Auctions

Episode 8

Baochun Li
Professor
Department of Electrical and Computer Engineering
University of Toronto
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Paying Per Click

- Ads in Google’s sponsored links are based on a **cost-per-click** model

  Advertisers only pay when a user actually clicks on the ad

- The amount that advertisers are willing to pay per click is often surprisingly high

  To occupy the most prominent spot for “calligraphy pens” costs about $1.70 per click

  For some queries, the cost per click can be stratospheric — $50 or more for a query on “mortgage refinancing”!
But how does a search engine set the prices per click for different queries? — it would be difficult to set these prices with so many keywords!
Auctions
(Chapter 9.1 — 9.6)
Let’s first focus on a few simple types of auctions, and see how they promote different kinds of behaviour among bidders.
Simple auctions

- Consider the case of a seller auctioning off one item to a set of buyers.

- Assumption: a bidder has an intrinsic value for the item being auctioned.

  She is willing to purchase the item for a price up to this value, but no higher.

  Also called the bidder’s true value for this item.
Four main types of auctions

- **Ascending-bid auctions (English auctions)**
  
  Carried out interactively in real-time
  
  The seller gradually raise the price
  
  Bidders drop out until one bidder remains — the winner at this final price
Four main types of auctions

- **Descending-bid auctions (Dutch auctions)**
  
  Carried out interactively in real-time
  
  Seller gradually lowers the price from some high initial value
  
  until the first moment when some bidder accepts and pays the current price
Four main types of auctions

- First-price sealed-bid auctions

  Bidders submit simultaneously “sealed bids” to the seller.
  The highest bidder wins the object and pays the value of her bid.
Four main types of auctions

- **Second-price sealed-bid** auctions (Vickrey auctions)
  
  Bidders submit simultaneous sealed bids to the sellers.
  
  The highest bidder wins the object and pays the value of the second-highest bid.
  
  William Vickrey, who proposed this type of auctions, were the first to analyze auctions with game theory (1961).
When are auctions appropriate?

- Auctions are generally used by sellers in situations where they do not have a good estimate of the buyers’ true values for an item, and where buyers do not know each other’s values.

  If the intrinsic value of the buyer is known, there’s no need for auctions.

  The seller (or the buyer) simply commit to a fixed price that is just below the intrinsic value of the buyer (or just above that of the seller).
The goal of auctions

- The goal of auctions is to elicit bids from buyers that reveal these values

  Assuming that the buyers have independent, private, true values for the item
Descending-Bid and First-Price Auctions

- In a descending-bid auction —

  As the seller lowers the price from its high initial starting point, no bidder says anything until finally someone actually accepts the bid and pays the current price.

  Bidders learn nothing while the auction is running, other than the fact that no one has accepted the current price yet.

  For each bidder $i$, there’s a first price $b_i$ at which she would be willing to break the silence and accept the item at price $b_i$.

- It is equivalent to a sealed-bid first-price auction: this price $b_i$ plays the role of bidder $i$’s bid.

  The item goes to the bidder with the highest bid value, and this bidder pays the value of her bid in exchange for the item.
In an ascending-bid auction —

Bidders gradually drop out as the seller steadily raises the price.
The winner of the auction is the last bidder remaining, and she pays the price at which the second-to-last bidder drops out.
For a bidder, it doesn’t make sense to stay after the price exceeds her true (intrinsic and private) value.
Or to leave before the current price reaches her true value.

A bidder stays in an ascending-bid auction up to the exact moment when the current price reaches her true value.
The item goes to the highest bidder at a price equal to the second-highest bid.
This is precisely the rule used in sealed-bid second-price auctions.
Second-Price Auctions

▫ **Main result**: With independent, private values, bidding your true value is a *dominant strategy* in a second-price sealed-bid auction

That is, the best choice of bid is exactly what the object is worth to you

▫ To show this, we need to formulate the second-price auction as a *game*

Bidders correspond to *players*

Let $v_i$ be bidder $i$’s true value for the object

Bidder $i$’s strategy is an amount $b_i$ to bid as a function of her true value $v_i$
Truthful bidding in second-price auctions

- The payoff to bidder $i$ with value $v_i$ and bid $b_i$ is defined as follows:

  If $b_i$ is not the winning bid, then the payoff to $i$ is 0. If $b_i$ is the winning bid, and some other $b_j$ is the second-place bid, then the payoff to $i$ is $v_i - b_j$.

- Claim: In a sealed-bid second-price auction, it is a dominant strategy for each bidder $i$ to choose a bid $b_i = v_i$. 
Proving the claim

- We need to show that if bidder $i$ bids $b_i = v_i$, then no deviation from this bid would improve her payoff, regardless of which strategy everyone else is using.

- Two cases to consider: deviations in which $i$ raises her bid, and deviations in which $i$ lowers her bid.

- In both cases, the value of $i$’s bid only affects whether $i$ wins or loses, but it never affects how much $i$ pays in the event that she wins.

  which is determined entirely by the other bids.
Deviating by raising or lowering her bid

Alternate bid \( b_i' \)

Raised bid affects outcome only if highest other bid \( b_j \) is in between.
If so, \( i \) wins but pays more than value.

Truthful bid \( b_i = v_i \)

Alternate bid \( b_i'' \)

Lowered bid affects outcome only if highest other bid \( b_k \) is in between.
If so, \( i \) loses when it was possible to win with non-negative payoff.

This completes the argument that truthful bidding is a dominant strategy in a sealed-bid second-price auction. The heart of the argument is the fact noted at the outset: in a second-price auction, your bid determines whether you win or lose, but not how much you pay in the event that you win. Therefore, you need to evaluate changes to your bid in light of this fact. This also further highlights the parallels to the ascending-bid auction. There too, the analogue of your bid – the point up to which you’re willing to stay in the auction – determines whether you’ll stay in long enough to win; however, the amount you pay in the event that you win is determined by the point at which the second-place bidder drops out.

The fact that truthfulness is a dominant strategy also makes second-price auctions conceptually very clean. Because truthful bidding is a dominant strategy, it is the best thing to do regardless of what the other bidders are doing. So, in a second-price
First-price auctions

- The payoff to bidder $i$ with value $v_i$ and bid $b_i$ is defined as follows:

  If $b_i$ is not the winning bid, then the payoff to $i$ is 0. If $b_i$ is the winning bid, then the payoff to $i$ is $v_i - b_i$.

- Bidding your true value is no longer a dominant strategy!
  A payoff of 0 if you lose (as usual), and a payoff of 0 if you win, too

- The optimal way to bid is to “shade” your bid slightly downward, in order to get a positive payoff if you win
  - If it’s too close to the true value, your payoff won’t be large if you win
  - If it’s too far below, you reduce the chance of winning