Multi-seller Combinatorial Spectrum Auction with Reserve Prices

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***Abstract*—Efficient spectrum allocation is becoming more and more important in wireless networking. Auction is believed to be an effective way to address the problem of spectrum shortage, by dynamically redistributing spare channels among service providers. Combinatorial auction gives buyers the freedom to place bids on combinations of channels rather than individual channels. For example, continuous channels are easier to operate on and more valuable to buyers. Unlike conventional commodities, spectrum features spatial reusability, which depends on various transmission ranges of heterogeneous channels, making combinatorial spectrum auction more challenging. Existing works on combinatorial spectrum auction only consider a single seller who tries to minimize her cost. However, in multi-seller markets, each seller has a reserve price, below which the seller is reluctant to sell her channel. In this paper, we propose a combinatorial auction mechanism for multiple sellers with specified reserve prices. We design an efficient greedy algorithm to determine auction winners by the average virtual bids, which is decided by buyers' real bids and the reserve prices of channels. Simulation results show that our proposed auction mechanism can achieve higher social welfare than existing auction mechanisms without reserve prices.**

I. Introduction

Thanks to the rapid development of wireless communication technologies in recent years, the shortage of radio spectrum is becoming a more and more serious problem. Dynamic spectrum access is believed to be an effective way to cope with the spectrum crunch. Conventionally, dynamic spectrum access is achieved through auction, one of the best-known market-based allocation mechanism, that achieves both perceived fairness and allocation efficiency. In double spectrum auctions, a third-party auctioneer first collects bids from all buyers and sellers, then allocates spectrum in a centralized manner.

However, design an efficient auction mechanism is challenging. Different from conventional goods, wireless spectrum features spatial reuse: buyers who are well separated geographically can operate on the same channel at the same time. In addition, due to different central frequencies, the transmission range is different for each channels, making it more difficult to decide interference relationship among buyers. Furthermore, buyers should be allowed to express their valuations for different combinations of channels rather than only placing bid on individual channels. For example, continuous channels are easier to manage, so that buyers may be willing to pay a higher price to attain a bundle of continuous channels. Unfortunately, it is well known that optimal allocation in combinatorial auction is often NP-hard.

The problem of channel allocation in spectrum auction has received much attention in recent years. However, few of these related works have fully addressed all the challenges. In TRUST [7], the authors considered spectrum spatial reuseability, but it is assumed that all channels are homogeneous and each buyer or seller is restricted to trade only one channel. Spectrum heterogeneity is considered in [9], yet buyers only bid on individual channels, but not combinations of channels. Combinatorial spectrum auction has been studied in [3]. These combinatorial auction mechanisms consider a single seller who aims at minimizing the cost of spectrum acquisition. [20] considers double auctions with a form of combinatorial bids while no spectrum heterogeneity nor reserved price. However, in this paper, each seller will have a reserve price, below which the seller is unwilling to sell her channels.

In this paper, we introduce a model of combinatorial (double) spectrum auction with reserve prices for heterogeneous spectrum redistribution among multiple sellers and buyers. Each buyer is allowed to express her preference over different combinations of channels by submitting a vector of bids for requested channel bundles. A trustworthy auctioneer will make the decision on channel allocation based on buyers' bids and sellers' reserve prices. To account for channel heterogeneity, different interference graphs are constructed to determine spectrum reuse. Reserve price that are imposed by a seller indicates the minimum price for the seller to sell her channel. Auction mechanisms that do not consider reserve prices may result in low social welfare [2], which should be avoided. To address this problem, we incorporate reserve prices into the combinatorial spectrum auction framework. We propose a novel algorithm to determine heterogeneous channel allocation subject to reserve price constraints. It is mainly based on greedy algorithm, which assigns channel bundles to buyers according to average virtual bids, which depend on buyers' real bids and reserve prices. Through extensive simulations, we domonstrate that our proposed algorithm outperforms other algorithms without reserve price consideration in terms of social welfare.

We make the following key contributions:

- We propose a combinatorial auction framework for multiple sellers, which considers the reserve price and its influence in channel allocation.
- We propose a greedy algorithm which assigns heterogeneous channels to buyers based on their requested bundles, bids and reserve prices.

- We conduct extensive simulation to verify that the proposed combinatorial auction mechanism with reserve prices outperforms existing combinatorial auction mechanisms without reserve prices in that higher social welfare can be achieved.

The rest of the paper is organized as follows. We describe the system model in details in Section II. In Section III, we present the greedy algorithm for spectrum allocation. Simulation results are shown in Section V. We briefly review the related work in Section VI, and finally summarize our work in Section VII.

II. System Model

In this paper, we consider a scenario in which there is a set of sellers $M = \{1, 2, ..., m\}$ and a set of buyers $N = \{1, 2, ..., n\}$. Each seller owns one channel, which can be sold to multiple non-interfering buyers. We model the channel trading process as a combinatorial auction in which buyers simultaneously submit their demands for channels to a trustworthy auctioneer. The auctioneer later makes decision on channel assignment and charge to the winners. In our model, we consider heterogeneous channels, which means that channels have different qualities and thus each buyer has her own preference over channels. Since wireless devices can be equipped with multiple radios, the buyers are allowed to request for more than one channel and submit multiple channel requests, among which only one of the requests can be granted. Other useful notations in the model of combinatorial channel auction are listed as follows:

Reserve Price R_i: Seller i has a reserve price of r_i, which reflects the value of using the channel herself. If the payment from the buyers is smaller than r_i, seller i is unwilling to sell her channel, and the transaction fails. Let $R = (R_1, R_2, ...R_m)$ represent the reserve prices of all sellers.

Channel Request S_j: Buyer j submits a vector of requested channel bundles $S_j = (s_{1,j}, s_{2,j}, ..., s_{k_j,j})$ to the auctioneer. Each one of the requested bundles can meet the need of the buyer, but no partial bundle is acceptable. Although buyer j submits a request vector S_j which consists of multiple channel bundles, only one will be granted by the auctioneer. Let $S = (S_1, S_2, ..., S_n)$ denote the channel request vectors of all buyers.

Channel Valuation V_j: Though each bundle in the request vector S_j can satisfy buyer j's QoS, she has different valuations $V_i = (v_{1,j}, v_{2,j}, ...v_{k,j})$ over different bundles. For example, two continuous channels are easier to manage than two non-continuous channels, so a buyer is willing to pay more for the former than the latter.

Channel Bid B_j:Buyer j submits the bid vector $B_j = (b_{1,j}, b_{2,j}, ..., b_{k_j,j}))$ to the auctioneer, meaning that if she wins bundle $s_{i,j}$, she would like to pay no more than $b_{i,j}$ for it.

Clearing Price q_i, p_j: A buyer is a winner if he is assigned at least one of his requested channels bundles. A winning buyer j will be charged p_j. A losing buyer will be charged nothing. A seller is a winner if her channel is assigned to at least one winning buyers. A winning seller i will be paid q_i. A losing seller will receive no payment.

Social Welfare: In this paper, we assume that buyers' bids and sellers' reserve prices equal their true valuations for channels[1]. Through combinatorial auction, the buyers' utility is their valuation of the winning channels minus the payment to the auctioneer; the sellers' utility is the received payment from the auctioneer minus the reserve price; the auctioneer's utility is the payment from buyers minus the payment to sellers. Since the payment flow among buyers, sellers and the auctioneer, the resulting social welfare is the difference between buyers' bids and sellers' reserve prices regarding winning transactions.

Interference Graph: The key feature of spectrum allocation is interference-restricted spacial reuse. To characterize interference heterogeneity of different channels, we construct a series of interference graphs $\{G^i = (U, E^i)\}_{i=1}^{M}$, in which each node $u \in U$ represents a buyer, and each edge $e^i \in E^i$ connects a pair of interfering buyers on channel i. If two virtual buyers originate from the same buyer, they are viewed as interfering buyers, since they should not be assigned the same channel. Let $e^i_{j,j'} \in \{0, 1\}$ denote the interference status between buyers j and j' regarding channel i.

III. Combinatorial Auction with Reserve Prices

In this section, we present a combinatorial spectrum auction with reserve prices. The auction mechanism takes the channel request $S = (S_1, S_2, ..., S_n)$, bids $B = (B_1, B_2, ..., B_n)$ and reserve price $R = (r_1, r_2, ...r_m)$ as inputs and determines two outputs: the channel allocation and the payments. Let $x_{s_{k,j}} = 1$ donate that the channel bundle $s_{k,j}$ is allocated to the buyer j and $x_{s_{k,j}} = 0$ otherwise. The process of winner determination can be modeled as a binary problem, and the objective is to maximize the *social welfare*.

$$\max \quad \sum_{j \in N} \sum_{k=1}^{k_j} b_{k,j} \times x_{s_{k,j}} \tag{1}$$

$$\text{Subject to} \quad \sum_{k=1}^{k_j} x_{s_{k,j}} \leq 1, \forall j \in N \tag{2}$$

$$\sum_{i \in N} x_{s_{k,j}} \times b_{k,j} \geq \sum_{j \in N} p_j \tag{3}$$

$$\sum_{i \in M} q_i \geq \sum_{i \text{ is winning}} r_i \tag{4}$$

$$x_{s_{k,j}} \in \{0, 1\}, \forall s_{k,j} \in S \tag{5}$$

Constraint (2) indicates that each buyer can win at most one bundle of channels in the request channel vector. Constraints (3) and (4) conform to individual rationality: buyers pay no more than their bids and sellers get paid no less than their reserve prices.

we also need to ensure the economic-robustness of the auction. Two properties: truthfulness and individual rationality are needed to design economic-robust double auctions. We define the these economic properties in our combinatorial double auctions:

[1]In this paper, we make the first attempt to address the problem of reserve price in combinatorial auction. In the future, we will analyze truthfulness, regarding both buyers' bids and sellers' reserve prices.

(1) **Truthfulness**. A double multi-seller auction is truthful if no seller *m* or buyer *n* can improve its own utility by bidding untruthfully no matter how other players bid. Truthfulness is essential to avoid market manipulation and ensure auction fairness and efficiency. In truthful auctions, the dominate strategy for bidders is to bid truthfully, eliminating the fear of market manipulation and the overhead of strategizing over others. With the true valuation, the auctioneer can allocate spectrum efficiently to buyers who value it the most.

(2)**Individual Rationality**. A double multi-seller auction is individual rational if no winning seller is paid less than its bid and no winning buyer pays more than its bid. This property guarantees non-negative utilities for bidders who bid truthfully, providing them incentives to participate.

Unfortunately, it is well known that optimal allocation in combinatorial auction is NP-hard. To deal with computational intractability, we propose a greedy algorithm for channel allocation to achieve efficient social welfare. The proposed combinatorial auction algorithm consists of three major parts: virtual bid calculation, channel assignment and price determination.

1) Virtual bid calculation: The key idea of virtual bid calculation is to incorporate reserve prices into buyers' bids. The same bid for a channel with high reserve price and a channel with low reserve price should have different chances of winning. Due to spectrum reusability, the same channel can be allocated to multiple non-interfering buyers, who can share the reserve price. To find non-interfering buyers is equivalent to finding independent set on the interference graph. Therefore, for buyer j whose requested bundles contains channel i, we first compute the size $n_{i,j}$ of the maximum independent set involving buyer j on the interference graph of G^i; then, we assume that buyer j will share $1/n_{i,j}$ of channel i's reserve price[2]; finally, we subtract the share of reserve price from buyer j's bid.

Toy Example. As shown in Fig.1, there are three channels and three buyers. Table I shows the requested bundles of each buyer and the corresponding bids before and after running Algorithm 1. For example, buyer 1 submits a vector of six requested channel bundles $S_1 = (\{i_1\}, \{i_2\}, \{i_3,\}, \{i_1, i_2\}, \{i_1, i_3\}, \{i_2, i_3\})$ and their corresponding bids $B_1 = (3, 4, 5, 9, 11, 12)$. The three conflict graphs show the interference relationship among buyers on three heterogeneous channels i_1, i_2 and i_3 and the channels have corresponding reserve prices $R = (3, 4, 5)$. The updated request bundles are showed in the lower part of Table I .

2) Channel assignment: As shown in Algorithm 2, firstly, we sort all the requested bundles according to their average virtual bids in non-decreasing order. The average virtual bid of bundle $s_{k,j}$ is the bid for the bundle $b_{k,j}$ divided by the number of channels in the bundle $|s_{k,j}|$. Then, we sequentially assign the bundle with the highest average vitual bid to its bidder. After assigning each bundle, we eliminate all the other bundles of the same bidder, since each buyer will only be granted one bundle. We also remove all the bundles that violates the interference constraint. For example, if $s_{k,j}$ is the selected bundle in the current iteration, we will first delete all the other bundles in B_j. Then, for each channel $i \in s_{k,j}$, we find buyer j's interfering neighbors on interference graph G^i, and check their vectors of requested bundles. If j' interferes with j, and $i \in s_{k',j'}$, we will remove $s_{k',j'}$. Such iteration will continue until there are no requested bundles.

Algorithm 1 Virtual Bid Calculation

Input: Interference graphs $G^i, i \in \mathcal{M}$, channel request vectors $S = (S_1, S_2, ..., S_n)$ and corresponding bid vectors $B = (B_1, B_2, ..., B_n)$, reserve price $R = (r_1, r_2, ..., r_m)$.
Output: Virtual bid vectors $B = (B_1, B_2, ...B_n)$.

```
for all j ∈ N do
    for all Channel i ∈ S_j do
        Get the size of the maximum independent set which
        contains buyer j on interference graph G^i as n_{i,j}.
    end for
    for all s_{k,j} ∈ S_j do
        b_{k,j} = b_{k,j} − Σ_{i∈s_{k,j}} r_i/n_{i,j}.
    end for
end for
```

Algorithm 2 Channel Assignment

Input: Virtual bid $B = (B_1, B_2, ...B_n)$, interference graphs $G^i, i \in \mathcal{M}$, channel request vectors $S = (S_1, S_2, ..., S_n)$.
Output: A channel assignment.

```
for all b_{k,j} ∈ B do
    b_{k,j} = b_{k,j}/|s_{k,j}|.
end for
Sort all b_{k,j} ∈ B in non-decreasing order, resulting in
bundle list L.
while L is non-empty do
    Assign the first bundle b_{k,j} in L to buyer j.
    Remove all bids from the same buyer b_{k',j} ∈ B_j, k' ≠ k
    from list L.
    for all Channel i ∈ b_{k,j} do
        Find buyer j's set of interfering neighbors A on
        channel i.
        for all Bundle b_{k',j'}, j' ∈ A do
            if i ∈ b_{k',j'} then
                Remove b_{k',j'} from list L.
            end if
        end for
    end for
end while
```

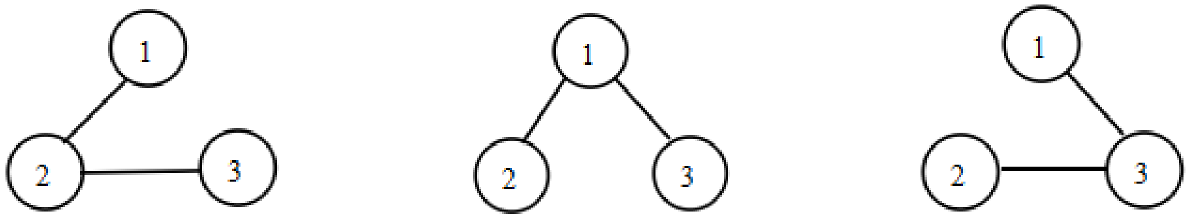
3) Price determination: In price determination step, a buyer j pays the amount according to a critical price $v^c_{k,j}$ such that: if buyer j bids higher than $v^c_{k,j}$ it wins and if j bids lower than $v^c_{k,j}$ it loses. The critical price can be determined by the following Algorithm 3. Intuitively, given other buyers' requests and bids, the algorithm finds the first buyer who makes buyer j's demand unsatisfied.

After the critical price calculation by Algorithm 3, if the

[2]It is hard to determine how many winning buyers will actually share the reserve price. To use the size of the maximum independent set is a conservative estimation. We will explore other approaches to address this problem in our future work.

TABLE I: A Toy Example

Buyer	$(S_{i,1},B_{i,1})$	$(S_{i,2},B_{i,2})$	$(S_{i,3},B_{i,3})$	$(S_{i,4},B_{i,4})$	$(S_{i,5},B_{i,5})$	$(S_{i,6},B_{i,6})$
1	$\{\{i_1\},3\}$	$\{\{i_2\},4\}$	$\{\{i_3\},5\}$	$\{\{\{i_1,i_2\},9\}$	$\{\{i_1,i_3\},11\}$	$\{\{i_2,i_3\},12\}$
2	$\{\{i_1\},3\}$	$\{\{i_2\},5\}$	$\{\{i_3\},3\}$	$\{\{i_2,i_3\},10\}$	$\emptyset$	$\emptyset$
3	$\{\{i_1\},4\}$	$\{\{i_2\},4\}$	$\{\{i_3\},5\}$	$\{\{i_1,i_2\},10\}$	$\{\{i_1,i_3\},10\}$	$\emptyset$
Buyer	$(S_{i,1},B_{i,1})$	$(S_{i,2},B_{i,2})$	$(S_{i,3},B_{i,3})$	$(S_{i,4},B_{i,4})$	$(S_{i,5},B_{i,5})$	$(S_{i,6},B_{i,6})$
1	$\{\{i_1\},1.5\}$	$\{\{i_2\},0\}$	$\{\{i_3\},2.5\}$	$\{\{\{i_1,i_2\},3.5\}$	$\{\{i_1,i_3\},7\}$	$\{\{i_2,i_3\},4.5\}$
2	$\{\{i_1\},0\}$	$\{\{i_2\},3\}$	$\{\{i_3\},0.5\}$	$\{\{i_2,i_3\},5.5\}$	$\emptyset$	$\emptyset$
3	$\{\{i_1\},2.5\}$	$\{\{i_2\},2\}$	$\{\{i_3\},0\}$	$\{\{i_1,i_2\},6.5\}$	$\{\{i_1,i_3\},3.5\}$	$\emptyset$



(a) (b) (c)

Fig. 1: Interference graph on channel (a) i_1; (b) i_2; (c) i_3.

total bids for all winning bundles is greater than the sum of reserve prices of all channels in the winning bundles, the entire transaction is successful. For example,if $s_{k,j}$ is a winning bundle, the auctioneer will charge buyer j the price $p_j = v^c_{k,j}$. Let β denote the total bids for all winning bundles. A seller is a winner if her channel is in at least one winning bundle. The auctioneer will pay each winning seller the price according to their reserve prices. Suppose that seller i is a winner, then she will receive a price of $q_i = \beta r_i / \sum_{i' \in \mathcal{M}} r_{i'}$.

Algorithm 3 Critical Price Determination

Input: Virtual bid $B = (B_1, B_2, ...B_n)$, interference graphs $G^i, i \in \mathcal{M}$, channel request vectors $S = (S_1, S_2, ..., S_n)$.
Output: Critical price for winner j

```
Delete buyer j's bid B_j and request S_j from B and S
for all b_{k,i} ∈ B do
    b_{k,i} = b_{k,i}/|s_{k,i}|.
end for
Sort all b_{k,i} ∈ B in non-decreasing order, resulting in bundle list L.
while L is non-empty do
    if Buyer i,j interfere in G^i then
        v^c_{k,j} = b_{k,i}, b_{k,i} is the first bundle bid in L
        Remove b_{k,i} from list L.
    else
        Remove b_{k,i} from list L.
        Check next b_{k',i'} according to list L.
    end if
end while
```

IV. Analysis

Proposition 1. *The auction is truthful in that each buyer maximizes its utility by submitting its truthful valuation.*

Proof. Let v_j be the truthful bidding, and b_j the bidding such that $b_j \neq v_j$.

We discuss the problem in following two cases:

In the first case, the buyer j gets utility $u_j(b_j) = v_j - p_j \geq 0$ when bidding truthfully, with $u_j(b_j) = v_j - p_j \geq 0$ for wining the bundle $s_{k,j}$. If buyer j treats and wins bundle $\widetilde{s}_{k,j}$ by bidding b_j, then its utility $\widetilde{u}_j(b_j)$ remains the same since the payment of critical price is the same:

$$\widetilde{u}_j = v_j - p_j = v_j - v^c_{k,j} \tag{6}$$

If the buyer j loses the auction when cheating in the bid ,$\widetilde{u}_j = 0$which is not better than bidding truthfully.

In the second case, The buyer j loses in the auction when bidding truthfully. Then its utility $u_j = 0$. If the buyer also loses when bidding untruthfully, the utilities are the same. But now we consider the case, in which the buyer wins a bundle $\widetilde{s}_{k,j} \neq \emptyset$ by cheating the bid b_j. Then we have $v_j \leq p_j \leq b_j$, because otherwise, she still cannot win any bundle.

$$\widetilde{u}_j = v_j - p_j = v_j - v^c_{k,j} \leq v_j - b_j = 0 \tag{7}$$

In both cases, bidding other than truthful valuation is not better than bidding truth valuation.

□

V. Simulation

A. Simulation Settings

We assume that buyers are randomly located in a 10×10 m area with the transmission range of each channel being randomly chosen in the range (0,5]m. The interference graph of each channel is set based on the channel's transmission range and buyers' locations. Buyers' offered bids are independently distributed following a uniform distribution in (0,10]. The maximum size of requested channel bundle is limited to 3.

We first implement the proposed algorithm in simulation scenarios with specified numbers of channels with various average reserve price. The number of buyers is set to one of 12 and 16, respectively and numbers of channels is 6.

Then we compare the performance of our proposed algorithm to the case without reserve price consideration. The number of buyers increases from 12 to 60 with increment of 6 while the number of channels is set to one 6.

Two metrics: *Social Welfare* and *Utilized Channel* are evaluated in the simulation. *Social Welfare* is the sum of winning buyers prices offered to their allocated bundles of channels. *Channel Utilized* is the channels matched to buyers.

B. Performance

We compare the performance of the proposed combinatorial auction algorithm with the existing combinatorial auction without consideration of the reserve price.

It is verified by Fig. 2(a), that the proposed algorithm with reserve prices outperforms the existing algorithm without reserve prices. Social welfare is improved. when the buyer number is large. As the number of buyers increases, social welfare goes up, since more buyers benefit from the channel transaction.

As shown in Fig. 2(b)(d), social welfare decreases as the average reserve price increases, because sellers ask for more compensation for their channels, and buyers have a lower chance of getting desirable channels. Correspondingly, channel utilization decreases, as shown in Fig. 2(c)(e) since there are fewer traded channels. As the number of buyers increases from 12 to 16, both social welfare and channel utilization increase as shown in Fig. 2(b)(c), for the same reason as our explanation for Fig. 2(a). As the number of available channels increases from 6 to 10, both social welfare and channel utilization also increase as shown in Fig. 2(d)(e). This is because buyers have more chances to obtain desirable channels and benefit from spectrum transaction.

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VI. Related Work

A. Channel Allocation

Many works on combinatorial auction have been studied in recent years. Some works deal with one-seller-multi-buyer forward auction . In [4] the authors propose auction mechanism without consideration of spectrum reusability. In [5], [6], the authors consider spatial reuseability of channel, while they ignore that the trading channels are heterogeneous and therefore the set of interfering neighbors is frequency dependent.

Some works copy with problems of multi-seller-multi-buyer double auction. [7] designs a truthful double spectrum auction while it limits that each seller or buyer to trade only one channel. In [8], the diversity of reserve prices and bids are considered but not the influence of channel heterogeneity on interference graph. The authors in [9] proposes an auction mechanism for heterogeneous spectrum. However, it is restricted that each buyer can only bid for one spectrum. Whats more, the auction in [9] will become untruthful when extended to multi-demand heterogeneous spectrum auction. Besides, there are some other related works on channel auction, such as works that targets at revenue maximization instead of truthfulness [10], [11] and works used interference temperature instead of interference graph [12], [13].

B. Combinatorial Auction

A number of works on combinatorial auction have been proposed during the last decades. However few of these combinatorial auction considers the spectrum spatial reusability. In [14]- [15], the authors proved that in general combinatorial auctions, achieving optimal social welfare and ensuring strategy-proofness cannot be obtained at the same time. [16] argues that there is no payment scheme to make greedy allocation algorithm strategy-proof in general combinatorial auctions. In [17]- [19], a number of auction mechanisms with well bounded approximation ratios were proposed considering the intractability of combinatorial auction.

VII. Conclusion and Future Directions

In this paper, we consider the problem of combinatorial spectrum auction with multiple sellers who impose reserve prices as the lowest price to sell their channels. Each buyer submits a vector of requested channel bundles and the corresponding bid for each bundle. In this way, we allow buyers to express their preferences for different combinations of channels. To account for heterogeneous interference relationships of all channels, we construct different interference graphs to determine spatial reuse. We propose a greedy algorithm for winner determination in the combinatorial spectrum auction. To begin with, we compute the virtual bid of each bundle by subtracting a fraction of reserve price from the real bid. Then, we assign channels to buyers according the average virtual bid and the interferent contraint. The simulation results show that our proposed auction mechanism can achieve higher social welfare and channel utilization compared with existing auction mechanisms without reserve prices.

There are various future directions. The price determination can be improve to achieve strategy-proofness. Since there are multiple sellers, not only the truthfulness of buyers but the truthfulness of sellers should be considered. Another possible direction is to compute virtual bid in different ways. In this paper, we conservatively use the size of the maximum independent set as the number of buyers who will share the reserve price, which is not necessary the case. Better ways of calculating virtual bids can be explored.

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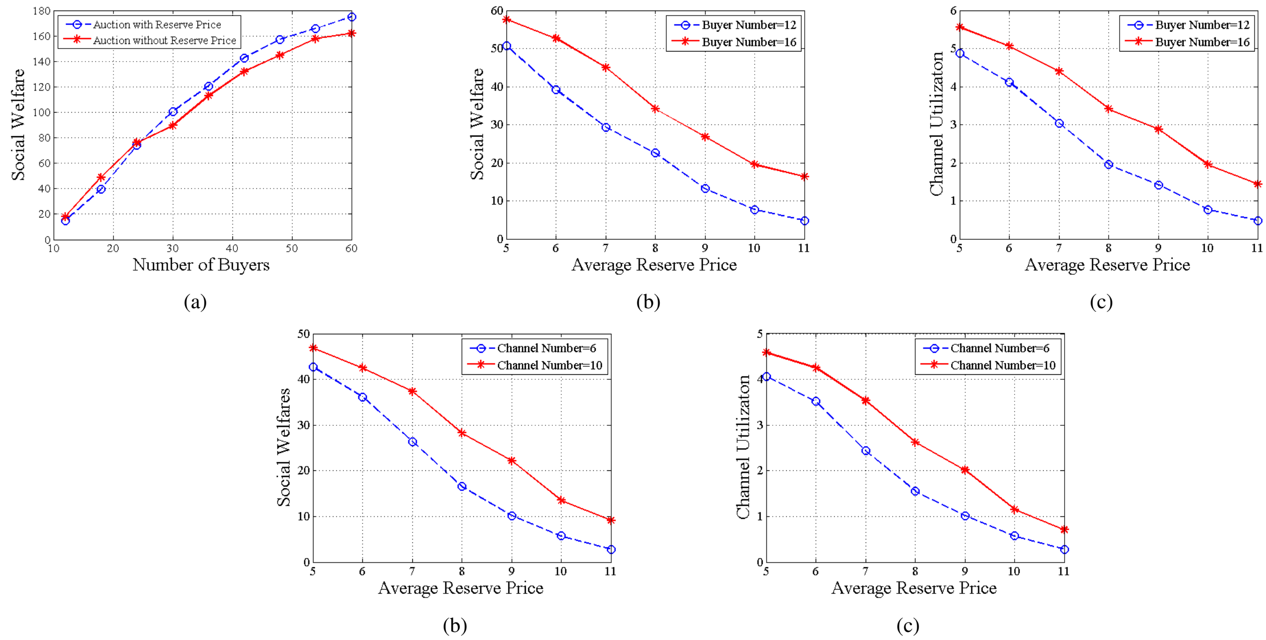

Fig. 2: Performance of the proposed combinatorial auction algorithm

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