference relationships. To capture such heterogeneity [10], we construct a series of interference graphs $\{G^s = (\mathcal{N}, E^s)\}_{s=1}^{M}$, in which $e^s \in E^s$

2. For simplicity, we assume that channels are independent from each other, so they can be considered separately. In combinatorial auctions, different combinations of channels may be different, making the spectrum allocation much more complicated. We will consider the combination of channels in future work.

connects a pair of interfering buyers on channel s. Let $e^s_{c,c'} \in \{0,1\}$ denote the interference status between buyers c and c' regarding channel s. If $e^s_{c,c'} = 0$, buyer c, c' can reuse the same channel.

Preferences of Buyers and Sellers. In a conventional matching framework, a buyer/seller expresses her preferences towards different sellers/buyers through a preference list, which is a complete, reflexive and transitive relation. Nevertheless, the preference list defined over individual buyer/seller is not enough for spectrum matching. For example, let $\succ_s$ denote the preference list of seller s, and we have $c_1 \succ_s c_2 \succ_s c_3 \succ_s c_4$. If buyer c_1, c_4 can reuse the same channel, and buyer c_2, c_3 can reuse the same channel, we cannot decide whether $\{c_1, c_4\}$ is more preferred than $\{c_2, c_3\}$. One possible solution is to define the preference list of a seller over all combinations of buyers, which is undesirable since the number of possible combinations is 2^N, and many combinations consisting interfering buyers are invalid. To address this problem, we borrow the concept of *bid* from spectrum auction.[3] Buyer c has a bid vector $B_c = (b^1_c, b^2_c, \ldots, b^M_c)$, in which b^s_c is the price she is willing to pay for channel s.[4] The preference list of buyer c can be simply constructed as follows:

$$s \succ_c s' \Leftrightarrow b^s_c > b^{s'}_c.$$

If $b^s_c = b^{s'}_c$, we can randomly determine their preference relation. A seller always prefers to sell her channel to a set of buyers whose aggregate price is higher, as long as they do not interfere with each other. Let $\mathcal{A}, \mathcal{B} \subseteq \mathcal{N}$ denote two subsets of buyers, seller s prefers $\mathcal{A}$ to $\mathcal{B}$ if: 1) $\mathcal{A}$ contains only non-interfering buyers, and the aggregate bid of $\mathcal{A}$ is higher than that of $\mathcal{B}$, or; 2) $\mathcal{A}$ contains only non-interfering buyers, but $\mathcal{B}$ does not

$$\mathcal{A} \succ_s \mathcal{B} \Longleftrightarrow$$
$$\begin{cases} \forall c, c' \in \mathcal{A}, e^s_{c,c'} = 0, \forall c, c' \in \mathcal{B}, e_{c,c'} = 0, \sum_{c\in\mathcal{A}} b^s_c > \sum_{c\in\mathcal{B}} b^s_c, \text{or} \\ \exists c, c' \in \mathcal{B}, e^s_{c,c'} = 1. \end{cases}$$

If $\forall c, c' \in \mathcal{A}, e^s_{c,c'} = 0, \forall c, c' \in \mathcal{B}, e_{c,c'} = 0, \sum_{c\in\mathcal{A}} b^s_c = \sum_{c\in\mathcal{B}} b^s_{c'}$, or both $\mathcal{A}$ and $\mathcal{B}$ contain interfering buyers, we can randomly determine their preference relation.

4 Spectrum Matching Framework

4.1 Problem Formulation

We formally define spectrum matching as follows.

Definition 1 (Spectrum Matching). *Given the set of sellers $\mathcal{M}$ and the set of buyers $\mathcal{N}$, a spectrum matching is a mapping μ: $\mathcal{M} \cup \mathcal{N} \rightarrow 2^{\mathcal{N}} \cup 2^{\mathcal{M}}$, such that:*

- *For every seller $s \in \mathcal{M}$, $\mu(s) \subseteq \mathcal{N}$.*
- *For every buyer $c \in \mathcal{N}$, $\mu(c) \subseteq \mathcal{M}$.*
- *For every seller s and buyer c, $s \in \mu(c)$ if and only if $c \in \mu(s)$.*

3. In spectrum auction, bids are important for determining the price paid by buyers to sellers, but the spectrum allocation in auction is bid-independent in order to guarantee truthfulness [10]. In this paper, we only target at a stable matching result, but not the price determination. We will consider it as a future direction.

4. If a channel is unacceptable to a buyer, she can simply set the bid for the channel as zero.

The major differences that separate spectrum matching from common goods matching [7] are interference constraint and corresponding spectrum reusability. A common item can only be sold to one buyer, hence we have $\mu(s) \in \mathcal{N}$ in [7]. However, in spectrum matching, we can match a set of non-interfering buyers to the same channel, i.e., $\mu(s) \subseteq \mathcal{N}$, which significantly improves spectral efficiency, but makes it more difficult to reach a stable matching.

A spectrum matching is feasible only if it satisfies the interference constraint, the maximum quota and the minimum requirement.

Definition 2 (Feasible Spectrum Matching). *A spectrum matching is feasible, if it satisfies:*

- *Interference constraint. For every seller $s \in \mathcal{M}$, $\forall c, c' \in \mu(s), e^s_{c,c'} = 0$, i.e., buyers matched to the same seller should be interference-free.*
- *Maximum quota and minimum requirement. For every buyer $c \in \mathcal{N}$, $l_c \leq |\mu(c)| \leq h_c$, i.e., the number of channels matched to a buyer should be no fewer than her minimum requirement and no greater than her maximum quota.*

4.2 Stable Matching

Buyers and sellers are selfish and rational individuals who will break off from the matching result if they have better choices. A stable spectrum matching features *individual rationality*, *fairness*, and *nonwastefulness*.

Definition 3 (Individual Rationality). *A feasible spectrum matching μ is individually rational if:*

- *Every buyer prefers the current set of matched channels to any of its subsets, i.e., $\forall c \in \mathcal{N}, \mathcal{A} \subset \mu(c), \mu(c) \succ_c \mathcal{A}$.*
- *Every seller prefers the current set of matched buyers to any of its subsets, i.e., $\forall s \in \mathcal{M}, \mathcal{B} \subset \mu(s), \mu(s) \succ_s \mathcal{B}$.*

Being individually rational is the basis of a stable matching. To define fairness, we have to introduce the concept of type I blocking pair, which is tailored under the framework of spectrum matching.

Definition 4 (Type I Blocking Pair). *Given a feasible spectrum matching μ, buyer c and seller s form a type I blocking pair (s, c), if:*

1) *Buyer c prefers channel s to one of her currently matched channels.*
2) *Seller s can let buyer c reuse her channel, i.e., buyer c does not interfere with any buyers currently matched to seller s.*

Mathematically speaking, buyer c and seller s form a type I blocking pair (s, c), if:

1) $\exists s' \in \mu(c), s \succ_c s'$.
2) $\forall c' \in \mu(s), e_{c,c'} = 0$.

Definition 5 (Fairness). *A feasible spectrum matching μ is fair if and only if there are no type I blocking pairs.*

The type I block pair makes a spectrum matching unstable because buyer c can replace a less-preferred channel s' with a more-preferred channel s, and seller s may gain a higher profit by letting buyer c reuse her channel without violating interference constraint. A matching with type I blocking pair is regarded as *unfair* because channel s' unfairly fill the quota (when matched to buyer c) over channel s. Different from the traditional definition of fairness [7], due to interference constraint, it is required that buyer c is interference-free from any buyers in $\mu(s) \setminus \mathcal{A}$.

Apart from the type I blocking pair, we also have the type II blocking pair.

Definition 6 (Type II Blocking Pair). *Given a feasible spectrum matching μ, buyer c and seller s form a type II blocking pair (s, c), if:*

1) *Buyer c can purchase channel s without violating her maximum quota.*
2) *Seller s can let buyer c reuse her channel, i.e., buyer c does not interfere with any buyers currently matched to seller s.*

Mathematically speaking, seller s and buyer c form a type II blocking pair (s, c), if:

1) $|\mu(c)| < h_c$.
2) $\forall c' \in \mu(s), e_{c,c'} = 0$.

Definition 7 (Non-wastefulness). *A feasible spectrum matching μ is non-wasteful if and only if there are no type II blocking pairs.*

The type II blocking pair makes a spectrum matching unstable because buyer c can acquire one more channel under her maximum quota, and the purchase can benefit seller s as well. Hence, a matching with type II blocking pair is considered as *wasteful* since the quota of buyer c is wasted if not filling it with channel s.

The difference between type I and type II blocking pairs is whether or not a buyer will abandon a currently matched channel for a more-preferred channel. If yes, this results in a type I blocking pair; otherwise, this leads to a type II blocking pair.

Definition 8 (Strong Stability). *A feasible spectrum matching μ is strongly stable if it is individual rational, fair and non-wasteful.*

Though strong stability is most desirable, the following proposition tells us that it may not exist.

Proposition 1 (Non-existence). *Strong stable matching does not always exist for spectrum matching with guaranteed minimum requirement.*

Proof. It has been proved in [7] that for common goods (without reusibility), a simultaneously fair and non-wasteful matching may not exist, when minimum requirement is considered. Consider a special case where all buyers interfere with each other, i.e., the interference graph is complete, spectrum matching is equivalent to common goods matching. Therefore, a strong stable matching may not exist. □

Therefore, we focus on designing matching algorithms that realize a partially stable spectrum allocation. In the following context, we first adapt the traditional deferred acceptance algorithm for spectrum allocation without considering minimum requirement in Section 5. Then, taking minimum requirement into account, we propose an individual rational and fair matching algorithm in Section 6, and an individual rational and non-wasteful matching algorithm in Section 7.

Algorithm 1. Adapted Deferred Acceptance (ADA) Algorithm for Spectrum Matching

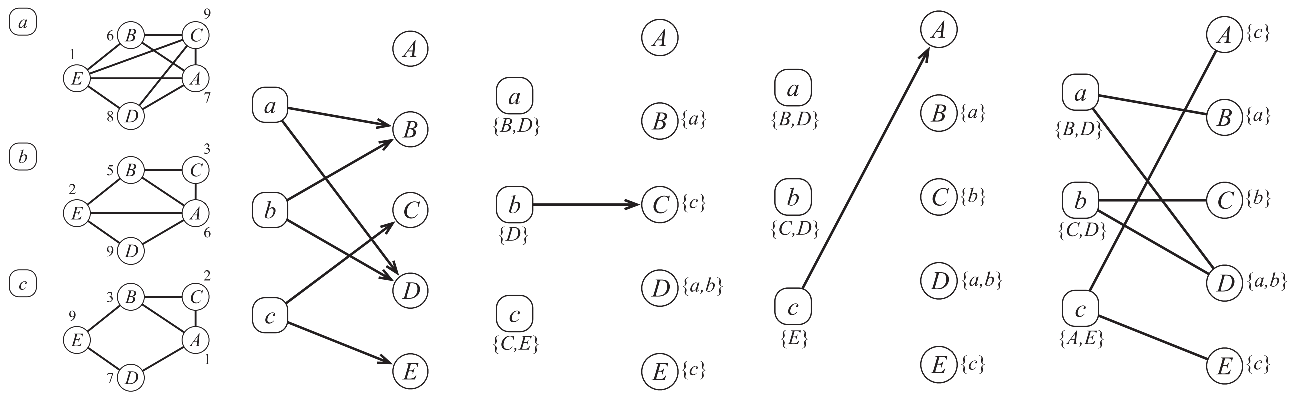
Input: Preference lists of all buyers $\succ_c, \forall c \in \mathcal{N}$, preference lists of all sellers $\succ_s, \forall s \in \mathcal{M}$, maximum quotas of buyers, $h_c, \forall c \in \mathcal{N}$, interference graph $G_s, \forall s \in \mathcal{M}$.
Output: A spectrum matching μ.

$\forall c \in \mathcal{N}, \mu(c) = \Phi$, the waiting list $\mathcal{W}_c = \Phi$.
$\forall s \in \mathcal{M}, \mu(s) = \Phi$, the candidate list $\mathcal{A}_s = \mathcal{N}$.
while $\exists \mathcal{A}_s \neq \Phi$ **do**
  for all Seller s with non-empty $\mathcal{A}_s$ **do**
    $\mathcal{Q}_s :=$ set of buyers that satisfies $c \in \mathcal{A}_s$, $\forall c' \in \mu(s), e_{c,c'} = 0$.
    Find the maximum weighted independent set on $\mathcal{Q}_s$ as $\mathcal{Q}_s^{max}$.
  end for
  if $\forall s, \mathcal{Q}_s^{max} = \Phi$ **then**
    Return μ.
  else
    for all Buyer $c \in \mathcal{Q}_s^{max}$ **do**
      Seller s applies for buyer c.
      Seller s removes buyer c from her candidate list, $\mathcal{A}_s = \mathcal{A}_s \setminus \{c\}$.
      Buyer c adds seller s to her waiting list, $\mathcal{W}_c = \mathcal{W}_c \cup \{s\}$.
    end for
  end if
  for all Buyer c with non-empty $\mathcal{W}_c$ **do**
    Buyer c accepts no more than h_c most-preferred channels in $\mathcal{W}_c \cup \mu(c)$ as the new $\mu(c)$, and rejects others.
    Clear the waiting list $\mathcal{W}_c = \Phi$.
  end for
  Every seller s updates her matching $\mu(s)$.
end while
Return μ.

5 Spectrum Matching Without Minimum Requirement

The traditional deferred acceptance algorithm, designed to solve the school admission problem, runs as follows [2]. There is a set of students to be admitted to a set of schools, each with a maximum quota. In the first round, each student applies for her favourite school. Among all applicants, a school with a maximum quota of h temporarily puts the top h most-preferred students in its waiting list, or all students if the number of applicants is smaller than h; other applicants are rejected. In the following rounds, each rejected student applies for her most-preferred school that has never rejected her before. Each school updates its waiting list by selecting the top h students among the current applicants and those in the previous waiting list. This process is repeated until all students have exhausted the schools that they can apply for.

We follow the convention of traditional matching frameworks [2] to let the sellers propose and the buyers decide



(a) Interference graph. (b) First round. (c) Second round. (d) Third round. (e) Final matching result.

Fig. 1. A toy example of the Adapted Deferred Acceptance (ADA) algorithm (Algorithm 1).

whether to accept or reject the proposals since it is the buyers who have the maximum quota and the minimum requirement. To adapt the original deferred acceptance algorithm to incorporate spectrum reusability, we assume that $l_c = 0$, $\forall c \in \mathcal{N}$, so that there is no minimum requirement for any buyer. We view sellers as students, and buyers as schools with maximum quotas. Instead of applying for only one buyer at each round, a seller can apply for a set of non-interfering buyers. As shown in Algorithm 1, we start with an empty matching. $\mathcal{A}_s$ denotes the candidate buyers that seller s has not applied for. The algorithm runs as follows.

(1) In each round, seller s first chooses a subset of buyers in $\mathcal{A}_s$ that do not interfere with any buyers in $\mu(c)$, denoted by $\mathcal{Q}_s$ (line 7), then finds the maximum weighted independent set among these buyers based on their bids, denoted by $\mathcal{Q}_s^{max}$ (line 8). $\mathcal{Q}_s^{max}$ is the best set of buyers that seller s can apply for in the current round. If $\mathcal{Q}_s^{max}$ is empty for all sellers, the algorithm terminates and returns the matching result (line 10$\sim$12); otherwise, proceed to the next step. Note that to find $\mathcal{Q}_s^{max}$ is equivalent to finding the maximum weighted independent set on the interference graph, which is NP-hard. Therefore, we adopt the polynomial time greedy algorithm in [29] to address this problem. The choice of different algorithms to find the maximum weighted independent set may lead to different spectrum matching results, but will not affect the stability of these matching results.

(2) Seller s applies for every buyers in $\mathcal{Q}_s^{max}$, and removes them from her candidate set (line 14$\sim$15). A buyer c who has received applications will add the corresponding sellers to her waiting list $\mathcal{W}_c$ (line 16).

(3) A buyer c with non-empty waiting list $\mathcal{W}_c$ will select no more than h_c most-preferred sellers from $\mathcal{W}_c \cup \mu(c)$, and reject others. Buyer c updates $\mu(c)$ to be these selected sellers (line 20).

(4) All sellers update their matching results according to those of the buyers (line 23).

Note that Algorithm 1 as well as the following proposed Algorithms 3 and 5 are distributed algorithms. Buyers and sellers need to be synchronized to accomplish each round of proposal and acceptance/rejection, and we have addressed this synchronization problem in our previous work [6].

Toy Example. As shown in Fig. 1, there are three sellers ($a \sim c$) and five buyers ($A \sim E$). The maximum quotas of buyers are $(h_A, h_B, h_C, h_D, h_E) = (1, 1, 1, 2, 1)$. The interference graph on each channel is shown in Fig. 1a, and the values beside each node are buyers' heterogeneous bids for different channels. A buyer's preference list can be constructed from her bids for all channels. Note that due to spectrum heterogeneity, the transmission range of different channels are different, leading to heterogeneous interference relationships. For example, buyer C and buyer E interfere with each other on the long-transmission-range channel a, but do not interfere with each other on the short-transmission-range channels b and c. The process of the adapted deferred acceptance algorithm is shown in Figs. 1b, 1c, 1d, and 1e. To begin with, seller a, b and c each apply for a set of non-interfering buyers with the maximum aggregate bid on their own interference graph (Fig. 1b). Buyer D accepts both channel a and b, since $h_D = 2$. Buyer B has to reject seller b, since $h_B = 1$, and buyer B prefers channel a to channel b. Buyer C and buyer E both accept channel c. In the second round, even though the candidate set of seller a is not empty ($\mathcal{A}_a = \{A, C, E\}$), she cannot apply for any buyer since they all interfere with buyer B or D, who are currently matched to seller a, i.e., $\mathcal{Q}_a^{max} = \Phi$. The same is true for seller c. In contrast, seller b can apply for buyer C who can reuse the channel with buyer D (Fig. 1c). Since the maximum quota of buyer C is $h_C = 1$, she has to give up channel c for the more-preferred channel b. Thanks to this, in the third round, seller c can apply for buyer A in her candidate set $\mathcal{A}_c = \{A, B, D\}$, who interferes with buyer C but not with buyer E (Fig. 1d). We can check that $\mathcal{Q}_s^{max}$ is empty for all sellers, and the final spectrum matching result is given in Fig. 1e.

Proposition 2 (Computational Complexity). *The proposed ADA algorithm converges with a computational complexity of $O(MN\tau)$, in which τ is the computational complexity of finding the maximum weighted independent set.*

Proof. Each time a seller makes applications, she will remove at least one buyer from her candidate list (line 15 in Algorithm 1). Eventually, for every seller, the set of buyers that do not interfere with her already-matched buyers will become empty, and Algorithm 1 will come to an end. Since we have M sellers, each of whom can apply to at most N buyers, and each time a seller chooses buyers by finding maximum weighted independent graph, the computational complexity of the ADA algorithm is $O(MN\tau)$, in which τ depends on the method used to find maximum weighted independent graph. □

Proposition 3 (Stability). *The matching result of the proposed ADA algorithm is individually rational, fair and non-wasteful.*

The proof is ignored due to page limitation. Interested readers can refer to [28] for detailed proof.

We can see that when minimum requirement is not considered, the ADA algorithm is strongly stable according to Definition 8, though strongly stable matching may not exist when minimum requirement is taken into account as we proved in Proposition 1. In the following sections, we will develop partially stable matching algorithm based on Algorithm 1.

6 Partially Stable Spectrum Matching Featuring Fairness

In this section, we design a fair spectrum matching algorithm with guaranteed minimum requirement. The basic idea is to divide each buyer into two dummy buyers dubbed the regular buyer and the extended buyer. The regular buyer undertakes the minimum requirement of the original buyer, and thus is indispensable. The extended buyer undertakes the remaining quota of the original buyer to help obtain extra channels without violating the maximum quota. Let c^r and c^e denote the regular and the extended buyer of the original buyer c, respectively. We set the maximum quota of the regular buyer c^r as $h_{c^r} = l_c$, and the maximum quota of the extended buyer c^e as $h_{c^e} = h_c - l_c$. There is no minimum requirement for all dummy buyers. For common goods, if we reserve at least $\sum_c l_c$ items for regular buyers (assign no more than $E = N - \sum_c l_c$ items to extended buyers), their maximum quotas will be filled up, which means that the minimum requirement of all original buyers will be satisfied. Nevertheless, to calculate how many channels to withhold for regular buyers in spectrum matching is non-trivial. For example, there are 4 buyers, none of whom interfere with each other, and each with a minimum requirement of 1. Instead of reserving 4 channels, we can just keep 1 channel for all buyers to reuse and fulfill their minimum requirement. In the rest of this section, we first assume that a number of E channels can be assigned to extended buyers ($N - E$ channels are reserved for regular buyers), and introduce the extended deferred acceptance algorithm to reach a feasible stable matching, then we explain how to compute E.

6.1 Matching Transformation

The transformation of the spectrum matching with minimum requirement into the spectrum matching without minimum requirement is elaborated in Algorithm 2. The seller set $\mathcal{M}$ is unchanged. The preference lists of the regular buyer and the extended user are the same as the original buyer (line 7). The preference list of each seller is reconstructed by inserting the extended buyer right after the regular buyer without changing the sequence of the original preference list (line 11). When rebuilding the interference graph of channel s, we create a regular and an extended nodes for each node in the original graph. These two nodes inherit all the interference relationship of the original nodes, and there is an edge between them, because they cannot share the same channel, as they essentially represent the same buyer.

Algorithm 2. Spectrum Matching Transformation

Input: Buyer set $\mathcal{N}$, seller set $\mathcal{M}$, preference lists of buyers $\succ_c, \forall c \in \mathcal{N}$, preference lists of sellers $\succ_s, \forall s \in \mathcal{M}$, maximum quota and minimum requirement of buyers $h_c, l_c, \forall c \in \mathcal{N}$, interference graph $G_s, \forall s \in \mathcal{M}$.

Output: Buyer set $\widetilde{\mathcal{N}}$, seller set $\widetilde{\mathcal{M}}$, preference lists of buyers $\widetilde{\succ}_c, \forall c \in \widetilde{\mathcal{N}}$, preference lists of sellers $\widetilde{\succ}_s, \forall s \in \widetilde{\mathcal{M}}$, maximum quotas of buyers $\widetilde{h}_c, \forall c \in \widetilde{\mathcal{N}}$, interference graph $\widetilde{G}_s, \forall s \in \widetilde{\mathcal{M}}$.

1: $\widetilde{\mathcal{M}} = \mathcal{M}$.
2: $\widetilde{\mathcal{N}}^e = \Phi, \widetilde{\mathcal{N}}^r = \Phi$.
3: **for all** $c \in \mathcal{N}$ **do**
4: $\quad \widetilde{\mathcal{N}}^e = \widetilde{\mathcal{N}}^e \cup \{c^e\}$.
5: $\quad \widetilde{\mathcal{N}}^r = \widetilde{\mathcal{N}}^r \cup \{c^r\}$.
6: $\quad h_{c^r} = l_c, h_{c^e} = h_c - l_c$.
7: $\quad \widetilde{\succ}_{c^r} := \widetilde{\succ}_{c^e} := \succ_c$.
8: **end for**
9: $\widetilde{\mathcal{N}} = \widetilde{\mathcal{N}}^e \cup \widetilde{\mathcal{N}}^r$.
10: **for all** $s \in \mathcal{M}$ **do**
11: $\quad$ Change $\succ_s := c_{s,1} \succ_s c_{s,2} \succ_s \dots$ into $\widetilde{\succ}_s := c^r_{s,1} \widetilde{\succ}_s c^e_{s,1} \widetilde{\succ}_s c^r_{s,2} \widetilde{\succ}_s c^e_{s,2} \widetilde{\succ}_s \dots$.
12: $\quad$ **for all** $c \in G_s$ **do**
13: $\quad\quad$ Add c^r and c^e to $\widetilde{G}_s$.
14: $\quad\quad$ c^r and c^e each inherits all edges of c in G^s.
15: $\quad\quad$ Create an edge between c^r and c^e.
16: $\quad$ **end for**
17: **end for**

6.2 Main Matching Algorithm

Given the transformed buyers and sellers, the spectrum matching algorithm is shown in Algorithm 3. The process of seller application is similar to that of Algorithm 1, but unlike Algorithm 1, we have different rules for regular buyers and extended buyers to decide whether to accept or reject a seller.

(1) If buyer c belongs to the regular buyer set $\widetilde{\mathcal{N}}^r$, she will pick up no more than $\widetilde{h}_c$ channels from $\mathcal{W}_c \cup \mu(c)$ (lines 17∼19), similar to that in Algorithm 1.

(2) If buyer c belongs to the extended buyer set $\widetilde{\mathcal{N}}^e$, we will consider all other extended buyers to rearrange their matching results. Let e be the counter to record the number of channels assigned to all extended buyers (line 21). First, we will put every extended buyer's matched channels $\mu(c)$ to her waiting list $\mathcal{W}_c$, and clear her matched channels (line 23). Then, we sequentially check the extended buyers in a specific order $c^e_1, c^e_2, \dots, c^e_N$. Let j denote the index of buyer that is being considered, and initially set $j = 1$.

  (a) If the number of channels matched to all extended buyers equals E (the maximum number of channels for all extended buyers), or every extended buyer either has an empty waiting list, or fulfills her maximum quota, the algorithm returns to step (1) (line 16); otherwise, the algorithm proceeds to step (b).

(b) Buyer c_j^e can choose her most-preferred channel from the waiting list (just one channel), as long as her maximum quota is not hit ($|\mu(c_j^e)| < h_{c_j^e}$) and her waiting list is non-empty ($\mathcal{P}_{c_j^e} \neq \Phi$). Increase j by 1, and return to step (a).

Given the matching result $\widetilde{\mu}$ of Algorithm 3, we can obtain the original matching as $\forall c \in \mathcal{N}, \mu(c) = \widetilde{\mu}(c^e) \cup \widetilde{\mu}(c^r)$, $\forall s \in \mathcal{M}$, if $c^e \in \widetilde{\mu}(s)$ or $c^r \in \widetilde{\mu}(s)$, then $c \in \mu(s)$.

Algorithm 3. Extended Deferred Acceptance (EDA) Algorithm for Spectrum Matching with Minimum Requirement

Input: Preference lists of buyers $\succ_c, \forall c \in \widetilde{\mathcal{N}}$, preference lists of sellers $\succ_s, \forall s \in \mathcal{M}$, maximum quotas of buyers $\widetilde{h}_c, \forall c \in \widetilde{\mathcal{N}}$, interference graph $\widetilde{G}_s, \forall s \in \widetilde{\mathcal{M}}$.
Output: Spectrum matching $\widetilde{\mu}$.

$\forall c \in \widetilde{\mathcal{N}}, \widetilde{\mu}(c) = \Phi$, the waiting list $\mathcal{W}_c = \Phi$.
$\forall s \in \widetilde{\mathcal{M}}, \widetilde{\mu}(s) = \Phi$, the candidate list $\mathcal{A}_s = \widetilde{\mathcal{N}}$.
while $\exists \mathcal{A}_s \neq \Phi$ **do**
  for all Seller s with non-empty $\mathcal{A}_s$ **do**
    Find $\mathcal{Q}_s^{max}$ as in Algorithm 1.
  end for
  if $\forall s, \mathcal{Q}_s^{max} = \Phi$ **then**
    Return $\widetilde{\mu}$.
  else
    for all Buyer $c \in \mathcal{Q}_s^{max}$ **do**
      Seller s applies for buyer c.
      Seller s removes buyer c from her candidate list, $\mathcal{A}_s = \mathcal{A}_s \setminus \{c\}$.
      Buyer c adds seller s to her waiting list, $\mathcal{W}_c = \mathcal{W}_c \cup \{s\}$.
    end for
  end if
  for all Buyer c with non-empty $\mathcal{W}_c$ **do**
    if $c \in \widetilde{\mathcal{N}}^r$ **then**
      Buyer c accepts no more than $\widetilde{h}_c$ channels in $\mathcal{W}_c \cup \widetilde{\mu}(c)$, and reject others.
      Clear the waiting list $\mathcal{W}_c = \Phi$.
    else
      $e = 0, j = 1$.
      for all $c \in \widetilde{\mathcal{N}}^e$ **do**
        $\mathcal{W}_c = \mathcal{W}_c \cup \mu(c), \mu(c) = \Phi$.
      end for
      while $e < E$ and $\exists c^e, \mathcal{W}_{c^e} \neq \Phi, |\widetilde{\mu}(c^e)| < \widetilde{h}_{c^e}$ **do**
        if $|\mu(c_j^e)| < \widetilde{h}_{c_j^e}$ and $\mathcal{W}_{c_j^e} \neq \Phi$ **then**
          Buyer c_j^e chooses her most-preferred channel s in $\mathcal{W}_{c_j^e}$, i.e., $\widetilde{\mu}(c_j^e) = \widetilde{\mu}(c_j^e) \cup \{s\}$.
          $\mathcal{W}_{c_j^e} = \mathcal{W}_{c_j^e} \setminus \{s\}$.
          $e = e + 1$.
        end if
        $j = j + 1 \bmod |\widetilde{\mathcal{N}}^e|$.
      end while
    end if
    $\mathcal{W}_c = \Phi$.
  end for
end while

6.3 Determine E

Now, we determine how many channels to reserve for regular buyers. Assume that the minimum requirement of every buyer is 1 and the channels are homogeneous (the interference graphs are the same), then finding the number of reserved channels is equivalent to the graph coloring problem: coloring the nodes of the interference graph such that no two adjacent nodes share the same color. To verify whether it is possible to color a graph with k colors is a $k-$coloring problem, which is NP-complete except for the cases $k \in \{0, 1, 2\}$. To overcome such computational hardness, we propose a greedy algorithm, as shown in Algorithm 4. First, since the minimum requirement of each buyer is h_c, which may be greater than 1, we create h_c virtual buyers to replace the original buyer in the interference graph.[5] Similar to that in Section 3, each virtual buyer inherits all interference relationship of the original buyer, and every two virtual buyers of the same original buyer share an edge between them. In this way, we reduce the original problem to the $k-$coloring problem. We rank all buyers in a specific order $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$,[6] and give every channel an index. Let $\mathcal{L}$ denote the set of channels that have already been used. In each iteration, we pick up a buyer v_i in $\mathcal{V}$ (line 9), and seek for the set of channels in $\mathcal{L}$ that are not used by any of v_i's neighbors among $v_1, \ldots, v_{i-1}$ (line 10). If the resulting channel set is non-empty, we assign the available channel with the lowest index to v_i (line 12); otherwise, we retrieve a new channel from $\mathcal{M}$ for v_i and add it to $\mathcal{L}$. This process continues until all buyers are assigned a channel. The number of reserved channels for regular buyers is $|\mathcal{L}|$, and the number of channels for extended buyers is $E = M - |\mathcal{L}|$.

Algorithm 4. Determining E

Input: Buyer set $\mathcal{N}$, seller set $\mathcal{M}$, minimum requirement of buyers $l_c, \forall c \in \mathcal{N}$, interference graph G.
Output: E.

1: **for all** node c in G **do**
2:   Create h_c nodes to replace c in G.
3:   Each of the h_c nodes inherits all edges of c in G.
4:   Create an edge between each pair of the h_c nodes.
5: **end for**
6: $\mathcal{L} := \Phi$.
7: $\mathcal{V} :=$ a random list of all buyers.
8: **while** $\mathcal{V}$ is not empty **do**
9:   Remove the first buyer v_i from the list.
10:   $\widetilde{\mathcal{L}} \subseteq \mathcal{L} :=$ the subset of channels that are not assigned to v_i's neighbors among $v_1, \ldots, v_{i-1}$.
11:   **if** $\widetilde{\mathcal{L}}$ is not empty **then**
12:     Assign the channel with the lowest index in $\widetilde{\mathcal{L}}$ to v_i.
13:   **else**
14:     Assign a new channel s with the lowest index in $\mathcal{M}$ to v_i.
15:     $\mathcal{M} = \mathcal{M} \setminus \{s\}, \mathcal{L} = \mathcal{L} \cup \{s\}$.
16:   **end if**
17: **end while**
18: $E = M - |\mathcal{L}|$.

5. We conservatively choose the interference graph of the channel with the longest transmission range (least chances for spectrum reuse) to calculate the number of reserved channels for regular buyers. In future works, we will consider heterogeneous interference graph for determining E.

6. The sequence of buyers may affect the performance of the algorithm, but it is difficult to determine the optimal sequence. We will leave it to future works.

TABLE 2
Toy Example

	$\succ_A$	$\succ_B$	$\succ_C$
	$f:6$	$b:6$	$e:6$
	$d:5$	$d:5$	$b:5$
	$a:4$	$e:4$	$c:4$
	$e:3$	$a:3$	$d:3$
	$b:2$	$c:2$	$a:2$
	$c:1$	$f:1$	$f:1$
l	2	1	2
h	3	3	3

	$\widetilde{\succ}_{A^r}$	$\widetilde{\succ}_{A^e}$	$\widetilde{\succ}_{B^r}$	$\widetilde{\succ}_{B^e}$	$\widetilde{\succ}_{C^r}$	$\widetilde{\succ}_{C^e}$
	f	f	b	b	e	e
	d	d	d	d	b	b
	a	a	e	e	c	c
	e	e	a	a	d	d
	b	b	c	c	a	a
	c	c	f	f	f	f
$\widetilde{h}_c$	2	1	1	2	2	1

	$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$	$\succ_f$
	A	B	C	A	C	A
	B	C	B	B	B	B
	C	A	A	C	A	C

	$\widetilde{\succ}_a$	$\widetilde{\succ}_b$	$\widetilde{\succ}_c$	$\widetilde{\succ}_d$	$\widetilde{\succ}_e$	$\widetilde{\succ}_f$
	A^r	B^r	C^r	A^r	C^r	A^r
	A^e	B^e	C^e	A^e	C^e	A^e
	B^r	C^r	B^r	B^r	B^r	B^r
	B^e	C^e	B^e	B^e	B^e	B^e
	C^r	A^r	A^r	C^r	A^r	C^r
	C^e	A^e	A^e	C^e	A^e	C^e

Toy Example. We assume that there are three buyers A, B, C and six sellers $a \sim f$. The preference lists of buyers and sellers are shown in Table 2. Using Algorithm 2, we can obtain the transformed buyers and sellers, also shown in Table 2. For simplicity, we assume that the interference graphs are the same for all channels. As the minimum requirement of buyers are $(2, 1, 2)$, we create virtual buyers as shown in Fig. 2a, and fix the order of buyers as A, A', B, C, C'. To start with, we assign channel a to buyer A, channel b to buyer A', and channel c to buyer B, resulting in $\mathcal{L} = \{a, b, c\}$. Then, channel c is assigned to buyer C, since it is not occupied by her neighbors A and A'. Finally, we have to add channel d to $\mathcal{L}$ for C'. We can see that 4 instead of $2 + 1 + 2 = 5$ channels need to be reserved for regular buyers. Therefore, we can compute E as $6 - 4 = 2$.

In the first round, as shown in Fig. 2b, seller f applies for buyer A^r, and all other sellers applies for buyer B^r and C^r. Buyer A^r and buyer B^r accepts seller f and seller b, respectively. Buyer C^r accepts seller b and e since her maximum quota is 2. In the second round, as shown in Fig. 2c, seller a, c and d applies for extended buyers B^e and C^e, and seller e applies for extended buyer C^e. Recall that all extended buyers can have no more than $E = 2$ channels. The waiting lists for buyer A^e, B^e, C^e are $\Phi, \{a, c, d, e\}, \{a, c, d\}$. We pass buyer A^e as her waiting list is empty. Buyer B^e accepts seller d, then buyer C^e accepts seller c, making $e = 2 = E$. In the third round, as shown in Fig. 2c, seller a applies for buyer A^r, who accepts her. The final spectrum matching result is shown in Fig. 3. We can check that all minimum requirement are meet.

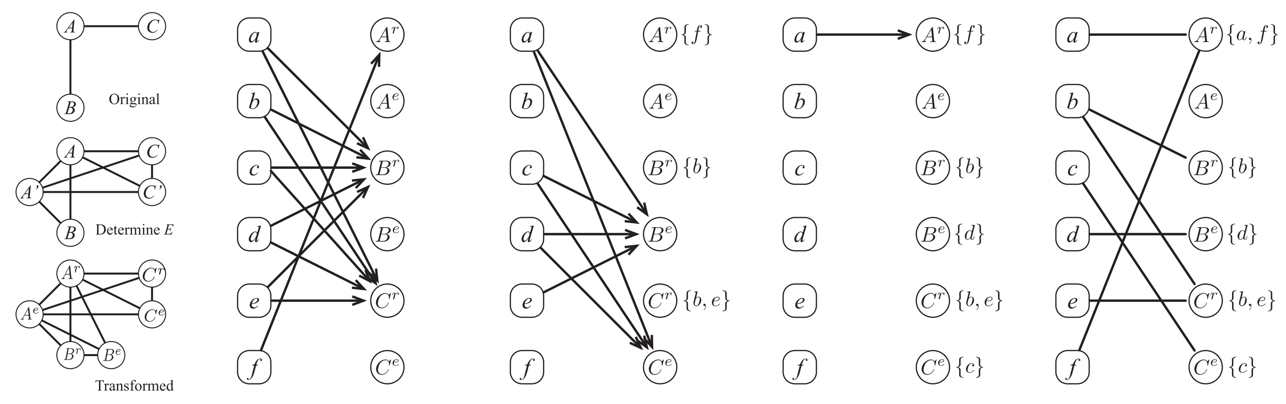
6.4 Theoretical Analysis

Proposition 4 (Computational Complexity). *The proposed EDA algorithm converges with a computational complexity of $O(MN^2\tau)$, in which τ is the computational complexity of finding the maximum weighted independent set.*

Proof. To transform the spectrum matching problem as in Algorithm 2 has a computational complexity of $O(M + N)$. To determine E as in Algorithm 4 has a computational complexity of $O(N)$ since it traverses all buyers to decide the reserved channels. In Algorithm 3, when considering regular buyers, the computational complexity is $O(MN\tau)$ (similar to Algorithm 1), and when considering extended buyers, the computational complexity is $O(MN^2\tau)$ because each time an extended buyer receives applications, all extended buyers have to be traversed to decide the temporary matching result as in line 25∼32 in Algorithm 3. In summary, the proposed EDA algorithm has a computational complexity of $O(MN^2\tau)$. □

Proposition 5. *The matching result of the proposed EDA algorithm is individually rational.*

Proof. Similar to the proof of Proposition 3. □



(a) Interference graph. (b) First round. (c) Second round. (d) Third round. (e) Final matching result.

Fig. 2. A toy example of the Extended Deferred Acceptance (EDA) algorithm (Algorithm 3).

To prove that the matching result of the EDA algorithm is fair, we first introduce the following lemma.

Lemma 1. *If a seller s is rejected by an extended buyer c^e, it must be true that $\forall s' \in \widetilde{\mu}(c^e)$, we have $s' \widetilde{\succ}_{c^e} s$.*

Proof. Assume that seller s is rejected by buyer c^e at round t, and $\widetilde{\mu}_t(s)$ denotes the (temporary) matching result at round t. We will first prove that $\forall t' > t, |\widetilde{\mu}_{t'}(c^e)| \leq |\widetilde{\mu}_t(c^e)|$. If at round t, the number of channels matched to buyer c reached her maximum quota, i.e., $|\widetilde{\mu}_t(c^e)| = h_{c^e}$, it is obviously true that $\forall t' > t, |\widetilde{\mu}_{t'}(c^e)| \leq |\widetilde{\mu}_t(c^e)|$. If $|\widetilde{\mu}_t(c^e)| < h_{c^e}$, consider round $t+1$. If no seller applies for buyer c^e, we naturally have $|\widetilde{\mu}_{t+1}(c^e)| = |\widetilde{\mu}_t(c^e)|$. So we consider that a seller applies for buyer c^e. Since all channels temporarily matched to extended buyers other than c^e at round t are still temporarily matched to these extended buyers at round $t+1$, according to lines 25$\sim$32 in Algorithm 3, the cap E will be hit before buyer c^e is able to accept the $(|\widetilde{\mu}_t(c^e)|+1)\text{th}$ seller. Therefore, we have $|\widetilde{\mu}_{t+1}(c^e)| \leq |\widetilde{\mu}_t(c^e)|$.

As seller s is rejected by c^e at round t, we know that $\forall s' \in \widetilde{\mu}_t(c^e), s' \succ_{c^e} s$. Now, we consider round $t+1$. Since we have proved that $|\widetilde{\mu}_{t+1}(c^e)| \leq |\widetilde{\mu}_t(c^e)|$, which means that the least-preferred seller in $\widetilde{\mu}_{t+1}(c^e)$ must be better than the least-preferred seller in $\widetilde{\mu}_t(c^e)$, we can derive that $\forall s'' \in \widetilde{\mu}_{t+1}(c^e), s'' \succ_{c^e} s$. Therefore, in the final matching, we must have $\forall s' \in \widetilde{\mu}(c^e), s' \widetilde{\succ}_{c^e} s$. □

Proposition 6 (Fairness). *The matching result of the proposed EDA algorithm is fair.*

Proof. Suppose there exists a type I blocking pair (s, c) such that $\forall c' \in \mu(s), e_{c,c'} = 0$. This indicates that s has applied for but been rejected by both c^r and c^e, because otherwise, seller s would have applied for c^r or c^e according to lines 4$\sim$6 in Algorithm 3. For any seller $s' \in \widetilde{\mu}(c^r)$, it must be true that $s' \succ_{c^r} s$, because s is rejected in favor of other h_{c^r} sellers who are more preferred than s. For any seller $s' \in \widetilde{\mu}(c^e)$, according to Lemma 1, we have $s' \succ_{c^e} s$. This infers that no type I blocking pair will exist. Therefore, the matching results of the proposed EDA algorithm is fair. □

Proposition 7 (Wasteful). *The matching result of the proposed EDA algorithm may be wasteful.*

Proof. Recall the matching result of the toy example in Fig. 2e, we can see that seller c and buyer B form a type II blocking pair, because 1) $|\mu(B)| = 2 < 3 = h_B$; 2) buyer B can reuse channel c with channel c's currently matched buyer C. The pair (c, B) satisfies both conditions of type II blocking pair, thus the matching result of the proposed EDA algorithm may be wasteful. □

In conclusion, the proposed EDA algorithm is partially stable, featuring individual rationality and fairness, but not non-wastefulness.

7 Partially Stable Spectrum Matching Featuring Non-Wastefulness

In this section, we design a non-wasteful spectrum matching algorithm with guaranteed minimum requirement. The basic idea is to reserve enough channels to meet the minimum requirement of buyers, and conduct the deferred acceptance algorithm with the rest of the channels. The number of channels to reserve can also be calculated with Algorithm 4.

Algorithm 5. Multistage Deferred Acceptance (MDA) Algorithm for Spectrum Matching with Minimum Requirement

Input: Preference lists of buyers $\succ_c, \forall c \in \mathcal{N}$, preference lists of sellers $\succ_s, \forall s \in \mathcal{M}$, maximum quotas of buyers $h_c, \forall c \in \mathcal{N}$, minimum requirement of buyers $l_c, \forall c \in \mathcal{N}$, interference graph $G_s, \forall s \in \mathcal{M}$.

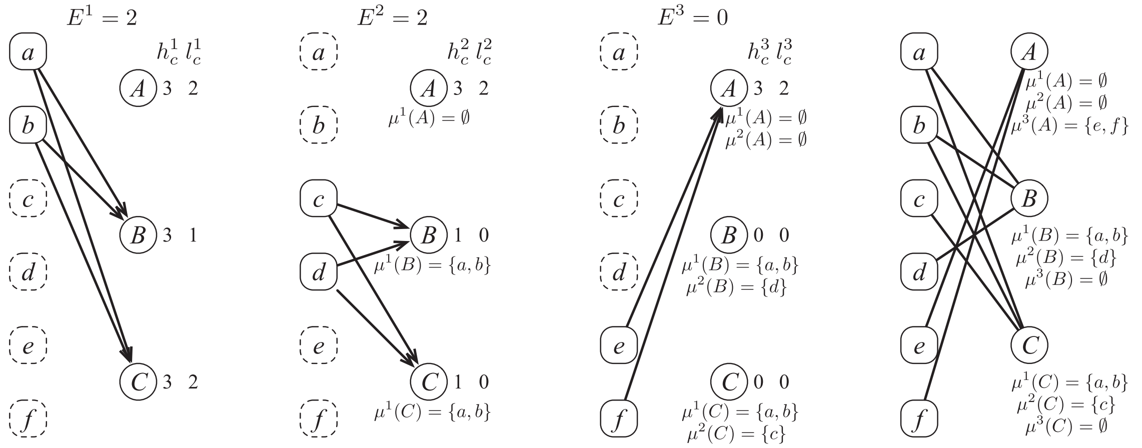
Output: Spectrum matching μ.

1: $t = 0$.
2: $\mathcal{M}^t = \mathcal{M}$. $h_c^t = h_c, l_c^t = l_c, \forall c \in \mathcal{M}$.
3: **while** $\mathcal{M}^t$ is not empty **do**
4:   Calculate the number of reserved channels E^t with $\mathcal{N}, \mathcal{M}^t, \{l_c^t\}_{c\in\mathcal{N}}$.
5:   $t = t + 1$.
6:   **if** $E^t > 0$ **then**
7:     $\widehat{\mathcal{M}}^t :=$ the first E^t sellers in $\mathcal{M}^t$.
8:     $\mathcal{M}^{t+1} = \mathcal{M}^t \setminus \widehat{\mathcal{M}}^t$.
9:     $\mu^t = \text{ADA}(\widehat{\mathcal{M}}^t, \mathcal{N}, \{h_c^t\}_{c\in\mathcal{N}})$.
10:     **for all** $c \in \mathcal{N}$ **do**
11:       $h_c^{t+1} = h_c^t - |\mu^t(c)|$.
12:       $l_c^{t+1} = \max\{0, l_c^t - |\mu^t(c)|\}$.
13:     **end for**
14:   **else**
15:     $\mu^t = \text{ADA}(\mathcal{M}^t, \mathcal{N}, \{l_c^t\}_{c\in\mathcal{N}})$.
16:   **end if**
17: **end while**
18: **for all** $c \in \mathcal{N}$ **do**
19:   $\mu(c) = \cup_t \mu^t(c)$.
20: **end for**
21: **for all** $s \in \mathcal{M}$ **do**
22:   $\mu(s) = \mu^{t(s)}(s)$, in which $t(s)$ is the round when channel s is assigned.
23: **end for**

7.1 Main Matching Algorithm

We assume that all sellers are ranked according to a predefined order. The spectrum matching algorithm is shown in Algorithm 3.

(1) At round t, if the remaining buyer/channel set $\mathcal{M}^t$ is empty, the algorithm terminates; otherwise, we calculate E^t, the number of channels to reserve, based on the buyer set $\mathcal{N}$, the remaining seller/channel set $\mathcal{M}^t$, and the unsatisfied minimum requirement $\{l_c\}_{c\in\mathcal{N}}$.
   (a) If $E^t \geq 0$, we can distribute E^t channels freely to all buyers by running the ADA algorithm based on the buyer set $\mathcal{N}$, the first E^t channel set $\widehat{\mathcal{M}}^t$, and the remaining maximum quota $\{h_c^t\}_{c\in\mathcal{N}}$ (lines 7$\sim$9). Then, we update the remaining maximum quota $\{h_c^t\}_{c\in\mathcal{N}}$ and minimum requirement $\{l_c^t\}_{c\in\mathcal{N}}$ by subtracting the number of matched channels (lines 10$\sim$13).
   (b) If $E^t = 0$, which means that the remaining channels can only satisfy the remaining minimum



(a) First round. (b) Second round. (c) Third round. (d) Final matching result.

Fig. 3. A toy example of the Multistage Deferred Acceptance (MDA) algorithm (Algorithm 5).

requirement of all buyers, we run the ADA algorithm based on the buyer set $\mathcal{N}$, the remaining channel set $\mathcal{M}^t$, and the remaining minimum requirement $\{l_c^t\}_{c\in\mathcal{N}}$ (line 15).

(2) At last, we integrate the matching results μ^t at each round to obtain the final matching result.

Toy Example. We use the same toy example as in Fig. 2 and Table 2. The spectrum matching process using the MDA algorithm is shown in Fig. 3. In the first round, similar to the toy example in Section 6, we can derive that 4 channels should be reserved, thus $E^1 = 6 - 4$. We run the ADA algorithm on two channels $\{a, b\}$ with regard to the maximum quotas $h_c^1, c \in \{A, B, C\}$. Channel a and b are matched to both buyer B and C. Therefore, the remaining maximum quota and minimum requirement are updated as shown in Fig. 3b. In the second round, 2 channels should be reserved to meet the remaining minimum requirement of buyer A, thus $E^2 = 4 - 2 = 2$. We run the ADA algorithm on two channels $\{c, d\}$, and get the corresponding matching result. In the third round, we have to reserve two channels for buyer A, so $E^3 = 2 - 2 = 0$. By running the ADA algorithm with the minimum requirement of all buyers, channel e and channel f are matched to buyer A. We aggregate the matching result in different rounds to obtain the final matching result.

It is obvious that the matching result of the MDA algorithm is different from that of the EDA algorithm. The matching result of the EDA algorithm is fair but wasteful, and we will prove that the matching result of the MDA algorithm is non-wasteful.

7.2 Theoretical Analysis

Proposition 8 (Computational Complexity). *The proposed MDA algorithm converges with a computational complexity of $O(M^2N\tau)$, in which τ is the computational complexity of finding the maximum weighted independent set.*

Proof. At each round, at least one channel will be used to run the ADA algorithm, so there are at most M rounds. Each round takes $O(MN\tau)$ time, the computational complexity of the ADA algorithm. The set of remaining channels will eventually be empty. Therefore, the MDA algorithm will converge, and its computational complexity is $O(M^2N\tau)$. □

Proposition 9. *The matching result of the proposed EDA algorithm is individually rational.*

Proof. Similar to the proof of Proposition 3. □

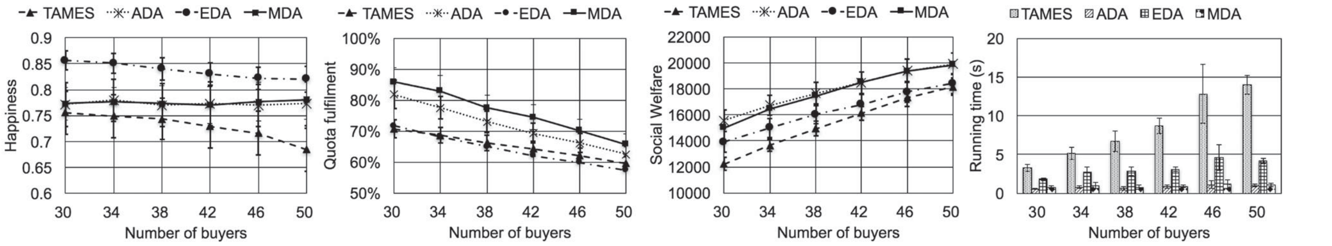
Proposition 10 (Non-wastefulness). *The matching result of the proposed MDA algorithm is non-wasteful.*

Proof. Assume there is a type II blocking pair (s, c), and seller s participates in the round t. If $E^t > 0$, ADA algorithm runs with $\{h_c\}_{\forall c\in\mathcal{N}}$. It must be true that seller s has applied for buyer c but been rejected as $|\mu^t(c)| = h_c^t$; otherwise, seller s can be accepted to be matched to buyer c. We can easily derive that in the final matching, $|\mu(c)| = h_c$, which violates the condition of type II blocking pair. If $E^t = 0$, ADA algorithm runs with $\{l_c\}_{\forall c\in\mathcal{N}}$. Since all the remaining channels are reserved to fulfill the minimum requirement, all channels assigned in this stage cannot be moved from their matched buyers without violating their minimum requirement. Therefore, no type II blocking pair exists, and the matching result of the proposed MDA algorithm is non-wasteful. □

Proposition 11 (Unfairness). *The matching result of the proposed MDA algorithm may be unfair.*

Proof. Recall the matching result of the toy example in Fig. 3d, we can see that seller e and buyer C form a type I blocking pair, because 1) buyer C prefers channel e to her matched channel a, i.e., $e \succ_C a, a \in \mu(C)$; 2) buyer C can reuse channel e with channel e's currently matched buyer B. The pair (e, C) satisfies both conditions of type I blocking pair, thus the matching result of the proposed MDA algorithm may be unfair. □

From the above counterexample, we can see that (e, C) forms a type I blocking pair mainly because channel a is scheduled for matching at an earlier round than channel e, since we sequentially select sellers according to a predefined order (line 7 in Algorithm 5). We can define a weak type I blocking pair, and prove that the matching result of the proposed MDA algorithm is weakly fair.



(a) Buyer happiness. (b) Quota fulfillment. (c) Social welfare. (d) Running time.

Fig. 4. Performance comparison. The number of sellers is fixed as $M = 60$. $l_c \sim \mathrm{unif}(5,6), h_c \sim \mathrm{unif}(7,8)$.

Definition 9 (Weak Type I Blocking Pair). *Given a feasible spectrum matching μ, assuming there is a pre-defined order of all sellers $\succ_{PD}$, buyer c and seller s form a weak type I blocking pair (s, c), if:*

1) *Buyer c prefers channel s to one of her currently matched channels, and channel s is ranked behind this channel according to the pre-defined order.*
2) *Seller s can let buyer c reuse her channel, i.e., buyer c does not interfere with any buyers currently matched to seller s.*

Mathematically speaking, buyer c and seller s form a type I blocking pair (s, c), if:

1) $\exists s' \in \mu(c), s \succ_c s' \& s \succ_{PD} s'$.
2) $\forall c' \in \mu(s), e_{c,c'} = 0$.

Definition 10 (Weak Fairness). *A feasible spectrum matching μ is weakly fair if and only if there are no weak type I blocking pairs.*

Proposition 12 (Weakly Fairness). *The matching result of the proposed MDA algorithm is weakly fair.*

Proof. Assume there is a weak type I blocking pair (s, c), and seller s participates in the round t. At round t, if buyer c begins with $h_c^t = 0$, every channel matched to buyer c satisfies $s' \succ_{PD} s, \forall s' \in \cup_{\tau=1}^{t-1} \mu^\tau(c)$. If buyer c can still accept channels, i.e., $h_c^t > 0$, it must be true that $s' \succ_c s, \forall s' \in \mu^t(c)$ thanks to the fairness of the ADA algorithm. Therefore, (s, c) cannot form a weak type I blocking pair, and the matching result of the proposed MDA algorithm is weakly fair. □

8 Simulation

In this section, we compare the performance of the proposed EDA algorithm with two benchmarks for comparison: one is the ADA algorithm without considering minimum requirement, and the other is adapted from the matching process in TAMES, a spectrum double auction mechanism [10]. The detailed adaptation procedure is as follows. For every buyer $c \in \mathcal{N}$, we create l_c virtual buyers, and transform the interference graphs according to the process in Section 6.3. Then, according to a specific order of channels, we sequentially find the independent set on the corresponding interference graph to match to the corresponding seller. When seeking for the independent set for a particular channel, we greedily pick the buyer with the highest $b_c^s/(d_{G^s}(c) + 1)$ and eliminate her interfering neighbors until the interference graph becomes empty [29]. Note that b_c^s is buyer c's bid for channel s, and $d_{G^s}(c)$ is the degree of buyer c on interference graph G_s. We will refer to this matching approach as TAMES in the following context.

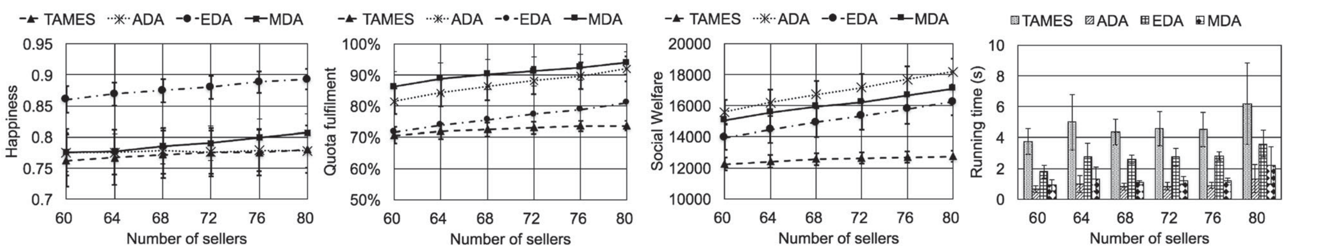
We assume that buyers uniformly locate within a $100\ \mathrm{m} \times 100\ \mathrm{m}$ area, and the transmission range of a channel is drawn randomly from the range $[40\ \mathrm{m}, 45\ \mathrm{m}]$. Buyers' bids for different channels follow independent identical distribution $unif(1{,}100)$. For performance evaluation, we focus on the following four metrics.

- *Buyer happiness*. The happiness of a buyer is the average rank percentile of her matched sellers [3].
- *Quota fulfillment*. Quota fulfillment, which we use to measure non-wastefulness, is defined as the ratio between the number of channels matched to a buyer to the buyer's maximum quota.
- *Social welfare*. Social welfare is defined as the sum of successful buyers' bids for their matched channels [10].
- *Running time*.

Each simulation runs for 500 times on a ThinkPad laptop with Intel(R) Core (TM) i7-5600U CPU at 2.60 GHz and 12.00 GB RAM

8.1 Buyer Happiness

The buyer happiness of the EDA algorithm is the highest, as shown in Figs. 4b and 5, indicating that a buyer are more likely to be matched to her more preferred channels. This



(a) Buyer happiness. (b) Quota fulfillment. (c) Social welfare. (d) Running time.

Fig. 5. Performance comparison. The number of buyers is fixed as $N = 30$. $l_c \sim \mathrm{unif}(5,6), h_c \sim \mathrm{unif}(7,8)$.

further confirms the fairness of the EDA algorithm: since buyers are matched with the channels they prefer, they are happy with the matching results. As the MDA algorithm is non-wasteful, a buyer will be matched with more channels, which degrades the buyer's average happiness. Nevertheless, since the MDA algorithm still features weakly fairness, the happiness of the MDA algorithm is higher than the ADA algorithm. With more buyers competing for channels, a buyer has a lower chance to be matched to her favorite channels, therefore, the buyer happiness drops as the number of buyers increases (Fig. 4a). For a similar reason, the buyer happiness will be enhanced if there are more channels (Fig. 5a).

8.2 Quota Fulfillment

As shown in Figs. 4a and 5, the quota fulfillment of the MDA algorithm is the highest as we expected, since the MDA algorithm is non-wasteful, which means that a buyer's maximum quota is filled as much as possible. The quota fulfillment of the EDA algorithm is relatively low as we showed in the toy example that the EDA algorithm may be wasteful. As shown in Fig. 4b, the quota fulfillment decreases as the number of buyers goes up, because more buyers compete for limited channels. On the contrary, more sellers provide more channels, which helps boost the quota fulfillment, as shown in Fig. 5b.

8.3 Social Welfare

The social welfare of ADA is the highest, as shown in Figs. 4c and 5c. By completely disregarding the constraint of minimum requirement, ADA matches buyers and seller solely according to their preference lists, which depend on the buyers' bid vectors (recall that social welfare is the sum of successful buyers' bids for their matched channels), therefore resulting in a high social welfare. The social welfare of the MDA algorithm approximates that of the ADA algorithm because each buyer is matched with more channels due to its non-wastefulness. The EDA algorithm sacrifices non-wastefulness for fairness, and its social welfare is lower than that of the MDA algorithm but higher than that of TAMES. It is naturally true that the increment in the number of either buyers or sellers will improve the social welfare.

8.4 Running Time

As shown in Figs. 4d and 5d, the running time of the MDA algorithm is relatively short, while the running time of the EDA algorithm is relatively long, but the time complexity is acceptable. This is because that the time complexity of the MDA and the EDA algorithms are $O(MN^2\tau)$ and $O(M^2N\tau)$, respectively. Since $N > M$ in our simulation, the running time of the MDA algorithm is higher than that of the EDA algorithm.

9 Conclusion and Future Works

In this paper, we present a spectrum matching framework for spectrum transactions where buyers have maximum quotas and minimum requirements. We first introduce the Adaptive Deferred Acceptance algorithm to tackle the problem of spectrum reusability. To fulfill the minimum requirements of all buyers, we further develop the Extended Deferred Acceptance algorithm and the Multistage Deferred Acceptance algorithm, which can generate an interference-free matching and meet different stability criterion. Unlike optimizing matching over a certain objective function, stable matching ensures that no buyers or sellers, albeit selfish, are willing to violate the matching result. The simulation results show that EDA and MDA perform better than the benchmark algorithms in terms of buyer happiness and quota fulfillment. Comparing the two algorithms, the EDA algorithm has a higher buyer happiness due to its fairness, while the MDA algorithm has a higher quota fulfillment due to its non-wastefulness.

There are various potential future directions. First, buyers may have higher valuations for combinations of continuous channels. Therefore, matching frameworks that allow buyers to specify their preferences on different combinations of channels are desirable. Second, the time dimension may be included in the matching algorithm to deal with dynamics of spectrum availability and demand. Third, money transfer should be incorporated in the matching framework to complete the spectrum exchange between buyers and sellers. Finally, more efforts are needed to explore a strongly stable spectrum matching algorithm that is both fair and non-wasteful.

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