Stable Matching for Spectrum Market with Guaranteed Minimum Requirement

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ABSTRACT

To enable dynamic spectrum access, service providers with spare spectrum (sellers) trade with those who are in need of additional spectrum (buyers). In a spectrum market, the transaction result is essentially a match between sellers and buyers. Though it is tempting to optimize the matching over certain utility functions, a stable matching is more desirable, since it takes into account a diverse set of preferences of buyers and sellers, and produces a matching result which no participants have incentives to deviate from. While existing works on spectrum matching only consider the *maximum* number of channels a buyer can purchase, in real-world scenarios, the *minimum* spectrum requirement should be satisfied to support the proper operation of wireless communication. To address this issue, in this paper, we present a new framework of spectrum matching with both maximum and minimum requirements. Different from conventional matching problems, the spectrum market poses distinctive challenges due to spectrum reusability. Instead of being sold exclusively to just one buyer, the same channel can be reused by multiple buyers who are not interfering with each other. To tackle this problem, we design a novel algorithm, called Extended Deferred Acceptance (EDA), that converges to an interference-free matching and guarantees the minimum spectrum requirement. We theoretically prove the stability of the matching result. Our simulation results show that EDA can achieve a 100% coverage on the minimum requirements, while alternative benchmark algorithms fail to do so, and buyers are more satisfied with the matching result of EDA than that of alternative algorithms.

CCS CONCEPTS

• **Networks → Network economics**; *Wired access networks*;

KEYWORDS

Spectrum allocation, stable matching, minimum requirement

ACM Reference format:
Yanjiao Chen, Yuxuan Xiong, Qian Wang, Xiaoyan Yin, and Baochun Li. 2017. Stable Matching for Spectrum Market with Guaranteed Minimum Requirement . In *Proceedings of Mobihoc '17, Chennai, India, July 10-14, 2017,* 10 pages.
https://doi.org/http://dx.doi.org/10.1145/3084041.3084062

1 INTRODUCTION

Spectrum is an indispensable resource for wireless communication, yet it is also a limited resource, strained for supporting the ever-increasing wireless traffic. To make the best use of available spectrum and to avoid under-utilization due to static spectrum assignments, dynamic spectrum access has emerged to allow wireless service providers to buy or sell spare channels according to their demands [5, 10].

In a spectrum market, the transaction of spectrum is essentially a matching between spectrum sellers and buyers. As opposed to optimizing the matching over certain utility functions, a stable matching is more desirable for a free spectrum market, due to two important reasons. First, buyers and sellers are selfish individuals, acting out of their own interests, which are not necessarily aligned with the system optimization. While optimal matching may only be enforced, stable matching takes into account individual preferences of buyers and sellers, producing a matching result that no participants have incentives to deviate from. Second, optimization solvers are usually computationally hard, suffering from scalability problems, whereas the running time of stable matching algorithms is polynomial.

Stable matching for college admission problems was first studied by Gale and Shapley in their pioneering work [12], in which the *Deferred Acceptance* (DA) algorithm was proposed to match students to schools subject to maximum quotas of the schools. Since

Mobihoc '17, July 10-14, 2017, Chennai, India

ACM ISBN 978-1-4503-4912-3/17/07…$15.00
https://doi.org/http://dx.doi.org/10.1145/3084041.3084062

then, stable matching has been widely applied to resource allocation in computer science, such as virtual machine management in the cloud [24], user association in small cells [22], and spectrum sharing in device-to-device communication [15]. However, unlike traditional matching problems, spectrum matching features reusability: in wireless communications, due to signal attenuation, two transmission pairs who are distant enough will not interfere with each other, thus can reuse the same channel[1]. This indicates that a seller is allowed to sell her channel to multiple buyers as long as they do not interfere with each other, which poses special challenges for stable matching.

Spectrum matching was first studied in [6], in which an adapted two-stage deferred acceptance algorithm was designed to accommodate spectrum reusability and to produce a stable matching result. However, in [6], it is only designated that every buyer has a maximum quota that cannot be exceeded, but in real-world scenarios, in order to support proper operations of wireless communications, buyers also have a minimum spectrum requirement that should be satisfied. Since the classic deferred acceptance algorithm cannot address the minimum requirement in matching, Fragiadakis *et al.* [11] proposed an extended deferred acceptance algorithm for the school admission problem where schools have maximum and minimum quotas. Nevertheless, as discussed before, the algorithm cannot be directly applied to spectrum matching due to its unique feature of spectrum reusability.

In this paper, we aim at designing a stable matching algorithm for the spectrum market, where buyers with a maximum quota and a minimum requirement intend to purchase channels, and sellers may sell the same channel to multiple non-interfering buyers (spectrum reuse). We start by clarifying the matching framework and the concept of stability in terms of spectrum matching, as an interference constraint should be imposed for the final matching result. To address the heterogeneity of spectrum, different interference graphs are constructed to determine spectrum reuse for different channels.

We first propose an Adapted Deferred Acceptance (ADA) algorithm for spectrum matching without the minimum requirement, which takes advantage of spectrum reusability, and yields an interference-free matching. Then, inspired by [11], we design an Extended Deferred Acceptance (EDA) algorithm for spectrum matching with a minimum requirement. The main idea is to first divide each buyer into a regular buyer and an extended buyer with carefully-set maximum quotas but no minimum requirements, and then tailor the deferred acceptance rules for the two types of buyers, respectively. We theoretically prove that the final matching result is stable.

We evaluate the performance of EDA through extensive simulations, comparing it to ADA and the matching process of a well-known double auction mechanism, called TAMES [7]. Unlike EDA, neither ADA nor TAMES can meet the minimum requirement of all buyers. Moreover, buyers are also happier with the matching result of EDA than those of ADA or TAMES.

The remainder of the paper is organized as follows. We first review related works in Section 2, and then describe the system model of the spectrum market in Section 3. In Section 4, we introduce the framework of spectrum matching with maximum and minimum requirements, as well as the concept of stability. In 5, we present the proposed algorithms to achieve the objective of stable spectrum matching. Simulation results are demonstrated in Section 6, and we summarize our work in Section 7.

[1]We refer to the divisible units of spectrum as channels.

2 RELATED WORK

Auction-based Spectrum Allocation. The double auction is also an important spectrum allocation paradigm for dynamic spectrum access. In double auctions, a third-party auctioneer executes certain auction mechanisms to decide the spectrum allocation based on the bids of buyers and the asks of sellers, which is indeed a matching process. Such a matching is enforced by the auctioneer, whose objective is usually revenue maximization or truthfulness. While matching results with these properties are desirable, they may not be achieved in the absence of a centralized controller, i.e., the auctioneer, because selfish buyers and sellers have incentives to deviate from the matching result if they have better choices. A truthful spectrum double auction was first proposed in [25]. To deal with spectrum heterogeneity, truthful auction mechanisms were designed in [7, 9]. Typical revenue maximization auction mechanisms include [1, 13, 17, 18].

Stable Matching. Stable matching has been extensively studied since 1962, when Gale and Shapley first analyzed the school admission problem [12] and proposed deferred acceptance algorithm to achieve a stable matching [21]. Variants of matching problems in economics have been examined in [4, 8, 19, 20]. Stable matching has been widely used for resource allocation in computer science. In [24], algorithms were proposed to match heterogeneous sized jobs to virtual machines in the cloud. In [22], matching framework was designed to associate users to small cells. In [15], device-to-device users were matched to cellular users for resource sharing. In [2, 3], a friendly jammer was matched to a transmission pair to help protect them from eavesdropping. Matching problems with minimum quotas were studied in [8, 11, 14, 16]. Specifically, an extended deferred acceptance algorithm was proposed in [11] to achieve stable matching results. However, these matching algorithms cannot be directly applied to spectrum matching, due to the unique feature of spectrum reusibility.

3 SYSTEM MODEL

Market participants. In a free spectrum market, service providers with spare channels serve as sellers, and service providers who need additional channels serve as buyers. Assume that seller s owns m_s channels. Inspired by the idea in [7], we create m_s dummies for seller s, each of whom possesses one channel[2]. Let $\sum_s m_s = M$ denote the total number of all available channels, and $\mathcal{M} = \{1, 2, ..., M\}$ denote the set of these channels. We use the index interchangeably for a seller and her channel, e.g., seller s's channel is referred to as channel s. Therefore, we have M virtual sellers and every seller trades exactly one channel. In the remainder of this paper, we omit the term "virtual" without confusion. Let $\mathcal{N} = \{1, 2, ..., N\}$ denote the set of buyers. To ensure the operation of her base station

[2]For simplicity, we assume that channels are independent from each other, so they can be considered separately. In combinatorial auctions, different combinations of channels may be different, making the spectrum allocation much more complicated. We will consider the combination of channels in future works.

or terminal device, buyer c has a minimum spectrum requirement of l_c. Although buyer c is willing to acquire as many channels as possible, she has a maximum quota of h_c due to limitations such as the budget constraint.

Interference relationship. The key feature of spectrum is reusability, which must conform to interference constraints. Interference relationship among buyers is usually characterized by interference graph, in which the nodes denote buyers, and two nodes share an edge if the two buyers interfere with each other. Different channels have different transmission ranges, resulting in a diversity of interference relationships. To capture such heterogeneity [7], we construct a series of interference graphs $\{G^s = (\mathcal{N}, E^s)\}_{s=1}^{M}$, in which $e^s \in E^s$ connects a pair of interfering buyers on channel s. Let $e^s_{c,c'} \in \{0, 1\}$ denote the interference status between buyers c and c' regarding channel s. If $e^s_{c,c'} = 0$, buyer c, c' can reuse the same channel.

Preferences of buyers and sellers. In a conventional matching framework, a buyer/seller expresses her preferences towards different sellers/buyers through a preference list, which is a complete, reflexive and transitive relation. Nevertheless, the preference list defined over individual buyer/seller is not enough for spectrum matching. For example, let $\succ_s$ denote the preference list of seller s, and we have $c_1 \succ_s c_2 \succ_s c_3 \succ_s c_4$. If buyer c_1, c_4 can reuse the same channel, and buyer c_2, c_3 can reuse the same channel, we cannot decide whether $\{c_1, c_4\}$ is more preferred than $\{c_2, c_3\}$. One possible solution is to define the preference list of a seller over all combinations of buyers, which is undesirable since the number of possible combinations is 2^N, and many combinations consisting interfering buyers are invalid. To address this problem, we borrow the concept of *bid* from spectrum auction[3]. Buyer c has a bid vector $B_c = (b^1_c, b^2_c, ..., b^M_c)$, in which b^s_c is the price she is willing to pay for channel s. The preference list of buyer c can be simply constructed as follows:

$$s \succ_c s' \Leftrightarrow b^s_c \geq b^{s'}_c.$$

If $b^s_c = b^{s'}_c$, we can randomly determine their preference relation. A seller always prefers to sell her channel to a set of buyers whose aggregate price is higher, as long as they do not interfere with each other. Let $\mathcal{A}, \mathcal{B} \subseteq \mathcal{N}$ denote two subsets of buyers, seller s prefers $\mathcal{A}$ to $\mathcal{B}$ if: 1) $\mathcal{A}$ contains only non-interfering buyers, and the aggregate bid of $\mathcal{A}$ is higher than that of $\mathcal{B}$, or; 2) $\mathcal{A}$ contains only non-interfering buyers, but $\mathcal{B}$ does not.

$$\mathcal{A} \succ_s \mathcal{B} \Longleftrightarrow \begin{cases} \forall c, c' \in \mathcal{A}, e^s_{c,c'} = 0, \sum_{c\in\mathcal{A}} b^s_c > \sum_{c\in\mathcal{B}} b^s_c, \text{or} \\ \exists c, c' \in \mathcal{B}, e^s_{c,c'} = 1. \end{cases}$$

If $\sum_{c\in\mathcal{A}} b^s_c = \sum_{c\in\mathcal{B}} b^s_c$, or both $\mathcal{A}$ and $\mathcal{B}$ contain interfering buyers, we can randomly determine their preference relation.

4 SPECTRUM MATCHING FRAMEWORK

4.1 Basic Model

We formally define spectrum matching as follows.

[3]In spectrum auction, bids are important for determining the price paid by buyers to sellers, but the spectrum allocation in auction is bid-independent in order to guarantee truthfulness [7]. In this paper, we only target at a stable matching result, but not the price determination. We will consider it as a future direction.

Definition 4.1 (Spectrum Matching). Given the set of sellers $\mathcal{M}$ and the set of buyers $\mathcal{N}$, a spectrum matching is a mapping μ: $\mathcal{M} \cup \mathcal{N} \rightarrow 2^{\mathcal{N}} \cup 2^{\mathcal{M}}$, such that:

- For every seller $s \in \mathcal{M}$, $\mu(s) \subseteq \mathcal{N}$.
- For every buyer $c \in \mathcal{N}$, $\mu(c) \subseteq \mathcal{M}$.
- For every seller s and buyer c, $s \in \mu(c)$ if and only if $c \in \mu(s)$.

The major differences that separate spectrum matching from common goods matching [11] are interference constraint and corresponding spectrum reusability. A common item can only be sold to one buyer, hence we have $\mu(s) \in \mathcal{N}$ in [11]. However, in spectrum matching, we can match a set of non-interfering buyers to the same channel, i.e., $\mu(s) \subseteq \mathcal{N}$, which significantly improves spectral efficiency, but makes it more difficult to reach a stable matching.

A spectrum matching is feasible only if it satisfies the interference constraint, the maximum quota and the minimum requirement.

Definition 4.2 (Feasible Spectrum Matching). A spectrum matching is feasible, if it satisfies:

- Interference constraint. For every seller $s \in \mathcal{M}$, $\forall c, c' \in \mu(s), e^s_{c,c'} = 0$, i.e., buyers matched to the same seller should be interference-free.
- Maximum quota and minimum requirement. For every buyer $c \in \mathcal{N}$, $l_c \leq |\mu(c)| \leq h_c$, i.e., the number of channels matched to a buyer should be no fewer than her minimum requirement and no greater than her maximum quota.

4.2 Properties

Buyers and sellers are selfish and rational individuals who will break off from the matching result if they have better choices. A stable spectrum matching features *individual rationality*, *fairness*, and *nonwastefulness*.

Definition 4.3 (Individual Rationality). A feasible spectrum matching μ is individually rational if:

- Every buyer prefers the current set of matched channels to any of its subsets.
- Every seller prefers the current set of matched buyers to any of its subsets.

Being individually rational is the basis of a stable matching. To define fairness, we have to introduce the concept of type I blocking pair, which is tailored under the framework of spectrum matching.

Definition 4.4 (Type I Blocking Pair). Given a feasible spectrum matching μ, buyer c and seller s form a type I blocking pair (s, c), if:

(1) Seller s prefers buyer c to some of her currently matched buyers, and buyer c does not interfere with other buyers currently matched to seller s.
(2) Buyer c prefers channel s to one of her currently matched channels.

Mathematically speaking, buyer c and seller s form a type I blocking pair (s, c), if:

(1) $\exists \mathcal{A} \subseteq \mu(s), \{c\} \succ_s \mathcal{A}$, and $\forall c' \in \mu(s) \setminus \mathcal{A}, e_{c,c'} = 0$.
(2) $\exists s' \in \mu(c), s \succ_c s'$.

The type I block pair makes a spectrum matching unstable because buyer c can replace a less-preferred channel s' with a more-preferred channel s, and seller s may gain a higher profit by evicting a certain set of buyers $\mathcal{A}$ and let buyer c reuse her channel with the remaining buyers. Different from the definition of type I blocking pair in [11], due to interference constraint, it is required that buyer c is not only superior to the buyer set $\mathcal{A}$, but also interference-free from all other buyers in $\mu(s) \setminus \mathcal{A}$.

Definition 4.5 (Fairness). A feasible spectrum matching μ is fair if and only if there are no type I blocking pairs.

Apart from the type I blocking pair, we also have the type II blocking pair.

Definition 4.6 (Type II Blocking Pair). Given a feasible spectrum matching μ, buyer c and seller s form a type II blocking pair (s, c), if:

(1) Seller s prefers buyer c to some of her currently matched buyers, and buyer c does not interfere with other buyers currently matched to seller s.
(2) Buyer c can purchase channel s without violating her maximum quota.

Mathematically speaking, seller s and buyer c form a type II blocking pair (s, c), if:

(1) $\exists \mathcal{A} \subseteq \mu(s), \{c\} \succ_s \mathcal{A}$, and $\forall c' \in \mu(s) \setminus \mathcal{A}, e_{c,c'} = 0$.
(2) $|\mu(c)| < h_c$.

The type II blocking pair makes a spectrum matching unstable because buyer c can acquire one more channel under her maximum quota, and the purchase can benefit seller s as well.

Definition 4.7 (Nonwastefulness). A feasible spectrum matching μ is nonwasteful if and only if there are no type II blocking pairs.

The difference between type I and type II blocking pairs is whether or not a buyer will abandon a currently matched channel for a more-preferred channel. If yes, this results in a type I blocking pair; otherwise, this leads to a type II blocking pair.

Definition 4.8 (Strong Stability). A feasible spectrum matching μ is strongly stable if it is individual rational, fair and non-wasteful.

5 STABLE SPECTRUM MATCHING

In this section, we first consider spectrum matching without minimum requirement, and adapt the deferred acceptance algorithm to account for spectrum reusability. Then, based on the adapted deferred acceptance algorithm, we develop a stable matching algorithm that fulfill the minimum requirement of buyers.

5.1 Spectrum Matching Without Minimum Requirement

The traditional deferred acceptance algorithm, designed to solve the school admission problem, runs as follows [12]. There is a set of students to be admitted to a set of schools, each with a maximum quota. In the first round, each student applies for her favorite school. Among all applicants, a school with a maximum quota of h temporarily puts the top h most-preferred students in its waiting list, or all students if the number of applicants is smaller than h; other applicants are rejected. In the following rounds, each rejected student applies for her most-preferred school that has never rejected her before. Each school updates its waiting list by selecting the top h students among the current applicants and those in the previous waiting list. This process is repeated until all students have exhausted the schools that they can apply for.

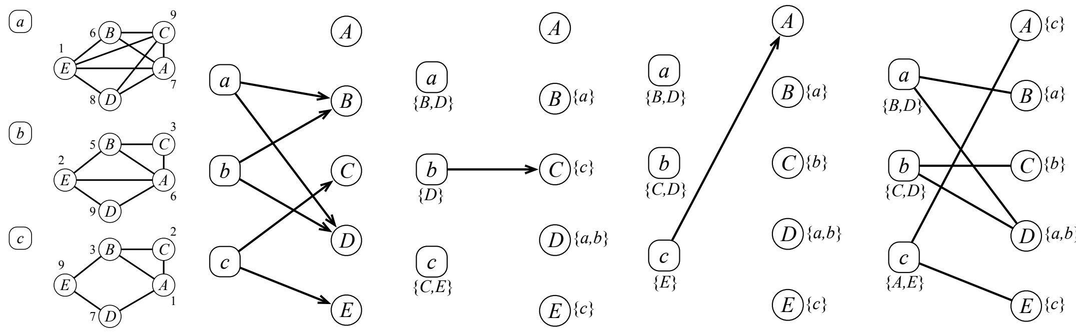
Algorithm 1 Adapted Deferred Acceptance (ADA) Algorithm for Spectrum Matching

Input: Preference lists of all buyers $\succ_c, \forall c \in \mathcal{N}$, preference lists of all sellers $\succ_s, \forall s \in \mathcal{M}$, maximum quotas of buyers, $h_c, \forall c \in \mathcal{N}$, interference graph $G_s, \forall s \in \mathcal{M}$.
Output: A spectrum matching μ.

1: $\forall c \in \mathcal{N}, \mu(c) = \Phi$, the waiting list $\mathcal{W}_c = \Phi$.
2: $\forall s \in \mathcal{M}, \mu(s) = \Phi$, the candidate list $\mathcal{A}_s = \mathcal{N}$.
3: **while** $\exists \mathcal{A}_s \neq \Phi$ **do**
4:   **for all** Seller s with non-empty $\mathcal{A}_s$ **do**
5:     Q_s := set of buyers that satisfies $c \in \mathcal{A}_s$, $\forall c' \in \mu(s), e_{c,c'} = 0$.
6:     Find the maximum weighted independent set on Q_s as Q_s^{max}.
7:   **end for**
8:   **if** $\forall s, Q_s^{max} = \Phi$ **then**
9:     Return μ.
10:   **else**
11:     **for all** Buyer $c \in Q_s^{max}$ **do**
12:       Seller s applies for buyer c.
13:       Seller s removes buyer c from her candidate list, $\mathcal{A}_s = \mathcal{A}_s \setminus \{c\}$.
14:       Buyer c adds seller s to her waiting list, $\mathcal{W}_c = \mathcal{W}_c \cup \{s\}$.
15:     **end for**
16:   **end if**
17:   **for all** Buyer c with non-empty $\mathcal{W}_c$ **do**
18:     Buyer c accepts no more than h_c most-preferred channels in $\mathcal{W}_c \cup \mu(c)$ as the new $\mu(c)$, and rejects others.
19:     Clear the waiting list $\mathcal{W}_c = \Phi$.
20:   **end for**
21:   Every seller s updates her matching $\mu(s)$.
22: **end while**
23: Return μ.

To adapt the original deferred acceptance algorithm to incorporate spectrum reusability, we assume that $l_c = 0, \forall c \in \mathcal{N}$, so that there is no minimum requirement for any buyer. We view sellers as students, and buyers as schools with maximum quotas. Instead of applying for only one buyer at each round, a seller can apply for a set of non-interfering buyers. As shown in Algorithm 1, we start with an empty matching. $\mathcal{A}_s$ denotes the candidate buyers that seller s has not applied for. The algorithm runs as follows.

(1) In each round, seller s first chooses a subset of buyers in $\mathcal{A}_s$ that do not interfere with any buyers in $\mu(c)$, denoted by Q_s (line 7), then finds the maximum weighted independent set among these buyers based on their bids, denoted by Q_s^{max} (line 8). Q_s^{max} is the best set of buyers that seller s can apply for in the current round. If Q_s^{max} is empty for all



(a) Interference graph. (b) First round. (c) Second round. (d) Third round. (e) Final matching result.

Figure 1: A toy example of the Adapted Deferred Acceptance (ADA) algorithm (Algorithm 1).

Algorithm 2 Spectrum Matching Transformation

Input: Buyer set $\mathcal{N}$, seller set $\mathcal{M}$, preference lists of buyers $\succ_c$, $\forall c \in \mathcal{N}$, preference lists of sellers $\succ_s, \forall s \in \mathcal{M}$, maximum quota and minimum requirement of buyers $l_c, h_c, \forall c \in \mathcal{N}$, interference graph $G_s, \forall s \in \mathcal{M}$.

Output: Buyer set $\widetilde{\mathcal{N}}$, seller set $\widetilde{\mathcal{M}}$, preference lists of buyers $\widetilde{\succ}_c, \forall c \in \widetilde{\mathcal{N}}$, preference lists of sellers $\widetilde{\succ}_s, \forall s \in \widetilde{\mathcal{M}}$, maximum quotas of buyers $\widetilde{h}_c, \forall c \in \widetilde{\mathcal{N}}$, interference graph $\widetilde{G}_s, \forall s \in \widetilde{\mathcal{M}}$.

$\widetilde{\mathcal{M}} = \mathcal{M}$. $\widetilde{\mathcal{N}}^e = \Phi, \widetilde{\mathcal{N}}^r = \Phi$.
for all $c \in \mathcal{N}$ **do**
  $\widetilde{\mathcal{N}}^e = \widetilde{\mathcal{N}}^e \cup \{c^e\}$. $\widetilde{\mathcal{N}}^r = \widetilde{\mathcal{N}}^r \cup \{c^r\}$.
  $h_{c^r} = l_c, h_{c^e} = h_c - l_c$.
  $\widetilde{\succ}_{c^r} := \widetilde{\succ}_{c^e} := \succ_c$.
end for
$\widetilde{\mathcal{N}} = \widetilde{\mathcal{N}}^e \cup \widetilde{\mathcal{N}}^r$.
for all $s \in \mathcal{M}$ **do**
  Change $\succ_s := c_{s,1} \succ_s c_{s,2} \succ_s \cdots$ into $\widetilde{\succ}_s := c^r_{s,1} \widetilde{\succ}_s c^e_{s,1} \widetilde{\succ}_s c^r_{s,2} \widetilde{\succ}_s c^e_{s,2} \widetilde{\succ}_s \cdots$.
  for all $c \in G_s$ **do**
    Add c^r and c^e to $\widetilde{G}_s$.
    c^r and c^e each inherits all edges of c in G^s.
    Create an edge between c^r and c^e.
  end for
end for

sellers, the algorithm terminates and returns the matching result (line 10~12); otherwise, proceed to the next step.

(2) Seller s applies for every buyers in Q_s^{max}, and removes them from her candidate set (line 14~15). A buyer c who has received applications will add the corresponding sellers to her waiting list $\mathcal{W}_c$ (line 16).

(3) A buyer c with non-empty waiting list $\mathcal{W}_c$ will select no more than h_c most-preferred sellers from $\mathcal{W}_c \cup \mu(c)$, and reject others. Buyer c updates $\mu(c)$ to be these selected sellers (line 20).

(4) All sellers update their matching results according to those of the buyers (line 23).

Toy Example. As shown in Fig. 1, there are three sellers ($a \sim c$) and five buyers ($A \sim E$). The maximum quotas of buyers are $(h_A, h_B, h_C, h_D, h_E) = (1, 1, 1, 2, 1)$. The interference graph on each channel is shown in Fig. 1(a), and the values beside each node are buyers' heterogeneous bids for different channels. A buyer's preference list can be constructed from her bids for all channels. Note that due to spectrum heterogeneity, the transmission range of different channels are different, leading to heterogeneous interference relationships. For example, buyer C and buyer E interfere with each other on the long-transmission-range channel a, but do not interfere with each other on the short-transmission-range channels b and c. The process of the adapted deferred acceptance algorithm is shown in Fig. 1(b)~(e). To begin with, seller a, b and c each apply for a set of non-interfering buyers with the maximum aggregate bid on their own interference graph (Fig. 1(b)). Buyer D accepts both channel a and b, since $h_D = 2$. Buyer B has to reject seller b, since $h_B = 1$, and buyer B prefers channel a to channel b. Buyer C and buyer E both accept channel c. In the second round, even though the candidate set of seller a is not empty ($\mathcal{A}_a = \{A, C, E\}$), she cannot apply for any buyer since they all interfere with buyer B or D, who are currently matched to seller a, i.e., $Q_a^{max} = \Phi$. The same is true for seller c. In contrast, seller b can apply for buyer C who can reuse the channel with buyer D (Fig. 1(c)). Since the maximum quota of buyer C is $h_C = 1$, she has to give up channel c for the more-preferred channel b. Thanks to this, in the third round, seller c can apply for buyer A in her candidate set $\mathcal{A}_c = \{A, B, D\}$, who interferes with buyer C but not with buyer E (Fig. 1(d)). We can check that Q_s^{max} is empty for all sellers, and the final spectrum matching result is given in Fig. 1(e).

5.2 Spectrum Matching With Minimum Requirement

To address the minimum requirement in matching, we are inspired by the extended deferred acceptance algorithm in [11] with minimum spectrum requirement of buyers. The basic idea is to divide each buyer into a regular buyer and an extended buyer. Let c^r and c^e denote the regular and the extended buyer of the original buyer c, respectively. By setting the maximum quota of the regular buyer c^r as $h_{c^r} = l_c$, and the maximum quota of the extended buyer c^e as $h_{c^e} = h_c - l_c$, we eliminate the minimum requirement of all buyers. For common goods, if we reserve at least $\sum_c l_c$ items for regular buyers (assign no more than $E = N - \sum_c l_c$ items to extended

Algorithm 3 Extended Deferred Acceptance (EDA) Algorithm for Spectrum Matching with Minimum Requirement

Input: Preference lists of buyers $\widetilde{\succ}_c, \forall c \in \widetilde{\mathcal{N}}$, preference lists of sellers $\widetilde{\succ}_s, \forall s \in \mathcal{M}$, maximum quotas of buyers $\widetilde{h}_c, \forall c \in \widetilde{\mathcal{N}}$, interference graph $\widetilde{G}_s, \forall s \in \widetilde{\mathcal{M}}$.
Output: Spectrum matching $\widetilde{\mu}$.

1: $\forall c \in \widetilde{\mathcal{N}}, \widetilde{\mu}(c) = \Phi$, the waiting list $\mathcal{W}_c = \Phi$.
2: $\forall s \in \widetilde{\mathcal{M}}, \widetilde{\mu}(s) = \Phi$, the candidate list $\mathcal{A}_s = \widetilde{\mathcal{N}}$.
3: **while** $\exists \mathcal{A}_s \neq \Phi$ **do**
4:   **for all** Seller s with non-empty $\mathcal{A}_s$ **do**
5:     Find Q_s^{max} as in Algorithm 1.
6:   **end for**
7:   **if** $\forall s, Q_s^{max} = \Phi$ **then**
8:     Return $\widetilde{\mu}$.
9:   **else**
10:     **for all** Buyer $c \in Q_s^{max}$ **do**
11:       Seller s applies for buyer c.
12:       Seller s removes buyer c from her candidate list, $\mathcal{A}_s = \mathcal{A}_s \setminus \{c\}$.
13:       Buyer c adds seller s to her waiting list, $\mathcal{W}_c = \mathcal{W}_c \cup \{s\}$.
14:     **end for**
15:   **end if**
16:   **for all** Buyer c with non-empth $\mathcal{W}_c$ **do**
17:     **if** $c \in \widetilde{\mathcal{N}}^r$ **then**
18:       Buyer c accepts no more than $\widetilde{h}_c$ channels in $\mathcal{W}_c \cup \widetilde{\mu}(c)$, and reject others.
19:       Clear the waiting list $\mathcal{W}_c = \Phi$.
20:     **else**
21:       $e = 0, j = 1$.
22:       **for all** $c \in \widetilde{\mathcal{N}}^e$ **do**
23:         $\mathcal{W}_c = \mathcal{W}_c \cup \mu(c), \mu(c) = \Phi$.
24:       **end for**
25:       **while** $e < E$ and $\exists c^e, \mathcal{W}_{c^e} \neq \Phi, |\widetilde{\mu}(c^e)| < \widetilde{h}_{c^e}$ **do**
26:         **if** $|\mu(c_j^e)| < \widetilde{h}_{c_j^e}$ and $\mathcal{W}_{c_j^e} \neq \Phi$ **then**
27:           Buyer c_j^e chooses her most-preferred channel s in $\mathcal{W}_{c_j^e}$, i.e., $\widetilde{\mu}(c_j^e) = \widetilde{\mu}(c_j^e) \cup \{s\}$.
28:           $\mathcal{W}_{c_j^e} = \mathcal{W}_{c_j^e} \setminus \{s\}$.
29:           $e = e + 1$.
30:         **end if**
31:         $j = j + 1 \mod |\widetilde{\mathcal{N}}^e|$.
32:       **end while**
33:     **end if**
34:     $\mathcal{W}_c = \Phi$.
35:   **end for**
36: **end while**

buyers), their maximum quotas will be filled up, which means that the minimum requirement of all original buyers will be satisfied. Nevertheless, to calculate how many channels to withhold for regular buyers in spectrum matching is non-trivial. For example, there are 4 buyers, none of whom interfere with each other, and each with a minimum requirement of 1. Instead of reserving 4 channels, we can just keep 1 channel for all buyers to reuse and fulfill their minimum requirement. In the rest of this section, we first assume that a number of E channels can be assigned to extended buyers ($N - E$ channels are reserved for regular buyers), and introduce the extended deferred acceptance algorithm to reach a feasible stable matching, then we explain how to compute E.

5.2.1 Matching Transformation. The transformation of the spectrum matching with minimum requirement into the spectrum matching without minimum requirement is elaborated in Algorithm 2. The seller set $\mathcal{M}$ is unchanged. The preference lists of the regular buyer and the extended user are the same as the original buyer (line 7). The preference list of each seller is reconstructed by inserting the extended buyer right after the regular buyer without changing the sequence of the original preference list (line 11). When rebuilding the interference graph of channel s, we create a regular and an extended nodes for each node in the original graph. These two nodes inherit all the interference relationship of the original nodes, and there is an edge between them, because they cannot share the same channel, as they essentially represent the same buyer.

5.2.2 Main Matching Algorithm. Given the transformed buyers and sellers, the spectrum matching algorithm is shown in Algorithm 3. The process of seller application is similar to that of Algorithm 1, but unlike Algorithm 1, we have different rules for regular buyers and extended buyers to decide whether to accept or reject a seller.

(1) If buyer c belongs to the regular buyer set $\widetilde{\mathcal{N}}^r$, she will pick up no more than $\widetilde{h}_c$ channels from $\mathcal{W}_c \cup \mu(c)$ (line 17~19), similar to that in Algorithm 1.
(2) If buyer c belongs to the extended buyer set $\widetilde{\mathcal{N}}^e$, we will consider all other extended buyers to rearrange their matching results. Let e be the counter to record the number of channels assigned to all extended buyers (line 21). Firstly, we will put every extended buyer's matched channels $\mu(c)$ to her waiting list $\mathcal{W}_c$, and clear her matched channels (line 23). Then, we sequentially check the extended buyers in a specific order $c_1^e, c_2^e, ..., c_N^e$. Let j denote the index of buyer that is being considered, and initially set $j = 1$.
   (a) If the number of channels matched to all extended buyers equals E (the maximum number of channels for all extended buyers), or every extended buyer either has an empty waiting list, or fulfills her maximum quota, the algorithm returns to step (1) (line 16); otherwise, the algorithm proceeds to step (b).
   (b) Buyer c_j^e can choose her most-preferred channel from the waiting list (just one channel), as long as her maximum quota is not hit ($|\mu(c_j^e)| < h_{c_j^e}$) and her waiting list is non-empty ($\mathcal{P}_{c_j^e} \neq \Phi$). Increase j by 1, and return to step (a).

Given the matching result $\widetilde{\mu}$ of Alg. 3, we can obtain the original matching as $\forall c \in \mathcal{N}, \mu(c) = \widetilde{\mu}(c^e) \cup \widetilde{\mu}(c^r), \forall s \in \mathcal{M}$, if $c^e \in \widetilde{\mu}(s)$ or $c^r \in \widetilde{\mu}(s)$, then $c \in \mu(s)$.

5.2.3 Determine E. Now, we determine how many channels to reserve for regular buyers. Assume that the minimum requirement of every buyer is 1 and the channels are homogeneous (the interference graphs are the same), then finding the number of reserved channels is equivalent to the graph coloring problem: coloring the

Algorithm 4 Determining E

Input: Buyer set $\mathcal{N}$, seller set $\mathcal{M}$, minimum requirement of buyers $h_c, \forall c \in \mathcal{N}$, interference graph G.
Output: E.

1: **for all** node c in G **do**
2:   Create h_c nodes to replace c in G.
3:   Each of the h_c nodes inherits all edges of c in G.
4:   Create an edge between each pair of the h_c nodes.
5: **end for**
6: $\mathcal{L} := \Phi$.
7: $\mathcal{V}$:= a random list of all buyers.
8: **while** $\mathcal{V}$ is not empty **do**
9:   Remove the first buyer v_i from the list.
10:   $\widetilde{\mathcal{L}} \subseteq \mathcal{L}$:= the subset of channels that are not assigned to v_i's neighbors among $v_1, ..., v_{i-1}$.
11:   **if** $\widetilde{\mathcal{L}}$ is not empty **then**
12:     Assign the channel with the lowest index in $\widetilde{\mathcal{L}}$ to v_i.
13:   **else**
14:     Assign a new channel s with the lowest index in $\mathcal{M}$ to v_i.
15:     $\mathcal{M} = \mathcal{M} \setminus \{s\}, \mathcal{L} = \mathcal{L} \cup \{s\}$.
16:   **end if**
17: **end while**
18: $E = M - |\mathcal{L}|$.

nodes of the interference graph such that no two adjacent nodes share the same color. To verify whether it is possible to color a graph with k colors is a $k-$coloring problem, which is NP-complete except for the cases $k \in \{0, 1, 2\}$. To overcome such computational hardness, we propose a greedy algorithm, as shown in Alg. 4. Firstly, since the minimum requirement of each buyer is h_c, which may be greater than 1, we create h_c virtual buyers to replace the original buyer in the interference graph[4]. Similar to that in Section 3, each virtual buyer inherits all interference relationship of the original buyer, and every two virtual buyers of the same original buyer share an edge between them. In this way, we reduce the original problem to the $k-$coloring problem. We rank all buyers in a specific order $\mathcal{V} = \{v_1, v_2, ..., v_N\}$[5], and give every channel an index. Let $\mathcal{L}$ denote the set of channels that have already been used. In each iteration, we pick up a buyer v_i in $\mathcal{V}$ (line 9), and seek for the set of channels in $\mathcal{L}$ that are not used by any of v_i's neighbors among $v_1, ..., v_{i-1}$ (line 10). If the resulting channel set is non-empty, we assign the available channel with the lowest index to v_i (line 12); otherwise, we retrieve a new channel from $\mathcal{M}$ for v_i and add it to $\mathcal{L}$. This process continues until all buyers are assigned a channel. The number of reserved channels for regular buyers is $|\mathcal{L}|$, and the number of channels for extended buyers is $E = M - |\mathcal{L}|$.

Toy Example. We assume that there are three buyers A, B, C and six sellers $a \sim f$. The preference lists of buyers and sellers are shown in Table 1. Using Algorithm 2, we can obtain the transformed buyers and sellers, also shown in Table 1. For simplicity, we assume that

Table 1: Toy example

	$\succ_A$	$\succ_B$	$\succ_C$
	$a:6$	$c:6$	$f:6$
	$e:5$	$e:5$	$c:5$
	$b:4$	$f:4$	$d:4$
	$f:3$	$d:3$	$b:3$
	$c:2$	$b:2$	$e:2$
	$d:1$	$a:1$	$a:1$
l	2	1	2
h	3	3	3

	$\widetilde{\succ}_{A^r}$	$\widetilde{\succ}_{A^e}$	$\widetilde{\succ}_{B^r}$	$\widetilde{\succ}_{B^e}$	$\widetilde{\succ}_{C^r}$	$\widetilde{\succ}_{C^e}$
	a	a	c	c	f	f
	e	e	e	e	c	c
	b	b	f	f	d	d
	f	f	d	d	b	b
	c	c	b	b	e	e
	d	d	a	a	a	a
$\widetilde{h}_c$	2	1	1	2	2	1

$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$	$\succ_e$	$\succ_f$
A	A	B	C	A	C
B	C	C	B	B	B
C	B	A	A	C	A

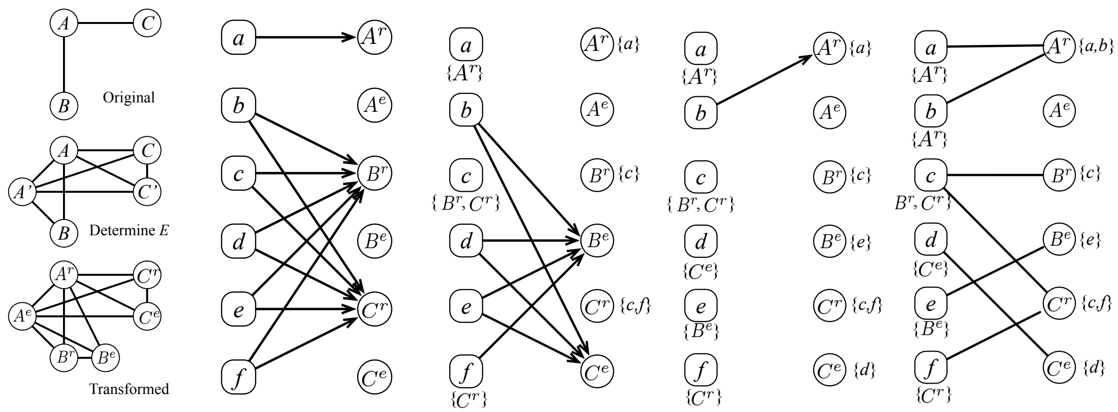
$\widetilde{\succ}_a$	$\widetilde{\succ}_b$	$\widetilde{\succ}_c$	$\widetilde{\succ}_d$	$\widetilde{\succ}_e$	$\widetilde{\succ}_f$
A^r	A^r	B^r	C^r	A^r	C^r
A^e	A^e	B^e	C^e	A^e	C^e
B^r	C^r	C^r	B^r	B^r	B^r
B^e	C^e	C^e	B^e	B^e	B^e
C^r	B^r	A^r	A^r	C^r	A^r
C^e	B^e	A^e	A^e	C^e	A^e

the interference graphs are the same for all channels. As the minimum requirement of buyers are $(2, 1, 2)$, we create virtual buyers as shown in Fig. 2(a), and fix the order of buyers as A, A', B, C, C'. To start with, we assign channel a to buyer A, channel b to buyer A', and channel c to buyer B, resulting in $\mathcal{L} = \{a, b, c\}$. Then, channel c is assigned to buyer C, since it is not occupied by her neighbors A and A'. Finally, we have to add channel d to $\mathcal{L}$ for C'. We can see that 4 instead of $2 + 1 + 2 = 5$ channels need to be reserved for regular buyers. Therefore, we can compute E as $6 - 4 = 2$.

In the first round, as shown in Fig. 2(b), seller a applies for buyer A^r, and all other sellers applies for buyer B^r and C^r. Buyer A^r and buyer B^r accepts seller a and seller c, respectively. Buyer C^r accepts seller c and f since her maximum quota is 2. In the second round, as shown in Fig. 2(c), seller b, d and e applies for extended buyers B^e and C^e, and seller f applies for extended buyer C^e. Recall that all extended buyers can have no more than $E = 2$ channels. The waiting lists for buyer A^e, B^e, C^e are $\Phi, \{b, d, e, f\}, \{b, d, e\}$. We pass buyer A^e as her waiting list is empty. Buyer B^e accepts seller e, then buyer C^e accepts seller d, making $e = 2 = E$. In the third round, as shown in Fig. 2(c), seller b applies for buyer A^r, who accepts her. The final spectrum matching result is shown in Fig. 3. We can check that all minimum requirement are meet.

[4]We conservatively choose the interference graph of the channel with the longest transmission range (least chances for spectrum reuse) to calculate the number of reserved channels for regular buyers. In future works, we will consider heterogeneous interference graph for determining E.

[5]The sequence of buyers may affect the performance of the algorithm, but it is difficult to determine the optimal sequence. We will leave it to future works.



(a) Interference graph. (b) First round. (c) Second round. (d) Third round. (e) Final matching result.

Figure 2: A toy example of the Extended Deferred Acceptance (EDA) algorithm (Algorithm 3).

5.3 Properties

Though strong stability is most desirable, the following proposition tells us that it may not exist.

Proposition 5.1. *Strong stable matching does not always exist.*

Proof. It has been proved in [11] that for common goods (without reusibility), a simultaneously fair and non-wasteful matching may not exist, when minimum requirement is considered. Consider a special case where all buyers interfere with each other, i.e., the interference graph is complete, spectrum matching is equivalent to common goods matching. Therefore, a strong stable matching may not exist. □

Therefore, we focus on a relaxed stability concept [24].

Definition 5.2. (Weak Stability). A matching is weakly stable if it is individual rational and fair.

Proposition 5.3. *The matching result of the proposed EDA algorithm is individually rational.*

Proof. According to Definition 4.3, in the final matching, every buyer prefers the current set of matched channels to any subset of these channels, because as long as the maximum quota is not exceeded, more channels will provide a buyer with a higher satisfaction; every seller prefers the current set of matched buyers to any subset of these buyers, because as long as the interference constraint is not violated, more buyers will provide a seller with a higher profit. □

To prove that the matching result of the EDA algorithm is fair, we first introduce the following lemma.

Lemma 5.4. *If a seller s is rejected by an extended buyer c^e, it must be true that $\forall s' \in \widetilde{\mu}(c^e)$, we have $s' \widetilde{\succ}_{c^e} s$.*

Proof. Assume that seller s is rejected by buyer c^e at round t, and $\widetilde{\mu}_t(s)$ denotes the (temporary) matching result at round t. We will first prove that $\forall t' > t, |\widetilde{\mu}_{t'}(c^e)| \leq |\widetilde{\mu}_t(c^e)|$. If at round t, the number of channels matched to buyer c reached her maximum quota, i.e., $|\widetilde{\mu}_t(c^e)| = h_{c^e}$, it is obviously true that $\forall t' > t, |\widetilde{\mu}_{t'}(c^e)| \leq |\widetilde{\mu}_t(c^e)|$. If $|\widetilde{\mu}_t(c^e)| < h_{c^e}$, consider round $t+1$. If no seller applies for buyer c^e, we naturally have $|\widetilde{\mu}_{t+1}(c^e)| = |\widetilde{\mu}_t(c^e)|$. So we consider that a seller applies for buyer c^e. Since all channels temporarily matched to extended buyers other than c^e at round t are still temporarily matched to these extended buyers at round $t+1$, according to line 25~32 in Algorithm 3, the cap E will be hit before buyer c^e is able to accept the $(|\widetilde{\mu}_t(c^e)|+1)th$ seller. Therefore, we have $|\widetilde{\mu}_{t+1}(c^e)| \leq |\widetilde{\mu}_t(c^e)|$.

As seller s is rejected by c^e at round t, we know that $\forall s' \in \widetilde{\mu}_t(c^e), s' \succ_{c^e} s$. Now, we consider round $t+1$. Since we have proved that $|\widetilde{\mu}_{t+1}(c^e)| \leq |\widetilde{\mu}_t(c^e)|$, which means that the least-preferred seller in $\widetilde{\mu}_{t+1}(c^e)$ must be better than the least-preferred seller in $\widetilde{\mu}_t(c^e)$, we can derive that $\forall s'' \in \widetilde{\mu}_{t+1}(c^e), s'' \succ_{c^e} s$. Therefore, in the final matching, we must have $\forall s' \in \widetilde{\mu}(c^e), s' \widetilde{\succ}_{c^e} s$. □

Proposition 5.5. *The matching result of the proposed EDA algorithm is fair.*

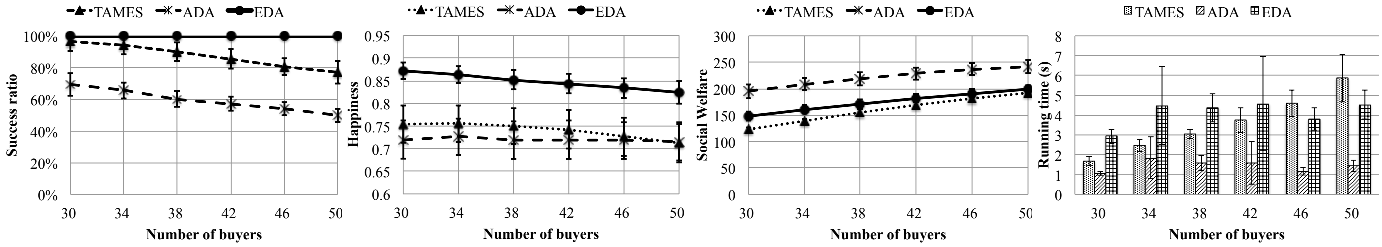
Proof. Suppose there exists a type I blocking pair (s, c) such that $\exists \mathcal{A} \subseteq \mu(s), \{c\} \succ_s \mathcal{A}$, and $\forall c' \in \mu(s) \setminus \mathcal{A}, e_{c,c'} = 0$. This indicates that s has applied for but been rejected by both c^r and c^e, because otherwise, seller s would have applied for c^r or c^e instead of buyers in $\mathcal{A}$. For any seller $s' \in \widetilde{\mu}(c^r)$, it must be true that $s' \succ_{c^r} s$, because s is rejected in favor of other h_{c^r} sellers who are more preferred than s. For any seller $s' \in \widetilde{\mu}(c^e)$, according to Lemma 5.4, we have $s' \succ_{c^e} s$. This infers that no type I blocking pair will exist. Therefore, the matching results of the proposed EDA algorithm is fair. □

In conclusion, we have the following theorem.

Theorem 5.6. *The matching result of the proposed EDA algorithm is weakly stable.*

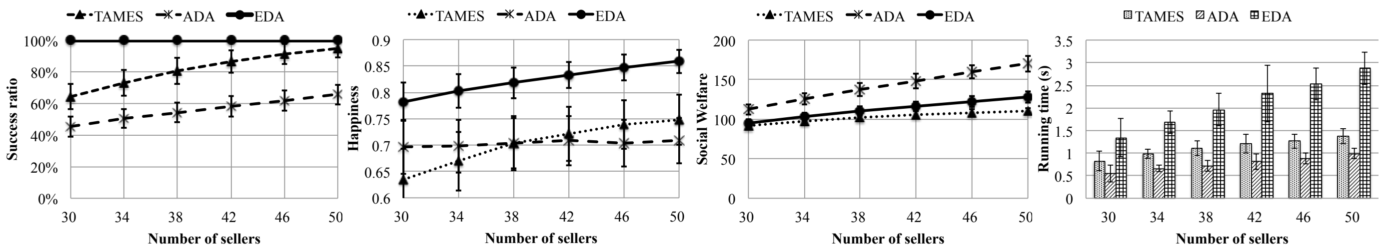
6 SIMULATION

In this section, we compare the performance of the proposed EDA algorithm with two benchmarks for comparison: one is the ADA algorithm without considering minimum requirement, and the other



(a) Success ratio. (b) Buyer happiness. (c) Social welfare. (d) Running time.

Figure 3: Performance of the proposed EDA algorithm. The number of sellers is fixed as $M = 60$. $l_c \sim$ unif$(5, 6)$, $h_c \sim$ unif$(10, 15)$.



(a) Success ratio. (b) Buyer happiness. (c) Social welfare. (d) Running time.

Figure 4: Performance of the proposed EDA algorithm. The number of buyers is fixed as $N = 30$. $l_c \sim$ unif$(4, 6)$, $h_c \sim$ unif$(10, 15)$.

is adapted from the matching process in TAMES, a spectrum double auction mechanism [7]. The detailed adaptation procedure is as follows. For every buyer $c \in \mathcal{N}$, we create l_c virtual buyers, and transform the interference graphs according to the process in Section 5.2.3. Then, according to a specific order of channels, we sequentially find the independent set on the corresponding interference graph to match to the corresponding seller. When seeking for the independent set for a particular channel, we greedily pick the buyer with the highest $b_c^s/(d_{G^s}(c)+1)$ and eliminate her interfering neighbors until the interference graph becomes empty [23]. Note that b_c^s is buyer c's bid for channel s, and $d_{G^s}(c)$ is the degree of buyer c on interference graph G_s. We will refer to this matching approach as TAMES in the following context.

We assume that buyers uniformly locate within a $100m \times 100m$ area, and the transmission range of a channel is drawn randomly from the range $[40m, 45m]$. Buyers' bids for different channels follow independent identical distribution $unif(1, 100)$. For performance evaluation, we focus on the following four metrics.

- *Success ratio.* A buyer whose matched channels meet her minimum requirement is considered to be successful in the spectrum matching process. We define the success ratio as the number of such buyers divided by N.
- *Buyer happiness.* The happiness of a buyer is the average rank percentile of her matched sellers [24].
- *Social welfare.* Social welfare is defined as the sum of successful buyers' bids for their matched channels [7].
- *Running time.*

Each simulation runs for 500 times on a ThinkPad laptop with Intel(R) Core (TM) i7-5600U CPU at 2.60GHz and 12.00GB RAM

6.1 Success Ratio

As shown in Fig. 3(a) and Fig. 4, the proposed EDA algorithm maintains a 100% success ratio, meaning that the minimum requirement of every buyer is satisfied. Even though TAMES transforms interference graphs based on the minimum requirements of buyers, it fails to achieve a 100% success ratio. This shows that EDA can better exploit spectrum reusability, and realizes a higher spectrum efficiency than TAMES. In comparison, the success ratio of ADA is the lowest. This confirms that the adaptation of the traditional deferred acceptance algorithm cannot be directly applied to spectrum matching where buyers have minimum requirements.

6.2 Buyer Happiness

The buyer happiness of the EDA algorithm outperforms those of TAMES and ADA, as shown in Fig. 3(b) and Fig. 4, indicating that a buyer are more likely to be matched to her more preferred channels. This further confirms the (weak) stability of EDA algorithm: since buyers are happier with the currently matched channels, they are less willing to deviate from the matching results. ADA ignores the minimum requirements of buyers, thus some buyers may suffer from spectrum starvation in the final matching results, hurting the average buyer happiness. TAMES merely groups buyers without considering their preferences for different channels, resulting in a low buyer happiness as well.

With more buyers competing for channels, a buyer has a lower chance to be matched to her favorite channels, therefore, the buyer happiness drops as the number of buyers increases (Fig. 3(b)). For a similar reason, the buyer happiness will be enhanced if there are more channels (Fig. 4(b)). Interestingly, as the number of seller increases, the buyer happiness of TAMES first falls below then surpasses that of ADA. The buyer happiness of ADA seems to be insensitive to either the number of buyers or sellers, the reason of which needs more exploration.

6.3 Social Welfare

The social welfare of ADA is the highest, and the social welfare of EDA is slightly better than that of TAMES, as shown in Fig. 3(c) and Fig. 4(c). By completely disregarding the constraint of minimum requirement, ADA matches buyers and seller solely according to their preference lists, which depend on the buyers' bid vectors (recall that social welfare is the sum of successful buyers' bids for their matched channels), therefore resulting in a high social welfare. It is naturally true that the increment in the number of either buyers or sellers will improve the social welfare.

6.4 Running Time

It is shown that EDA achieves a 100% success ratio and a higher buyer happiness at the cost of a longer running time than TAMES and ADA, as shown in Fig. 3(d) and Fig. 4(d), but the time complexity is acceptable. The running time of EDA is more sensitive to the number of sellers than the number of buyers. This is because every seller has to choose a set of non-interfering buyers to apply for, which is equivalent to selecting independent sets on interference graphs, the most time-consuming process in the algorithm.

7 CONCLUSION

In this paper, we present a spectrum matching framework for spectrum transactions where buyers have maximum quotas and minimum requirements. We first introduce the Adaptive Deferred Acceptance algorithm to tackle the problem of spectrum reusability, where multiple buyers who are not interfering with each other can be matched to the same channel. To fulfill the minimum requirements of all buyers, we further develop the Extended Deferred Acceptance (EDA) algorithm, which can generate an interference-free matching that is theoretically proved to be (weakly) stable. Unlike optimizing matching over a certain objective function, stable matching ensures that no buyers or sellers, albeit selfish, are willing to violate the matching result. The simulation results confirm that EDA can reach a 100% success ratio guaranteeing minimum requirements of buyers, and EDA can achieve a higher buyer happiness, with an acceptably longer running time than the benchmark algorithms.

8 ACKNOWLEDGMENT

This work was supported in part by National Natural Science Foundation of China under Grant No. 61373167, U1636219, Grant No. 61202393, the CPSF under Grant No. 2012M521797, the International Cooperation Foundation of Shaanxi Province, China under Grant No. 2013KW01-02, and the International Postdoctoral Exchange Fellowship Program 2013 under Grant No. 57 funded by the office of China Postdoctoral Council. The co-authors would also like to acknowledge the generous research support from a NSERC Discovery Research Program and a NSERC Strategic Partnership Grant titled "A Cloud-Assisted Crowdsourcing Machine-to-Machine Networking Platform for Vehicular Applications" at the University of Toronto. Qian Wang is the corresponding author.

REFERENCES

[1] M. Al-Ayyoub and H. Gupta. 2011. Truthful Spectrum Auctions with Approximate Revenue. In *IEEE International Conference on Computer Communications*.

[2] Siavash Bayat, Raymond HY Louie, Zhu Han, Yonghui Li, and Branka Vucetic. 2012. Distributed Stable Matching Algorithm for Physical Layer Security With Multiple Source-Destination Pairs and Jammer Nodes. In *IEEE Wireless Communications and Networking Conference (WCNC)*.

[3] Siavash Bayat, Raymond HY Louie, Zhu Han, Branka Vucetic, and Yonghui Li. 2013. Physical-Layer Security in Distributed Wireless Networks Using Matching Theory. *IEEE Transactions on Information Forensics and Security* 8, 5 (2013), 717–732.

[4] Elizabeth Bodine-Baron, Christina Lee, Anthony Chong, Babak Hassibi, and Adam Wierman. 2011. Peer Effects and Stability in Matching Markets. *Algorithmic Game Theory* (2011), 117–129.

[5] Xu Chen and Jianwei Huang. 2013. Database-Assisted Distributed Spectrum Sharing. *IEEE Journal on Selected Areas in Communications* 31, 11 (2013), 2349–2361.

[6] Yanjiao Chen, Linshan Jiang, Haofan Cai, Jin Zhang, and Baochun Li. 2016. Spectrum Matching. In *IEEE International Conference on Distributed Computing Systems (ICDCS)*.

[7] Yanjiao Chen, Jin Zhang, Kaishun Wu, and Qian Zhang. 2014. TAMES: A Truthful Double Auction for Multi-Demand Heterogeneous Spectrums. *IEEE Transactions on Parallel and Distributed Systems (TPDS)* 25, 11 (2014), 3012–3024.

[8] Lars Ehlers, Isa E Hafalir, M Bumin Yenmez, and Muhammed A Yildirim. 2014. School Choice with Controlled Choice Constraints: Hard Bounds Versus Soft Bounds. *Journal of Economic Theory* 153 (2014), 648–683.

[9] Xiaojun Feng, Yanjiao Chen, Jin Zhang, Qian Zhang, and Bo Li. 2012. TAHES: A Truthful Double Auction Mechanism for Heterogeneous Spectrums. *IEEE Transactions on Wireless Communications (TWC)* 11, 11 (2012), 4038–4047.

[10] Xiaojun Feng, Jin Zhang, and Qian Zhang. 2011. Database-Assisted Multi-AP Network on TV White Spaces: Architecture, Spectrum Allocation and AP Discovery. In *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks*.

[11] Daniel Fragiadakis, Atsushi Iwasaki, Peter Troyan, Suguru Ueda, and Makoto Yokoo. 2016. Strategyproof Matching with Minimum Quotas. *ACM Transactions on Economics and Computation* 4, 1 (2016), 6.

[12] David Gale and Lloyd S Shapley. 1962. College Admissions and the Stability of Marriage. *Amer. Math. Monthly* (1962), 9–15.

[13] L. Gao, J. Huang, Y. J. Chen, and B. Shou. 2012. ContrAuction: An Integrated Contract and Auction Design for Dynamic Spectrum Sharing. In *IEEE Annual Conference on Information Sciences and Systems (CISS)*.

[14] Masahiro Goto, Naoyuki Hashimoto, Atshushi Iwasaki, Yujiro Kawasaki, Suguru Ueda, Yosuke Yasuda, and Makoto Yokoo. 2014. Strategy-proof Matching with Regional Minimum Quotas. In *ACM International Conference on Autonomous Agents and Multi-agent Systems*.

[15] Yunan Gu, Yanru Zhang, Miao Pan, and Zhu Han. 2014. Cheating in Matching of Device to Device Pairs in Cellular Networks. In *IEEE Global Communications Conference (GLOBECOM)*.

[16] Koki Hamada, Kazuo Iwama, and Shuichi Miyazaki. 2011. The Hospitals/Residents Problem with Quota Lower Bounds. In *European Symposium on Algorithms*. Springer Berlin Heidelberg.

[17] H. Huang, X. Y. Li, Y. e. Sun, H. Xu, and L. Huang. 2015. PPS: Privacy-Preserving Strategyproof Social-Efficient Spectrum Auction Mechanisms. *IEEE Transactions on Parallel and Distributed Systems (TPDS)* 26, 5 (2015), 1393–1404.

[18] M. Parzy and H. Bogucka. 2011. Non-identical Objects Auction for Spectrum Sharing in TV White Spaces - The perspective of service providers as secondary users. In *IEEE International Symposium on Dynamic Spectrum Access Networks*.

[19] Marek Pycia. 2005. Many-to-One Matching Without Substitutability. *MIT Industrial Performance Center Working Paper* 8 (2005), 2005.

[20] Alvin E Roth. 1985. The College Admissions Problem Is Not Equivalent to the Marriage Problem. *Journal of economic Theory* 36, 2 (1985), 277–288.

[21] Alvin E Roth. 2008. Deferred Acceptance Algorithms: History, Theory, Practice, and Open Questions. *International Journal of Game Theory* 36, 3-4 (2008), 537–569.

[22] Walid Saad, Zhu Han, Rong Zheng, Mérouane Debbah, and H Vincent Poor. 2014. A College Admissions Game for Uplink User Association in Wireless Small Cell Networks. In *IEEE International Conference on Computer Communications*.

[23] Shuichi Sakai, Mitsunori Togasaki, and Koichi Yamazaki. 2003. A Note on Greedy Algorithms for the Maximum Weighted Independent Set Problem. *Discrete Applied Mathematics* 126, 2 (2003), 313–322.

[24] Hong Xu and Baochun Li. 2013. Anchor: A Versatile and Efficient Framework for Resource Management in the Cloud. *IEEE Transactions on Parallel and Distributed Systems (TPDS)* 24, 6 (2013), 1066–1076.

[25] X. Zhou and H. Zheng. 2009. TRUST: A General Framework for Truthful Double Spectrum Auctions. In *IEEE International Conference on Computer Communications*.