Network Coding Aware Dynamic Subcarrier Assignment in OFDMA Based Wireless Networks

Xinyu Zhang, Baochun Li

***Abstract*—Orthogonal Frequency Division Multiple Access (OFDMA) has been integrated into emerging broadband wireless systems such as the 802.16 wirelessMAN. A critical problem in OFDMA is to assign multiple frequency bands (called subcarriers) to different users. Taking advantage of the frequency diversity and multiuser diversity in OFDMA systems, dynamic subcarrier assignment mechanisms have shown to be able to achieve much higher downlink capacity than static assignment. A rich literature exists that proposes MAC and physical layer schemes aiming at exploiting the diversity gain with low implementation complexity. In this paper, we propose a cross layer approach that explores the joint advantage of network coding and dynamic subcarrier assignment. With network coding, it becomes possible to assign the same subcarriers to different downlinks without causing any interference. Consequently, our coding-aware assignment scheme improves the bandwidth efficiency and increases the downlink throughput by a substantial margin. In designing the scheme, we identify a tradeoff between diversity gain and the network coding advantage, which is critical to the network performance in terms of throughput and fairness. To explore the tradeoff, we formulate the coding-aware assignment scheme as a mixed integer program, and design a polynomial time approximation algorithm that can be used in practical systems. We prove the asymptotic performance bound of the algorithm, and demonstrate that it closely approximates the optimum under realistic experimental settings.**

***Index Terms*—Network coding, OFDMA networks, WiMax/802.16**

I. Introduction

THE emerging generation of wireless standards such as 802.16 [1] have identified OFDMA (Orthogonal Frequency Division Multiple Access) as a promising technology enabling broadband wireless access. In OFDMA systems, the prescribed frequency band is divided into hundreds of orthogonal subbands called *subcarriers*. The base station (BS) assigns disjunctive sets of subcarriers to mobile stations (MS) which multiplex the available downlink capacity. In the original 802.16 PHY specification, subcarriers are either statically or randomly allocated to the MSs, oblivious of their diverse channel conditions. In reality, however, the path-loss and fading profiles vary across the whole frequency band, and even the same subcarrier experiences independent attenuation when assigned to MSs at different locations. Such frequency diversity and multiuser diversity have motivated **d**ynamic **s**ubcarrier **a**ssignment (DSA) mechanisms, which deliberately match each downlink to the set of subcarriers supporting higher throughput. It has been observed that an optimal DSA algorithm can achieve up to twice higher downlink throughput compared with static assignment schemes [2]. A large body of work has also focused on suboptimal algorithms aiming at achieving similar performance at lower implementation complexity [2].

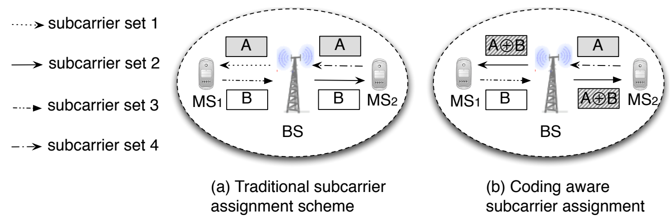

Fig. 1. The motivating scenario for coding aware subcarrier assignment in OFDMA wireless networks.

In this paper, we add a new dimension to the literature of DSA, proposing a cross layer approach towards **c**oding **a**ware **d**ynamic **s**ubcarrier **a**ssignment (CADSA). Taking advantage of network coding, our CADSA algorithm combines the downlink data frames [1] heading towards different MSs, and delivers them through the same set of subcarriers. As a result, it improves the bandwidth efficiency of OFDMA systems by a significant margin. As an intuitive justification, consider the scenario in Fig. 1, where two MSs are exchanging information with each other via the BS, creating an opportunity for network coding (henceforth referred to as *coding opportunity*). Traditional assignment algorithms will allocate disjoint sets of subcarriers to the downlinks. In contrast, the CADSA algorithm XORs the two uplink frames and multicasts the combined frame via the two downlinks. The corresponding MSs receive the same frame, but can decode different information by XORing the combined frame with one that is known *a priori*. For instance, through the operation $B \oplus (A \oplus B)$, MS_1 directly obtains frame A, which originated from MS_2.

In an ideal case where all downlinks have coding opportunities and the subcarriers have uniform channel gains for all MSs, it is straightforward that CADSA can save half of the subcarriers, achieving a two-fold increase in capacity, compared with traditional assignment algorithms. However, the benefits of network coding diminish in case of high multiuser diversity, when sharing the same subcarrier may result in underutilized bandwidth. For instance, if MS_1 in Fig. 1 is farther to the BS than MS_2 and has much lower channel gain, the throughput of both downlinks is capped by

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[1] We use the term "frame" and "packet" synonymously, to denote a group of information bits transmitted through the wireless link.

the achievable rate of the downlink to MS_1. In such cases, network coding may result in underutilized bandwidth, and it is nontrivial to determine whether CADSA still outperforms separate assignment in terms of total throughput.

To quantify the benefits of network coding in practical wireless fading environment, we formulate an optimization framework that provides upper bounds on the performance of CADSA. In view of its high complexity, we propose an approximation algorithm that achieves similar downlink capacity and fairness. We analyze the worst case approximation ratio of the algorithm, and further verify its performance in realistic simulation environment. Compared with traditional assignment schemes, the CADSA algorithm demonstrates much higher downlink rates, especially when the downlinks experience uniformly high SNR. Towards pracitcal implementation of the algorithm, we also design a scheduling and coding mechanism to CADSA such that no additional overhead is induced compared with existing dynamic mechanisms without network coding.

The remainder of this paper is organized as follows. In Sec. II, we review existing work on subcarrier assignment algorithms as well as network coding protocols in wireless networks. Sec. III introduces the PHY and MAC models, as well as the scheduling and coding algorithm for CADSA. In Sec. IV, we formulate the optimization framework, design our approximation algorithm and analyze its theoretical performance bound. Sec. V presents our simulation results that quantify the performance of the CADSA framework in comparison with other related work. Finally, Sec. VI concludes the paper.

II. Related Work

Dynamic mechanisms for resource allocation in OFDMA downlink have been extensively investigated in literature (see [2] for a comprehensive survey). Existing algorithms are centered around two optimization frameworks: maximizing the sum capacity subject to power and fairness constraints, or minimizing the power budget subject to per-link rate and fairness constraints. Both optimization problems are essentially mixed-integer programs which are NP-hard in general [2]. Instead of tracking the optimal solution with exponential complexity, many suboptimal algorithms have been proposed. These algorithms generally involve two aspects: the subcarrier assignment and the power allocation. Subcarrier assignment schemes match each downlink with a set of subcarriers with high channel gains (see, *e.g.*, [3], [4]). Power allocation algorithms adaptively assigns transmission power to each subcarrier, which adjusts its modulation type according to the SNR at the receiver side. Existing work on adaptive power allocation mostly assumes a continuous relation between achievable rate and the SNR of each subcarrier [2]. In practical MAC and PHY standard like 802.16 [1], however, the relation is a stepwise function determined by the adaptive modulation and coding (AMC) protocols. In addition, most of the above algorithms reside in the MAC and PHY layers, instead of employing the network level paradigms, such as the scenario in Fig. 1. Our previous work [5] proposed a joint design and optimization of network coding and subcarrier assignment, and demonstrated its advantage through simulation experiments. In the present paper, we not only design an approximation scheme for CADSA, but also quantify its performance bound with theoretical rigor. In addition, we evaluate the CADSA scheme under the partial coding case (*i.e.*, not all packets can be encoded), instead of the ideal full-coding scenario.

Coding based information exchange was first proposed by Wu *et al.* [6], and then exploited to improve the unicast throughput of 802.11 based wireless mesh networks [7]. The basic idea is to locally search for coding opportunities, and XOR packets heading towards different next-hops, based on prior knowledge of whether they can be decoded. Following the seminal work, many other analysis and protocols have been proposed. For example, [8] studied the joint design of network coding and routing, and computed the optimal performance using optimization software. [9] leveraged the MS's self-information to enable a joint design of network coding and PHY interference cancellation. This line of research has mostly focused on the 802.11 single-channel models. Some recent works have also extended CADSA from different perspectives. Xu *et al.* [10] analyzed the benefits of power-aware, coding-aware subcarrier assignment. [11] further discussed the scenario with multiple WiMax relays, and [12] takes into account the relay selection problem in WiMax such networks. A survey of how network coding is applied to various relay networks is provided in [13].

III. System Models

In this section, we introduce the underlying network models for CADSA. In addition, we introduce the coding and scheduling algorithm that enables network coding in OFDMA based wireless networks.

A. Network Models

We consider a cell-like *wireless switching network* [9], where the base station serves as an intermediate relay for MSs located in the same cell. Frames are transmitted from one MS (the source) to the BS through the uplink, and then switched to another MS (the destination) via the downlink. We refer to such an end-to-end network flow as a *session*. A session may deliver an entire file or data stream consisting of many frames. When multiple sessions (corresponding to multiple source-destination pairs) co-exist, it becomes critical to allocate subcarriers to the uplink and downlink of each session, in order to maximize the total network throughput while maintaining fairness. Such single-cell switching network models can be seen as a decomposition of multi-hop multi-cell OFDMA networks [9], such as 802.16j based wireless mesh network and its extensions.

We model the wireless fading environment by large scale path-loss and shadowing, along with small scale Rayleigh fading effects. The resulting channel gain changes with time, and varies across the whole frequency band for each MS. The time variation and frequency selectivity are characterized by the doppler spread and delay spread respectively, which are associated with the velocity of the MS and the multipath effects caused by obstacles [14]. Due to frequency diversity and multiuser diversity, the achievable rate of a subcarrier depends not only on its fading profile, but also on which link it is assigned to, and how much power it has been allocated by the BS. It has been observed that dynamic power allocation schemes achieve marginal performance gain [2], [3]. Therefore, we only focus on the CADSA with equal power allocation, *i.e.*, all subcarriers equally share the power budget, and perform adaptive modulation and coding (AMC) according to the received SNR.

B. Frame Scheduling and Network Coding Algorithm for CADSA

We assume the system is operating at TDD mode in 802.16, *i.e.*, the uplinks and downlinks are activated alternately. In both uplink and downlink phase, the entire set of subcarriers are allocated to all sessions. As in most existing work [2], however, we only focus on subcarrier allocation for the downlink. Specifically, before each downlink phase, the BS performs subcarrier assignment and XOR network coding simultaneously using the CADSA algorithm. The input to the CADSA algorithm include the identity of each frame's destination MS and the channel gain of each subcarrier. The destination identity can be found in the network layer header field for each frame. The subcarrier's channel gain on the downlink is estimated at the MS using the built-in training sequence in OFDMA systems [15], and then transmitted to the BS through the uplink. To reduce the overhead, the feedback information only contains the best modulation type that a subcarrier can achieve given the current SNR.

Given the above information, the BS first searches for potential coding opportunities between each pair of frames heading towards different MSs. A coding opportunity exists for frames A and B if $D_A = S_B$ and $D_B = S_A$, where S_K and D_K denote frame K's source and destination, respectively. In this case, the CADSA encodes A and B into one frame, allowing the two links BS $\rightarrow D_B$ and BS $\rightarrow D_A$ to share the same set of subcarriers. At the receiver side, the mobile station D_A extracts frame A through the operation $B \oplus (B \oplus A)$. Similar decoding algorithm applies for D_B.

For successful decoding, each receiver must determine the identities of the encoded sessions. Such information is implicit in CADSA. Since exactly two sessions (if any) can be encoded, the pairs of sessions that share the same downlink subcarriers are exactly the encoded pairs. The subcarrier assignment information can be found in the signaling field (DL-MAP and UL-MAP [1]) in each downlink frame. In addition, the receiver needs to determine the identity of the key frame that can decode the encoded frame. When the BS always has no more than one backlogged frames (a reasonable assumption for TDMA-scheduled OFDMA systems like 802.16), then the key is just the latest frame that the receiver sent out. Otherwise, the receiver needs to maintain a historical frame queue, a FIFO queue, to store the latest uplink frames it sent. To decode a downlink frame that is encoded, the receiver needs to dequeue one frame in the historical frame queue and use it as the key. With the above measure, the CADSA frame becomes self-contained — it introduces no additional overhead compared with the general DSA without network coding.

Admittedly, a dynamic subcarrier allocation scheme (either CADSA or general DSA) introduces non-negligible overhead compared with static assignment, which is caused by the feedback information from each MS to the BS indicating the downlink modulation type. It has been observed that the overhead may compromise the benefits of adaptive subcarrier allocation, especially when a large number of subcarriers are involved [15]. Fortunately, it can be significantly reduced by coarse-grained adaptations, as demonstrated in existing DSA algorithms [15]. Such overhead reduction techniques apply to our CADSA algorithm as well.

IV. Subcarrier Assignment Algorithms

The subcarrier assignment algorithm is the core component of CADSA. We formulate the optimal subcarrier assignment scheme for CADSA as a mixed-integer linear program (MILP), and then derive a suboptimal approximate solution with polynomial time complexity. As a benchmark comparison, we also introduce the corresponding assignment problems without network coding.

A. The optimal CADSA

Before formulating the optimization problem, we introduce the following notations. Denote ζ, Ω, and ϕ as the set of subcarriers, sessions, and coding opportunities, respectively. Each element in ϕ is a two-element set $\{s,t\}$, indicating that frames from session s and t satisfy the network coding condition, and thus can be combined into one frame. In addition, we define function $R(c,m)$ as the achievable rate of subcarrier c when assigned to mobile station m. Given the feedback about modulation type, it can be obtained by $R = \frac{b_m c_r}{T_s}$, where b_m is the number of bits in a modulated symbol; T_s and c_r are the symbol period and error control coding rate, respectively.

Our main objective is to assign an appropriate set of subcarriers to the downlink of each session, such that the total downlink capacity (*i.e.*, aggregate downlink throughput) of the switching network is maximized while no session is starved. To avoid starvation of weak sessions (*i.e.*, sessions with low average channel gain), we enforce the max-min fairness constraint, which essentially minimizes the throughput differences between the weak sessions and the strong sessions. Denote the throughput of session s as λ_s, our objective can be expressed as $\max\ \min_s \lambda_s$, or equivalently:

$$\max \quad \lambda \tag{1}$$

$$\text{subject to: } \lambda \quad \leq \quad \lambda_s, \forall s \in \Omega \tag{2}$$

The downlink traffic of each session s consists of two classes: $b_{\{s,t\}}$, which is the amount contributed by subcarriers transmitting XORed frames for session s and t, $\forall \{s,t\} \in \phi$; and u_s, which is the amount of uncoded traffic carried by subcarriers uniquely assigned to session s. Therefore, we have:

$$\lambda_s = \sum_{t \neq s} b_{\{s,t\}} + u_s, \forall s \in \Omega, \{s,t\} \in \phi \tag{3}$$

If two downlinks share one subcarrier, then the subcarrier's rate must conform to the one with lower achievable rate, *i.e.*, the XORed traffic rate equals to the lower rate of the two encoded sessions. Denote x_{cs} as a 0-1 decision variable, which is set to 1 if subcarrier c is assigned to the downlink of session s, and 0 otherwise. Then, a subcarrier c is shared by two sessions s and t if and only if $x_{cs} \cdot x_{ct} = 1$. Therefore, $\forall \{s,t\} \in \phi$,

$$b_{\{s,t\}} = \sum_{c \in \zeta} \min(R(c, D_s), R(c, D_t)) \cdot x_{cs} \cdot x_{ct}, \quad (4)$$

where D_s is the destination MS for session s. The multiplication of two variables x_{cs} and x_{ct} results in a nonlinear constraint. To simplify the problem, we introduce an additional variable $y^c_{\{s,t\}}$ and reformulate the constraint into a linear one. Let $y^c_{\{s,t\}} \in \{0, 1\}$ and $y^c_{\{s,t\}} = x_{cs}x_{ct}$, then for each pair $\{s,t\} \in \phi$, the constraint (4) is equivalent to:

$$b_{\{s,t\}} = \sum_{c \in \zeta} \min(R(c, D_s), R(c, D_t)) \cdot y^c_{\{s,t\}}, \quad (5)$$

$$y^c_{\{s,t\}} \leq x_{cf}, \forall c \in \zeta, f \in \{s,t\} \quad (6)$$

Furthermore, the amount of uncoded traffic can be obtained by subtracting the coded traffic from the total rate allocated to each session. Here we need to subtract the sum rate of all subcarriers used for broadcasting coded frames, *i.e.*, $\forall s \in \Omega$ and $\{s,t\} \in \phi$,

$$u_s = \sum_{c \in \zeta} R(c, D_s)x_{cs} - \sum_{c \in \zeta} \sum_{t \neq s} R(c, D_s)y^c_{\{s,t\}} \quad (7)$$

Finally, except for those carrying coded traffic, one subcarrier can only be allocated to at most one session. Therefore, we have the following constraint:

$$\sum_{s \in \Omega} x_{cs} - \sum_{\{s,t\} \in \phi} y^c_{\{s,t\}} \leq 1, \forall c \in \zeta \quad (8)$$

If a subcarrier c is allocated to a session with no coding opportunity, then the first sum in the above inequality equals 1, while the second equals 0. In contrast, if c is shared by a pair of codable sessions $\{s,t\}$, then the first sum equals 2 while the second equals 1. In any case, the left hand side is either 1 or 0, and thus the constraint (8) always holds. When $x_{cs}x_{ct} = 1$, constraint (8) also enforces that the session s and t must be encoded via subcarrier c, *i.e.*, $y^c_{st} = 1$. Hence it also ensures the correctness of the linearization from constraint (4) to (6).

In consequence, the CADSA optimization becomes a mixed-integer linear program (MILP), with the objective (1), subject to constraints (2), (3), (5), (6), (7) and (8).

B. The Approximate CADSA

The above CADSA mixed-integer program is NP-hard in general. In effect, the NP-hardness can be proved following the similar line of analysis in traditional OFDMA subcarrier assignment problem [2]. Conventional exact solutions to MILP, such as branch and bound [16], can only handle small scale problems with tens of sessions and subcarriers. Although meta-heuristics like simulated annealing [16] may provide acceptable approximate solutions to large scale problems, they typically take a long time to converge, which is undesirable since in practice the subcarrier allocation algorithm needs to be called every few milliseconds. Here we propose a polynomial time approximate algorithm that can be applied to the base station of real wireless switching networks.

Our basic idea is to assign subcarriers to each session in a round based manner. In each round, we employ an *assignment algorithm* to maximize the downlink capacity, and a *penalty algorithm* to ensure fairness.

In the *assignment algorithm*, we group the sessions into those with coding opportunities, and those requiring a unique set of subcarriers. For ease of exposition, we first formulate a graphical model for the assignment mechanism for the former group (graph A in Fig. 2). This graph contains three sets of nodes: the set of sessions Ω, the coding opportunities ϕ and the subcarriers ζ. A link assumes zero weight unless it is from ϕ to ζ, where the weight equals to the achievable rate when the link is matched to a specific coding opportunity. For instance, the weight of $P_1 \rightarrow C_1$ equals to $\min(R(C_1, D_{S_1}), R(C_1, D_{S_2}))$. All links have unity rate, since a link is either fully used or discarded in the assignment within each round (note that these links are different from the actual wireless links). Further, to enhance fairness, a session can choose at most one coding opportunity (and correspondingly at most one subcarrier) within each round. To represent this constraint, we add a *virtual source* S in the graph A, which has a unit-capacity link to each session. S does not represent any network link, subcarrier or session. It is only used to complete the graphical modeling of the assignment algorithm. An additional constraint in the algorithm is that each subcarrier can be assigned to at most one pair of sessions in ϕ. This constraint is represented by adding a *virtual sink* T, which has unit-capacity link to each subcarrier in ζ.

Given the above graphical setup, the objective of the *assignment algorithm* in each round is equivalent to pushing the maximum units of flows from the virtual source S to destination T, and choosing the paths in such a way that maximizes the total link weights. Note that links from Ω to ϕ are many-to-one, and only those from ϕ to ζ have non-zero weights. In addition, one session can be encoded with at most one other session, since information exchange happens only for pairwise sessions. With such observations, we can eliminate nodes representing sessions in graph A, and assign subcarriers to coding opportunities directly. Consequently, we transform the original problem into a max-weight max-flow problem on graph B (Fig. 2).

For sessions without coding opportunities, the assignment is a straightforward max-weight max-flow problem problem that matches the sessions to the subcarriers directly (see graph C in Fig. 2). To complete the *assignment algorithm*, we merge the set ϕ in graph B with the set Ω in graph C, allowing both the codable and uncodable sessions to be matched to subcarriers. As a result, the assignment problem becomes weighted bipartite matching (WBM) in graph D (Fig. 2), which can be easily solved using existing network flow algorithms such as the cost scaling algorithm [17]. Once a subcarrier is occupied after the WBM procedure, it will be permanently removed from ζ. The algorithm terminates when

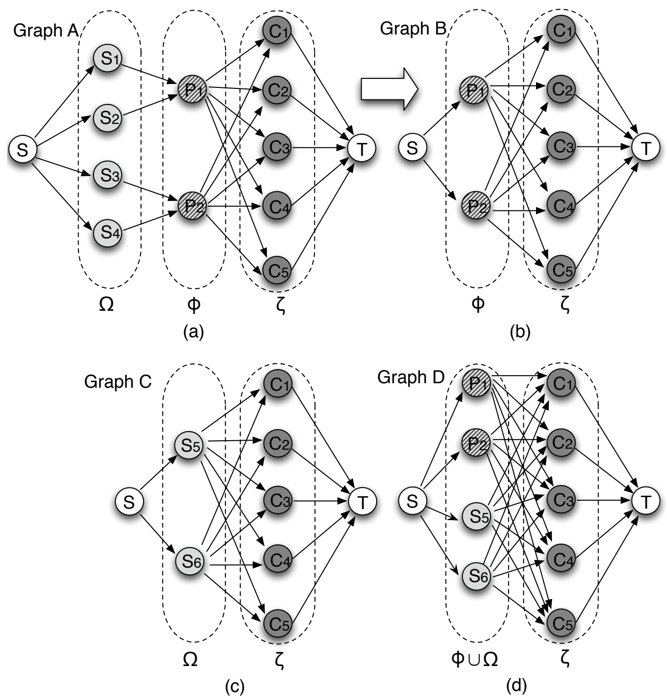

Fig. 2. The network flow model for the CADSA problem. (a) CADSA for sessions with coding opportunities. (b) Simplify the CADSA for sessions with coding opportunities. (c) CADSA for sessions without coding opportunities. (d) Merge sessions with and without coding opportunities. The assignment problem in each round is equivalent to weighted bipartite matching in the graph D.

no more subcarriers can be assigned in a round.

In the *penalty algorithm*, we aim at providing a fair share of bandwidth for each session. It is well known that the max-min fairness constraint seeks to minimize the difference between resource competitors. Naturally, in each round of resource allocation, lower priority will be given to the strong sessions. Therefore, we enforce the following penalty condition for $\forall s \in \Omega$:

$$T_h - \frac{1}{|\Omega|}\sum_{r\in\Omega} T_r > R_{min} \tag{9}$$

where T_h is the downlink throughput of session s in the current round. R_{min} is the achievable rate of a subcarrier when using the modulation type with the lowest rate. Sessions satisfying the penalty condition are gaining advantages over the average, and will be prohibited from the next-round's assignment.

In summary, we describe the suboptimal CADSA in **Algorithm 1**. The computational load of the algorithm is dominated by the the WBM algorithm, whose complexity is $O(\sqrt{n}m\log(n))$, where n and m are the number of nodes and links in the corresponding graph [17]. Since we call the algorithm for at most $|\zeta|$ rounds, the overall complexity is: $O(\lceil|\zeta|\cdot\sqrt{n}m\log(n))$. Such a polynomial time algorithm is well suited for implementation in the base station of real OFDMA systems.

C. Asymptotic Performance Analysis

The intuitions of the above CADSA approximation originate from the objective and constraints of the optimization framework in Sec. IV-A, but how well does it perform in comparison with the optimum? In this section, we answer this question with theoretical rigor by deriving the worst case performance bound of the suboptimal CADSA. We then analyze the average case performance of CADSA compared with the optimum.

Algorithm 1 The approximate Coding Aware Dynamic Subcarrier Assignment (CADSA) algorithm.

repeat
  Assignment algorithm:
  Construct the graph A and transform it into graph B.
  Construct graph C and merge it with graph B.
  Solve the corresponding WBM in graph D using the cost scaling algorithm.
  Penalty algorithm:
  Enforce the penalty condition. Exclude sessions satisfying the penalty condition. Include all other sessions in the next-round's assignment.
until No subcarrier is allocated in the above round.

1) Worst-case performance bound: Recall that Sec. I posited a dilemma, asking whether it is preferable to employ the coding opportunity (*i.e.*, to assign the same subcarriers to codable sessions) or the diversity advantage (*i.e.*, to assign subcarriers to codable sessions separately). In the approximation algorithm, we exploit all possible coding opportunities. In practical OFDMA networks such as WiMax, each subcarrier has a discrete set of modulation schemes to choose from, corresponding to a discrete set of data rates. The difference between maximum and minimum data rate is limited. By encoding two sessions that have different achievable rates, the session with a high rate may loose its advantage, compared with separate subcarrier allocation. However, the saved subcarrier can be allocated to an additional session, *e.g.*, a session that has low link quality. Therefore, network coding is more preferable under our objective function, *i.e.*, maximizing the minimum throughput among all sessions. In the following lemma, we first prove that the loss of diversity is bounded when using network coding. Then we prove the approximation ratio of the proposed algorithm.

Denote $R_{\max}$ and $R_{\min}$ as the achievable rate of a subcarrier when using the modulation type with the highest rate and lowest rate, respectively. Throughout the analysis, we assume $\forall s \in \Omega, \forall k \in \zeta, R(s,k) \geq R_{\min}$, *i.e.*, each sessions is able to support at least a data rate of $R_{\min}$ for all subcarriers. In practice, if a session rides on a weak link that cannot support $R_{\min}$, then it may be rejected by the network level admission control mechanism. Denote λ_n as the objective value generated by the optimal assignment algorithm corresponding to the MILP in Eqn.(2), and λ_c as that generated by an optimal assignment algorithm which exploits all coding opportunities. Let M be the total number of rounds used in the CADSA Algoirthm 1. Then we have:

Lemma 1. $\lambda_n - \lambda_c \leq M(R_{\max} - R_{\min})$.

Proof: Denote ζ_s as the set of subcarriers assigned to a session $s \in \Omega$. Suppose after an optimal assignment, $\exists\{s,t\} \in \phi$, such that $\zeta_s \neq \zeta_t$.

There are two possible cases under this presumption. In the first case, $\exists a \in \zeta$, such that $a \in \zeta_s$, but $a \notin \zeta_t$. If we employ the coding opportunity and allow $a \in \zeta_t$ as well, then the throughput of s increases by at least $R_{\min}$, whereas the throughput of session t decreases by at most $(R_{\max} - R_{\min})$. Since Algorithm 1 runs for at most M rounds, and at least one subcarrier is assigned in each round, the throughput loss of a single session in the approximate CADSA (compared with the optimum) is at most $M(R_{\max} - R_{\min})$. If t is the critical session that has the minimum throughput among all sessions, then the objective λ_c only depends on the throughput of t. Therefore, by encoding instead of separation, the minimum downlink rate of all sessions is decreased by at most $M(R_{\max} - R_{\min})$.

In the second case, $\exists a \in \zeta$, such that $a \in \zeta_t$, but $a \notin \zeta_s$. With a symmetric argument, it is easy to observe that similar result applies as in case 1. Lemma 1 follows directly after summarizing these two cases. $\square$

With Lemma 1, we are now ready to present the theoretical performance bound of the approximation algorithm. Define approximation ratio as the minimum throughput of all sessions of the CADSA divided by that of the optimal CADSA, then we have:

Theorem 1. *In the worst case, the approximation ratio of the suboptimal CADSA is* $\frac{R_{\min}}{2R_{\max} - R_{\min}}$.

Proof: Due to the penalty constraint imposed by the approximate CADSA, in an arbitrary round, the maximum throughput among all sessions exceeds the average throughput by at most the rate of a single subcarrier. Also, in each round except the final round, the rate of the weakest session is increased by one and only one subcarrier, otherwise we can further increase the total network throughput in the *assignment algorithm* without violating the penalty condition in the *penalty algorithm*. In the final round, there may not be enough subcarriers remaining to be allocated to each session.

Following the above reasoning, the optimization objective λ, *i.e.*, the throughput of the weakest session satisfies: $\lambda \le M \cdot R_{\max}$ and $\lambda \ge (M-1) \cdot R_{\min}$. In addition, Algorithm 1 trades at most $M(R_{\max} - R_{\min})$ throughput for coding opportunities according to Lemma 1.

Denote the optimal objective as λ^*. Note that $|\zeta|/|\Omega|$ is a lower-bound to M since each session is allocated at most one subcarrier in each round. In WiMax, ζ and Ω are chosen such that each session is allocated at least 48 subcarriers. Therefore, $M \ge \frac{|\zeta|}{|\Omega|} > 48 \gg 1$ in practice. Consequently, we have $\frac{\lambda}{\lambda^*} \ge \frac{(M-1)R_{\min}}{MR_{\max} + M(R_{\max} - R_{\min})} \approx \frac{R_{\min}}{2R_{\max} - R_{\min}}$. This completes the proof. $\square$

It should be noted that in the special case of $R_{\min} = R_{\max}$, the approximation ratio is $\frac{M-1}{M} \approx 1$. In this case, it is straightforward that all network coding opportunities should be exploited (following the argument in the proof for Lemma 1). As a result, the original optimization problem becomes a weighted bipartite matching problem, which has an exact solution.

In addition, the above is just a worst case bound. According to the 802.16 specification [1], $\frac{R_{\min}}{2R_{\max} - R_{\min}} = \frac{3417}{2 \times 13176 - 3417} \approx \frac{1}{7}$. In fact, however, we will show in the simulation experiments that the CADSA achieves a performance level quite close to the optimum. The main reason lies in the proof for Lemma 1: in the second case, since t is allocated more subcarriers than the set shared with s, t is usually the session with lower downlink capacity, *i.e.*, $R(a, D_s) > R(a, D_t)$. Therefore, by encoding these two sessions, the throughput of t is maintained whereas that of s is increased. In other words, no diversity loss happens in the common case.

2) Average case analysis: In typical cellular wireless networks, signal attenuation is dominated by large-scale path-loss due to link distance, rather than small-scale fading due to doppler spread or multipath reflection [14]. To pinpoint the average case, we only consider large-scale fading effects. With this simplification, the channel gain of different MSs depends on their relative distance to the BS, and for each MS, the channel gain remains stable over time and across different subcarriers. Then we can prove:

Theorem 2. *If channel gain only depends on link distance, then the approximation ratio of the suboptimal CADSA is* $\frac{M-1}{M}$.

Proof: The proof follows a similar line of analysis to Lemma 1 and Theorem 1, but with the special assumption that channel gain only depends on link distance, we first extend Lemma 1 and show $\lambda_n = \lambda_c$, *i.e.*, coding opportunity should always be exploited.

The analysis for the first case in Lemma 1 still holds. Suppose in an *optimal* CADSA solution, $\exists a \in \zeta$, such that $a \in \zeta_t$ and $a \notin \zeta_s$. If we employ the coding opportunity and allow $a \in \zeta_s$ as well, then the throughput of s increases, whereas the throughput of session t decreases by $(R' - R_{\min})$, where $R' > R_{\min}$. This means t is the session with minimum throughput but higher channel gain than s. By reallocating one subcarrier from s to t without coding, we can improve the throughput of t by $(R' - R_{\min})$, but only reduce the throughput of s by $R_{\min}$. Since $(R' - R_{\min}) > R_{\min}$ (due to the discrete data rates in 802.16), the λ_n is improved via this reallocation, which contradicts the optimality of the solution. Hence, the existence of a is invalidated, *i.e.*, CADSA achieves the same performance as the optimum in one round.

Under the presumption in Theorem 2, the throughput of the weakest session satisfies: $\lambda \le M \cdot R_{\min}$, since all subcarriers for this session has rate $R_{\min}$, and at most one subcarrier is assigned to each session in a round. Furthermore, in the last round of assignment, there may not be sufficient subcarriers to assign to each session, thus we have $\lambda \ge (M-1)R_{\min}$. Following similar line of reasoning in Theorem 1, we have $\frac{\lambda}{\lambda^*} \ge \frac{(M-1)R_{\min}}{MR_{\min}} = \frac{M-1}{M}$. $\square$

Theorem 2 essentially justifies the intuition that when channel gain is dominated by large-scale path-loss, the worst-case in Theorem 1 rarely occurs, and the performance of the heuristic CADSA approximates the optimum.

3) Extension to other fairness measure: Recall that the above CADSA algorithms aim at providing max-min fairness. Such an objective usually leads to similar performance among all sessions. In case when the sessions have diverse traffic demands and priority, the weighted max-min fairness claims to be a better metric. The corresponding objective is to maximize the minimum normalized throughput, *i.e.*,

$$\begin{aligned} \max \quad & \lambda \\ \text{subject to: } \lambda_s \quad & \geq \quad \lambda \cdot d_s \end{aligned}$$

where d_s denotes the traffic demand of session s; λ is the minimum satisfied portion of throughput for all sessions.

To extend the suboptimal CADSA to a weighted max-min fair algorithm, we revise the penalty mechanism, such that the sessions with higher $\frac{\lambda_s}{d_s}$ are punished. Denote $d_{\min}$ and $d_{\max}$ as the minimum and maximum demand of all sessions, then the corresponding penalty threshold equals $\frac{R_{\min}}{d_{\max}}$. Following similar analysis to Theorem 1, we can prove that the worst case approximation ratio is $\Theta(\frac{R_{\min} d_{\min}}{2R_{\max} d_{\max}})$.

The proposed CADSA can achieve max-min fairness or its variants, because it enforces a penalty condition to balance the throughput of all sessions. It cannot be straightforwardly used to achieve other objectives such as proportional fairness due to their non-linearity. The development of new algorithms to achieve these objectives is an interesting direction and is left for our future work.

D. The General DSA Algorithms

As a benchmark, we inspect the general DSA algorithm, *i.e.*, the dynamic subcarrier assignment algorithm without network coding. Such schemes have been extensively explored in the literature. Here we consider the optimization based solution with equal power allocation (see, *e.g.*, [3], [4]), which is formulated as:

$$\max \quad \lambda \tag{10}$$

$$\text{subject to:} \quad \lambda \leq \lambda_s, \forall s \in \Omega \tag{11}$$

$$\lambda_s = \sum_{c \in \zeta} R(c, D_s) \cdot x_{cs}, \forall s \in \Omega \tag{12}$$

$$\sum_{s \in \Omega} x_{cs} \leq 1, \forall c \in \zeta \tag{13}$$

The objective (10), together with the constraint (11), guarantees the max-min fairness for per-session throughput. Constraint (12) bounds the downlink throughput by the achievable rate of all subcarriers allocated to it. Constraint (13) dictates that one subcarrier can be assigned to at most one link of all sessions.

Though the formulation is much simpler than CADSA, finding the optimal solution is still an NP-hard problem [2]. Various approximations have been proposed for this problem. A typical approach is the greedy algorithm [2]–[4] (henceforth referred to as *DSA heuristic*) which selects one subcarrier with the highest channel gain for each session iteratively, until no more subcarriers can be assigned. We provide more extensive evaluation of it together with the CADSA heuristic in the following section.

V. Performance Evaluation

In this section, we investigate the performance of the approximate CADSA **Algorithm 1** in comparison with an upperbound to the the optimal solution, as well as the tranditional non-coding schemes.

A. Experiment Setup

The key of our experiment settings is to derive the achievable data rate of a subcarrier when it is allocated to an arbitrary MS. This requires computing the corresponding SNR value, and mapping the SNR to an achievable rate. To generate realistic results, we adopt empirical parameters to model the wireless fading environment, and configure the OFDMA system according to the 802.16 specification [1]. We developed a C++ based simulator that models the mobile fading environment. The channel model in our simulator is built atop teh Chsim module in OMNeT++ [18], but with configurations specific to the 802.16 OFDMA channel.

First, the signal attenuation due to large scale fading follows the log-normal equation [14]:

$$\text{Channel gain (dB)} = K + 10\alpha \log(d) + X \tag{14}$$

where d denotes the distance between the BS and the MS; K is a constant equal to 46.7dB in 5GHz outdoor environment; the path loss exponent α is set to 2.4; X is a zero-mean Gaussian random variable with empirical standard deviation 5.4dB [15]. We assume that the shadowing loss varies on the time scale of 0.1 second.

The small scale fading effects are caused by movement of the MS in multipath environment, and modeled by the Rayleigh fading process. The inherent frequency selective property is characterized by an exponential power delay profile with delay spread 15 μs. The time selective nature is captured by the doppler spread, which depends on the MS's speed (throughout the simulation, the MSs are moving at pedestrian speed 2m/s, according to the random waypoint model with pause period 0.01s). The combined complex gain is generated using an improved Jakes-like method introduced in [14], which models the frequency correlation between adjacent subcarriers and the time correlation for each subcarrier.

Without loss of generality, we choose the following set of configurations from the 802.16d wirelessMAN-OFDMA specifications [1]. The system bandwidth is 7 MHz, centered around the 5 GHz frequency, and equally shared by all subcarriers. The maximum number of data subcarriers is 1536; subcarrier spacing is $3\frac{29}{32}$ kHz; symbol period T_s is $264\mu s$; downlink frame length T_f is 2 ms. Available modulation schemes include QPSK$\frac{1}{2}$ (error control coding rate), QPSK$\frac{3}{4}$, 16QAM$\frac{1}{2}$, 16QAM$\frac{3}{4}$, 64QAM$\frac{1}{2}$, and 64QAM$\frac{3}{4}$. The corresponding SNR thresholds are 6.0dB, 8.5dB, 11.5dB, 15dB, 19dB and 21dB [1]. When computing SNR, the BS transmission power, noise temperature and noise figure are 1W, 290K and 7dB, respectively. Both the BS and the MSs use omnidirectional single-antenna transceivers.

B. Experiment Results

We compare three subcarrier allocation schemes: the coding aware dynamic subcarrier assignment (CADSA) algorithm, dynamic subcarrier assignment without network coding (DSA), and the randomized subcarrier allocation mechanism (referred to as RAND). Similar to the scheme in 802.16, the RAND algorithm randomly allocates an equal number of subcarriers to each downlink, and chooses the modulation for

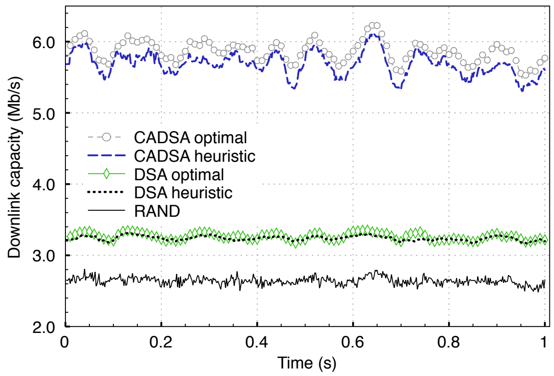

Fig. 3. The total downlink capacity as a function of time.

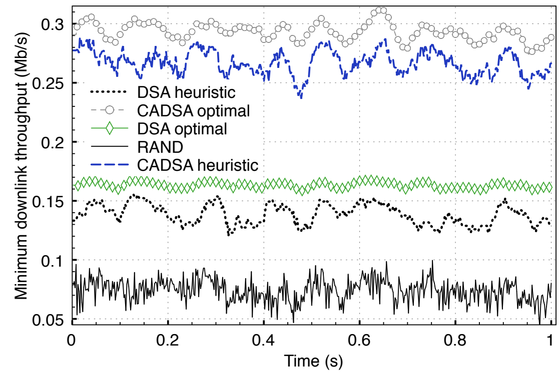

Fig. 5. The minimum downlink throughput as a function of time.

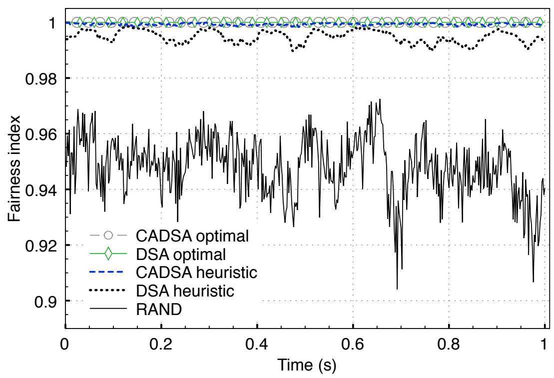

Fig. 4. The fairness index of each scheme as a function of time.

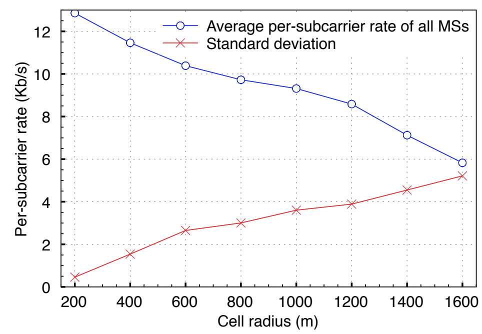

Fig. 6. The variation of per-subcarrier rate with the cell radius.

each subcarrier according to its SNR value. Since the optimal solution for CADSA and DSA cannot be obtained for large scale scenarios using optimization software, we evaluate their LP-relaxations instead. We relax the integer constraints on the variables x_{cs} and y^c_{st}, allowing them to be real numbers in $[0, 1]$. The resulting linear-programming solution is infeasible since it assumes subcarriers can be fractionally assigned. However, the LP-relaxation imposes an upper bound on the original mixed-integer linear program (and thus a loose upperbound on the suboptimal CADSA), since the solution space of the MILP is a subspace of the LP. Therefore, the LP-relaxation is used to understand the performance gap between the suboptimal CADSA and the optimal solution.

1) Throughput comparison: We focus on the scenario where 8 mobile MSs are moving in a circular cell with 0.6 km radius. We randomly start 20 pairwise sessions with constant bit rate traffic, assuming that the downlink of each session always has data to transmit. Due to the limitation of our linear programming software, we only use 256 data subcarriers (consecutively located around the central frequency) of the entire frequency band. We compute the downlink capacity, *i.e.*, the aggregate downlink throughput of all sessions, over a period of one second.

As shown in Fig. 3, the performance gain of CADSA over DSA keeps consistently around 75%. The downlink capacity of the suboptimal CADSA approximates the optimum well. Both CADSA and DSA outperform RAND by a significant margin. Notably, the throughput of the heuristic DSA can approach or even exceed the optimal values. This is at the cost of fairness, *i.e.*, there can be a certain gap between the max and min throughput of all sessions when running the heuristics. To quantify the difference in fairness, we compute the Jain's fairness index [19] for all the above schemes. Denote the throughput of session i as F_i, then the fairness index is $F = \frac{(\sum_{i=1}^{|\Omega|} F_i)^2}{|\Omega| \sum_{i=1}^{|\Omega|} F_i^2}$. From Fig. 4, we see that the optimal LP solutions tend to achieve full fairness (*i.e.*, $F = 1$). The intuition behind is that the optimal algorithm can reduce the difference in throughput by switching subcarriers from high-throughput sessions to low-throughput sessions. In contrast, the heuristic DSA and RAND tend to deviate from the optimal fairness index. Remarkably, the fairness of the approximate CADSA is quite close to the optimum, owning to its penalty mechanism. As a result, the minimum throughput of all sessions remains around 90% of the optimal value (Fig. 5).

2) Influence of multi-user diversity: Generally, multi-user diversity is reduced when we decrease the cell radius, since the MSs' difference in distances to the base station is reduced. This is justified in Fig. 6, where we define the per-subcarrier rate of an MS as the average rate over both time and frequency domain. The evaluation stops at 1.6km since the BS's transmission range is around 1.5km in our experiment.

In Fig. 7 and Fig. 8, we explore the influence of multi-user diversity on time-averaged downlink capacity and fairness. In these and the experiments below, we have 512 subcarriers assigned to 40 random sessions that are running among 10 MSs. As we increase the cell radius, the average channel condition deteriorates, resulting in lower downlink capacity. Meanwhile, the multi-user diversity becomes larger, making it harder for the heuristic DSA and RAND to ensure fairness. With the penalty mechanism, however, the approximate CADSA keeps near-optimal fairness and yet much higher capacity, even under

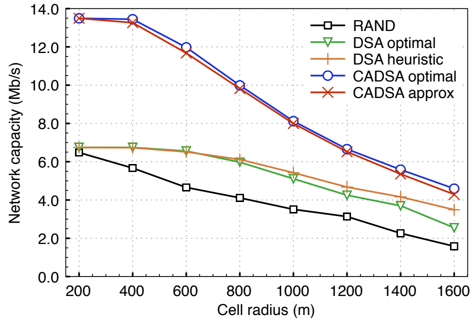

Fig. 7. Influence of attenuation spread on the downlink capacity.

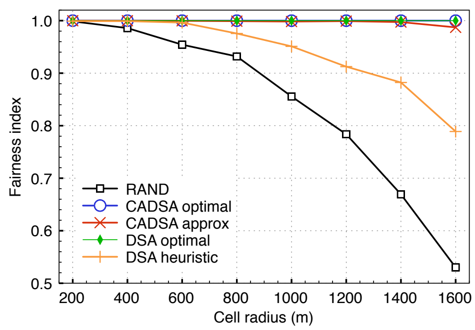

Fig. 8. Influence of attenuation spread on fairness.

severe channel conditions.

In general, the dynamic subcarrier assignment algorithms outperform RAND in the scenarios with large multi-user diversity [15], *i.e.*, the channel gains of different MSs vary substantially. However, to exploit the network coding advantage, it is preferable to encode the downlinks with similar channel gains, and assign the same subcarriers to them. Otherwise the downlink with a worse channel condition will undermine the shared downlink rate. Obviously, there is a trade-off between the diversity advantage and network coding advantage, governed by the level of multi-user diversity.

To quantitatively explore this trade-off, we adopt the *minimum throughput* of all sessions as the performance metric, which is essentially the optimization objective of DSA and CADSA. We define *diversity gain* as the performance gain of the optimal DSA over RAND (*i.e.*, dynamic assignment over the static assignment algorithm), and *coding gain* as the performance gain of the optimal CADSA over DSA (*i.e.*, coding aware assignment over non-coding based assignment algorithm). Formally, let λ_X denote the minimum session

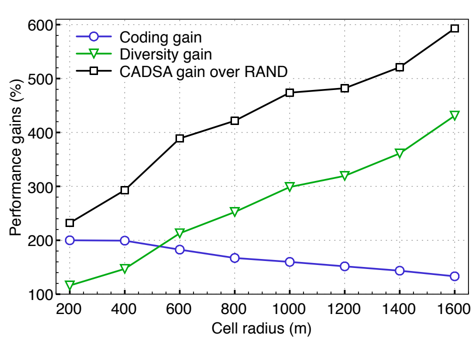

Fig. 9. Influence of attenuation spread on performance gains.

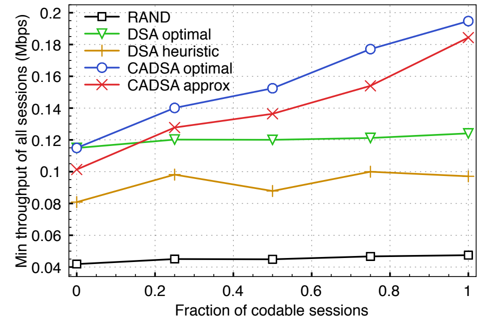

Fig. 10. Minimum throughput when the fraction of codable sessions varies.

throughput resulting from scheme X ($X \in \{$DSA, RAND, CADSA$\}$), then:

$$\text{Diversity gain} = \frac{\lambda_{\text{DSA}}}{\lambda_{\text{RAND}}}, \qquad \text{Coding gain} = \frac{\lambda_{\text{CADSA}}}{\lambda_{\text{DSA}}} \quad (15)$$

Fig. 9 illustrates the variation of diversity gain and coding gain, as a function of the cell radius. We observe that with small cell radius (hence less multi-user diversity), the coding gain approaches the 100% bound. When the MSs experience considerably different channel conditions (hence larger multi-user diversity), the coding gain diminishes, whereas the diversity gain increases. By balancing a trade-off between both schemes, the CADSA mechanism achieves up to 6x performance improvement over the RAND. Different from Fig. 7, the performance metric here is the minimum throughput of all sessions, which implicitly accounts for fairness. RAND tends to starve those sessions with small channel gain, thus resulting in much lower performance than CADSA.

3) Partially codable sessions: Note that in the above experiments, we assumed the sessions are paired so that each session is interested in exchanging information with another one, thus a coding opportunity exists for each session. In practice, not all sessions may have coding opportunities, and therefore the gains of network coding also depend on the fraction of sessions that can be encoded.

To explore the influence of such practical factors, we run CADSA with variable fraction of coding opportunities. Specifically, we deploy 10 MSs and 40 random sessions in a cell with 1km radius. We vary the number of codable sessions from 0 to 40. Fig. 10 and Fig. 11 plot the minimum throughput of all sessions and the corresponding performance gains resulting from network coding and dynamic assignment. Since the cell radius remains stable, the diversity gain does not vary. However, the coding gain increases monotonically with the fraction of codable sessions, and the overall performance gain of CADSA depends a lot on the number of coding opportunities available.

VI. Conclusion

In this paper, we have designed a cross layer scheme that integrates network coding and dynamic subcarrier assignment in OFDMA wireless networks. We have formulated the optimal coding aware subcarrier assignment scheme, and proposed a polynomial time suboptimal algorithm with provably good performance. Our simulations in the frequency selective fading environment and under 802.16-like settings have

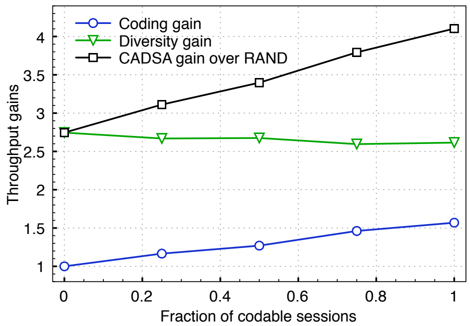

Fig. 11. Performance gains when the fraction of codable sessions varies.

demonstrated that network coding can more efficiently utilize the available subcarriers. The coding-aware scheme results in considerably higher network throughput without causing additional overhead when compared with adaptive assignment algorithms without network coding. In addition, we identified an important tradeoff between the coding advantage and the diversity gain, which may need further exploration from an information theoretic perspective.

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