Games

Episode 6
Part III: “Dynamics”

Baochun Li
Professor
Department of Electrical and Computer Engineering
University of Toronto
“Dynamics” — Motivation for a new “chapter”

- Sending packets through the Internet —
  What are the design principles to make this happen?
  How do we make it **fair** to all best-effort connections?
  How do we support **performance guarantees** to those who need them?

- But intuitively, those who wish to have more “priorities,” “weights,” or “guarantees” need to, somehow, pay a price!

- **But how?**
This involves game-theoretic reasoning!

- All “peers” in a network make their individual decisions to maximize their own benefits
  
  A BitTorrent “peer” may be the best example

- To make it more general —

  Rather than simply choosing a route in isolation, individual sender/receiver pairs can evaluate routes in the presence of the congestion resulting from the decisions made by themselves and everyone else.

  In this “chapter” of the course, we try to develop models for network traffic using game-theoretic ideas.

  And show that adding capacity can sometimes **slow down** the traffic on a network!
Viewing networks from a different perspective

- Traditionally, we view networks from the perspective of its underlying structure and architecture.
- Now, we switch to a look at an interdependence in the behaviour of the individuals who inhabit the system.
  The outcome for any one depends on the combined behavior of all.
- Such interconnectedness at the level of behaviour can be studied in the language of game theory.
Another example: Google

96% of the revenue ($22 billion in a quarter) is derived from advertising
Jewelry - Timeless Creations with Crystals - swarovski.com
Ad www.swarovski.com/Jewelry ▼
Shop Swarovski.com Today!
Product Warranty · Free Shipping from $120 · Secure Online Payment · Free Customer Help
Types: Necklaces, Bracelets, Rings, Pendants, Jewelry Sets, Figurines, Watches
📍 2 Bloor Street West - (416) 850-6072 - Open today · 10:00 AM – 8:00 PM ▼

Jewelry - Toronto's Best Custom Jeweller - Randor.com
Ad www.randor.com/Toronto ▼
We Make Your Dream Ring a Reality!
In Business Since 1988 · Book A Consultation
Diamond Education Centre · Women's Wedding Bands · Loose Diamond Listings
📍 27 Queen Street East #605, Toronto, ON - Open today · 10:00 AM – 5:00 PM ▼

Jewelry Rings - Peoplesjewellers.com
Ad www.peoplesjewellers.com/Rings ▼
Declare Your Diamond Kind of Love and Shop Jewellery at Peoples.
Types: Diamond, Birthstone, Amethyst, Blue Topaz, Aquamarine…
Clearance 50% + 10% Off · Arctic Brilliance Jewelry · Vera Wang Love Collection
📍 220 Yonge St, Toronto - (416) 977-8466 - Open today · 10:00 AM – 9:30 PM ▼
Adwords: keyword-based advertising
How does Google decide how much to charge for each ad?
To understand how ads are being priced, we need to understand the fundamentals of auctions.
To understand auctions, again, we need to understand the fundamentals of games.
Textbook
Networks, Crowds, and Markets

(D. Easley and J. Kleinberg, Cambridge University Press, July 2010)

Starting from Chapter 6

Freely downloadable from:
What is a **game**? — A first example

- Suppose you are a college student
- Two pieces of work due tomorrow: an exam and a presentation
What is a game? — A first example

- You need to decide: study for the exam or prepare for the presentation?
  - Assumption 1: You don’t have time to do both
  - Assumption 2: You can accurately estimate the grade
- Exam: 92 if you study, 80 if you don’t
- Presentation: You need to do it with a partner
  - If both of you prepare for it, both get 100
  - If one of you prepares, both get 92
  - If neither of you prepares, both get 84
Basic ingredients of a game

- There is a set of participants, called **players**
  - You and your partner
- Each player has a set of options for how to behave, referred to as the player’s possible **strategies**
  - “Study for the exam” or “prepare for the presentation”
- For each choice of strategies, each player receives a **payoff**
  - The average grade you get on the exam and the presentation
How do players select their strategies?

A few simplifying assumptions —

Everything the player cares about is summarized in the player’s payoffs.

Each player knows everything about the structure of the game:
- his own list of strategies
- who the other player is
- the strategies available to the other player
- who her payoff will be for any choice of strategies

Each player chooses a strategy to maximize his/her own payoff, given his beliefs about the strategy used by the other player — this is called rationality, and it implicitly includes two ideas:
- each wants to maximize payoff
- each player actually succeeds in selecting the optimal strategy.
Exam or presentation?
Exam or presentation?

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<tr>
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<th>Your Partner</th>
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</thead>
<tbody>
<tr>
<td><strong>Presentation</strong></td>
<td>90, 90</td>
</tr>
<tr>
<td><strong>Exam</strong></td>
<td>92, 86</td>
</tr>
</tbody>
</table>

**Figure 6.1.** Exam or Presentation?
Strictly dominant strategy

A player has a strategy that is strictly better than all other options, regardless of what the other player does.

In our example, studying for the exam is the strictly dominant strategy.

A player will definitely play the strictly dominant strategy.

This will be the outcome of the game.

There is something striking about this easy solution —

If you and your partner could somehow agree that you would both prepare for the presentation, you will each get 90 as an average, and be better off.

But, despite that both of you understand this, the payoff of 90 cannot be achieved by rational play of this game! — why?
A related story: the Prisoner’s Dilemma
A related story: the Prisoner’s Dilemma

<table>
<thead>
<tr>
<th></th>
<th>Suspect 1</th>
<th></th>
<th>Suspect 2</th>
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<tbody>
<tr>
<td></td>
<td><strong>NC</strong></td>
<td><strong>C</strong></td>
<td></td>
</tr>
<tr>
<td>Suspect 1</td>
<td>-1, -1</td>
<td>-10, 0</td>
<td></td>
</tr>
<tr>
<td>Suspect 2</td>
<td>0, -10</td>
<td>-4, -4</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 6.2.** Prisoner’s Dilemma
The “arms race” between competitors
The “arms race” between competitors

![Decision Matrix for Performance-Enhancing Drugs](image)

**Figure 6.3.** Performance-Enhancing Drugs
Best responses

- If \( S \) is a strategy chosen by Player 1, and \( T \) is a strategy chosen by Player 2

- \( P_1(S, T) \) denotes the payoff to Player 1 as a result of this pair of strategies (written in the payoff matrix in previous examples)

- A strategy \( S \) for Player 1 is a best response to a strategy \( T \) for Player 2, if \( S \) produces at least as good a payoff as any other strategy paired with \( T \): \( P_1(S, T) \geq P_1(S', T) \)

- It is a strict best response if: \( P_1(S, T) > P_1(S', T) \)
Dominant strategies

- We say that a **dominant strategy** for Player 1 is a strategy that is a **best response** to every strategy of Player 2.

- We say that a **strictly dominant strategy** for Player 1 is a strategy that is a **strict best response** to every strategy of Player 2.

- In the Prisoner’s Dilemma, both players had strictly dominant strategies.
  
  But this is not always the case!
The game of the marketing strategies

- People who prefer a low-priced version account for 60% of the population, and people who prefer an upscale version account for 40% of the population.
- If a firm is the only one to produce a product for a given market segment, it gets all the sales.
- Firm 1 is the much more popular brand, and so when the two firms directly compete in a market segment, Firm 1 gets 80% of the sales and Firm 2 gets 20% of the sales.
Only one player has a strictly dominant strategy.

**Figure 6.5.** Marketing Strategy

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Low-Priced</th>
<th>Upscale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-Priced</td>
<td>.48, .12</td>
<td>.60, .40</td>
</tr>
<tr>
<td>Upscale</td>
<td>.40, .60</td>
<td>.32, .08</td>
</tr>
</tbody>
</table>

Assumption: the players have common knowledge about the game: they know its structure, they know that each of them knows its structure, and so on.
What if neither player has a strictly dominant strategy?

- Two firms and three clients: A, B and C

  If the two firms approach the same client, the client will give half its business to each.

  Firm 1 is too small to attract clients on its own, so if it approaches one client while Firm 2 approaches a different one, then Firm 1 gets a payoff of 0.

  If Firm 2 approaches client B or C on its own, it will get their full business. However, A is a larger client, and will only do business with both firms.

  Because A is a large client, doing business with it is worth 8, whereas doing business with B or C is worth 2.
The three-client game
The three-client game

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th></th>
<th></th>
<th>Firm 2</th>
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<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>4, 4</td>
<td>0, 2</td>
<td>0, 2</td>
<td></td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>0, 0</td>
<td>1, 1</td>
<td>0, 2</td>
<td></td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>0, 0</td>
<td>0, 2</td>
<td>1, 1</td>
<td></td>
</tr>
</tbody>
</table>

Neither player has a **strictly dominant strategy**.

Figure 6.6. Three-Client Game
The main idea of the Nash Equilibrium is: even when there are no dominant strategies, we should expect players to use strategies that are best responses to each other.
Nash Equilibrium

- Suppose that Player 1 chooses a strategy \( S \) and Player 2 chooses a strategy \( T \).
- We say that this pair of strategies, \( (S, T) \), is a Nash equilibrium if \( S \) is a best response to \( T \), and \( T \) is a best response to \( S \).
- This concept is an equilibrium concept:
  - If the players choose strategies that are best responses to each other, then no player has an incentive to deviate to an alternative strategy.
  - The system is in an equilibrium state, with no force pushing it toward a different outcome.
- The only Nash equilibrium in the example: \( (A, A) \).
Multiple Equilibria: a coordination game

**Figure 6.7. Coordination Game**

<table>
<thead>
<tr>
<th>Your Partner</th>
<th>PowerPoint</th>
<th>Keynote</th>
</tr>
</thead>
<tbody>
<tr>
<td>PowerPoint</td>
<td>1, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>Keynote</td>
<td>0, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>
Multiple Equilibria: a coordination game

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<tr>
<th></th>
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<tbody>
<tr>
<td>PowerPoint</td>
<td>1, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>Keynote</td>
<td>0, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

**Figure 6.7. Coordination Game**

Two Nash equilibria: (PowerPoint, PowerPoint) and (Keynote, Keynote)
An unbalanced coordination game

Still two Nash equilibria: (PowerPoint, PowerPoint) and (Keynote, Keynote)

But both may choose Keynote, as strategies to reach the equilibrium that gives higher payoffs to both will be selected.

Figure 6.8. Unbalanced Coordination Game
What if you don’t agree with your partner?

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<tr>
<th>Your Partner</th>
<th>PowerPoint</th>
<th>Keynote</th>
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</thead>
<tbody>
<tr>
<td>You</td>
<td>1, 2</td>
<td>0, 0</td>
</tr>
<tr>
<td>PowerPoint</td>
<td>0, 0</td>
<td>2, 1</td>
</tr>
</tbody>
</table>

**Figure 6.9.** Battle of the Sexes
**Multiple Equilibria: The Hawk-Dove Game**

<table>
<thead>
<tr>
<th></th>
<th>Animal 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Animal 1</strong></td>
<td>D</td>
</tr>
<tr>
<td>D</td>
<td>3, 3</td>
</tr>
<tr>
<td>H</td>
<td>5, 1</td>
</tr>
</tbody>
</table>

**Figure 6.12.** Hawk-Dove Game
Multiple Equilibria: The Hawk-Dove Game

- Two Nash equilibria: (D, H) and (H, D)
- The concept of Nash equilibrium helps to narrow down the set of reasonable predictions, but it does not provide a unique prediction!

$$\begin{array}{ccc}
\text{Animal 1} & D & H \\
\hline
D & 3, 3 & 1, 5 \\
H & 5, 1 & 0, 0 \\
\end{array}$$

**Figure 6.12.** Hawk-Dove Game
There is no Nash equilibrium for this game, if we treat each player as simply having the two strategies, H or T!

In real life, players try to make it hard for their opponents to predict what they will play — randomization.

**Figure 6.14. Matching Pennies**
Mixed strategies

- Each player chooses a probability $p$ ($q$) with which he or she will play H (and $1 - p$ ($1 - q$) for T)

- We now changed the game to allow a set of strategies corresponding to the interval of numbers between 0 and 1 — mixed strategies

The previous examples show pure strategies

- But how do we evaluate the payoffs?
The expected value of the payoff

- If Player 1 chooses the pure strategy H while Player 2 chooses a probability of $q$ (to play H), as before, then the expected payoff to Player 1 is $(-1)(q) + (1)(1 - q) = 1 - 2q$.

- Similarly, if Player 1 chooses the pure strategy T while Player 2 chooses a probability of $q$, then the expected payoff to Player 1 is $(1)(q) + (-1)(1 - q) = 2q - 1$.

- We assume that each player is seeking to maximize his expected payoff from the choice of a mixed strategy.

- The definition of Nash equilibrium for the mixed strategy version remains the same.

  The pair of strategies is now $(p, q)$. 
Revisiting the matching pennies game

- No pure strategies can be part of a Nash equilibrium — why?
- What is Player 1’s best response to strategy q used by Player 2?
- If $1 - 2q \neq 2q - 1$
  - then one of the pure strategies H or T is in fact the unique best response by Player 1 to a play of q by Player 2
  because one of $(1 - 2q)$ or $(2q - 1)$ is larger in this case, and so there is no point for Player 1 to put any probability on her weaker pure strategy
  But we just said pure strategies cannot be part of a Nash equilibrium!
- So we must have $1 - 2q = 2q - 1$
  $(0.5, 0.5)$ is the unique Nash equilibrium for the game
Can a game have both mixed and pure-strategy equilibria?

- You will be indifferent between PowerPoint and Keynote if
  \[ (1)(q) + (0)(1 - q) = (0)(q) + (2)(1 - q) \]

- Each of you chooses PowerPoint with probability 2/3!

<table>
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<tr>
<td>PowerPoint</td>
<td>1, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>Keynote</td>
<td>0, 0</td>
<td>2, 2</td>
</tr>
</tbody>
</table>

**Figure 6.17.** Unbalanced coordination game.
What’s good for the society?

- In a Nash equilibrium, each player’s strategy is a **best response** to the other player’s strategy — they optimize **individually** but we have shown that, as a group, the outcome may not be the best.

- We wish to classify outcomes in a game by whether they are “**good for society**” but we need a precise definition of what we mean by this!
A choice of strategies — one by each player — is **Pareto-optimal** if there is no other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff.
Which choice of strategies is Pareto optimal?

Players can construct a binding agreement to play the superior pair of strategies — “cooperative” vs. “non-cooperative” games.

But without the binding agreement, one player would want to switch, even though both realize that there exists a superior pair.

![Figure 6.1. Exam or Presentation?](image-url)

<table>
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<th>Exam</th>
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<tbody>
<tr>
<td><strong>You</strong>&lt;br&gt;<strong>Presentation</strong></td>
<td>90, 90</td>
<td>86, 92</td>
</tr>
<tr>
<td><strong>Exam</strong></td>
<td>92, 86</td>
<td>88, 88</td>
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</table>
Examples of Pareto optimality

- Players can construct a binding agreement to play the superior pair of strategies — “cooperative” vs. “non-cooperative” games

But without the binding agreement, one player would want to switch, even though both realize that there exists a superior pair

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<tbody>
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<td>90, 90</td>
<td>86, 92</td>
</tr>
<tr>
<td>Exam</td>
<td>92, 86</td>
<td>88, 88</td>
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</tbody>
</table>

**Figure 6.1.** Exam or Presentation?

Nash equilibrium
Social optimality

A choice of strategies — one by each player — is **socially optimal** if it maximizes the sum of the players’ payoffs.

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<tr>
<td><strong>Exam</strong></td>
<td>92, 86</td>
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</table>

**Figure 6.1.** Exam or Presentation?
If an outcome is socially optimal, it must be Pareto-optimal, but not the other way around.

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<th>Your Partner</th>
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<tbody>
<tr>
<td><strong>Presentation</strong></td>
<td><strong>Exam</strong></td>
</tr>
<tr>
<td>You: <strong>Presentation</strong></td>
<td>90, 90</td>
</tr>
<tr>
<td>You: <strong>Exam</strong></td>
<td>92, 86</td>
</tr>
</tbody>
</table>

**Figure 6.1.** Exam or Presentation?
Multiplayer games

- A game with $n$ players, named $1, 2, ..., n$, each with a set of possible strategies

An **outcome** (or joint strategy) is a choice of a strategy for each player.

Each player $i$ has a payoff function $P_i$ that maps outcomes of the game to a numerical payoff for $i$:

For each outcome consisting of strategies $(S_1, S_2, ..., S_n)$, there is a payoff $P_i(S_1, S_2, ..., S_n)$ to player $i$.
Multiplayer games

- A strategy $S_i$ is a best response by Player $i$ to a choice of strategies $(S_1, S_2, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n)$ by all the other players if:

$$P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n) \geq P_i(S_1, S_2, \ldots, S_{i-1}, S_i', S_{i+1}, \ldots, S_n)$$

for all other possible strategies $S_i'$ available to player $i$.

- An outcome consisting of strategies $(S_1, S_2, \ldots, S_n)$ is a Nash equilibrium if each strategy it contains is a best response to all the others.
Strictly dominated strategies

- We understand that if a player has a strictly dominant strategy, it will play it — but this is pretty rare!

- Even if a player does not have a dominant strategy, she may still have strategies that are dominated by other strategies.

A strategy is strictly dominated if there is some other strategy available to the same player that produces a strictly higher payoff in response to every choice of strategies by the other players.

Strategy $S_i$ for player $i$ is strictly dominated if there is another strategy $S'_i$ for player $i$ such that:

$$P_i(S_1, S_2, \ldots, S_{i-1}, S'_i, S_{i+1}, \ldots, S_n) > P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n)$$

for all choices of strategies $(S_1, S_2, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n)$ by the other players.

- Makes sense to study this when there are multiple strategies.
Two firms are each planning to open a store in one of six towns.

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
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<tbody>
<tr>
<td></td>
<td>B</td>
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<tr>
<td>1, 5</td>
<td>2, 4</td>
</tr>
<tr>
<td>4, 2</td>
<td>3, 3</td>
</tr>
<tr>
<td>3, 3</td>
<td>2, 4</td>
</tr>
</tbody>
</table>

The Facility Location Game: dominated strategies

- Two firms are each planning to open a store in one of six towns.
Iterated deletion of strictly dominated strategies

- With A and F eliminated, B and E becomes strictly dominated strategies!

- The outcome of the game is (C, D) — which can be proved to be a Nash equilibrium.

- Obtained by going through a process called iterated deletion of strictly dominated strategies.
Weakly dominated strategies

- A strategy is weakly dominated if there is another strategy available that does at least as well no matter what the other players do, and does strictly better against some joint strategy of the other players.

Strategy $S_i$ for player $i$ is weakly dominated if there is another strategy $S_i'$ for player $i$ such that:

$$P_i(S_1, S_2, \ldots, S_{i-1}, S_i', S_{i+1}, \ldots, S_n) \geq P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n)$$

for all choices of strategies $(S_1, S_2, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n)$ by the other players, and

$$P_i(S_1, S_2, \ldots, S_{i-1}, S_i', S_{i+1}, \ldots, S_n) > P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n)$$

for at least one choice of strategies $(S_1, S_2, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n)$ by the other players.
Deleting weakly dominated strategies

- Deleting weakly dominated strategies may destroy Nash equilibria!

**Figure 6.23.** Stag Hunt: a version with a weakly dominated strategy.

<table>
<thead>
<tr>
<th></th>
<th>Hunter 1</th>
<th>Hunter 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hunt Stag</td>
<td>✗ 3, 3</td>
<td>0, 3</td>
</tr>
<tr>
<td>Hunt Hare</td>
<td>3, 0</td>
<td>3, 3</td>
</tr>
</tbody>
</table>

Both outcomes are Nash equilibria!
Dynamic Games

- Dynamic games are games played over time: some player or set of players moves first, other players observe the choice(s) made, and then they respond
  - Negotiations that involve a sequence of offers and counteroffers
  - Bidding in an auction

- An example: Two firms decide which region they should advertise in
  - Firm 1 moves first
  - Region A is bigger with a market size of 12, Region B is smaller with 6
  - First mover advantage: it will get 2/3 of the region’s market if both firms are in the same region
  - Assumes each player knows the complete history — “perfect information”
Extensive-form representation of the game

Figure 6.24. A simple game in extensive form.

A play corresponds to a path in the tree
Conversion to normal form

"AB" means “play A if Firm 1 plays B”

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<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>8, 4</td>
<td>6, 12</td>
<td>12, 6</td>
<td>12, 6</td>
</tr>
<tr>
<td>AA,AB</td>
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<td>4, 2</td>
<td>6, 12</td>
<td>4, 2</td>
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<td>6, 12</td>
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<td>BA,AB</td>
<td>12, 6</td>
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<tr>
<td>BA,AB</td>
<td>12, 6</td>
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</table>

Firm 2

Figure 6.25. Conversion to normal form.
More complex example: The Market Entry Game

- Consider a region where Firm 2 is currently the only serious participant in a given line of business, and Firm 1 is considering whether to enter the market.

- The first move in this game is made by Firm 1, which must decide whether to stay out of the market or to enter it.

  - If Firm 1 chooses to stay out, then the game ends, with Firm 1 getting a payoff of 0 and Firm 2 keeping the payoff from the entire market.
  
  - If Firm 1 chooses to enter, then the game continues to a second move by Firm 2, who must choose whether to cooperate and divide the market evenly with Firm 1 or retaliate and engage in a price war.

    - If Firm 2 cooperates, then each firm gets a payoff corresponding to half the market.
    
    - If Firm 2 retaliates, then each firm gets a negative payoff.
Extensive-form representation of the game

Player 1

Stay Out

Enter

Player 2

0

2

Retaliate

Cooperate

Firm 1

Firm 2

RC

S

E

0

2

-1

-1

1

1

Figure 6.26.

Figure 6.27.

Normal form of the Market Entry game.
Conversion to normal form

What’s the outcome of this game?

What does this outcome correspond to?

<table>
<thead>
<tr>
<th></th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
</tr>
<tr>
<td><strong>Firm 1</strong></td>
<td><strong>S</strong></td>
</tr>
<tr>
<td>S</td>
<td>0, 2</td>
</tr>
<tr>
<td>E</td>
<td>−1, −1</td>
</tr>
</tbody>
</table>

Surprisingly, both outcomes are pure-strategy Nash equilibria!
Important points about extensive vs. normal form

- The premise behind our translation from extensive to normal form — that each player commits ahead of time to a complete plan for playing the game — is not really equivalent to our initial premise in defining dynamic games:

  that each player makes an optimal decision at each intermediate point in the game, based on what has already happened up to that point.

- In the Market Entry Game, if Firm 2 can truly “precommit” to the plan to “Retaliate,” then the equilibrium \((S, R)\) makes sense, since Firm 1 will not want to provoke the retaliation that is encoded in Firm 2’s plan:

  For example, suppose that before Firm 1 had decided whether to enter the market, Firm 2 were to advertise an offer to beat any competitor’s price by 10%.
Concluding remarks

- The style of analysis we developed is based on games in normal form.

- To analyze dynamic games in extensive form, we chose to:
  - First find all Nash equilibria of the translation to normal form;
  - Then treat each as a candidate prediction of play in the dynamic game;
  - Finally go back to the extensive-form version to see which make sense as actual predictions.

- We can also directly work with extensive-form representation:
  - From the terminal nodes upward.
Chapter 6