Cooperative Resource Management in Cognitive WiMAX with Femto Cells

Jin Jin, Baochun Li
Department of Electrical and Computer Engineering
University of Toronto
{jinjin, bli}@eecg.toronto.edu

***Abstract*—WiMAX with femto cells is a cost-effective next-generation broadband wireless communication system. Cognitive Radio (CR) has recently emerged as a promising technology to improve spectrum utilization by allowing dynamic spectrum access. There will be large potential benefits by applying the CR technique to WiMAX with femto cells, which are barely explored in the literature.**

In this paper, we propose a novel *cognitive* WiMAX architecture with femto cells, where the base station and users are equipped with CRs and intelligently adjusts power, channel, and other resources to accommodate the entire network ecosystem. In this new design, we develop an optimization framework for *location-aware cooperative* resource management, by jointly employing multi-hop cooperative communication, power control, channel assignment, primary user protection, buffer management, and fairness, and incorporating user, channel, and cooperative diversities. To achieve optimality, it is designed based on stochastic Lyapunov optimization, aiming to take advantage of the radio flexibility and fully utilize the spectrum. Evaluated by the rigorous analysis and extensive simulations, our resource management protocol is near-optimal with closed-form bounds, with which cognitive WiMAX achieves substantial performance improvement.

I. Introduction

WiMAX is an emerging technology to facilitate broadband wireless mobile access in metropolitan area [1], and has been commonly referred as 4G. In WiMAX, femto cells are a cost-effective means of providing ubiquitous connectivity. Users that reside in femto cells experience increased throughput due to the shorter ranges. Fig. 1 shows a typical WiMAX network consisting of one macro base station (BS) and six femto cells, serving two classes of users: primary user (PU) and secondary user (SU). PUs communicate with the corresponding femto BSs with dedicated channels, enjoying guaranteed quality of services (QoS). SUs are highly dynamic and communicate directly with macro BS with best effort services.

As the power used by femto BSs is an order of magnitude less than macro BS, the serving area of each femto cell is quite limited (shown by shadow circle areas). The smaller size of femto cells creates abundant opportunities for spatial reuse: the transmissions outside the femto cells are able to be executed over the same channels used inside femto cells. Thus, they work in a completely distributed fashion, and the channel availability in the network is *location-dependent* and *dynamic* for SUs due to the bursty channel use by PUs and SU mobility.

This work is supported by NSERC Discovery, CRD and Strategic Grants (RGPIN 238994-06, CRDPJ 379623-08, STPGP 364910-08).

However, traditional WiMAX architectures and MAC-layer protocols are hobbled by the holdover from cellular networks: they lack dynamic utilization of spectrum holes and are essentially based on *single-hop* transmissions, requiring *globally* available channel resources. The existing state-of-the-art resource management protocols have to carefully coordinate the transmissions of macro and femto cells in a time-sharing mode [2], which have inherent weakness on overlooking the special network characteristics and hence missing the bulk of channel reuse opportunities. For example shown in Fig. 1, channel 1 is used by PU 1. Macro BS then can not use this channel to transmit data to SU 2 in order to avoid interference to PU 1, although SU 2 resides outside the interference region of PU 1. This is due to the single-hop transmission schedule with a fixed power, leading to resource under-utilization.

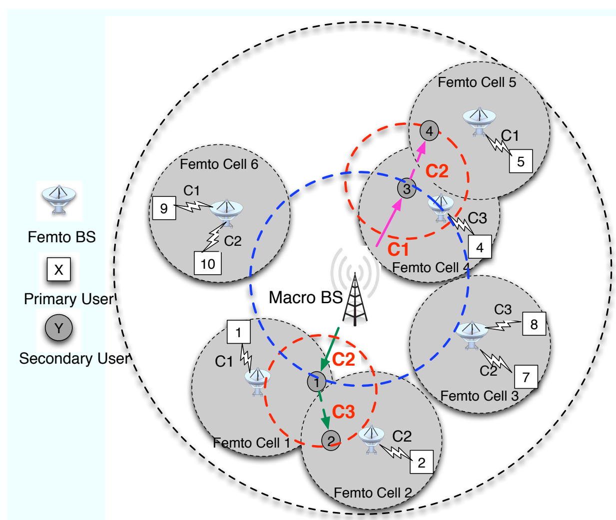

Fig. 1. An illustrative example of cognitive WiMAX with femto cells.

Therefore, there is a compelling need to re-design the resource management scheme in WiMAX with femto cells in order to tightly integrate with the network architecture and fully utilize the spectrum. Cognitive radio (CR) [3] has emerged as an important technology to exploit high-degree spectrum reuse, by allowing spectrum sensing and dynamical spectrum access. Such a technique brings much flexibility and potentially generates benefits if employed in WiMAX femto cell networks, especially with the proliferation of powerful cognitive wireless devices as well as the surge of demand on service varieties and qualities in WiMAX. However, the collaboration of WiMAX, femto cells, and CRs is barely investigated in the literature.

In this paper, we propose a novel architecture of *cognitive* WiMAX with femto cells. Different from traditional architectures, wireless devices in cognitive WiMAX are equipped with frequency-agile CRs that bring convenience for spectrum sensing and adjusting the frequency, power, range, and other variables to accommodate the entire wireless ecosystem accordingly. With such a flexible radio, we further design a novel resource management framework to optimize the performance by efficiently utilizing the scarce network resources. Rather than confining to a single-hop transmission, we advocate to perform *multi-hop* cooperative communication, aiming to fully exploit the spectrum holes. The key observation is that, the set of accessible channels for different users are different depending on their locations. With CRs and our specially designed resource management scheme, the requirement for *globally* available channels is relaxed, and users use *locally* accessible channels within one hop to perform communication, providing abundant transmission opportunities with channel reuse, and thus substantially improving the channel utilization.

Such an architecture naturally fits WiMAX femto cell networks: PUs can use dedicated resources to enjoy guaranteed QoS, while SUs opportunistically take advantage of spectrum holes to get best effort services without generating interference to PUs. The intuition is shown in Fig. 1, where the transmission from macro BS to SU 2 is not feasible if all channels are occupied in femto cells. With CRs, the macro BS carefully adjusts its transmission power. In the consequentially tuned transmission range (the inner circle from macro BS in Fig. 1), the macro BS sends data to SU 1 via channel 2 without generating interference to PUs. In tandem, SU 1 relays data to SU 2 using cooperative transmissions performed on channel 3 which is commonly available for both the sender and receiver. Similarly, the original infeasible communication from the macro BS to SU 4 can be performed in two-hop transmission with SU 3 as the relay. Essentially, we take advantage of the *location-dependent* characteristics of WiMAX femto cell, and data issued from BS are propagated via multiple paths and hops over spectrum holes supported by CRs. Intuitively, wireless channels are effectively utilized by incorporating user, channel, and cooperative diversities.

The salient highlight of our design on cognitive WiMAX and resource management is to jointly consider power control, flow routing, cooperative scheduling, interference avoidance, and buffer management. Our contributions are three fold:

- ▷ We advocate a cognitive WiMAX architecture with femto cells, and provide the corresponding system models.
- ▷ We design a location-aware cooperative resource management protocol, including flow control policy, buffer management strategy, and channel and power allocation scheme. It is based on stochastic Lyapunov optimization with performance guarantees.
- ▷ We apply generalized expectation maximization algorithm to efficiently solve the optimization problems required in the resource management protocol, by fully exploiting the unique problem structure and network characteristics.

To our knowledge, it is the first work studying cognitive WiMAX with femto cells and resource management problems. Our proposed protocol is analyzed theoretically and evaluated via simulations. Corroborating our intuition, system performance is substantially improved with our design.

The remainder of the paper is organized as follows. In Sec. II, we present the design of the cognitive WiMAX with femto cells and the corresponding network models. In Sec. III, we describe the design of our resource management protocol, and analyze its performance. In Sec. IV, the generalized EM algorithm to solve the optimization problems is provided and discussed. We conduct simulations to evaluate the performance in Sec. V. In Sec. VI, we review the related work. We finally conclude our paper in Sec. VII.

II. System Models

A. Architecture of Cognitive WiMAX with Femto Cells

In cognitive WiMAX with femto cells, both macro BS and SUs are equipped with ultra-sensitive cognitive radios to perform spectrum sensing and power and frequency adjustment. The network consists of one macro BS and F femto cells with A PUs and N SUs, sharing C orthogonal channels supported by OFDMA. Each PU resides in a dedicated femto cell and communicates with the corresponding femto BS over one pre-allocated channel to support guaranteed QoS. SUs are fully mobile and served opportunistically by the macro BS without generating interference to PUs. The entire network operates in a time-slotted fashion, where channel conditions and user actions remain the same during a given time slot, and vary independently from one time slot to another. Without loss of generality, we set the time slot duration as 1.

Let $\mathbf{S(t)} = \{S_a^c(t)\}_{A\times C}$ represent the channel states on each time slot t. $S_a^c(t) = 0$ means PU a is using channel c. Otherwise, $S_a^c(t) = 1$. We assume the channel availability state process evolves according to a finite state ergodic Markov chain. Within a time slot, a SU can access a subset of the channels, potentially depending on its current location and channel state $\mathbf{S(t)}$. This channel accessibility information is concisely represented by $\mathbf{H(t)} = \{h_n^c(t)\}_{N\times C}$. $h_n^c(t) = 1$ if SU n can access channel c. Otherwise, $h_n^c(t) = 0$.

Macro BS obtains the channel availability information in the entire network via channel sensing with CRs, and the channel state information can be expressed by a probability vector $\mathbf{Y(t)} = \{Y_a^c(t)\}_{A\times C}$ according to the sensing results. Each element captures the probability that channel c is *not* occupied by PU a at time slot t. Intuitively, the closer $\mathbf{Y(t)}$ is to $\mathbf{S(t)}$ (better sensing techniques employed), the smaller interference that can be potentially generated to PUs.

CRs make it possible for the macro BS and SUs to adaptively use network resources. We denote the macro BS's transmission power on each channel as $\mathbf{P_{BS}(t)} = \{P_{BS}^c(t)\}_C$. $\mathbf{U_{BS}(t)} = \{\mu_n^c(t)\}_{N\times C}$ represents the channel allocation to SUs in a macro cell, where $\mu_n(t)$ is the binary variable capturing the assignment of channel c to SU n for the transmission from macro BS. We denote SU power allocation using $\mathbf{P_{SU}(t)} =$

$\{P_n^c(t)\}_{N\times C}$, where $P_n^c(t)$ is the amount of power that SU n uses on channel c. The cooperative transmission scheduling is described by $\mathbf{U_{SU}(t)} = \{\mu_{mn}^c(t)\}_{NN\times C}$, each element of which is a $0-1$ variable to capture the allocation of channel c to the cooperative transmission from SU m to SU n.

B. Models of Resource Management

Spectrum and power resources can be finely tuned and dynamically allocated to macro and cooperative transmissions, in order to fully utilize the spectrum and take advantage of channel reuse and diversity. According to the network model given above, we have the following four groups of constraints for resource management.

Power Constraints:

$$\sum_{c=1}^{C} P_{BS}^c(t) \le P^{max} \tag{1}$$

$$\sum_{c=1}^{C} P_n^c(t) \le P_n^{max} \qquad \forall n \tag{2}$$

$$P_{BS}^c(t)\cdot g_a^c \cdot S_a^c(t) \le \beta \qquad \forall a, c \tag{3}$$

Inequality (1) shows that the total transmission power of the macro BS has an upper bound P^{max}. The power constraints on SUs are represented by (2). To avoid interference to PUs, the power received by PUs on each channel should not exceed the tolerable level β, if the corresponding channel is being used. Inequality (3) describes this set of constraints, where g_a^c is the propagation gain from macro BS to PU a at channel c and it can be calculated by $g_a^c = d_a^{-j}$. $d_a \ge 1$ is the distance between BS and PU a, where j is the path loss index [4].

Channel Constraints:

$$0 \le \sum_{n=1}^{N} \mu_n^c(t) \le 1 \qquad \forall c \tag{4}$$

$$\mu_{mn}^c(t) \le h_m^c(t), \mu_{mn}^c(t) \le h_n^c(t) \qquad \forall m,n,c \tag{5}$$

$$\mu_{mn}^c(t) \le l_m^c(t), \mu_{mn}^c(t) \le l_n^c(t) \qquad \forall m,n,c \tag{6}$$

Inequality (4) indicates that the macro BS can not use the same channel to transmit data to multiple SUs. (5) shows that cooperative communication is constrained by the channel accessibility represented by $\mathbf{H(t)}$ with the dynamic spectrum access technique. (6) shows the constraint imposed by the channel availability on each SU with regards to the transmission from macro BS to SUs. Similar to $\mathbf{H(t)}$, we use $\mathbf{L(t)} = \{l_n^c(t)\}_{N\times C}$ to capture this information:

$$l_n^c(t) = \begin{cases} 1 & \text{If } P_{BS}^c(t)\cdot g_n^c(t) \le \gamma \\ 0 & \text{Otherwise} \end{cases} \tag{7}$$

The definition of $\mathbf{L(t)}$ indicates whenever the multicast power received by a SU exceeds the threshold γ on a channel, this channel should be considered to be unavailable, and can not be assigned for cooperative transmission. $g_n^c(t)$ in (7) is the propagation gain from BS to SU n on channel c at time t.

Cooperative Constraints:

$$0 \le \sum_{m=1}^{N} \mu_{mn}^c(t) \le 1 \qquad \forall c, n \tag{8}$$

$$0 \le \mu_n^c(t) + \sum_{m=1}^{N} \mu_{nm}^c(t) \le 1 \qquad \forall c, n \tag{9}$$

$$0 \le \sum_{c=1}^{C} \mu_{mn}^c(t) + \sum_{m'=1}^{N} \mu_{nm'}^c(t) \le 1 \qquad \forall n, m \tag{10}$$

Inequality (8) shows that one SU can not be helped by multiple SUs via the same channel. (9) indicates the incoming and outgoing transmissions on each SU can not be performed on the same channel. With respect to the multi-hop mode of transmission, we constrain it in two hops described by (10). Cooperative communication is performed concurrently via multiple channels, which is supported by multiple radios equipped on SUs. Only a small number of radios are requires which is practically feasible, as the distribution of SUs is sparse and dynamic, and the probability that multiple SUs are within the interference region of each other is very low.

Flow Constraints:

With relays enabled, we perform *multi-path* transmission with perfect flow splitting at the relays, due to its ability for load balancing and flexibility. Denote $f_n^c[m](t)$ as the flow rate that the macro BS transmits to SU n over channel c with the data destined for SU m in time slot t, which means SU n should relay the data to SU m. If $m = n$, SU n gets its own data. Similarly, let $f_{mn}^c(t)$ be the flow rate of the cooperative transmission from SU m to SU n at channel c in time period t. Thus, we have the throughput on each SU as:

$$U_n(t) = \sum_{c=1}^{C} \mu_n^c(t) f_n^c[n](t) + \sum_{c=1}^{C}\sum_{m=1}^{N} \mu_{mn}^c(t) f_{mn}^c(t) \tag{11}$$

$\mathbf{U} = (U_1, \cdots, U_N)$ denotes the throughput vector.

The flow routing is subject to the following constraints:

$$\sum_{c=1}^{C} \mu_{nm}^c(t) f_{nm}^c(t) = \sum_{c=1}^{C} \mu_n^c(t) f_n^c[m](t) \quad \forall m \ne n \tag{12}$$

$$\sum_{m=1}^{N} f_n^c[m](t) \le \mu_n^c(t)\omega_n^c(t) \qquad \forall n \tag{13}$$

$$f_{nm}^c(t) \le \omega_{nm}^c(t) \qquad \forall n, m, c \tag{14}$$

Eq. (12) shows the flow balance requirement. The flow rate should be scheduled and optimized at the macro BS, and be guaranteed to be feasible. (13) and (14) indicates that the aggregate flow rate on each link can not exceed the link capacity. $\omega_n^c(t)$ and $\omega_{nm}^c(t)$ denote the capacities of macro transmission link (macro BS to SU n) and cooperative transmission link (SU n to SU m) on channel c, respectively:

$$\omega_n^c(t) = B\cdot \log_2(1 + \frac{P_{BS}^c(t) g_n^c(t)}{N_0}) \qquad \forall n, c \tag{15}$$

$$\omega_{nm}^c(t) = B\cdot \log_2(1 + \frac{P_n^c(t) g_{nm}^c(t)}{N_0}) \qquad \forall n, m, c \tag{16}$$

where $g_{nm}^c(t)$ is the propagation gain from SU n to SU m and B is the channel bandwidth. We denote the upper bound of the channel capacity as ω_{max} due to the power constraint and noise (denoted as N_0).

It is easy to prove that the capacity of each channel is achieved for macro transmissions. Otherwise, the macro BS can transmit more data to SUs to increase the aggregate throughput. Thus, the inequality in (13) can be turned to equality. In the network, the relays fully utilize the cooperative links if channel resources are allocated, and the cooperative link capacity is much smaller than the transmission link from the macro BS due to highly constrained power on SUs. Hence, the capacities of cooperative links are achieved as well in (14).

C. Impact of Resource Management and Problem Hardness

The resource management protocol should be tightly integrated with the architecture of cognitive WiMAX with femto cells. The objective is to maximize the aggregate throughput on all SUs under a fairness criteria while keeping the interference to PUs within a tolerable level. Three design factors are taken into consideration as follows.

Power Control. Essentially, the power control scheme is to tune the transmission and interference ranges, making high-degree spatial reuse and spectrum hole utilization in the location-dependent WiMAX femto cells. In the absence of adjustable power, there is hardly much we can do once we encounter an infeasible transmission scenario in scheduling. In our network, we seek the optimal power allocation for both the macro BS and SUs that can be continually tuned.

Multi-hop Channel Allocation. Multi-hop transmission significantly reduces the transmission requirement on *global* spectrum availability, making resource allocation feasible with *locally* available channel resources when direct single-hop transmission is infeasible. In tandem, cooperative communication [5] exploits user, channel, and cooperative diversities that benefit the network performance.

Flow Routing. Multi-path transmission makes the problem more challenging as the transmitter has to schedule which packet is sent to which node via which relay. The allocation of data flows should not cause channel overflow and packet loss, and at the same time fully consider the efficiency of resource utilization.

To achieve an optimal resource management, we first consider a greedy centralized optimization framework to maximize the aggregate throughput utilities at each time slot:

$$\begin{aligned} \max_{\mathbf{P_{BS}(t)},\mathbf{P_{SU}(t)},\mathbf{U_{BS}(t)},\mathbf{U_{SU}(t)}} \quad & \sum_{n=1}^{N} \theta_n U_n(t) \\ \text{subject to:} \quad & (1) - (16). \end{aligned}$$

where $\theta_n > 0$ describes the SU priority (fairness).

Derived from (13), we have the following fact:

$$\sum_{m=1}^{N} f_n^c[m](t) = \mu_n^c(t)\omega_n^c(t) \qquad \Rightarrow$$

$$\sum_{c=1}^{C}\sum_{m=1}^{N} \mu_n^c(t) f_n^c[m](t) = \sum_{c=1}^{C} \mu_n^c(t)\omega_n^c(t) \qquad \Rightarrow$$

$$\sum_{c,m} \mu_{nm}^c(t) f_{nm}^c(t) = \sum_{c=1}^{C} \mu_n^c(t)\omega_n^c(t) - \sum_{c=1}^{C} \mu_n^c(t) f_n^c[n](t) \Rightarrow$$

$$\sum_{n=1}^{N} U_n = \sum_{n=1}^{N}\sum_{c=1}^{C} \omega_n^c(t)\mu_n^c(t) \tag{17}$$

Thus, the greedy optimization can be rewritten to be:

$$\begin{aligned} \max_{\mathbf{P_{BS}(t)},\mathbf{P_{SU}(t)},\mathbf{U_{BS}(t)},\mathbf{U_{SU}(t)}} \quad & \sum_{n=1}^{N} \theta_n U_n(t) \\ \text{subject to:} \quad & (17) \text{ and } \mathbf{U} \in \Lambda \end{aligned}$$

where Λ represents the achievable throughput region of SUs. It is easy to solve the optimization problem if the set Λ is known in advance. However, in practice, this region is unknown. Blindly transmitting data will lead to channel overflow or under-utilization, and flow routing for each SU will be out of control. Moreover, greedy optimization can not guarantee the optimality in the long term. To address these challenges, we next present our online resource management protocol.

III. Resource Management With Stochastic Lyapunov Optimization

In this section, we propose an online location-aware cooperative resource management protocol, based on stochastic Lyapunov optimization without the requirement of the knowledge on SU throughput region. With rigorous proof, we show that it is able to achieve near-optimal throughput performance over time. We also provide deterministic worst case bounds of the interference to PUs and the maximum data buffer backlog.

A. Stochastic Network Model

The macro BS maintains a data buffer for each SU, and $B_n(t)$ denotes the buffer backlog. In each time slot, new packets are admitted into the buffer with a rate of $R_n(t)$ and the macro BS transmits the data buffered to the corresponding SU (directly or via relay) as long as channel resources are allocated. Essentially, $R_n(t)$ reflects the throughput performance if we carefully tune this rate and manage resources to make the buffer backlog bounded and stable. R_{max} is the achievable maximum rate due to the computation and bandwidth limit of SUs. Then, we have the following data buffer dynamics:

$$B_n(t+1) = \max\{B_n(t) - U_n(t), 0\} + R_n(t) \tag{18}$$

Let r_n denote the time average rate of SU n. We have,

$$r_n = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} R_n(\tau) \tag{19}$$

$\mathbf{r} = (r_1, \cdots, r_N)$ denotes the rate vector on all SUs.

In cooperative communication, relays may generate interference to PUs due to sensing errors. If one cooperative transmission link causes interference to a PU, we count it as one collision of the PU. We use $E_a^c(t)$ to capture the total number of such collisions for PUs as defined:

$$E_a^c(t) = \sum_{m=1}^{N}\sum_{n=1}^{N} \mu_{mn}^c(t) I_m^a(t)\big(1 - S_a^c(t)\big) \tag{20}$$

where $I_m^a(t)$ is the binary variable indicating whether the cooperative communication issued by SU m possibly generate interference to PU a. This information can be captured according to location information (if PU a is in the transmission range of SU m, then $I_m^a(t) = 1$). It is intuitive that the more interference incurred, the more severely PUs would suffer from the packet losses. Let e_a^c denote the time average rate of collisions:

$$e_a^c = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E_a^c(\tau) \tag{21}$$

In the network, this interference information of each PU can be tracked using an interference buffer, and all SUs are aware of it. The buffer backlog, denoted as $X_a^c(t)$, reflects the interference level, which can not exceed a time average tolerable rate ρ_a^c. Thus, we have the following interference buffer dynamics:

$$X_a^c(t+1) = \max\{X_a^c(t) - \rho_a^c, 0\} + E_a^c(t) \tag{22}$$

Overall, we aim to maximize the aggregate throughput of SUs under the fairness criteria (consistent with the centralized greedy optimization problem):

$$\begin{aligned} \max \quad & \sum_{n=1}^{N} \theta_n r_n \\ \text{subject to:} \quad & (1) - (22). \end{aligned} \tag{23}$$

B. Resource Management Policies

We design the online resource management protocol based on stochastic optimization to solve the problem (23). It includes three policies stated as follows:

(i) **Flow Control:** At each time slot t, the macro BS controls the data rate admitted to the data buffer of each SU as the solution to the following problem:

$$\begin{aligned} \min \quad & R_n(t)\big(B_n(t) - V\theta_n\big) \\ \text{subject to:} \quad & 0 \le R_n(t) \le R_{max} \end{aligned} \tag{24}$$

where $V \ge 0$ is a constant parameter, which can be tuned according to the system requirement. The above problem has the threshold-based solution:

$$R_n(t) = \begin{cases} 0, & \text{if } B_n(t) > V\theta_n, \\ R_{max}, & \text{otherwise.} \end{cases}$$

(ii) **Macro Allocation:** At each time slot t, the power and channel allocation for the macro transmissions issued by the macro BS to SUs should follow the policy by solving the following problem:

$$\begin{aligned} \max \quad & \sum_{n=1}^{N}\sum_{c=1}^{C} B_n(t)\omega_n^c(t)\mu_n^c(t) \\ \text{subject to:} \quad & (1), (3), (4), (15) \end{aligned} \tag{25}$$

This allocation policy reflects two intuitive designs: (a) The link with a higher capacity has a higher priority to get channel resources, which is helpful to achieve higher aggregate throughput (represented by $\omega_n^c(t)$). (b) Resource allocation favors SUs with a large data buffer backlog (represented by $B_n(t)$). More data in the buffer implies higher urgency to transmit the data to avoid buffer overflow. From the fairness point of view, a larger backlog also indicates the SUs have obtained a smaller share of channel resources to transmit data in the previous time slots. Thus, they should be given a higher priority to obtain channel resources in the current time slot.

(iii) **Cooperative Allocation:** At each time slot t, the power and channel allocation for cooperative communication should follow the policy by solving the following problem:

$$\begin{aligned} \max \quad & \sum_{a,m,n,c} \mu_{mn}^c(t)\big\{\big(B_n(t) - B_m(t)\big)\omega_{mn}^c(t) - \\ & \qquad X_a^c(t) I_m^a(t)\big(1 - Y_a^c(t)\big)\big\} \\ \text{subject to:} \quad & (2), (5) - (7), (16) \end{aligned} \tag{26}$$

Three factors are taken into account for cooperative allocation. (a) *Buffer backlog.* More buffered data of SU n than SU m implied higher urgency to transmit the data of SU n than m, leading to a higher priority that SU m helps SU n via cooperative communication (represented by $B_n(t) - B_m(t)$). (b) *Channel rate.* The higher rate a cooperative link is able to achieve (represented by $\omega_{mn}^c(t)$), the higher chance channel resources are allocated on the link. (c) *Interference level.* The channel allocation favors cooperative transmissions that will *not* potentially generate interference to PUs (represented by $I_m^a(t)(1 - Y_a^c(t))$), especially the ones who already have high interference levels (represented by $X_a^c(t)$). Note that cooperative allocation is performed after the macro allocation with a fixed power and channel allocation for macro transmissions.

C. Performance Analysis

We now characterize the performance of our scheduling policies with the following bounds.

(i) **Backlog Performance**. Initialize $B_n(0) = 0$. The data buffer backlogs are bounded as:

$$B_n(t) \le B_{max} \triangleq V\theta_{max} + R_{max} \quad \forall n, t \tag{27}$$

Proof: $B_n(0) = 0 < B_{max}$. Now, suppose that $B_n(t) \le B_{max}$. We show the same holds for $B_n(t+1)$. We have two cases. (*a*) $B_n(t) \le B_{max} - R_{max}$. Obviously, $B_n(t+1) \le B_{max}$ according to Eq. (18). (*b*) $B_n(t) > B_{max} - R_{max}$, then $B_n(t) > V\theta_n - R_{max} + R_{max} = V\theta_n$. Thus, we will choose $R_n(t) = 0$ according to our *macro allocation* policy, so that $B_n(t+1) \le B_n(t) \le B_{max}$. Overall, (27) is proved. ■

(ii) **Interference Performance**. Initialize $X_a^c(0) = 0$. $\forall t > 0$, if $Y_a^c(t) < 1$, set $0 < \varepsilon < 1$ and $Y_a^c(t) \le 1 - \varepsilon$. Then the worst case of the interference buffer backlogs for all PUs is upper bounded by:

$$X_a^c(t) \le X_{max} \triangleq \frac{B_{max}\omega_{max}}{\varepsilon} + \lfloor \frac{N}{2} \rfloor \quad \forall c, a, t \tag{28}$$

Proof: $X_a^c(0) = 0 < X_{max}$. Now, suppose that $X_a^c(t) \le X_{max}$. We show the same holds for $X_a^c(t+1)$. First, suppose $Y_a^c(t) = 1$. Then, there will be no interference to PU a as it does not occupy channel c. Thus, we get $X_a^c(t+1) \le X_{max}$ according to (22) with $E_a^c(t) = 0$. Next, suppose $Y_a^c(t) < 1$, and we have two cases. (*a*) $X_a^c(t) \le X_{max} - \lfloor \frac{N}{2} \rfloor$. Note that

$\lfloor \frac{N}{2} \rfloor$ represents the maximum number of cooperative transmission links (SU pairs) in the network, which is also the maximum value of $E_a^c(t)$. Obviously, $X_a^c(t+1) \le X_{max}$ under this case. (*b*) $X_a^c(t) > X_{max} - \lfloor \frac{N}{2} \rfloor = \frac{B_{max}\omega_{max}}{\varepsilon}$. Then, $X_a^c(t)\varepsilon > B_{max}\omega_{max}$. Thus, we have $X_a^c(t)(1 - Y_a^c(t)) > (B_n(t) - B_m(t))\omega_{mn}^c$. If $I_m^a(t) = 1$, according to our *cooperative allocation* policy, choose $\mu_{mn}^c(t) = 0$, which means there is no cooperative communication on channel c. If $I_m^a(t) = 0$, the transmissions issued by all SUs can not reach PU a. Thus, $X_a^c(t+1) \le X_a^c(t) \le X_{max}$. Overall, (28) is proved. ■

(iii) **Utility performance**. Initialize $B_n(0) = 0, X_a^c(t) = 0$. The time average throughput utility achieved by our protocol is within $\tilde{B}/V$ of the optimal value:

$$\lim_{t\to\infty} \inf \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{n=1}^{N} \theta_n \mathbb{E}\big\{R_n(\tau)\big\} \ge \sum_{n=1}^{N} \theta_n r_n^* - \frac{\tilde{B}}{V} \qquad (29)$$

where r_n^* is the optimal achievable rates of problem (23), and $V, \tilde{B} > 0$ are constants.

We use the technique of *Stochastic Lyapunov Optimization* to prove it. Let $\mathbf{Q}(t) = (Q_1(t), \cdots, Q_K(t))$ be a vector of queue lengths for a discrete time stochastic queueing network. Let $W(\mathbf{Q})$ be any non-negative scalar valued function of the queue lengths, called a Lyapunov function. Define the *Lyapunov drift* $\Delta(t)$ as follows:

$$\Delta(t) \triangleq \mathbb{E}\big\{W(\mathbf{Q}(t+1)) - W(\mathbf{Q}(t))\big\} \qquad (30)$$

The network accumulates *utility* every time slot with bounded value. We have the stochastic process $f(t)$ to represent the utility earning with over-time optimum f^*.

Theorem 1: Suppose there exist $V > 0, \tilde{B} > 0, d > 0$, and a non-negative function $W(\mathbf{Q})$ such that $\mathbb{E}\{W(\mathbf{Q}(d))\} < \infty$. For $t > d$, if the Lyapunov drift satisfies:

$$\Delta(t) - V\mathbb{E}\{f(t)\} \le \tilde{B} - Vf^* \qquad (31)$$

then we have:

$$\lim_{t\to\infty} \inf \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\big\{f(\tau)\big\} \ge f^* - \frac{\tilde{B}}{V} \qquad (32)$$

Proof: Refer to [6]. ■

In our resource management problem, we set $\mathbf{Q}(t) = (B_1(t), \cdots, B_N(t), X_1^1(t), \cdots, X_1^C(t), \cdots, X_A^1(1), \cdots, X_A^C(t))$. Define $f(t) \triangleq \sum_{n=1}^{N} \theta_n R_n(t)$ as the aggregated throughput utility earning at each time slot according to (23), and thus $f^* \triangleq \sum_{n=1}^{N} \theta_n r_n^*$ as the over-time optimal utility accordingly. We further define the Lyapunov function as follows:

$$W(\mathbf{Q}(t)) \triangleq \frac{1}{2} \sum_{a=1}^{A} \Big(\sum_{n=1}^{N} (B_n(t))^2 + \sum_{c=1}^{C} (X_a^c(t))^2 \Big)$$

Now, we calculate the Lyapunov drift as follows:

$$\Delta(t) \le B - \mathbb{E}\Big\{ \sum_{a=1}^{A} \sum_{n=1}^{N} \big\{B_n(t)\big(U_n(t) - R_n(t)\big)\big\} \Big\}$$
$$- \mathbb{E}\Big\{ \sum_{a=1}^{A} \sum_{c=1}^{C} X_a^c(t)\big(\rho_a^c - E_a^c(t)\big) \Big\} \qquad (33)$$

where $B \triangleq \frac{1}{2}\big(A \cdot N \cdot (B_{max})^2 + \sum_{a=1}^{A} \sum_{c=1}^{C} (\rho_a^c)^2 + A \cdot C\big)$.

Now we subtract $V\mathbb{E}\{\sum_{n=1}^{N} \theta_n R_n(t)\}$ from both sides of the drift inequality (33) and substitute (20) into (33). We have:

$$\Delta(t) - V\mathbb{E}\Big\{ \sum_{n=1}^{N} \theta_n R_n(t) \Big\} \le B - \sum_{a=1}^{A} \sum_{c=1}^{C} \rho_a^c \mathbb{E}\Big\{X_a^c(t)\Big\}$$
$$+ A \cdot \mathbb{E}\Big\{ \sum_{n=1}^{N} R_n(t)\big(B_n(t) - V\theta\big) \Big\}$$
$$- \mathbb{E}\Big\{ \sum_{a=1}^{A} \sum_{n=1}^{N} B_n(t)U_n(t) - \sum_{a=1}^{A} \sum_{c=1}^{C} X_a^c(t)E_a^c(t) \Big\} \qquad (34)$$

We then derive the following equation by substituting (11), (13), and (20) into the last term of (34).

$$\mathbb{E}\Big\{ \sum_{a=1}^{A} \sum_{n=1}^{N} B_n(t)U_n(t) - \sum_{a=1}^{A} \sum_{c=1}^{C} X_a^c(t)E_a^c(t) \Big\} =$$
$$\mathbb{E}\Big\{ \sum_{a,n,c} B_n(t) f_n^c[n](t) \mu_n^c(t) \Big\} + \mathbb{E}\Big\{ \sum_{a,n,m,c} \mu_{mn}^c(t) B_n(t) \omega_{mn}^c \Big\}$$
$$- \mathbb{E}\Big\{ \sum_{a,n,m,c} \mu_{mn}^c(t) X_a^c(t) I_m^a(t)(1 - S_a^c(t)) \Big\} \qquad (35)$$

Further, we have the following fact (derived from Eq. (13)):

$$\sum_{m=1, m\ne n}^{N} f_n^c[m](t) + f_n^c[n](t) = \omega_n^c(t)\mu_n^c(t) \quad \Rightarrow$$
$$f_n^c[n](t)\mu_n^c(t) = \omega_n^c(t)\mu_n^c(t) - \sum_{m=1, m\ne n}^{N} f_n^c[m](t)\mu_n^c(t) \qquad (36)$$

Using (36) and (12), we have:

$$\mathbb{E}\Big\{ \sum_{a,n,c} B_n(t) f_n^c[n](t) \mu_n^c(t) \Big\} =$$
$$\mathbb{E}\Big\{ \sum_{a,n,c} B_n(t)\omega_n^c(t)\mu_n^c(t) + \sum_{a,n,m,c} B_n(t)\mu_{nm}^c(t)\omega_{nm}^c(t) \Big\} \qquad (37)$$

Substitute (37) into (35) and put (35) into (34). We have:

$$\Delta(t) - V\mathbb{E}\Big\{ \sum_{n=1}^{N} \theta_n R_n(t) \Big\} \le B - \sum_{a=1}^{A} \sum_{c=1}^{C} \rho_a^c \mathbb{E}\Big\{X_a^c(t)\Big\}$$
$$+ A \cdot \mathbb{E}\Big\{ \sum_{n=1}^{N} R_n(t)\big(B_n(t) - V\theta\big) \Big\}$$
$$- A \cdot \mathbb{E}\Big\{ \sum_{n=1}^{N} \sum_{c=1}^{C} B_n(t)\omega_n^c(t)\mu_n^c(t) \Big\}$$
$$- \mathbb{E}\Big\{ \sum_{a,m,n,c} \mu_{mn}^c(t) \big\{ \big(B_n(t) - B_m(t)\big)\omega_{mn}^c(t) -$$
$$X_a^c(t) I_m^a(t) \big(1 - S_a^c(t)\big) \big\} \Big\} \qquad (38)$$

The last three terms in the right side of (38) are exactly our resource management policies (replace $S_c(t)$ as $Y_c(t)$ by considering the sensing errors on the macro BS). Note that the macro transmission is dominant in the aggregate throughput on SUs according to (17). Thus, we can optimize the last two terms separately although they have common constraints (7) and (9). Then, it is clear to see that our management policies minimize the right side of (38) over all alternate feasible scheduling policies at each time slot.

We now define the stationary, randomized policy *SR*, that chooses a feasible resource allocation at every time slot as a function of only the channel state information $\mathbf{S}(\mathbf{t})$ and $\mathbf{P}(\mathbf{t})$, which will yield the following steady state values [7]:

$$\mathbb{E}\{R_n^{SR}(t)\} = r_n^* \tag{39}$$

$$e_a^{c,SR} \triangleq \lim_{t\to\infty} \sum_{\tau=0}^{t-1} \mathbb{E}\{E_a^{c,SR}(\tau)\} \leq \rho_a^c \tag{40}$$

Note that our policies minimize the right side of (38) including the *SR* policy [7]. Thus, we can show (from (33)):

$$\Delta(t) - V\mathbb{E}\{f(t)\} \leq B - \mathbb{E}\Big\{\sum_{a=1}^{A}\sum_{c=1}^{C} X_a^c(t)\big(\rho_a^c - E_a^{c,SR}(t)\big)\Big\}$$
$$-A\cdot\mathbb{E}\Big\{\sum_{n=1}^{N}\big\{B_n(t)\big(U_n(t) - R_n^{SR}(t)\big)\big\} - Vf^* \tag{41}$$

Elaborated in the Appendix, we are finally able to obtain:

$$\Delta(t) - V\mathbb{E}\Big\{\sum_{n=1}^{N}\theta_n R_n(t)\Big\} \leq \tilde{B} - V\sum_{n=1}^{N}\theta_n r_n^* \tag{42}$$

The form fits (31). Thus, by applying Theorem 1, we are able to prove (29).

IV. Optimization Solution

Macro and *cooperative* allocation policies in Sec. III-B require us to solve optimization problems (25) and (26), which are non-linear integer programming (NIP) and thus NP-hard. We can use the traditional *branch-and-bound* algorithm to solve these problems optimally. However, it does not exploit the special structure of these optimization problems, and has a high complexity due to LP relaxation and inefficient search. In this section, we propose to apply the Generalized **Expectation Maximization (EM)** algorithm [8] to our problems, which specifically exploits special problem structures and cognitive WiMAX network characteristics, and reduces the complexity.

A. Generalized EM Algorithm

Generalized EM is an iterative method to optimize two sets of variables (λ, θ). We obtain the optimal solutions by iteratively updating the variables via two steps:

E step: $\theta^{(k+1)} = \arg\max_\theta \mathcal{F}(\theta, \lambda^{(k)})$

M step: $\lambda^{(k+1)} = \arg\max_\lambda \mathcal{F}(\theta^{(k+1)}, \lambda^{(k)})$

Successive application of generalized EM maximizes the lower bound of $\mathcal{F}$, *i.e.*,

$$\mathcal{F}(\theta^{(k+1)}, \lambda^{(k)}) \geq \mathcal{F}(\theta^{(k)}, \lambda^{(k)})$$
$$\mathcal{F}(\theta^{(k+1)}, \lambda^{(k+1)}) \geq \mathcal{F}(\theta^{(k+1)}, \lambda^{(k)})$$

Accordingly, in the *macro allocation* problem (25), we divide the variables into two sets: $\mathbf{P_{BS}}(\mathbf{t})$ and $\mathbf{U_{BS}}(\mathbf{t})$. We iteratively solve the problem with two steps. First, take $\mathbf{P_{BS}}(\mathbf{t})$ as the variable and $\mathbf{U_{BS}}(\mathbf{t})$ as a fixed value (referred to as the *BS Power Optimization* step). Then, take $\mathbf{U_{BS}}(\mathbf{t})$ as the variable but $\mathbf{U_{BS}}(\mathbf{t})$ as a fixed value (referred to as the *Macro Channel Assignment* step). The optimal solution can be obtained by repeating these two steps until convergence.

Surprisingly, by separating the problem into two steps, the complexity of the optimization problem is largely reduced due to the special problem structure. With a fixed channel allocation, the *BS Power Optimization* step is actually a LP. The *macro Channel Assignment* step, with a fixed power allocation, can be considered as a maximum weighted bipartite matching (WBM) problem, which can be solved optimally with **polynomial** time complexity. Construct a bipartite graph $A = (\Phi \times \chi, E)$. The vertices in Φ denote all SUs, and the vertices in χ denote all channels. The edge set E corresponds to $|\Phi| \times |\chi|$ edges connecting all possible pairs, with weight $B_n(t)\omega_n^c(t)$. Run the WBM algorithm to obtain the matched pairs, providing corresponding channel assignment. The WBM problem can be solved in a centralized fashion using network flow algorithms such as the cost scaling algorithm [9], and can also be solved in distributed approximation algorithms [10].

In the *cooperative allocation* problem (26), we can also divide the variables into two sets: $\mathbf{P_{SU}}(\mathbf{t})$ and $\mathbf{U_{SU}}(\mathbf{t})$. Then, the problem is separated into two steps: the *SU Power Allocation* step and the *Cooperative Channel Allocation* step. The *SU Power Allocation* step is a LP. The *Cooperative Channel Allocation* step can be formulated into a similar WBM, where Φ includes all cooperative links [1] and χ contains all available channels, excluding the ones that can not be used according to the constraints. The weight of each edge in E carries $\big(B_n(t) - B_m(t)\big)\omega_{mn}^c(t) - X_a^c(t)I_m^a(t)\big(1 - Y_a^c(t)\big)$. With this graph setup, the problem can be solved optimally.

B. Complexity Analysis

The Generalized EM algorithm converges to a local maximum of the original optimization problem [8]. We can carefully select the initial conditions, resulting in the global maximum. One efficient approach to set the initial values is to solve the LP relaxation of the original problem, and get the feasible solution by randomized rounding. Running the algorithm several times with different initial conditions is also helpful.

We perform a set of simulations to evaluate the Generalized EM algorithm in our problems by comparing with the traditional *branch-and-bound* algorithm, running over Intel Core Duo machine at 1.83 GHz and a memory of 2 GB. The results are listed in Table I. With respect to the performance of average throughput over SUs, the *Generalized EM* algorithm performs very close to the *branch-and-bound* algorithm, within a 1% difference on average. Further, we observe that the *Generalized EM* algorithm is able to converge within 1 ms on average which is much faster than the *branch-and-bound* algorithm. Thus, it is suitable for practical WiMAX systems.

V. Performance Evaluation

We are now ready to resort to extensive simulations to study the performance of cognitive WiMAX with femto cells. To be

[1] For example, $(1, 2)$ indicates the transmission link from SU 1 to SU 2. Note that it is different from $(2, 1)$ representing the link from SU 2 to SU 1.

TABLE I
EVALUATION OF THE *Generalized EM* ALGORITHM.

Algorithm	Ave. Throughput	Ave. running time
generalized EM	1.74 Mbps	1 ms
branch-and-bound	1.75 Mbps	6 ms

TABLE II
SIMULATION PARAMETERS.

Channel Type	Rayleigh fading and AWGN
Path loss Model	COST-HATA-231
Transmitter Power (macro BS)	25 dBm
Transmitter Power (SU)	5 dBm
Noise Power	-129.5 dBW
Adaptive Modulation	used

realistic, the simulations are conducted over a long term, where practical settings of WiMAX and CR configuration are adopted according to [3]. In our simulations, there are a total of 20 PUs located across 8 femto cells sharing 12 channels. Around the service region, a number of SUs are randomly moving with random initial locations. The channel availability state evolves according to a Markov chain with symmetric transition probabilities between the ON and OFF states given by 0.5. The simulation parameters are listed in Table II.

We simulate our proposed protocol with different numbers of active SUs, denoted as "Coop-X" ("X" represents the number of SUs). For comparison, we simulate the traditional resource management protocol in cognitive WiMAX with power control in a coarse granularity, by simply using the maximum feasible power for macro transmission (follow the constraint (3)) without cooperative communication and flow control, referred to as "NOCoop." Further, to specifically examine the advantages of the cognitive WiMAX architecture, we simulate the resource management protocol in traditional WiMAX networks without the CR technique, referred to as "Trad," where the transmission is only performed when feasible channels exist across the entire macro area and is provisioned under maximum power.

We first examine the throughput performance. Fig. 2 shows the results on average throughput over SUs via a 15000-second simulation. We observe that "Trad" performs worst, indicating the advantage of the new architecture by applying CR technique. Even "NOCoop" outperforms "Trad" with a substantial gain (21%) by exploiting spectrum reuse in a higher degree. Further, "Coop-30," "Coop-40," and "Coop-50" defeat "NOCoop" by 36%, 50%, and 63% respectively, and of course outperform "Trad" with much higher margins. It coincides with our intuition that resource management with cooperative communication, power control, flow routing, and other important cross-layer designs naturally fits in the design of cognitive WiMAX with femto cells, and is able to achieve significant throughput improvement due to its effective use of the wireless spectrum. Another trend to notice is that the margin that "Coop" outperforms "NOCoop" and "Trad" becomes more substantial with an increasing number of SUs. This observation indicates that a larger number of SUs create a higher degree of cooperation, which is beneficial for throughput performance.

Regarding the fairness performance, we capture the variance on the average throughput over SUs. At each time slot t, we calculate, for each SU, the average throughput over time horizon

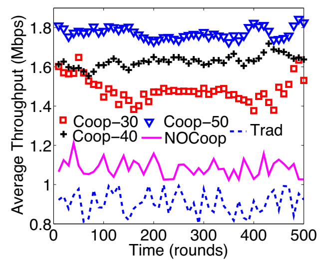

Fig. 2. Average throughput performance of all protocols.

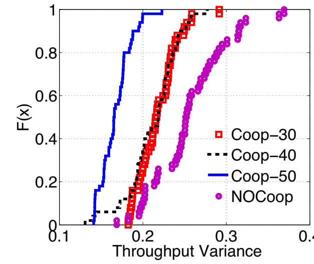

Fig. 3. CDF of throughput variance, which indicates fairness performance.

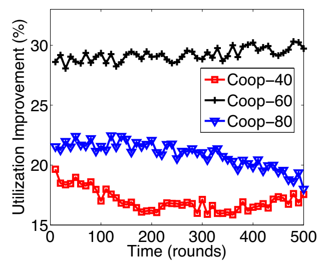

Fig. 4. Performance on the channel utilization improvement.

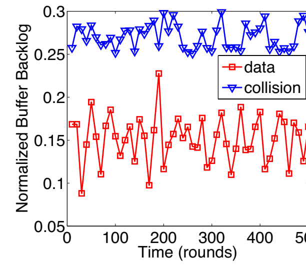

Fig. 5. Performance on the buffer backlog.

$[1, t]$, and then compute the throughput variance as the ratio between the standard deviation of the average throughput over time and the time average throughput itself. Fig. 3 plots the CDF of this throughput variance for all protocols. Not surprisingly, "Coop"s outperform "NOCoop," which shows the improvement of our protocol on fairness performance.

To obtain a deeper understanding of the advantages of our proposed protocol, we investigate the channel utilization performance, which is calculated as the sum throughput of all SUs over the aggregate throughput in the network including both SUs and PUs. This value accurately reflects the improvement on the spectrum utilization. Evident from the results shown in Fig. 4, the increase of the channel utilization reaches 30% in the best case ("Coop-60"). It demonstrates that the spectrum can be more efficiently utilized with our protocol. Another observation from the results is that the performance will degrade when the number of SUs is overly large ("Coop-80"), since the interference effect begins to dominate. A *sweet spot* may exist with respect to the number of SUs in cognitive WiMAX. We will further study it in our future work.

Finally, we track the buffer backlogs of both data and collision queues. The results are shown in Fig. 5, and the curves capture the normalized buffer backlog, calculated by the ratio between the backlogs and the bounds (obtained in (27) and (28)). The results show that the buffer remains bounded over the long term, which is desirable in the system design.

VI. RELATED WORK

During the past decade, broadband wireless mobile services have been the most remarkable growth areas in the communication industry. WiMAX is considered as the next generation technology to provide ubiquitous broadband wireless connections, and the IEEE 802.16 family of standards are the state-of-the-art wireless communication standards accordingly [1]. Further, WiMAX adopts OFDMA and the femto-cell architecture, providing a rich set of features and a high degree of flexibility. Cognitive radio (CR) is a revolution in radio technology to efficiently utilize spectrum resources in wireless networks. IEEE 802.22 [3] is the first standardization effort to define CR and so far has drawn much research attention in both academia and industry. Dynamic spectrum access (DSA) [11] is one of the key issues and has driven most of the CR research. [6] develops an opportunistic spectrum access framework that maximizes the throughput utility of the SUs. [4] and [12] study the dynamic access issues in the ad hoc mode of CR networks, where scheduling and routing are jointly considered.

In our work, we take advantage of the favorable properties of both WiMAX and CR techniques, and investigate the benefits of their collaboration. The concept of *Cognitive WiMAX* was proposed in [13]. However, our work radically differs from it in a number of aspects. First, we study cognitive WiMAX with femto cells employed, which provide potentials on spectrum reuse and represent the direction that WiMAX evolves to [1]. [13] only studies regular WiMAX scenarios. Second, we advocate cooperative and multi-path multi-channel communication, which is more efficient and hence works in a substantially different architecture. Third, we propose a novel location-aware resource management protocol with cross-layer designs, while [13] just uses CRs to perform channel sensing without DSA. Last but not least, we specifically provide a rigorous analysis on network performance, which is not discussed in [13].

VII. Concluding Remarks

In this paper, we propose cognitive WiMAX with femto cells and study the resource management problem in the network. Tightly integrated with the novel cognitive WiMAX architecture, our cross-layer resource management protocol is designed to apply power control, multi-hop cooperative communication and flow management techniques, achieving near-optimal performance. It is based on a sound theoretical foundation using stochastic Lyapunov optimization, but not without careful considerations of the practicality, feasibility, and efficiency of implementing these solutions. With this paper, we are convinced that it is a win-win approach by applying the CR technique to WiMAX with the employment of our resource management protocol by fully exploiting spectrum reuse and incorporating user, channel, and cooperative diversities.

Appendix

We aim to prove of inequality (42) from (41), by getting the constant lower bounds of the second and third last terms in (41). First, as $B_n(t) \geq 0$ and $U_n(t) \geq 0$, we have $\mathbb{E}\big\{\sum_{n=1}^{N} B_n(t)U_n(t)\big\} \geq 0$. Further, $0 \leq B_n(t) \leq B_{max}$ and $0 \leq R_n^{SR} \leq R_{max}$. Thus, (41) turns to be:

$$\Delta(t) - V\mathbb{E}\{f(t)\} \leq B + A\cdot N\cdot B_{max}R_{max} - Vf^* \\ -\mathbb{E}\Big\{\sum_{a=1}^{A}\sum_{c=1}^{C} X_a^c(t)\big(\rho_a^c - E_a^{c,SR}(t)\big)\Big\} \tag{43}$$

We then use "delayed" queue backlogs to formulate it. Clearly, for $t > d$, we have:

$$X_a^c(t-d) + d\cdot\lfloor\frac{N}{2}\rfloor \geq X_a^c(t) \geq X_a^c(t-d) - d\rho_a^c \tag{44}$$

Now, we substitute $X_a^c(t)$ in (41) with (44):

$$\Delta(t) - V\mathbb{E}\{f(t)\} \leq B + A\cdot N\cdot B_{max}R_{max} + Z - Vf^* \\ -\mathbb{E}\big\{\textstyle\sum_{a=1}^{A}\sum_{c=1}^{C} X_a^c(t-d)(\rho_c - E_a^{c,SR}(t))\big\} \tag{45}$$

where $Z \triangleq d\sum_{a=1}^{A}\sum_{c=1}^{C}\Big(\lfloor\frac{N}{2}\rfloor + (\rho_a^c)^2\Big)$.

Using iterative expectations, we have the following:

$$\mathbb{E}\big\{\sum_{c=1}^{C} X_a^c(t-d)E_a^{c,SR}(t)\big\} = \\ \mathbb{E}\big\{\sum_{c=1}^{C} X_a^c(t-d)\cdot\mathbb{E}\{E_a^{c,SR}(t)|\mathbb{T}(t-d)\}\big\} \tag{46}$$

$\mathbb{T}$ represents the system state at time slot t on the primary channel availability $\mathbf{H}(\mathbf{t})$ and $\mathbf{Y}(\mathbf{t})$, which can be considered as a Markov process. By the property of Markov processes, any functions of these states $\mathbf{H}(\mathbf{t})$ and $\mathbf{Y}(\mathbf{t})$ converge exponentially fast to their steady state values. Recall that the stationary, randomized policy is only based on the system states. Thus, there exists $\alpha > 0, 0 < \sigma < 1$ such that (using (39) and (40)):

$$\mathbb{E}\big\{E_a^{c,SR}(t)|\mathbb{T}(t-d)\big\} \leq e_a^{c,SR} + \alpha\sigma^d \leq \rho_a^c + \alpha\sigma^d \tag{47}$$

Now, substitute (47) into (45), then (41) finally can be expressed as follows, which fits to the form of (31):

$$\Delta(t) - V\mathbb{E}\{f(t)\} \leq \tilde{B} - Vf^* \\ \tilde{B} = B + A\cdot N\cdot B_{max}R_{max} + Z + A\cdot C\cdot X_{max}\alpha\sigma^d.$$

References

[1] C. Eklund, R. B. Marks, K. L. Stanwood, and S. Wang, "IEEE Standard 802.16: A Technical Overview of The WirelessMAN™ Air Interface for Broadband Wireless Access," *IEEE Communications Magzine*, vol. 40, no. 6, pp. 98 – 107, 2002.

[2] K. Sundaresan and S. Rangarajan, "Efficient Resource Management in OFDMA Femto Cells," in *Proc. of ACM Mobihoc*, 2009.

[3] C. Cordeiro, K. Challapali, D. Birru, and N. S. Shankar, "IEEE 802.22: The First Worldwide Wireless Standard Based on Cognitive Radios," in *Proc. of 1st IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN 05)*, 2005.

[4] Y. Shi and Y. T. Hou, "Optimal Power Control for Multi-hop Software Defined Radio Networks," in *Proc. of IEEE INFOCOM*, 2007.

[5] V. Mahinthan, L. Cai, L. W. Mark, and X. Shen, "Maximizing Cooperative Diversity Energy Gain for Wireless Networks," *IEEE Transactions on Wireless Communications*, vol. 6, no. 7, pp. 2530–2539, 2007.

[6] R. Urgaonkar and M. J. Neely, "Opportunistic Scheduling with Reliabilitty Guarantees in Cognitive Radio Networks," in *Proc. of IEEE INFOCOM*, 2008.

[7] M. J. Neely, E. Modiano, and C. Li, "Dynamic Power Allocation and Routing for Time Varying Wireless Networks," *IEEE Journal on Selected Areas in Communications*, 2005.

[8] J. R. Movellan, "Tutorial on Generalized Expectation Maximization," available at http://mplab.ucsd.edu/tutorials/EM.pdf.

[9] R. K. Ahuja, T. L. Magnati, and J. B. Orlin, *Network Flows: Theory, Algorithms, and Applications*. Prentic Hall, February 1993.

[10] J. Hoepman, "Simple Distributed Weighted Matchings," available at http://arxiv.org/abs/cs/0410047.

[11] Q. Zhao, S. Geirhofer, L. Tong, and B. Sadler, "Opportunistic Spectrum Access via Periodic Channel Sensing," *IEEE Transactions on Signal Processing*, 2008.

[12] Y. Shi and Y. T. Hou, "A Distributed Optimization Algorithm for Multi-hop Cognitive Radio Networks," in *Proc. of IEEE INFOCOM*, 2008.

[13] A. E. Leu, B. L. Mark, and M. A. McHenry, "A Framework for Cognitive WiMAX with Frequency Agility," *Proceedings of the IEEE*, 2009.