Price-based Resource Allocation
in Wireless Ad Hoc Networks

Yuan Xue, Baochun Li, Klara Nahrstedt [*]

Abstract. The shared-medium multi-hop nature of wireless ad hoc networks poses fundamental challenges to the design of an effective resource allocation algorithm to maximize the aggregated utility of flows, while maintaining basic fairness among multiple flows. When previously proposed scheduling algorithms have been shown to perform well in providing fair shares of bandwidth among *single-hop* wireless flows, they did not consider *multi-hop* flows with an end-to-end perspective. Moreover, the resource allocation strategies employed in the wireline network can not be applied directly in the context of ad hoc networks due to the unique characteristic of location dependent contention and spatial reuse of the shared wireless channel. In this paper, we propose a price-based resource allocation model to achieve maximized aggregated utility (*i.e.,* social welfare) of flows. Our original contributions are: First, we propose to use *maximal clique-associated* shadow prices for wireless channel access coordination, rather than *link-associated* price for wireline link access arbitration. Second, we present a new pricing policy for end-to-end multi-hop flow. Using this model, different fairness goals can be realized in ad hoc networks for end-to-end flows. With a two-tier distributed and iterative algorithm, scarce channel capacity is allocated fairly among multi-hop flows from an end-to-end perspective, using shadow prices as the mechanism to arbitrate channel access. Through extensive analysis and simulation results, we show that our proposed algorithm is able to fairly distribute resources among multi-hop flows, while simultaneously maximizing the aggregated utility of flows globally.

1 Introduction

A wireless ad hoc network consists of a collection of wireless nodes without a fixed infrastructure. Each node in the network serves as a router that forwards packets for other nodes. Each flow from the source to the destination traverses multiple hops of wireless links. Compared with wireline networks where flows contend only at the router with other simultaneous flows through the same router (contention in the time domain), the unique characteristics of multi-hop wireless networks show that, flows also compete

for shared channel bandwidth if they are within the transmission ranges of each other (contention in the spatial domain). This presents the problem of designing an appropriate topology-aware resource allocation algorithm, such that contending multi-hop flows fairly share the scarce channel capacity, while the total aggregated utility of all flows is maximized.

In the context of wireline networks, *pricing* (*e.g.,* [1–5]) has been extensively used in the literature as a means to arbitrate resource allocation. In order to adopt a price-based approach, *utility functions* are used to characterize the resource requirements and the degree of satisfaction of individual users. The goal of the network is to appropriately allocate resources to maximize an objective function that depends on user utilities. For example, the total aggregated utility over all users may be maximized (called the social welfare) subject to certain resource constraints. Kelly *et al.* [3] have shown that, to achieve such a goal, the network may use price as a signal to reflect the traffic load on the wireline links, and users can choose a transmission rate to respond to such price so that their net benefit can be maximized. It is shown that at equilibrium, such a price-based resource allocation scheme maximizes the social welfare and achieves a (weighted) proportional fair rate allocation, if the logarithmic utility function is used.

Unfortunately, there exist fundamental differences in multi-hop wireless ad hoc networks compared with traditional wireline networks, preventing verbatim application of the existing pricing theories. First, in multi-hop wireless networks, flows that traverse the same geographical *vicinity* contend for the same wireless channel capacity. This is in sharp contrast with wireline networks, where the network resource is modeled as a set of links (or *edges* in the topological graph) connecting nodes, and only flows that traverse the same link contend for the capacity of this link. When it comes to pricing, we may conveniently associate *shadow prices* with individual links in wireline networks, such that the price of a flow is the aggregate of shadow prices of the links it traverses. However, we may not be able to accommodate such conveniences when pricing ad hoc networks. Second, as opposed to wireline networks, algorithms designed for wireless ad hoc networks may not rely on the convenience of any centralized management or authority. The search for a fully distributed algorithm further exacerbates the problem.

In this paper, we address these unique characteristics of wireless ad hoc networks, and follow a price-based approach to allocate channel bandwidth availability to competing multi-hop flows. The fundamental question we attempt to answer is: how much bandwidth should we allocate to each of the flows, so that the aggregated utilities over all users can be maximized? Towards this goal, our original contribution is the following: (1) We associate shadow prices with *maximal cliques* in the corresponding *wireless link contention graph* of the ad hoc network, rather than individual links as in wireline networks; (2) Based on this new model, we present a new pricing policy for end-to-end multi-hop flows. In this policy, the price of an end-to-end multi-hop flow is the aggregate of prices of all its subflows[1], while the price of each subflow is the sum of shadow prices of *all* maximal cliques that it belongs to. We present a distributed algorithm that determines shadow prices such that the aggregated utility of all flows is maximized. We show that by choosing appropriate utility functions, different fairness models can

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[1] When each multi-hop flow is considered as multiple *single-hop flows*, each single-hop flow is referred to as a *subflow*.

be realized for multi-hop flows, including proportional fairness and max-min fairness. The algorithm consists of two tiers. The first tier constitutes an iterative algorithm to determine per-clique shadow prices. It can be shown that this algorithm converges to the unique system equilibrium. The second tier completes the details of the algorithm within the same maximal clique.

The remainder of this paper is organized as follows. Sec. 2 and Sec. 3 present related work and the network model, respectively. We propose our price-based resource allocation model in Sec. 4, and present our distributed algorithm in Sec. 5. Finally, we show simulation results in Sec. 6, and conclude in Sec. 7.

2   Related Work

Price-based resource allocation strategies, as well as the analysis of their fairness properties, have received much attention in recent years in the setting of wireline networks (*e.g.*, [1–5]). For example, in the work proposed in [1, 3, 2], a shadow price is associated with each wireline link. The network uses these prices as signals to users which reflect the traffic load on the links along their route, and users choose a transmission rate to optimize their net benefit. Nevertheless, the fundamental differences in contention models between ad hoc and wireline network deserve a fresh treatment to this topic. The resource allocation strategies employed in the wireline network can not be applied directly in the context of ad hoc networks due to the unique characteristic of location dependent contention and spatial reuse of the shared wireless channel. In this paper, we propose a new pricing model for wireless ad hoc networks to address such uniqueness. We propose using clique-associated shadow prices for channel access, rather than the traditional link-associated price for wireline link access arbitration. Based on this model, we present a new pricing policy with respect to end-to-end multi-hop flows.

In the setting of wireless LANs, the use of pricing has also been studied in the context of efficient power control (*e.g.*, [6]) and service differentiation [7]. These solutions focus on single-hop infrastructure wireless networks, while we consider multi-hop wireless networks.

There also exists work to use pricing as incentives to encourage packet relays in wireless ad hoc networks (*e.g.*, [8]). Our work is fundamentally different from these results in the following aspects. (1) In [8], a simplified ad hoc network model is used, where each node i in the network has a capacity of C_i, which is independent from other nodes. We will later show that such a network model can not correctly characterize the unique characteristics of location-dependent contention in ad hoc networks. (2) In [8], a user is assumed to have limited transmission resources. Thus, the user might not volunteer to forward packets for other users as this impacts the ability to transmit their own traffic. In light of this problem, the role of pricing is to provide adequate user incentives to forward packets for other users. The goal of optimal price setting at each node is to maximize its net benefit, which reflects its utility gain, its revenue from packet relays and its cost for paying other nodes to relay its own packets. In contrast, the role of pricing in our work is to regulate channel access and provide globally optimal resource allocation in the sense of maximizing aggregated utility. This goal is complementary to any incentives to packet relays.

Resource allocation, using MAC-layer fair scheduling for single-hop MAC layer flows, is also studied in ad hoc networks [9–12]. In comparison, we address end-to-end multi-hop flows. It can be shown with examples that, optimal resource allocation among single-hop flows may not be optimal for multi-hop flows, due to the unawareness of bottlenecks. Moreover, global optimal resource allocation among multi-hop flows can not be completely reached by MAC-layer scheduling, which is only based on local information. Pricing is needed as a signal to coordinate the global resource demand and supply.

3   Network Model and Resource Constraints

In this paper, we consider a *static* multi-hop wireless network with channel capacity C. In this network, only nodes that are within the transmission range of each other can communicate directly and form a *wireless link*. Data transmissions over a single wireless link are referred to as a *subflow*. Two subflows contend with each other if either the source or destination of one subflow is within the transmission range of the source or destination of the other[2]. Let l_{ij} denote the subflow from node i to j. For the example in Fig. 1(A), we may observe that l_{12} contends with l_{62}, l_{23} and l_{34}. The locality of wireless transmissions implies that the degree of contention for the shared medium is location-dependent.

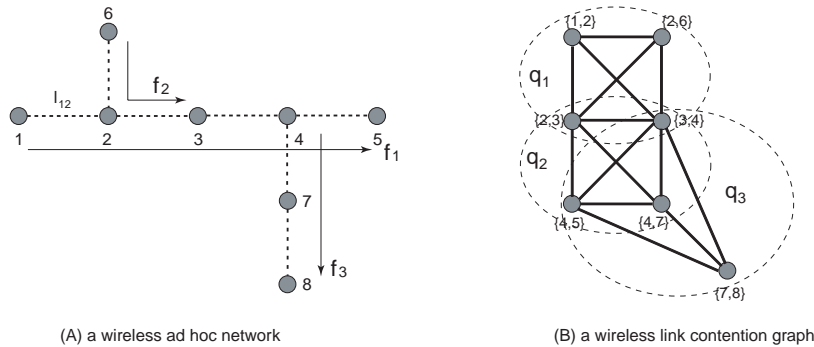(A) a wireless ad hoc network   (B) a wireless link contention graph

Fig. 1. Location-dependent contention: an example.

We first illustrate resource constraints due to such location-dependent contention by using the example shown in Fig. 1, where there are three multi-hop flows f_1, f_2 and f_3, with data rates x_1, x_2 and x_3, respectively. Further, we use y_{ij} to denote the aggregated data rate of *all* subflows l_{ij} between node i and j. For example, $y_{12} = x_1$, $y_{23} = x_1 + x_2$. The channel capacity C gives the upper bound on the aggregated data rate of all mutually contending subflows. In this example, there are three contending subflow sets

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[2] If we assume that the interference range is greater than the transmission range, the contention model can be straightforwardly extended.

$\{l_{12}, l_{23}, l_{34}, l_{26}\}$, $\{l_{23}, l_{34}, l_{45}, l_{47}\}$ and $\{l_{34}, l_{45}, l_{47}, l_{78}\}$, where all subflows within a set contend with each other. We then have

$$y_{12} + y_{26} + y_{23} + y_{34} \leq C \tag{1}$$

$$y_{23} + y_{34} + y_{45} + y_{47} \leq C \tag{2}$$

$$y_{34} + y_{45} + y_{47} + y_{78} \leq C \tag{3}$$

This leads to $3x_1 + 2x_2 \leq C$, $3x_1 + x_2 + x_3 \leq C$ and $2x_1 + 2x_3 \leq C$, which show the resource constraints when it comes to end-to-end flow rate allocation.

Motivated by this example, we consider a static network topology modeled by a undirected graph $\mathcal{G}_T = (\mathcal{V}_T, \mathcal{E}_T)$, where $\mathcal{V}_T$ is the set of nodes in the network. A wireless link $\{i, j\} \in \mathcal{E}_T$, if nodes i and j are within the transmission range of each other.

We now introduce the concept of *wireless link contention graph*. In a wireless link contention graph, each vertex represents a wireless link. Each edge between two vertices denotes that subflows, if there exists any, on the wireless links corresponding to the two vertices contend with each other. A wireless link contention graph is different from a *flow contention graph* [11] in that each vertex in a flow contention graph represents a backlogged MAC layer flow, thus the topology of a flow contention graph depends on the traffic patterns and routes in the network. On the other hand, the wireless link contention graph only depends on the topology of the original network and is not related to the traffic patterns and routes of flows. As an example, Fig. 1(B) shows the wireless link contention graph of the ad hoc network in Fig. 1(A).

Formally, a graph $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{E}_C)$ is a wireless link contention graph of network $\mathcal{G}_T$, if there exists a mapping function $\varphi : \mathcal{E}_T \to \mathcal{V}_C$ that satisfies

(1) $\varphi(l) \in \mathcal{V}_C$, *if and only if* $l \in \mathcal{E}_T$;

(2) $\{\varphi(l), \varphi(l')\} \in \mathcal{E}_C$, *if* for $l, l' \in \mathcal{E}_T$, $\exists l'' \in \mathcal{E}_T$, so that $l \cap l'' \neq \emptyset$ and $l' \cap l'' \neq \emptyset$.

In a graph, a complete subgraph is referred to as a *clique*. A *maximal clique* is referred as a clique that is not contained in any other cliques[3]. In a wireless link contention graph, the vertices in a maximal clique represent a maximal set of wireless links where subflows along any two links contend with each other. Intuitively, each maximal clique in a wireless link contention graph represents a "maximal distinct contention region", since at most one subflow in the clique can transmit at any time and adding any other flows into this clique will introduce the possibility of simultaneous transmissions. Formally, we denote a maximal clique of a wireless link contention graph $\mathcal{G}_C$ as $\mathcal{G}_q = (\mathcal{V}_q, \mathcal{E}_q)$, and the set of all maximal cliques in a wireless link contention graph as $\mathcal{Q}$. For simplicity, we use the vertices set $\mathcal{V}_q$ of a clique $\mathcal{G}_q$ to represent the clique, and sometimes simply denote it as q. For the example in Fig. 1, the set of maximal clique is $\mathcal{Q} = \{q_1, q_2, q_3\}$ where $q_1 = \{\{12\}, \{23\}, \{34\}, \{26\}\}$, $q_2 = \{\{23\}, \{34\}, \{45\}, \{47\}\}$ and $q_3 = \{\{34\}, \{45\}, \{47\}, \{78\}\}$. To this end, we show the role of maximal cliques in resource allocation in the following definition and lemma.

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[3] Note that *maximal clique* has a different definition from *maximum clique* of a graph, which is the maximal clique with the largest number of vertices.

Definition 1 (feasibility). A wireless link bandwidth allocation $y = (y_l, l \in \mathcal{E}_T)$ is *feasible*, if there exists a collision-free transmission schedule that allocates bandwidth y_l to wireless link l.

Lemma 1. A wireless link bandwidth allocation $y = (y_l, l \in \mathcal{E}_T)$ is *feasible*, if and only if the following condition is satisfied:

$$\forall q \in \mathcal{Q}, \sum_{l \in q} y_l \leq C \tag{4}$$

Proof: It is trivial to show that Eq. (4) is a *necessary* condition for feasible bandwidth allocation, as flows within a clique q can not transmit simultaneously. Now, we show that the condition Eq. (4) is also *sufficient* for a feasible bandwidth allocation. We assume the converse is true, *i.e.,* when Eq. (4) holds, there exists no feasible bandwidth allocation. This means that under any schedule, there always exists a set of mutually contending wireless links whose bandwidth allocation exceeds the channel capacity C. Naturally, this set of mutually contending wireless links constitutes a clique q' so that $\sum_{l \in q'} y_l > C$. If q' is a maximal clique, a contradiction exists. If q' is not a maximal clique, then there exists another maximal clique $q'' \in \mathcal{Q}$ that contains q'. It is obvious to see $\sum_{l \in q''} y_l > C$, which also leads to a contradiction. $\square$

This lemma shows that each maximal clique can be regarded as an independent resource with capacity C. It motivates the use of a maximal clique as a basic resource unit for pricing.

We associate each user in the network with an end-to-end multi-hop flow (or simply *flow*) f, and denote the set of flows as $\mathcal{F}$. We assume each flow has a fixed path which passes a set of wireless links. We use this set of wireless links to represent the flow f (*i.e.,* $f \subset \mathcal{E}_T$). In the example shown in Fig. 1, $f_1 = \{\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}\}$, $f_2 = \{\{2,6\}, \{2,3\}\}$, and $f_3 = \{\{4,7\}, \{7,8\}\}$. We use the vector $\mathbf{x} = (x_f, f \in \mathcal{F})$ to denote the rate of flows in the set $\mathcal{F}$.

We now proceed to define a clique-flow matrix $\mathcal{R}$, where $\mathcal{R}_{qf} = |q \cap f|$ is the number of wireless links that flow f passes in clique q. If we treat a maximal clique as a singular resource, then the clique-flow matrix represents the "resource usage pattern" of each flow. In the example, the clique-flow matrix is shown in Fig. 2.

	f_1	f_2	f_3
q_1	3	2	0
q_2	3	1	1
q_3	2	0	2

$q_1 = \{\{1,2\},\{2,6\},\{2,3\},\{3,4\}\}$ $f_1 = \{\{1,2\},\{2,3\},\{3,4\},\{4,5\}\}$

$q_2 = \{\{2,3\},\{3,4\},\{4,5\},\{4,7\}\}$ $f_2 = \{\{2,6\},\{2,3\}\}$

$q_3 = \{\{3,4\},\{4,5\},\{4,7\},\{7,8\}\}$ $f_3 = \{\{4,7\},\{7,8\}\}$

Fig. 2. An example of the clique-flow matrix.

Finally, we use the vector $\mathcal{C} = (C_q, q \in \mathcal{Q})$ as the vector of channel capacities in each of the cliques, where C_qs are equal to the physical channel capacity C in the ideal case. Now, we give the constraints of resource allocation in an ad hoc network in the following theorem.

Theorem 1. In an ad hoc network $\mathcal{G}_T$, given the set of flows $\mathcal{F}$ that uses this network, there exists a feasible rate allocation $\mathbf{x} = (x_f, f \in \mathcal{F})$, if and only if $\mathcal{R}\mathbf{x} \leq \mathcal{C}$, where $\mathcal{R}$ is the clique-flow matrix defined on the network $\mathcal{G}_T$ and the flow set $\mathcal{F}$.

Proof: It is obvious that $\mathcal{R}\mathbf{x} \leq \mathcal{C} \Leftrightarrow \forall q \in \mathcal{Q}, \sum_{f \in \mathcal{F}} \mathcal{R}_{qf} x_f \leq C$. By the definition of $\mathcal{R}$, we have $\sum_{f \in \mathcal{F}} \mathcal{R}_{qf} x_f = \sum_{l \in q} y_l$. The result follows naturally from Lemma 1. □

4 Pricing Model in Wireless Ad hoc Networks

4.1 Optimal resource allocation: problem formulation

We associate each flow $f \in \mathcal{F}$ with an *utility function* $U_f(x_f)$, where x_f is the rate of flow f. We make the following assumption regarding the utility function.

Assumption: For each flow $f \in \mathcal{F}$, the function $U_f \colon \mathfrak{R}_+ \to \mathfrak{R}_+$ satisfies the following conditions:

(1) On the interval $I_f = [m_f, M_f]$, the utility functions U_f are increasing, strictly concave and twice continuously differentiable.

(2) U_f is additive so that the aggregated utility of rate allocation $\mathbf{x} = (x_f, f \in \mathcal{F})$ is $\sum_{f \in \mathcal{F}} U_f(x_f)$.

(3) The curvatures of U_f are bounded away from zero on I_f.

Following the work of resource allocation in wireline networks [1, 2], we investigate the optimal rate allocation in the sense of maximizing the aggregated utility function. As we will show in Sec. 4.2, by specifying appropriate utility functions, such an objective can enforce different fairness models, including proportional fairness and max-min fairness. We now formulate the problem of optimal resource allocation in wireless ad hoc networks as the following constrained non-linear optimization problem:

SYSTEM$(U, \mathcal{R}, \mathcal{C})$:

$$\textbf{maximize} \sum_{f \in F} U_f(x_f) \tag{5}$$

$$\textbf{subject to } \mathcal{R}\mathbf{x} \leq \mathcal{C} \tag{6}$$

$$\textbf{over} \quad \mathbf{x} \geq 0 \tag{7}$$

We observe that the objective function in Eq. (5) of the optimization problem is differentiable and strictly concave, while the feasible region of Eq. (6) and (7) is compact. By non-linear optimization theory, there exists a maximizing value of argument $\mathbf{x}$ for the above optimization problem, and we can apply the Lagrangian method to solve such a problem. Let us consider the Lagrangian form of this optimization problem:

$$
\begin{aligned}
L(\mathbf{x}, z; \mu) &= \sum_{f \in F} U_f(x_f) + \mu^T(\mathcal{C} - \mathcal{R}\mathbf{x} - \mathbf{z}) \\
&= \sum_{f \in F} \left(U_s(x_f) - x_f \sum_{q \in \mathcal{Q}} \mu_q \mathcal{R}_{qf} \right) + \sum_{q \in \mathcal{Q}} \mu_q \mathcal{C}_q - \sum_{q \in \mathcal{Q}} \mu_q z_q
\end{aligned}
\tag{8}
$$

where $\mu = (\mu_q, q \in \mathcal{Q})$ is a vector of Lagrange multipliers and $\mathbf{z} = (z_q, q \in \mathcal{Q})$ is a vector of slack variables. Hence, at a maximum of L over $\mathbf{x}, \mathbf{z} \geq 0$, the following conditions hold:

$$U'_f(x_f) = \sum_{q \in \mathcal{Q}} \mu_q \mathcal{R}_{qf}, \quad \text{if } x_f > 0$$

$$\leq \sum_{q \in \mathcal{Q}} \mu_q \mathcal{R}_{qf}, \quad \text{if } x_f = 0 \tag{9}$$

$$\sum_{q \in \mathcal{Q}} \mu_q = 0, \quad \text{if } z_q > 0$$

$$\geq 0, \quad \text{if } z_q = 0 \tag{10}$$

Thus, given that the system knows the utility functions U_f of all the flows, this optimization problem can be mathematically tractable through the above procedure. However, in practice, the system is not likely to know all the U_f, and it is also infeasible for an ad hoc network to compute and allocate resource in a centralized fashion.

In the Lagrange form shown in Eq. (8), the Lagrange multipliers μ_q can be regarded as the implied cost of unit flow accessing the channel within the maximal clique q. In other words, μ_q is the *shadow price* of clique q. Motivated by the concept of shadow price of cliques, we decompose the *SYSTEM* problem into the following two problems and seek a distributed solution where the ad hoc network does not need to know the utility functions of individual flows.

Suppose that each flow f is given the price per unit flow λ_f. Then f chooses an amount to pay per unit time w_f, and receives in return a rate x_f given by $x_f = \frac{w_f}{\lambda_f}$. The utility optimization problem for flow f becomes the following:

$FLOW_f(U_f; \lambda_f)$:

$$\textbf{maximize } U_f(\frac{w_f}{\lambda_f}) - w_f \tag{11}$$

$$\textbf{over} \quad w_f \geq 0 \tag{12}$$

The network, given the amounts that the flows are willing to pay, $w = (w_f, f \in \mathcal{F})$, attempts to maximize the function $\sum_{f \in \mathcal{F}} w_f \log(x_f)$. So the network's optimization problem can be formulated as follows:

$NETWORK(\mathcal{R}, \mathcal{C}; w)$:

$$\textbf{maximize } \sum_{f \in \mathcal{F}} w_f \log(x_f) \tag{13}$$

$$\textbf{subject to } \mathcal{R}\mathbf{x} \leq \mathcal{C} \tag{14}$$

$$\textbf{over} \quad \mathbf{x} \geq 0 \tag{15}$$

Note that in the *NETWORK* problem, the utility functions are not required to carry out the resource allocation computation. Similar to [3], we have the following results.

Theorem 2: There exist vectors $\lambda = (\lambda_f, f \in \mathcal{F})$, $w = (w_f, f \in \mathcal{F})$ and $\mathbf{x} = (x_f, f \in \mathcal{F})$ such that

(1) w_f solves $FLOW_f(U_f; \lambda_f)$, for $f \in \mathcal{F}$; (2) $\mathbf{x}$ solves $NETWORK(\mathcal{R}, \mathcal{C}; w)$; (3) $w_f = \lambda_f x_f$.

We omit the proofs due to space constraints. Refer to [13] for a detailed proof.

Corollary 1. The price λ_f of a flow f and the shadow price of channel clique μ_q satisfy the following relation:

$$\lambda_f = \sum_{q:f \cap q \neq \emptyset} \mu_q \mathcal{R}_{qf} \tag{16}$$

$$= \sum_{l:l \in f} \sum_{q:l \in q} \mu_q \tag{17}$$

This relation shows that the flow f needs to pay for *all the maximal cliques* that it traverses. For each clique, the price to pay is the product of the number of wireless links that f traverses in this clique and the shadow price of this clique as in Eq. (16). Alternatively, we can also explain the pricing model in the following way: (1) the price of a flow is *the aggregated price of its subflows*; (2) the price of a subflow is *the aggregated price of all the maximal cliques* that the subflow belongs to as in Eq. (17).
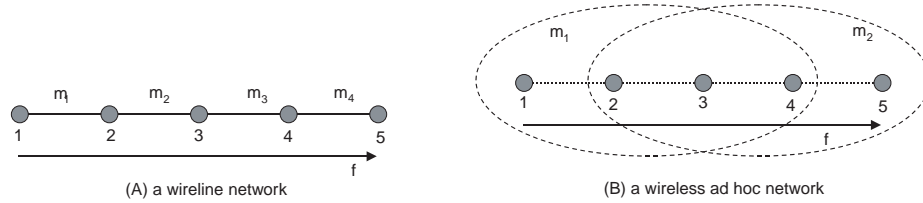

Fig. 3. Wireline and wireless ad hoc networks: a comparison.

We now show an example to illustrate the difference between the proposed pricing model for wireless ad hoc networks and the existing model in wireline networks. In Fig. 3(A), the wireline network has a chain topology consisting of four links, which are associated with prices μ_i, $i = 1, \ldots, 4$. The price of flow f in this wireline network is $\lambda_f = \sum_{i=1}^{4} \mu_i$. On the other hand, although the wireless ad hoc network in Fig. 3(B) has the same topology as the wireline network, its resource is no longer "links". Instead, two maximal cliques $q_1 = \{l_{12}, l_{23}, l_{34}\}$ and $q_2 = \{l_{23}, l_{34}, l_{45}\}$ constitute the resources. Suppose that the shadow price of these two cliques are μ_1 and μ_2, respectively. Then the price of flow f in this ad hoc network is given by $\lambda_f = 3\mu_1 + 3\mu_2$, which is the number of subflows of f in this clique multiplied by the shadow price of this clique. Alternatively, the price can be written as $\lambda_f = \mu_1 + (\mu_1 + \mu_2) + (\mu_1 + \mu_2) + \mu_2$, which is the sum of its subflows' prices, with each subflow paying the aggregated price of all the maximal cliques that it belongs to.

4.2 Achieving fairness among multi-hop flows

By choosing different utility functions for flows, the optimal resource allocation can enforce different fairness models among multi-hop flows in wireless ad hoc networks.

Definition 2 (proportional fairness). In a wireless ad hoc network, a vector of rates $\mathbf{x}^* = (x_f^*, f \in \mathcal{F})$ is *weighted proportionally fair* with weight vector w_f, if it is feasible, *i.e.*, $x_f^* \geq 0$ and $\mathcal{R}\mathbf{x}^* \leq \mathcal{C}$, and for any other feasible vector $\mathbf{x} = (x_f, f \in \mathcal{F})$, the aggregate of proportional change is zero or negative:

$$w_f \sum_{f \in \mathcal{F}} \frac{x_f - x_f^*}{x_f^*} \leq 0 \tag{18}$$

Proposition 1. A rate allocation $\mathbf{x}$ is weighted proportional fair with weight vector w_f in a wireless ad hoc network, if and only if it solves $SYSTEM(U, \mathcal{R}, \mathcal{C})$, with $U_f(x_f) = w_f \log x_f$ for $f \in \mathcal{F}$.

Definition 3 (max-min fairness). In a wireless ad hoc network, a vector of rates $\mathbf{x}^* = (x_f^*, f \in \mathcal{F})$ is *max-min fair*, if it is feasible, *i.e.*, $\mathbf{x}^* \geq 0$ and $\mathcal{R}\mathbf{x}^* \leq \mathcal{C}$, and if for any $f \in \mathcal{F}$, increasing x_f^* can not be done without decreasing the fair share $x_{f'}^*$ of another flow $f' \in \mathcal{F}$ which satisfies $x_f^* \geq x_{f'}^*$.

Proposition 2. A rate allocation $\mathbf{x}$ is max-min fair if and only if it solves $SYSTEM(U, \mathcal{R}, \mathcal{C})$, with $U_f(x_f) = -(-\log x_f)^\alpha$, $\alpha \to \infty$ for $f \in \mathcal{F}$.

These results straightforwardly follow its counterpart in wireline networks [3]. We omit the proofs due to space constraints. Refer to [13] for a detailed proof.

5 Algorithm: Pricing and Resource Allocation

The decomposition of $SYSTEM(U, \mathcal{R}, C)$ problem into $FLOW_f(U_f; \lambda_f)$ and $NETWORK(\mathcal{R}, \mathcal{C}; w)$ problems suggests that solving $SYSTEM(U, \mathcal{R}, C)$ can be achieved by solving $FLOW_f(U_f; \lambda_f)$ and $NETWORK(\mathcal{R}, \mathcal{C}; w)$ problems via an iterative algorithm. In each iteration, the source node of flow f individually solves its willingness to pay w_f in Eq. (11), adjusts its sending rate and notifies the network about this change. After the new sending rate is observed by the clique q, it updates its price μ_q accordingly and communicates the new prices to the flow, and the cycle repeats.

To illustrate how flow f adjusts its willingness to pay, we define the demand function $D_f: \Re_+ \to \Re_+$ of flow f as follows. $D_f(\lambda_f)$ is the optimal solution w_f^* to the $FLOW(U_f; \lambda_f)$ problem. That is,

$$D_f(\lambda_f) = w_f^* = \arg \max_{w_f \geq 0} \{U_f(\frac{w_f}{\lambda_f}) - w_f\} \tag{19}$$

The iterative algorithm that computes the price and resource allocation is then given as follows.

Algorithm I (First Tier) – Per-clique price calculation and resource allocation

– *Clique q's algorithm at iteration k*
  (1) receives sending rate $x_f(k)$ from all flows f that go through clique q;
  (2) computes a new price according to the following formula

$$\mu_q(k+1) = [\mu_q(k) + \alpha(\sum_{f: f \cap q \neq \emptyset} \mathcal{R}_{qf} x_f(k) - C)]^+ \tag{20}$$

where $\alpha > 0$ is a small step size parameter, and $[z]^+ = \max\{z, 0\}$. This algorithm is consistent with the law of supply and demand: if the demand for bandwidth at clique q exceeds the channel capacity supply C, which is the channel capacity, then the price μ_q is raised; otherwise, the price is reduced;

(3) communicates new price $\mu_q(k + 1)$ to all flows f that go through clique q.

– *Flow f's algorithm at iteration k*

(1) receives from the network the path price $\lambda_f(k)$ along its path, where

$$\lambda_f(k) = \sum_{q:f\cap q\neq\emptyset} \mu_q(k)\mathcal{R}_{qf} \tag{21}$$

(2) chooses a new willingness to pay $w_f(k + 1)$ to maximize its net benefit under the price $\lambda_f(k)$ according to

$$w_f(k + 1) = D_f(\lambda_f(k)) \tag{22}$$

(3) calculates rate $x_f(k + 1)$ according to the following formula and transmits at rate $x_f(k + 1)$, where

$$x_f(k + 1) = \frac{w_f(k + 1)}{\lambda_f(k)}. \tag{23}$$

We now show the convergence and the optimality of the above iterative algorithm through the following theorem.

Theorem 3 (Convergence). Provided that the stepsize α is sufficiently small, then Algorithm I has a unique equilibrium point $\mathbf{x}^*$, to which all trajectories converge.

Theorem 4 (Optimality). The equilibrium point $\mathbf{x}^*$ of Algorithm I is the optimal solution to the *SYSTEM* problem.

We omit the proofs due to space constraints. Refer to [13] for a detailed proof.

In the above iterative algorithm, a maximal clique is regarded as a network element that can carry out certain network functions. In particular, it assumes that a maximal clique q can perform the following tasks for price calculation and resource allocation: (1) obtain the aggregated bandwidth demand $\sum_{f:f\cap q\neq\emptyset} x_f\mathcal{R}_{qf}$ within the clique q; (2) calculate the per-clique shadow price μ_q; and (3) notify the price μ_q to the flows that pass through it.

However, a maximal clique is only a concept defined based on wireless link contention graph. To deploy the algorithm in an actual ad hoc network, the above tasks of a maximal clique need to be carried out by the nodes (hosts) that constitute the clique in a distributed fashion. To achieve this goal, the very first questions that need to be addressed are: (1) Which nodes constitute a maximal clique; (2) How many maximal cliques a node belongs to. The answer to these questions require an algorithm to find all maximal cliques in a graph. Although several such algorithms have been proposed in the existing literatures of graph theory [14], they can not be applied directly to our situation. This is because that these algorithms are all centralized and have high computational complexity, while the ad hoc network requires a full distributed solution.

Nevertheless, the unique graphical properties of the wireless link contention graph may have the potential to facilitate clique calculations. We proceed to propose a distributed maximal clique construction algorithm, in which the entire topology is decomposed into subgraphs. Maximal cliques can then be constructed only based on local

topology information within each subgraph. Our algorithm can significantly reduce the communication and computational overhead. Using this algorithm, each node computes the maximal cliques it belongs to and carries out part of the tasks of these cliques. In particular, the tasks of maximal cliques are distributed among nodes in the following way.

- All nodes need to provide their local connectivity information to their neighborhood nodes.
- If a node i is sending packets to node j, then for all such j, i needs to
  (1) calculate *all cliques* q, which include link $\{i, j\}$;
  (2) collect rate information and calculate the price μ_q for all cliques q;
  (3) notify all the passing flows (for which node i forwards packets) of the updated price.

For example, in Fig. 1, node 2 needs to calculate all the cliques that contain link $\{2, 3\}$; node 4 needs to calculate all the cliques that contain link $\{4, 5\}$ and all the cliques that contain link $\{4, 7\}$, respectively.

First, we study the problem of how much information node i needs to know in order to construct *all maximal cliques* that contains wireless link $\{i, j\}$. To do that, we introduce the following definitions and denotations.

Definition 4 (neighbor sets).
(1) A neighbor set of link l, $N(l)$ is defined as $N(l) = \{l' | l \cap l' \neq \emptyset\}$;
(2) A neighbor set of link set L, $N(L)$, is defined as $N(L) = \cup_{l \in L} N(l)$;
(3) A k neighbor set of link l, denoted as $N^k(l)$, is defined by induction: (i) $N^1(l) = N(l)$, and (ii) $N^k(l) = N(N^{k-1}(l))$ for $k > 1$.

Lemma 2. Any clique that contains link l, denoted as $q_{<l>}$, satisfies condition $q_{<l>} \subseteq \mathcal{G}(N^2(l))$, where $\mathcal{G}(N^2(l))$ is a spanning subgraph of G_T whose edge set is $N^2(l)$.

By definitions of the *wireless link contention graph* and *clique*, Lemma 2 can be derived. This lemma shows that $\mathcal{G}(N^2(l))$ contains all the links to construct all maximal cliques $q_{<l>}$. As an example, in Fig. 1, $N^2(\{2, 3\}) = \{\{1, 2\}, \{2, 6\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{4, 7\}\}$, which consists of all the links that are required to construct maximal cliques $q_{<2,3>}$.

In order to establish such a view of $\mathcal{G}(N^2(l))$ at node i, which sends packets along l, each node in the network distributes its connectivity and the traffic information to two hops away. Based on these information, node i constructs its local view of the network and uses the Bierstone algorithm [14] to find all maximal cliques that contain link $\{i, j\}$.

The iterative algorithm also requires communication between flows and the network. This is implemented via inbound signaling between the source nodes (in the role of flows) and the relaying nodes (in the role of network). In particular, there are two special fields in the packet header of each data packet. One field carries the sending rate of this flow; the other field carries the aggregated price of this flow along its path. The source of a flow fills in its sending rate x_f in the first field so that all the relaying nodes of flow f can distribute this rate to facilitate the price calculation. Similarly, each relaying node of f will fill in the price field of f's data packets. When the packets arrive at the destination, the destination node sends an acknowledgment packet to the source node to notify it about the possible price changes.

To summarize, each node i in an ad hoc network performs the following task locally in a cooperative manner to support global optimal resource allocation.

Algorithm II (Second Tier) - Per-node price calculation and resource allocation

– *Every node*. Every node i in the network sends its connectivity information $N(i) = \{j|\{i,j\} \in \mathcal{E}_T\}$ to two hops away.
– *Relaying node*. If node i transmits to node j, relaying packets for end-to-end flows $f_1, f_2, \ldots, f_m$, which means flow f_k, $k = 1, 2, \ldots, m$ is passing wireless link $\{i, j\}$, then i performs the following operations:
(1) retrieves the sending rates x_k of flows f_k, $k = 1, 2, ..., m$, from their packet headers, calculates the aggregated rate $y_{ij} = \sum_{k=1}^{m} x_k$ and distributes the aggregated rate information;
(2) collects connectivity information and constructs $G(N^2(\{i, j\}))$;
(3) collects aggregated rate information $y_{l'}$, where $l' \in N^2(\{i, j\})$;
(4) calculates all maximal cliques q, and their prices μ_q according to the iterative algorithm Eq. (20). Note that $\sum_{f:f \cap q \neq \emptyset} \mathcal{R}_{qf} x_f = \sum_{l' \in q} y_{l'}$.
(5) updates the aggregated price $\lambda_f = \lambda_f + \lambda_{\{i,j\}}$, where $\lambda_{\{i,j\}} = \sum_{q:\{i,j\} \in q} \mu_q$, in the packet header for all flows f_k;
– *Destination node*. If node i is a destination node of an end-to-end flow f, then it observes the change of the aggregated price λ_f. If there is a change in λ_f, i sends a packet to the source to explicitly notify it about the change.
– *Source node*. If node i is a source node of an end-to-end flow f, then it
(1) receives price update packets from the destination and retrieves path aggregated price λ_f from the packet.
(2) calculates x_f according to Eq. (23);
(3) updates its sending rate as x_f, and insert x_f into its packet header.

6 Simulation Results

In this section, we validate the performance of our price-based resource allocation algorithm through a detailed numerical study. In our simulation, the channel capacity of wireless networks and the bandwidth of wireline networks are both 1 Mbps. The utility function in use is $U_f(x_f) = \log(x_f)$, which will enforce the proportional fair resource allocation in both wireline and wireless ad hoc networks.

6.1 Resource allocation in wireline and wireless networks: a comparison

We first compare the effects of resource allocations in wireless ad hoc networks with wireline networks, using identical topologies. We consider a simple example. The network topology is shown in Fig. 1(A), where the network is shared by three flows. The resulting rate allocation of two types of networks is listed in Table 1. The corresponding price vectors when the systems are converged are shown in Table 2.

From these results, we have the following observations: (1) In both networks, the price μ_j, at the bottleneck resource j (which is the clique q_2 in the ad hoc network, and the wireline link $\{2, 3\}$ in the wireline network) is larger than 0. In comparison,

	x_1	x_2	x_3
wireline network (Mbps)	0.5	0.5	1
wireless ad hoc network (Mbps)	0.111	0.333	0.333

Table 1. Rate allocation in the example topology shown in Fig. 1

|	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7
wireline network	0	2	0	0	0	0	0
wireless ad hoc network	0	3	0	N/A	N/A	N/A	N/A

Table 2. Equilibrium prices in the example topology shown in Fig. 1

at non-bottleneck resources, they are equal to 0. This shows the role of shadow price to arbitrate resource allocation. When the demand (sending rate) exceeds the supply (channel capacity), which happens in the bottleneck resource, the shadow price, which reflects the marginal cost, is increased. When the demand is below supply, there is no such marginal cost, thus the price equals to 0. (2) Due to the contention in the spatial domain for the shared wireless medium, the rate allocation in the ad hoc network for each flow is less than the rate allocation for the corresponding flow in wireline networks.

We then proceed to show a more detailed comparison between the two types of networks using chain topologies, on which flows with different numbers of hops are sharing the resources. As shown in Fig. 4(A), a chain topology is shared among 5 flows. Four of them are single-hop flows, one of them is a flow with 4 hops. The rate allocation and the equilibrium prices for this scenario are given in Table 3 and Table 4. We also conduct simulations in a chain topology with 5 hops as shown in Fig. 4(B) for comparison purposes. The results are shown in Table 5 and Table 6.
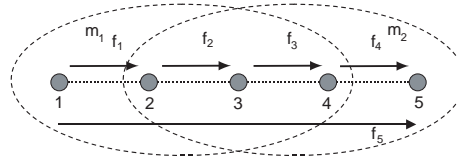
(A) 4-hop chain topology
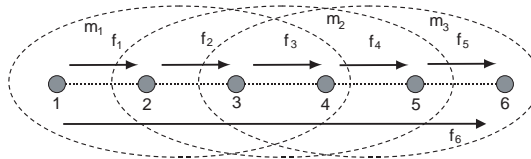
(B) 5-hop chain topology

Fig. 4. Two example chain topologies with 4 hops and 5 hops

	x_1	x_2	x_3	x_4	x_5
wireline network (Mbps)	0.8	0.8	0.8	0.8	0.2
wireless ad hoc network (Mbps)	0.4	0.2	0.2	0.4	0.067

Table 3. Rate allocation in the 4-hop chain topology shown in Fig. 4(A)

	μ_1	μ_2	μ_3	μ_4
wireline network	1.25	1.25	1.25	1.25
wireless ad hoc network	2.5	2.5	N/A	N/A

Table 4. Equilibrium prices in the 4-hop chain topology shown in Fig. 4(A)

	x_1	x_2	x_3	x_4	x_5	x_6
wireline network (Mbps)	0.833	0.833	0.833	0.833	0.833	0.167
wireless ad hoc network (Mbps)	0.333	0.333	0.167	0.333	0.333	0.056

Table 5. Rate allocation in the 5-hop chain topology shown in Fig. 4(B)

From the above results, we have the following observations.

In the wireline network, the rates of all single hop flows are the same, *e.g.*, in the 4-hop chain topology, $x_1 = x_2 = x_3 = x_4 = 0.8$. In wireless ad hoc networks, however, the rates of these flows are different. For example, in the 4-hop chain topology, $x_1 \neq x_2$. This is because, in wireline network flow $f_1 - f_4$ enjoy the same amount of resource in the network, while in the wireless ad hoc network, due to location-dependent contention, f_2 suffers higher contention than f_1. Alternatively, we can explain such resource allocation through the price that they need to pay. For f_1, the price is $\lambda_1 = \mu_1$, which is equal to 2.5 at equilibrium; while the price for f_2 is $\lambda_2 = \mu_1 + \mu_2$, which is equal to 5.0.

From the results in the 5-hop topology example, we can show another interesting observation in the ad hoc network scenario. In Table 5, we observe that $x_1 = x_2$, although it seems that these two flows suffer different degrees of contention. This result can be explained by the price of this flow, where $\lambda_1 = \mu_1 = 3$, and $\lambda_2 = \mu_1 + \mu_2 = 3$. Because the prices they need to pay are the same and they have the same utility function, they reach the same bandwidth allocation.

The reason why μ_2 is 0 in Table 6 can be intuitively explained as follows. The three cliques q_1, q_2 and q_3 have the same traffic demand. When q_1 and q_3 are appropriately priced, sufficient price has been charged on the network to arbitrate the resource usage, thus no price is needed for clique q_2. On the other hand, if only q_2 is priced, then a price is still required to regulate the channel access at wireless link $\{1, 2\}$ and $\{5, 6\}$.

	μ_1	μ_2	μ_3	μ_4	μ_5
wireline network	1.2	1.2	1.2	1.2	1.2
wireless ad hoc network	3.0	0.0	3.0	N/A	N/A

Table 6. Equilibrium prices in the 5-hop chain topology shown in Fig. 4(B)

Moreover, these two prices also regulate the subflows within q_2, causing a decrease of μ_2. Eventually at equilibrium, $\mu_2 = 0$.

When we compare the rate allocation of the long flow which has a large number of hops and the rate allocation with short flows, we notice that the long flow is penalized compared to the short flows. This observation is consistent with the previously presented observations in wireline networks under proportional fairness model [4]. However, the degree of penalties is different in these two networks. In the wireline network, the ratio of rate allocation for short flows and long flows are equal, *e.g.*, in the 4-hop topology, $\frac{x_1}{x_5} = \frac{x_2}{x_5} = \frac{x_3}{x_5} = \frac{x_4}{x_5} = 4$. In the ad hoc network with the 4-hop topology, such ratio depends on the location of the flows. For example, in the ad hoc network $\frac{x_1}{x_5} = 6$, $\frac{x_2}{x_5} = 3$.

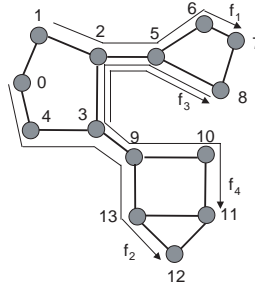### 6.2 Convergence of the iterative algorithm

Fig. 5. A random topology

We now show the convergence behavior of our proposed algorithm using three example networks. They are the simple network in Fig. 1(A), the chain topology network shown in Fig. 4(B) and a random generated network shown in Fig. 5. For each example, we have plotted the time-varying values of the transmission rate of each flow and shadow price at each clique in Fig. 6, Fig. 7 and Fig. 8.

We note that the system converges to an equilibrium rate allocation and an equilibrium price vector within 800 iterations. This result is comparable to other relay-based pricing scheme in ad hoc networks [8]. In a mobile environment, such a convergence time will be orders of magnitude lower than the topology changing time.
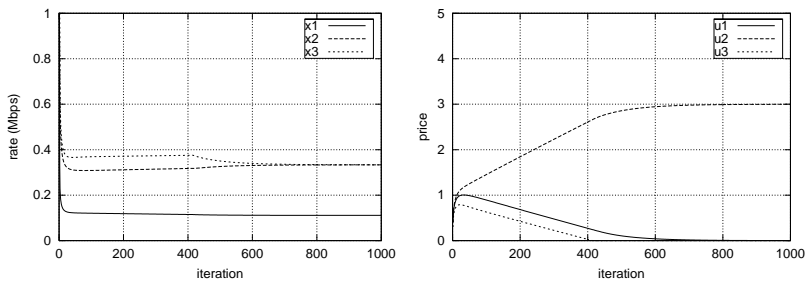
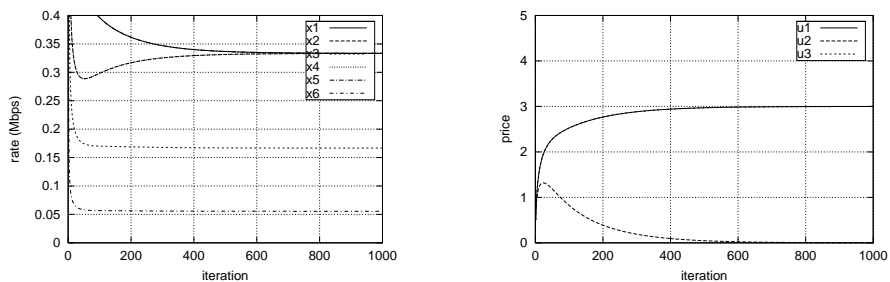**Fig. 6.** Trajectories of the transmission rate and price of the network in Fig. 1(A)

Fig. 7. Trajectories of transmission rate and price of the network in Fig. 4(B)

7 Concluding Remarks

In this paper, we have presented a novel pricing-based resource allocation algorithm based on an analytical pricing model that is specifically designed for the unique characteristics of multi-hop wireless ad hoc networks. The original contribution incorporated in the pricing model is the association of shadow prices with the *maximal cliques* in the contention graph model, rather than with individual links as in wireline networks. Based on insights brought forth by such strategies, the algorithms proposed are fully distributed, and arbitrate the contention among end-to-end *multi-hop* flows with respect to fair resource allocation. The frequently used fairness constraints, such as weighted proportional or max-min fairness, may be straightforwardly supported by assigning their corresponding utility functions in the pricing model. The validity of our claims is supported by both theoretical studies and extensive simulation results. To the best of our knowledge, there does not exist any previous work that addresses the problem of enforcing fairness among multi-hop flows, especially when a pricie-based approach is utilized to design fully distributed algorithms to achieve this goal.
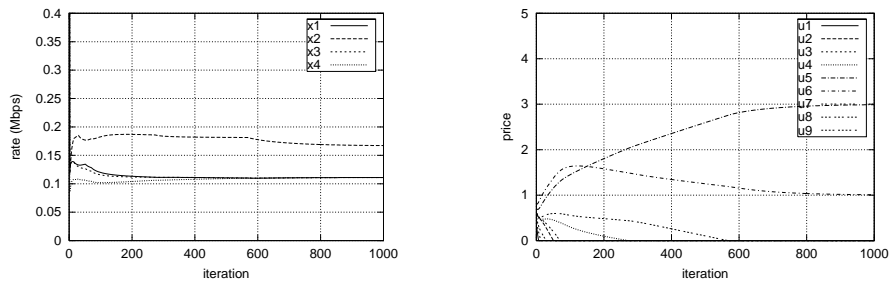
8 Acknowledgments

Fig. 8. Trajectories of transmission rate and price of the network in Fig. 5

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