Stable Combinatorial Spectrum Matching

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***Abstract*—The use of a combinatorial auction is believed to be an effective way to distribute spectrum to buyers who have diversified valuations for different spectrum combinations. However, the allocation of spectrum with combinatorial auctions mainly aims at optimizing over certain utility functions, e.g., social welfare, but ignores individual preferences of buyers and sellers, who have incentives to deviate from globally optimal allocation results to improve their own utility. In this paper, we explore the possibility of designing a new stable matching algorithm for combinatorial spectrum allocations. Starkly different from existing efforts on spectrum matching mechanism design, our proposed combinatorial spectrum matching framework not only allows buyers to express preferences towards spectrum combinations (rather than individual channels), but also computes the payment that should be transferred from buyers to sellers. Payment determination, while essential in spectrum exchange, has never been addressed in existing spectrum matching frameworks. We design a novel algorithm to achieve a stable combinatorial spectrum matching and to compute the corresponding payment profiles. We conducted an extensive array of experiments to compare the performance of stable combinatorial spectrum matching with spectrum auctions. It is shown that the combinatorial spectrum matching sacrifices little allocation efficiency in terms of social welfare and spectrum utilization, but achieves a much higher individual buyer utility, which will incentivize buyers to participate and comply with the allocation results.**

I. Introduction

The shortage of spectrum poses a significant obstacle to the rapid development of the wireless communications industry. The growing demand from wireless services and applications is restrained by the limited supply of spectrum resources, despite recent advances in wireless communication technologies. Traditional static spectrum allocation issues long-term spectrum licenses to wireless service providers, resulting in spectrum underutilization due to undulating traffic on different networks. To improve spectrum utilization, dynamic spectrum access has been proposed to redistribute spectrum resources so that service providers with idle channels can trade with those in need of additional channels [1], [2].

The use of auctions is deemed an effective way of resource redistribution, and spectrum auction mechanism design has been extensively studied. In the family of spectrum auctions, we are particularly interested in combinatorial spectrum auctions, which allow buyers to express diversified preferences for different channel combinations. The valuation for a spectrum combination may be higher (or lower) than the sum of valuations for individual channels in the combination. This is especially true for spectrum, since channels with contiguous frequencies are easier to operate on than those on non-contiguous frequencies. However, the allocation of spectrum with combinatorial auctions usually aims for maximizing the social welfare [3], [4] globally, but ignores individual preferences of buyers and sellers, who are rational and selfish market participants and care more about their own utility rather than the overall social welfare. Therefore, a globally optimal spectrum allocation with combinatorial auctions may not be implemented when buyers and sellers have incentives to defect and seek for alternative opportunities to improve their own utilities. To address this problem, in this paper, we explore the possibility of utilizing stable matching theory to realize combinatorial spectrum allocation.

Stable matching was first studied by Gale and Shapley for college admission problems in their pioneering work [5]. Since then, stable matching has been widely applied to resource allocation in computer science, such as virtual machine management in the cloud [6], user association in small cells [7], and spectrum sharing in device-to-device communication [8]. Unlike other common goods, the right of spectrum usage is not exclusive, but is subject to interference relationship among buyers. Due to transmission path loss, geographically distant buyers can reuse the same channel as long as they are out of the interference range of each other. While it is a promising way to boost utilization, such spectrum reusability challenges conventional matching frameworks, which feature fixed quotas rather than graph-based interference constraints.

As a new paradigm of spectrum allocation, several spectrum matching frameworks have been proposed. In [9], Chen *et al.* proposed a two-stage deferred acceptance algorithm to reach a stable matching, where every buyer has a maximum quota on channel purchases. To address the issue that a minimum number of channels may be needed for proper operation, in [10], an extended deferred acceptance algorithm is designed to achieve a stable matching while ensuring that the minimum spectrum requirement is met for all buyers. In the context of this work, perhaps the most closely related work is [11], a many-to-many matching framework proposed for combinatorial spectrum allocation. The main distinction of our work lies in the determination of payment from buyers to sellers, making our new combinatorial spectrum matching framework more applicable in real-world spectrum markets, where spectrum trading cannot be accomplished without money transfer.

We summarize the challenges in designing our new combinatorial spectrum matching framework as follows:

1) *Spatial reuse* is what differentiates the design of either auction or matching frameworks for spectrum and other common goods.
2) *Combinatorial expressiveness* enables buyers to pursue favourable spectrum combinations more aggressively.
3) *Payment determination* helps the matching framework to close the deal in real-world spectrum markets.
4) *Stability* ensures that no buyers or sellers are willing to deviate from the final spectrum allocation.

In this paper, we present the design of a new combinatorial spectrum matching framework that addressed all four challenges. A comparison of our design with conventional matching, existing spectrum matching and combinatorial spectrum auctions is shown in Table I. More specifically, in our matching framework, the buyers initiate their payment profiles as sellers' reserve prices, *i.e.*, the minimum payment sellers will accept for selling their channels. Then, every buyer iteratively proposes to a combination of sellers who maximizes her utility function, and every seller accepts a set of non-interfering buyers with the highest aggregate payment. After each iteration, each buyer adjusts her payment profile by augmenting the payments for sellers who reject her proposals, and maintaining the payments for sellers who accept her proposals. The adjustment of payment will alter buyers' preferences for spectrum/seller combinations, and the same is true for the sellers. Such a process continues until all buyers' proposals are accepted, indicating that the matching is stable at the current payment profiles.

We evaluate the performance of the proposed stable combinatorial spectrum matching framework by comparing it with a well-known combinatorial auction mechanism, called SMASHER [13], from a variety of perspectives, including social welfare, total payment, buyer utility, and channel utilization. Simulation results show that the gap of social welfare and channel utilization between the combinatorial spectrum matching and spectrum auctions is quite small, while the combinatorial spectrum matching achieves a much higher buyer utility, which will motivate buyers to participate and abide by the final spectrum allocation results.

II. Related Work

In this section, we briefly review related works on spectrum auction and stable matching, as well as existing spectrum matching frameworks.

Spectrum Auction. Auction mechanisms have long been developed for spectrum allocation. The most common spectrum auction models include forward auction [14], double auction [15], and online auction [16]. Combinatorial auction differentiates from other auction models by allowing buyers to bid on combinations of items [17]. However, this makes it far more difficult (usually NP-hard) to realize an optimal allocation that maximizes over certain metrics, e.g., social welfare [18], [19], [20]. Combinatorial auction has been applied to spectrum allocation. Dong et al. [21] leveraged combinatorial auction for spectrum allocation in cognitive radio networks, but they modelled the spectrum opportunity as orthogonal time-frequency slots rather than considering spatial reuse. Similarly, Yi et al. [4] utilized combinatorial auction and Stackelberg game for spectrum sharing in cognitive radio networks , but did not realize spectrum reuse. Zheng et. at. [3] successfully tackled the problem of spectrum reuse by transforming combinatorial spectrum auctions to classic combinatorial auctions through virtual channel creation for interfering buyers. As an extension, Cai et al. [22] further incorporated reserve prices of sellers into the combinatorial spectrum auction model. The objective of spectrum allocation in auction is usually social welfare maximization, while we are trying to attain a stable allocation where individual preferences of buyers and sellers are accommodated.

Stable Matching. In their pioneering work [5], Gale and Shapley studied the marriage and college admission problems, and proposed deferred acceptance algorithm to achieve a stable matching. As a useful tool for resource allocation, stable matching has been applied to many areas in computer science. For security protection, Bayat et al. [23] used the matching theory to pair source-destination nodes with friendly jammers to avoid eavesdropping. In wireless communications, the matching framework is adopted for small cell user association [7], and device-to-device resource sharing [8]. In cloud computing, Xu et al. [6] matched jobs to virtual machines, adapting the deferred acceptance algorithm to cater to heterogeneous job sizes. Money transfer in matching framework was studied in [24] for labor market, followed by many variants on matching with contracts between two parties [25], [26].

Spectrum Matching. Different from common goods, spectrum allocation features spatial reuse. Matching framework for spectrum allocation was first proposed by [12], where a two-stage algorithm based on deferred acceptance was proposed to realize spatial reuse and ensure stability. As an extension, Chen et al. [10] proposed an extended deferred acceptance algorithm to ensure both maximum spectrum quota and minimum spectrum requirement of buyers. In [11], Jiang et al. proposed a stable many-to-many matching framework for combinatorial spectrum allocation. However, none of the above spectrum matching frameworks could determine how much buyers should pay sellers for acquiring the spectrum, which is essential for the final deal in the spectrum market.

III. System Model

In this section, we present the system model for the problem of spectrum allocation, then formulate it as a combinatorial spectrum matching framework, and formally define the concept of stability under such frameworks.

A. Market Participants

Service providers with idle channels to lease are regarded as sellers, and service providers seek to purchase extra channels are regarded as buyers. Without loss of generality, we assume that each seller owns one channel[1] Let $\mathcal{M} = \{1, 2, ..., M\}$

[1]If a seller owns multiple channels, we can create multiple dummy sellers, each possesses one channel [27]. This transformation will not affect the matching result.

TABLE I
OUR COMBINATORIAL SPECTRUM MATCHING FRAMEWORK VS. EXISTING SOLUTIONS: A COMPARISON.

	Spatial Reuse	Combinatorial Expressiveness	Payment Determination	Stability
Traditional matching	✗	✗	✗	✓
One-to-many spectrum matching [12]	✓	✗	✗	✓
Many-to-many spectrum matching [11]	✓	✓	✗	✓
Combinatorial spectrum auctions [13], [3]	✓	✓	✓	✗
Combinatorial spectrum matching	✓	✓	✓	✓

denote the set of sellers/channels[2]. Let $\mathcal{N} = \{1, 2, ..., N\}$ denote the set of buyers. There is no limit on the number of channels that a buyer can purchase, yet it will be bounded by the buyer's valuation for different spectrum combinations and the corresponding payment.

B. Interference Relationship

Spectrum can be shared among non-interfering buyers. To determine the interference relationship, we construct heterogeneous interference graphs for different channels [27]. Let $\{G^i = (\mathcal{N}, E^i)\}_{i=1}^M$ represent the interference graph of channel i, in which the set of nodes are the set of buyers $\mathcal{N}$. If two buyers j and j' are within the interference range of each other when transmitting on channel i, there is an edge in the interference graph, i.e., $e_{j,j'} \in E^i$. We made the simplified condition that the interference relationship is symmetric, thus the interference graph is undirected. In principle, the same channel can be reused by an unlimited number of buyers who are not linked on the interference graph.

C. Utility of Buyers and Sellers

Let p_{ij} denote the payment of buyer j to seller i for channel i, and $\mathbf{p}^j = (p_{1j}, p_{2j}, ..., p_{mj})$ is the payment profile of buyer j. A buyer can purchase any combinations of channels, but different combinations bring different utilities to the buyer. Assume that buyer j is allocated the combination of channels $C^j \subseteq \mathcal{M}$, and her valuation for this combination is $V^j(C^j)$. It is obviously true that buyer j has zero valuation for an empty combination, i.e., $V^j(\emptyset) = 0$. Buyer j's utility $u^j(C^j, \mathbf{p}^j)$ is her valuation for the allocated combination minus the payment for the combination:

$$u^j(C^j, \mathbf{p}^j) = V^j(C^j) - \sum_{i \in C^j} p_{ij}. \tag{1}$$

Different from common goods that can only be exclusively sold to a single buyer, a channel can be assigned to multiple buyers as long as they do not interfere with each other. Let $D^i \subseteq \mathcal{N}$ denote the set of buyers that is matched to seller i, and $\mathbf{p}^i = (p_{i1}, p_{i2}, ..., p_{iN})$ denote the payment profile of seller i. Seller i's utility $w^i(D^i, \mathbf{p}^i)$ depends on the interference condition of buyer set D^i and the received payment $\mathbf{p}^i$.

$$w^i(D^i, \mathbf{p}^i) = \mathbf{I}_{\forall j,j' \in D^i, e^i_{j,j'}=0} \sum_{j \in D^i} p_{ij}. \tag{2}$$

[2]For simplicity, with a little abuse of notations, we use the same set to represent both sellers and their channels

in which $\mathbf{I}_{\forall j,j' \in D^i, e^i_{j,j'}=0} = 1$ if the interference constraint $\forall j, j' \in D^i, e^i_{j,j'} = 0$ is satisfied, otherwise, we impose a prohibitive penalty $\mathbf{I}_{\forall j,j' \in D^i, e^i_{j,j'}=0} = -\infty$ to prevent sellers from selling the same channel to interfering buyers.

We have the following assumptions regarding the utility function of buyers and sellers.

- **Reserve price**. Each seller has a reserve price, representing the minimum payment the seller will accept for the channel. Let r_i denote the reserve price of seller i, and we have $r_i = w^i(\emptyset, \mathbf{0}) \geq 0$. The reserve price reflects alternative options of a seller, for example, the seller may use the channel herself even if the traffic of the network is light.
- **Positive gain**. We assume that purchasing an additional channel at its reserve price always brings positive gain to a buyer. This is intuitively true, because no buyer would purchase a channel of which the reserve price is higher than its valuation. Therefore, We have:

$$V^j(C^j \cup \{i\}) - V^j(C^j) - r_i \geq 0, \forall i \in \mathcal{M}, C^j \subset \mathcal{M}, i \notin C^j. \tag{3}$$

- **Gross substitute**. Assume that channel i is in the combination that maximizes buyer j's utility. If the payments for other channels increase but the payment for channel i stays the same, then, there exists a combination containing channel i that maximizes the utility of buyer j. More specifically, let $C^{j*}(\mathbf{p}^j) = \arg_{C^j} u^j(C^j, \mathbf{p}^j)$ denote the combination that maximizes buyer j's utility under the payment profile $\mathbf{p}^j$. If $i \in C^{j*}(\mathbf{p}^j)$, then $i \in C^{j*}(\widetilde{\mathbf{p}}^j)$, where $p_{ij} = \widetilde{p}_{ij}, p_{i'j} \geq \widetilde{p}_{i'j}, \forall i' \neq i$. The property of gross substitute implies that the payment rise in other channels will not cause a buyer to abandon a channel whose payment is unchanged.

D. Stable Matching

We formally define a combinatorial spectrum matching framework as follows.

Definition 1. *(Combinatorial Spectrum Matching). Given the set of sellers $\mathcal{M}$ and the set of buyers $\mathcal{N}$, a combinatorial spectrum matching is a function μ from $\mathcal{M} \cup \mathcal{N}$ to subsets of $\mathcal{M} \cup \mathcal{N}$, such that:*

- *For every buyer $j \in \mathcal{N}$, $\mu(j) \subseteq \mathcal{M}$;*
- *For every seller $i \in \mathcal{M}$, $\mu(i) \subseteq \mathcal{N}$;*
- *For every seller i and buyer j, $i \in \mu(j)$ if and only if $j \in \mu(i)$.*

Now, we will characterize the stability of a combinatorial spectrum matching.

Definition 2 (Individual Rationality)**.** *A combinatorial spectrum matching μ is individually rational if:*

1) *Every seller is matched to a set of non-interfering buyers, and obtains a utility that is greater than her reserve price, i.e., $\forall j, j' \in \mu(i), e^i_{j,j'} = 0, \sum_{j\in\mu(i)} p_{ij} \geq r_i, \forall i \in \mathcal{M}$;*
2) *Every buyer obtains a positive utility, i.e., $u^j(\mu(j), \mathbf{s}^j) = V^j(\mu(j)) - \sum_{i\in\mu(j)} p_{ij} \geq 0, \forall j \in \mathcal{N}$.*

Being individually rational is the basis of a stable combinatorial spectrum matching. It ensures that sellers prefer selling their channels to alternative options (e.g., using the channel themselves), and the benefit of purchased channels worth the money paid by buyers.

Before defining stability, we first introduce the concept of blocking pairs.

Definition 3 (Blocking Pair)**.** *Given a combinatorial spectrum matching μ and the payment profile $\mathbf{p}$, seller set C and buyer j form a blocking pair (C, j), if there exists a payment profile $\widetilde{\mathbf{p}}^j$ that satisfies the following conditions:*

1) *Buyer j prefers channel set C under payment profile $\widetilde{\mathbf{p}}^j$ to her currently matched channel set under the current payment profile $\mathbf{p}^j$.*
2) *Every seller in C prefers to include buyer j in their matched buyer set under payment profile $\widetilde{\mathbf{p}}^j$ rather than being matched to their currently matched buyer set under payment profile $\mathbf{p}^j$.*

Mathematically speaking, seller set C and buyer j form a blocking pair (C, j), if there exists a payment profile $\widetilde{\mathbf{p}}^j$, and:

1) *$u^j(C, \widetilde{\mathbf{p}}^j \cup \mathbf{p}^{-j}) > u^j(\mu(j), \mathbf{p})$, in which $\mathbf{p}^{-j}$ is the payment profile of all other buyers except buyer j;*
2) *$\forall i \in C, \exists D^i \subseteq \mu(i)$, such that $w^i(D^i \cup \{j\}, \widetilde{\mathbf{p}}^j \cup \mathbf{p}^{-j}) \geq w^i(\mu(i), \mathbf{p})$.*

The buyer and sellers in the blocking pair have incentives to deviate from the matching result, and to be matched to each other with improved utilities, thus making the matching result unstable.

Definition 4 (Stable Combinatorial Spectrum Matching)**.** *A combinatorial spectrum matching μ is stable if it is individual rational and contains no blocking pairs.*

IV. Stable Combinatorial Spectrum Matching

In this section, we first introduce the classic deferred acceptance algorithm for stable matching in traditional matching frameworks, which inspires our design of a novel algorithm for stable combinatorial spectrum matching. After a detailed description of the proposed matching algorithm, we present extensive theoretical analysis on its properties including convergence and stability.

A. Algorithm Design

The traditional deferred acceptance algorithm is designed to solve the college admission problem, where there is a set of students to be admitted to a set of colleges, each with a fixed quota [5]. In the first round, each student proposes to her favourite college. Among all applicants, a college with a quota q temporarily adds the top q students in the waiting list, or all students if the number of applicants is smaller than q, and rejects others. In the following rounds, each rejected student applies to her most-preferred college which has never rejected her before. Each college updates its waiting list by selecting the top q students among the current applicants and those in the previous waiting list. This process is repeated until no students have colleges to propose to.

Similar to the deferred acceptance algorithm, our proposed combinatorial spectrum matching algorithm lets buyers propose to preferred combinations of sellers, and sellers decide whether or not to accept the proposal. But instead of selecting buyers based on a fixed quota, sellers will resort to spectrum reuse based on interference graphs. Furthermore, during the matching process, buyers will gradually increase their payments for sellers who have rejected their proposals. Due to the changes in payments, the utility of buyers and sellers will change as well. A buyer will not cease proposing to a seller because the seller has rejected her, but because the payment for the seller has increased so much that it is not profitable to buy the channel.

Let $p_{ij}(t)$ denote the provisional payment of buyer j for seller i at round t, and $\mu(t)$ denote the provisional matching result at round t. As shown in Alg. 1, the detailed process of the proposed algorithm works as follows.

- At round $t = 0$, the payment of every buyer to every seller equals the reserve price, i.e., $p_{ij}(0) = r_i, \forall i \in \mathcal{M}, j \in \mathcal{N}$. Every buyer will propose to the combination of sellers that maximize the buyer's utility. According to the *positive gain* assumption in Section III-C, at round $t = 0$, every buyer will propose to all sellers. Each seller will temporarily accept a set of non-interfering buyers with the highest aggregate payment, and reject other buyers. If there are multiple such non-interfering buyer set, the seller will randomly choose one to accept. To find the set of non-interfering buyers with the maximum total payment is equivalent to finding the maximum weighted independent set on the interference graph of channel i, which is NP-hard. We adopts the approximate algorithm in [28], which greedily picks the buyer with the highest $p_{ij}/(d_{G^i}(j) + 1)$ and eliminate her interfering neighbors until the interference graph becomes empty . Note that p_{ij} is buyer j's payment for channel i, and $d_{G^i}(j)$ is the degree of buyer j on interference graph G^i.
- At round $t \geq 1$,
  - For every buyer, say buyer j, if a proposal to seller i at round $t-1$ is rejected, buyer j increases the payment to seller i by δ_p, i.e., $p_{ij}(t) = p_{ij}(t-1) + \delta_p$. The payments to other sellers, including those who accept buyer j's proposal and those to whom buyer j does not propose, remain unchanged. The increment of δ_p will result in non-continuous offered payment, but it

TABLE II
BUYERS' VALUATION FOR DIFFERENT CHANNEL COMBINATIONS

Combination	Buyer A	Buyer B	Buyer C
$\{1\}$	6	6	3
$\{2\}$	3	7	9
$\{3\}$	2	10	4
$\{4\}$	8	5	6
$\{1,2\}$	10	15	13
$\{1,3\}$	9	17	8
$\{1,4\}$	15	13	10
$\{2,3\}$	7	18	15
$\{2,4\}$	12	13	16
$\{3,4\}$	11	16	11
$\{1,2,3\}$	14	26	20
$\{1,2,4\}$	19	22	21
$\{1,3,4\}$	18	24	16
$\{2,3,4\}$	16	25	22
$\{1,2,3,4\}$	24	33	27

conforms to the real-world case, for example, in the ascending price auction, there is a minimum amount by which the next bid must exceed the previous bid.

 - Given the payment profile $\mathbf{p}^j(t)$, buyer j will propose to the combination of sellers $C^j(t)$ that maximizes $u^j(C^j(t), \mathbf{p}^j(t))$. If there are multiple seller combinations that maximize $u^j(C^j(t), \mathbf{p}^j(t))$, the buyer will randomly choose one combination to propose to. Note that according to the *gross substitute* assumption in Section III-C, buyer j will continue to propose to the sellers who accept buyer j at round $t-1$ since their payment remains unchanged.
 - Given the proposals from all buyers, each seller will temporarily accept a set of non-interfering buyers $D^i(t)$ with the highest aggregate payment and reject others. If there are multiple such non-interfering buyer set, the seller will randomly choose one set to accept.

- The process continues until proposals from all buyers are accepted at the current payment profiles.

Toy Example. Assume that there are three buyers $\{A, B, C\}$ and four sellers $\{1, 2, 3, 4\}$. Buyers' valuations for different spectrum combinations are shown in Table II. We can see that the valuation for a combination may not equal the sum of valuations for individual channels in the combination. The reserve prices for all channels are assumed to be $[3, 3, 2, 5]$. The interference graphs on every channel are shown in Fig. 1. At round $t = 0$, as shown in Fig. 2(a), every buyer proposes to all sellers at their reserve prices. Buyer A's payments for seller 1 and 4 remain unchanged at round $t = 1$, since seller 1 and 4 accept buyer A at round $t = 0$. However, since buyer A is rejected by seller 2 and 3, the corresponding payments increase at round $t = 1$. Following the procedure of Alg. 1, we can reach the final matching result as shown in Fig. 2(j), where all the proposals from buyers are accepted by sellers. With the final matching result, the utilities of buyers A, B, C can be computed as $1, 12, 11$, and the utilities of sellers $1, 2, 3, 4$ are $8, 6, 6, 12$.

Algorithm 1 Stable Combinatorial Spectrum Matching

Input: Buyer utility function $u^j(C^j, \mathbf{p}^j), \forall j \in \mathcal{N}$.
Seller utility function $w^i(D^i, \mathbf{p}^i), \forall i \in \mathcal{M}$.
Payment adjustment step size δ_p.
Output: Matching result μ.
Payments $p_{ij}, \forall i \in \mathcal{M}, j \in \mathcal{N}$.

$t = 0$.
for all $i \in \mathcal{M}, j \in \mathcal{N}$ **do**
  $\mu(i) = \emptyset, \mu(j) = \emptyset$.
  Calculate $r_i = u^i(\emptyset, \mathbf{0})$.
  $p_{ij}(t) = r_i$.
end for
for all Seller $i \in \mathcal{M}$ **do**
  Current buyer waiting list $\mathcal{L}_i = \emptyset$.
end for
$flag = 1$.
while $flag$ **do**
  $t = t + 1$.
  for all Buyer $j \in \mathcal{N}$ **do**
    Find the channel combination that maximizes $u^j(C^j(t), \mathbf{p}^j(t))$, denoted by $C^{j*}(t)$.
    for all seller $i \in C^{j*}$ **do**
      Propose to seller i.
      Seller i adds buyer j to waiting list $\mathcal{L}_i = \mathcal{L}_i \cup \{j\}$.
    end for
  end for
  for all Seller $i \in \mathcal{M}$ **do**
    Find the buyer set $D^{i*}(t)$ that maximizes $w^i(D^i(t), \mathbf{p}^i(t))$.
    Reject buyers $j \in \mathcal{L}_i \setminus D^{i*}(t)$.
    Update the waiting list $\mathcal{L}_i = D^{i*}(t)$.
  end for
  for all $i \in \mathcal{M}, j \in \mathcal{N}$ **do**
    if Seller i has rejected buyer j's proposal **then**
      $p_{ij}(t+1) = p_{ij}(t) + \delta_p$.
    end if
  end for
  $flag = \sum_{ij}(p_{ij}(t+1) - p_{ij}(t))$.
end while
for all Seller $i \in \mathcal{M}$ **do**
  for all Buyer $j \in \mathcal{L}_i$ **do**
    $\mu(i) = \mu(i) \cup \{j\}, \mu(j) = \mu(j) \cup \{i\}$.
    $s_{ij} = s_{ij}(t)$.
  end for
end for

B. Theoretical Analysis

Proposition 1 (Convergence). *The proposed algorithm will converge in finite time.*

Proof. According to Alg. 1, if seller i accepts buyer j's proposal, the payment p_{ij} will stay unchanged; if seller i keeps rejecting buyer j's proposal, the payment p_{ij} will keep increasing. Since $V^j(C)$ is finite, buyer j will finally stop proposing to seller i. This is true for all buyers and

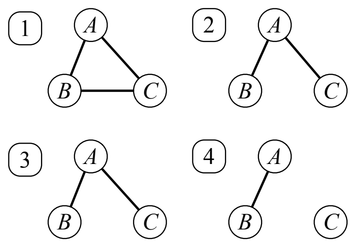


Fig. 1. Heterogeneous interference graphs.

sellers. Eventually, each seller will lose all proposals but the ones that she will accept, leading to the termination of the algorithm. □

Corollary 1 (Spectrum Utilization). *Every channel will be matched to at least one buyer, namely, at the final payment profiles, all channels will be matched.*

Proof. At round $t = 0$, when the payment profiles equal the reserve prices of all channels, every buyer will propose to all sellers. At round $t \geq 1$, every seller accepts some of buyers, whose payment for the seller will not change at round $t + 1$. According to the gross substitute assumption in Section III-C, these accepted buyers will continue to propose to the seller. Therefore, the set of buyers matched to the seller will not be empty. □

Proposition 2 (Individual Rationality). *The matching result of the proposed algorithm is individual rational.*

Proof. For any seller, the starting payment of each buyer equals her reserve price. Since the payment profiles keep increasing during the iteration, the final payment received by the seller will be greater than her reserve price.

For any buyer, at each round, the buyer will choose the spectrum combination to maximize the utility $u^j(C^j, \mathbf{p}^j)$. Since the buyer can always choose to buy nothing, which leads to a utility of $u^j(\emptyset, \mathbf{p}^j) = V^j(\Phi) = 0$, the final utility of the buyer will be no less than zero. □

Proposition 3 (Stability). *The matching result of the proposed algorithm is stable.*

Proof. Given the final matching result μ and payment profile $\mathbf{p}$, assume that seller set C and buyer j form a blocking pair with a payment profile $\widetilde{p}^j$, which satisfies 1) $u^j(C, \widetilde{\mathbf{p}}^j \cup \mathbf{p}^{-j}) > u^j(\mu(j), \mathbf{p})$, and 2) $\forall i \in C, \exists D^i \subseteq \mu(i), w^i(D^i \cup \{j\}, \widetilde{\mathbf{p}}^j) \geq w^i(\mu(i), \mathbf{p})$.

For any seller i in set C, it must be true that buyer j never proposes to seller i at payment $\widetilde{p}_{ij}$, otherwise, seller i would have accepted buyer j (along with some of the currently matched buyer D^i), since $w^i(D^i \cup \{j\}, \widetilde{\mathbf{p}}^j) \geq w^i(\mu(i), \mathbf{p})$. This means that $\forall i \in C, \widetilde{p}_{ij} \geq p_{ij}$. Therefore, we have:

$$u^j(C, \mathbf{p}) \geq u^j(C, \widetilde{\mathbf{p}}^j \cup \mathbf{p}^{-j}) > u^j(\mu(j), \mathbf{p}).$$

This indicates that under the payment profile $\mathbf{p}$, the seller combination that maximizes buyer j's utility should be C but not $\mu(j)$, which contradicts the process of Alg. 1. Therefore, no blocking pair exists, and the matching result of the proposed algorithm is stable. □

In the traditional deferred acceptance algorithm, the matching results favor the party who makes proposals[3]. In other words, the matching result is at least as good for every proposing entity as any other stable matching result. Define a channel as $p-$affordable to a buyer if the channel is matched to the buyer in a stable combinatorial spectrum matching. We have the following proposition.

Proposition 4 (Buyer Bias). *Given that there are no ties in the utilities of buyers and sellers, and the interference graph is complete on all channels, the matching result of the proposed algorithm is buyer biased, i.e., if a seller is $p-$affordable for a buyer, the seller will never reject the buyer with the payment no less than p.*

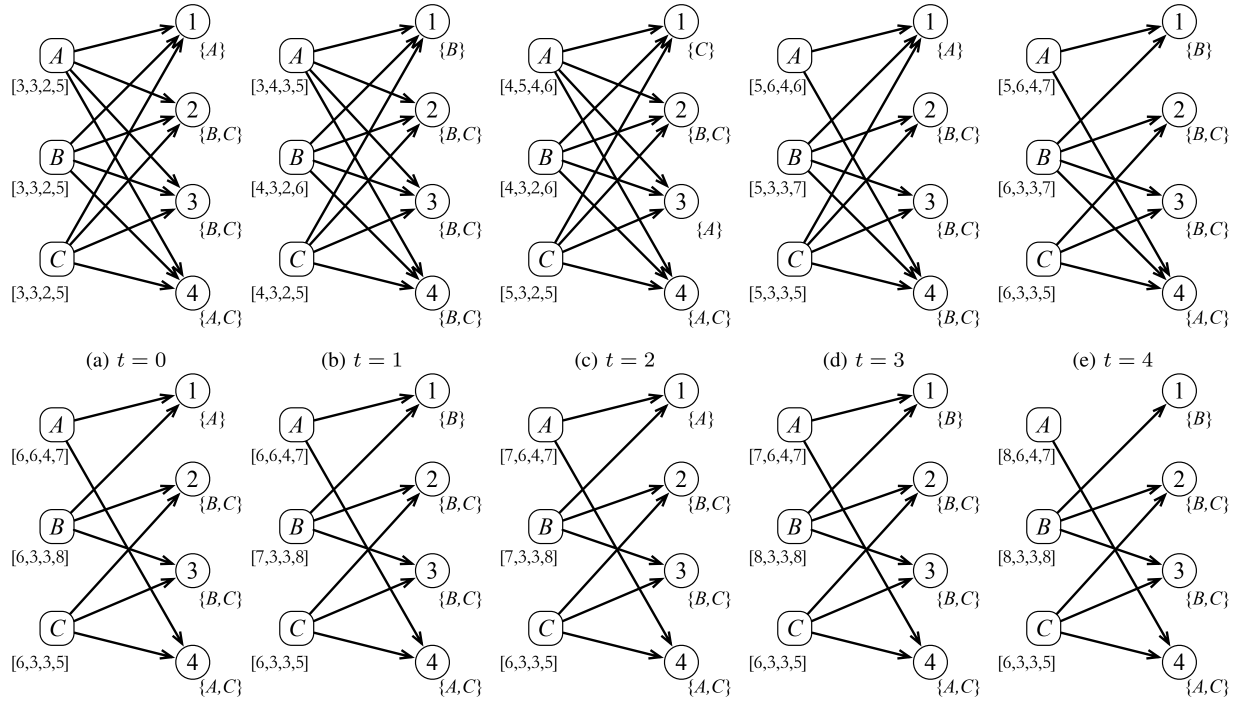
Proof. Consider seller i who is $p_{ij}-$affordable for buyer j. Assume that before round t, no seller rejects any buyer with a payment at which the seller is affordable for the buyer. Then, at round t, seller i rejects buyer j at p_{ij} for another buyer k whose payment is p_{ik}, meaning that $w^i(\{k\}, p_{ik}) > w^i(\{j\}, p_{ij})$ (note that there is no tie in utility functions, and that the interference graph is complete so that each channel can only be sold to one buyer). Since in any stable matching result, the payment profile of buyer k is at least as high as the payment profile at round t, according to the assumption of gross substitute, buyer k would like to add seller i to any set of sellers in a stable matching at price p_{ik}.

Consider any stable matching μ with payment profile $\mathbf{p}$, where seller i is matched to buyer j with payment p_{ij}, we will show that buyer k and seller set $\mu(k) \cup \{i\}$ form a blocking pair with payment profile $p_{ik} \cup \mathbf{s}^{-k}$. Firstly, we have $\forall i' \in \mu(k), w^{i'}(\{k\}, p_{ik} \cup \mathbf{p}^{-k}) = w^{i'}(\{k\}, \mathbf{p})$, and $w^i(\{k\}, p_{ik}) > w^i(\{j\}, p_{ij})$. Then, we have $u^j(\mu(k) \cup \{i\}, p_{ik} \cup \mathbf{p}^{-k}) > u^j(\mu(j), \mathbf{p})$, because buyer k would like to add seller k to any set of sellers in a stable matching with payment p_{ik}. Since in any stable matching that matches seller i to buyer j, a blocking pair exists, seller i is not $p_{ij}-$affordable for buyer j, which contradicts our hypothesis. Therefore, we have proved the buyer bias of the matching result of our proposed algorithm.

Unfortunately, we cannot prove that the matching result is buyer biased when spectrum reusability is taken into consideration. This is because whether a seller rejects or accepts a buyer depends not only on the payment profile of this buyer but also on the payment profiles of other buyers who can reuse the channel with the buyer.

□

[3]In the traditional deferred acceptance algorithm, the roles of the party who makes proposals and who decides acceptance can be switched, thus the bias of the matching results may be changed. To alter the role of buyers and sellers in the combinatorial spectrum matching framework is our future work.



(a) $t = 0$ (b) $t = 1$ (c) $t = 2$ (d) $t = 3$ (e) $t = 4$

(f) $t = 5$ (g) $t = 6$ (h) $t = 7$ (i) $t = 8$ (j) $t = 9$, final matching result.

Fig. 2. A toy example of the stable combinatorial spectrum matching (Algorithm 1).

V. Simulation

In this section, we compare the performance of the proposed stable combinatorial spectrum matching framework with the benchmark combinatorial spectrum auction mechanism [3] that achieves approximate maximum social welfare based on a greedy algorithm.

We have implemented the stable matching and the combinatorial auction on a desktop an Intel Core CPU of 8-core operating at 3.50 GHz processor and 32 GB RAM, and running a Windows 10 operating system with Python 3.6.1.

The number of sellers varies from 4 to 9. Since the number of possible channel combinations grow exponentially with the number of available channels, we restrict that the size of each combination is no more than three. Note that a buyer can still buy more than three channels by joining different combinations. The number of buyers varies from 50 to 100, randomly distributed in a $2000m \times 2000m$ area, and the transmission range of a channel is drawn randomly from the range $[250m, 450m]$. A buyer's valuations for individual channels are randomly chosen in the range $(0, 100]$, and the valuation for a channel combination is the sum of valuations of individual channels in the combination plus a random value in the range $[0, 100]$. We set the default value of number of sellers, number of buyers, unit payment increment as $6, 80$ and 1, respectively. All results are averaged over 100 runs.

For performance evaluation, we focus on the following four metrics.

- *Social welfare*. Social welfare is defined as the sum of utilities of all buyers and sellers [27].
- *Average buyer utility*. According to equation (1), a buyer's utility is her valuation for the purchased channel minus the payment to sellers.
- *Buyer payment*. The payment from buyers to sellers does not affect social welfare, as it merely transfers utility from buyers to sellers.
- *Channel utilization*. Channel utilization is the average number of buyers reusing the same channel.

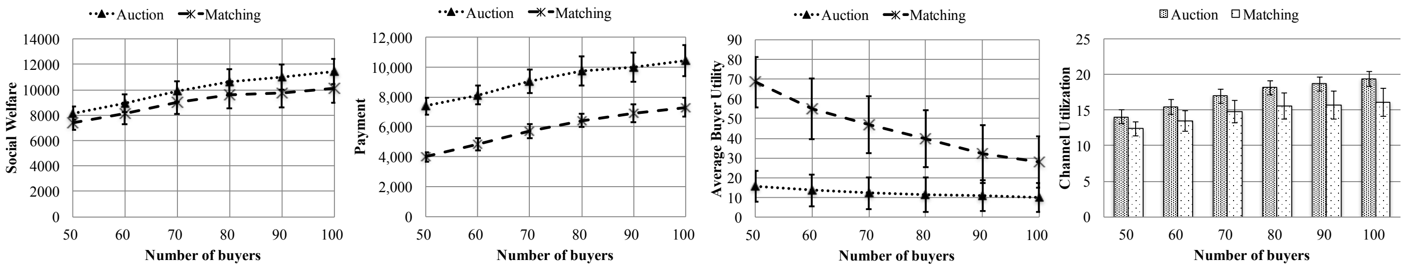
A. Social Welfare

Fig. 3(a) and Fig. 4(a) shows the comparison of social welfare. As the auction targets at (approximate) social welfare maximization, its social welfare is higher than the proposed matching framework, but the gap is very small (no more than 13.2%). This indicates that combinatorial matching only sacrifices a little allocation efficiency in order to achieve a stable matching result.

It is obvious that as the number of buyers increases, social welfare will also increase since there are more winning buyers, but the growing rate is rather slow due to limited spectrum resources. As the number of available channels increases, buyers are more likely to obtain their preferred channel (combinations), thus the social welfare also increases.

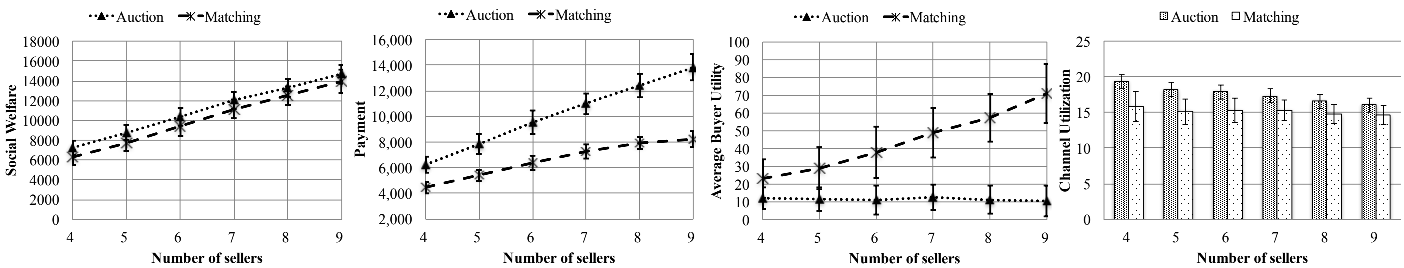
B. Buyer Payment

Fig. 3(b) and Fig. 4(b) shows the comparison of buyers' payment. It can be seen that the payment in the matching model is significantly lower than the payment in the auction model, which means that less utility is transferred from the buyer to the seller. As the matching framework lets buyers to



(a) Social welfare. (b) Buyer payment. (c) Average buyer utility. (d) Channel utilization.

Fig. 3. Comparison of combinatorial spectrum matching and combinatorial spectrum auction. The number of sellers is fixed as 6.



(a) Social welfare. (b) Buyer payment. (c) Average buyer utility. (d) Channel utilization.

Fig. 4. Comparison of combinatorial spectrum matching and combinatorial spectrum auction. The number of buyers is fixed as 80.

choose their favourite channels, they are more likely to spend less on a few most preferred channels, rather than buying as many channel as possible to maximize the social welfare. A lower payment also indicates that the sellers obtain a lower utility (recall that a seller's utility is the total payment of non-conflict matched buyers), confirming the Proposition 4 that the matching result is biased towards buyers, since it is the buyers' decision of which sellers to propose to.

When the number of buyers grows, there will be more winning buyers, thus the total payment will increase. Similarly if there are more channels, each buyer may have more choices, and they will buy and pay more for the channels.

C. Average Buyer Utility

Fig. 3(c) and Fig. 4(c) shows the comparison of average buyer utility. Since matching aims at stability (no buyer or seller is willing to deviate from the result) rather than social welfare maximization, individual buyer's utility of the matching is significantly higher than that of the auction. The low utility of the auction result may discourage buyers to participate or even cause buyers to disobey the allocation.

Naturally, average buyer utility will decrease as there are more buyers competing for a fixed pool of channels. Interestingly, if the number of available channels increases, auction and matching exhibit starkly different trends. Under the matching framework, buyer utility will significantly improve as buyers have more options, and thus are able to achieve a higher utility. In comparison, buyer utility generally remains the same under the auction framework, which indicates that even though there are more channels, most of the benefit is grabbed by the sellers through payment, while buyers obtain more channels but the same even slightly lower overall utility.

D. Spectrum Utilization

Fig. 3(d) and Fig. 4(d) shows the comparison of spectrum utilization. We can see that the channel utilization of matching is slightly lower than that of auction, which shows that the matching roughly maintains the allocation efficiency. It is intuitively true that if there are more buyers, channel utilization will go up as more buyers will reuse the same channel. The reason is similar for a lower channel utilization when there are more channels.

E. Impact of Discrete Payment Increment

The major parameter in the proposed stable combinatorial spectrum matching algorithm is the payment increment δ_p (see line 27 in Alg. 1). Ideally, δ_p is infinitesimal so that buyers may gradually raise their payment for channels to seek acceptance from the seller. In reality, δ_p is non-negligible, and the payment increment is discrete. As shown in Table III, we can see that, in general, with a smaller increment factor δ_p, a higher allocation efficiency (social welfare and channel utilization) and higher buyer utility can be achieved. But the discrepancy under different δ_p is very small, which means that the combinatorial matching algorithm is relatively robust against the choice of δ_p.

VI. Conclusion

In this paper, we have presented a stable combinatorial spectrum matching framework for spectrum allocation, which renders buyers the flexibility to express diversified valuations towards spectrum combinations. As opposed to combinatorial spectrum auctions, our proposed matching framework features stability instead of optimality, ensuring that no buyers or sellers are willing to deviate from the matching results.

TABLE III
IMPACT OF DISCRETE PAYMENT INCREMENT

	Social welfare			Buyer utility		
# of buyers	50	70	90	50	70	90
$\delta_p = 0.1$	7505.9	8899.2	10130.3	70.2	45.8	35.0
$\delta_p = 1$	7417.8	9005.1	9778.8	68.5	46.9	32.0
$\delta_p = 10$	7137.9	8678.2	9520.2	66.3	43.7	31.2
	Total payment			Channel utilization		
# of buyers	50	70	90	50	70	90
$\delta_p = 0.1$	3998.9	5692.0	6977.5	12.6	14.1	15.9
$\delta_p = 1$	3993.4	5724.5	6900.3	12.4	14.8	15.7
$\delta_p = 10$	3822.5	5618.5	6707.9	12.0	14.2	15.2

The proposed matching framework also differentiates from conventional ones by involving payment as a utility transfer between buyers and sellers, which is indispensable for the two parties to seal the deal of spectrum transaction. We have developed a novel algorithm that converges to a stable combinatorial spectrum matching result with the corresponding payment profile. We have conducted extensive simulations to compare the performance of the proposed combinatorial spectrum matching framework with combinatorial auctions. The simulation results show that while spectrum auctions have slightly higher social welfare and channel utilization, combinatorial spectrum matching can achieve a higher buyer utility, which confirms its stability for individual buyers to abide by the spectrum allocation results.

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