Markets with Intermediaries

Episode 10

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Network Models of Markets with Intermediaries

(Chapter 11)
Who sets the prices?

- **Matching markets** embody a number of basic principles —
  
  People naturally have different *preferences* for different kinds of goods
  
  *Prices* can decentralize the allocation of goods to people
  
  Such prices can in fact lead to *allocations* that are socially optimal

- But who sets the prices, who trades with whom, if there are many buyers and many sellers?
Trade with intermediaries

- In a wide range of markets, buyers and sellers do not interact directly with each other, but instead trade through intermediaries
  
  Think real estate brokers, stock markets

- There are many kinds of markets, each serving a purpose

- A particular good can be traded in multiple markets

- How do buyers and sellers interact in a stock market?
To get a sense for how markets with intermediaries typically work, let’s focus on the latter example and consider how buyers and sellers interact in the stock market. In the United States, buyers and sellers trade more than a billion shares of stock daily. But there is no single market for trade in stocks in the United States. Instead, trade occurs on multiple exchanges such as the New York Stock Exchange (NYSE) or the NASDAQ-OMX, as well as on alternative trading systems, such as those run by Direct Edge, Goldman Sachs, or Investment Technologies Group (ITG), which arrange trades in stocks for their clients. These markets operate in various ways: some (such as the NYSE or the NASDAQ-OMX) determine prices that look very much like our market-clearing prices from Chapter 10, while others (like Direct Edge, Goldman Sachs, or ITG) simply match up orders to buy and sell stocks at prices determined in other markets. Some have people (called specialists in the NYSE) directly involved in setting prices, while others are purely electronic markets with prices set by algorithms; some trade continuously throughout the day, while others trade less frequently as they wait for batches of buy and sell orders to arrive; and some allow anyone at least indirect access to the market, while others restrict the group of buyers and sellers that they will deal with (often to large institutional traders).

Many of these markets create something called an order book for each stock that they trade. An order book is simply a list of the orders that buyers and sellers have submitted for that stock. A trader may, for instance, submit an order to sell 100 shares if the price is at $5.00 or more per share; another trader may submit an order to sell 100 shares if the price is at $5.50 or more per share. Two other traders may submit orders to buy 100 shares if the price is no more than $4.00 per share, and to buy 100 shares if the price is no more than $3.50 per share. Orders of this type are called limit orders, because they are commitments to buy or sell only once the price reaches some limit set by the trader. If these were the only orders that existed, then the order book for this stock would look like Figure 11.1(a).
A network structure with multiple traders

- With multiple buyers, sellers, and traders (intermediaries) connected in a network structure, what is the trading behaviour at equilibrium?

![Diagram of a network structure with nodes labeled B (buyers), S (sellers), and T (traders).]
Assumptions in our simple network model

- Do not consider multiple goods for sale, or multiple quantities
- Each seller \( i \) holds one unit of the good, willing to sell it at any price that’s at least \( v_i \)
- Each buyer \( j \) values one unit of the good at \( v_j \), willing to pay no more than this value — all buyers have the same valuation
- No one wants more than one unit
- All buyers, sellers and traders are assumed to know all the valuations
- Network structure is fixed — not affected by valuations
Figure 11.3. A standardized view of the trading network from Figure 11.2. Sellers are represented by circles on the left, buyers are represented by circles on the right, and traders are represented by squares in the middle. The value that each seller and buyer places on a copy of the good is written next to the respective node.

Each edge represents an opportunity for trade. Since we assume that the traders act as intermediaries for the possible seller–buyer transactions, we require that each edge connects a buyer or seller to a trader. In Figure 11.3, we depict the same graph from Figure 11.2, but redrawn to emphasize these features of the network model. (In all of our figures depicting trading networks we will use the following conventions. Sellers are represented by circles on the left, buyers are represented by circles on the right, and traders are represented by squares in the middle. The value that each seller and buyer places on a copy of the good is written next to the respective node that represents them.)

Beyond the fact that we now have intermediaries, there are a few other differences between this model and our model of matching markets from Chapter 10. First, we assume that buyers have the same valuation for all copies of a good, whereas in matching markets we allowed buyers to have different valuations for the goods offered by different sellers. The model in this chapter can be extended to allow for valuations that vary across different copies of the good; things become more complicated, but the basic structure of the model and its conclusions remain largely the same. A second difference is that the network here is fixed and externally imposed by constraints such as geography (in agricultural markets) or eligibility to participate (in different financial markets). In matching markets, we began the chapter with fixed graphs, but then focused the core of the analysis on preferred-seller graphs that were determined not by external forces but by the preferences of buyers with respect to an evolving set of prices.

Prices and the Flow of Goods. The flow of goods from sellers to buyers is determined by a game in which traders first set prices, and then sellers and buyers react to these prices. Specifically, each trader offers a bid price to each seller that he is connected to; we will denote this bid price by $b_{ti}$ (then notation indicate that this is a price for a transaction between $t$ and $i$). This bid price is an offer by $t$ to buy $i$'s copy of the good at a value of $b_{ti}$. Similarly, each trader offers an ask price to each buyer to which he...
Model it as a game

- Two stages —
  The traders simultaneously announce their ask and bid prices to the sellers and buyers connected.
  Each seller and buyer chooses at most one trader (but doesn’t have to) — all sellers and buyers simultaneously make the choice.

- Let’s model it as a dynamic game —
  Maximizing the payoffs: a buyer $j$ maximizes $v_j - a_{tj}$; a trader maximizes its profit; a seller $i$ maximizes $b_{ti} - v_i$. 
Figure 11.4. (a) Each trader posts bid prices to the sellers to which he is connected, and ask prices to the buyers to which he is connected. (b) These prices determine a flow of goods as sellers and buyers each choose the offer that is most favorable to them.

Once traders announce prices, each seller and buyer chooses at most one trader to deal with; each seller sells his copy of the good to the trader he selects (or keeps his copy of the good if he chooses not to sell it), and each buyer purchases a copy of the good from the trader she selects (or receives no copy of the good if she does not select at a trader). This determines a flow of goods from sellers, through traders, to buyers; Figure 11.4(b) depicts such a flow of goods, with the sellers' and buyers' choices of traders indicated by the edges with arrows on them.

Because each seller has only one copy of the good, and each buyer only wants one copy, at most one copy of the good moves along any edge in the network. On the other hand, there is no limit on the number of copies of the good that can pass through as in a direct trade network. Note that a trader can only sell any goods she receives from sellers; we will include in the model a large penalty imposed on a trader who defaults on an offer to sell to a buyer as a result of not having enough goods on hand. Therefore, a trader has strong incentives not to produce bid and ask prices that cause more buyers than sellers to accept his offers. There are also incentives for a trader not to be caught in the reverse difficulty, with more sellers than buyers accepting his offers; in this case, he ends up with excess inventory that he cannot sell. We will find that neither of these outcomes happens in the solutions we consider; traders will choose bid and ask prices such that the number of goods they receive from sellers is equal to the number of goods they pass on to buyers.

Finally, notice something else about the flow of goods in this example: seller S3 accepts the bid even though it is equal to his value, and likewise buyer B3 accepts the ask even though it is equal to his value. In fact, each of S3 and B3 is indifferent between accepting and rejecting the offer. Our assumption in this model is that, when a seller or buyer is indifferent between accepting or rejecting, then we (as the modelers)
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When a seller or buyer is indifferent between accepting or rejecting, then we (as the modellers) have a choice.
Best responses and equilibrium

- T1 is making several bad decisions —

  He is losing the opportunity of dealing with S2 and B2 to T2.

  If he were to raise his bid to seller S2 to 0.4, and lower his ask to buyer B2 to 0.6 —

![Diagram showing traders and transactions between sellers and buyers]

Figure 11.5. Relative to the choice of strategies in Figure 11.4(b), trader T1 has a way to improve his payoff by undercutting T2 and performing the transaction that moves S2's copy of the good to B2.

For us, this two-stage structure will not make things too complicated, particularly because the second stage is extremely simple: the best response for each seller and buyer is always simply to choose the trader with the best offer, and so we can essentially view the sellers and buyers as "drones" who are hard-wired to follow this rule. Still, we will have to take the two-stage structure into account when we consider the equilibria for this game, which we do next.

Best Responses and Equilibrium.

Let's think about the strategies that the two traders have chosen in Figure 11.4(b). The upper trader, T1, is making several bad decisions.

First, because of the offers he is making to seller S2 and buyer B2, he is losing out on this deal to the lower trader, T2. If, for example, he were to raise his bid to seller S2 to 0.4, and lower his ask to buyer B2 to 0.6, then he'd take the trade away from trader T2: seller S2 and buyer B2 would both choose him, and he'd make a profit of 0.2.

Second, and even more simply, there is no reason for trader T1 not to lower his bid to seller S1 and raise his ask to buyer B1. Even with worse offers, S1 and B1 will still want to deal with T1, since they have no other options aside from choosing not to transact. Given this, T1 will make more money with a lower bid to S1 and a higher ask to B1.

Figure 11.5 shows the results of a deviation by the upper trader that takes both of these points into account; his payoff has now increased to (1 + 0.6 - 0 - 0.4) = 1.2. Note that seller S1 and buyer B1 are now indifferent between performing the transaction or not, and as discussed earlier, we give ourselves (as the modelers) the power to break ties in determining the equilibrium for such situations.

This discussion motivates the equilibrium concept we will use for this game, which is a generalization of Nash equilibrium. As in the standard notion of Nash equilibrium from Chapter 6, it is based on a set of strategies such that each player is choosing a best response to what all the other players are doing. However, the definition also needs to take the two-stage structure of the game into account.

To do this, we first think about the problem faced by the buyers and sellers in the second stage, after traders have already posted prices. Here, we have a standard
What are bid and ask prices at equilibrium?

- First think about the sellers and buyers — they choose optimally given whatever prices that traders have posted — and the traders know this.

- Then think about the problem faced by the traders, in deciding what prices to post in the first stage.
  
  Each trader chooses a strategy that is a best response to both the strategies the sellers and buyers will use and the strategies the other traders will use.

- At equilibrium (called a subgame perfect Nash equilibrium), what are the bid and ask prices?
  
  Let’s start with something simple.
Monopoly

Figure 11.6. A simple example of a trading network in which the trader has a monopoly and extracts all of the surplus from trade.

To see why this is the only equilibrium, we simply notice that, for any other bid and ask between 0 and 1, the trader could slightly lower the bid or raise the ask, thereby performing the transaction at a higher profit.

Perfect Competition.

Now let's look at a basic example showing perfect competition between two traders, as depicted in Figure 11.7.

In Figure 11.7 there is competition between traders T1 and T2 to buy the copy of the good from S1 and sell it to B1. To help in thinking about what would form an equilibrium, let's first think about things that are out of equilibrium, in a manner similar to what we saw in Figure 11.5. In particular, suppose trader T1 is performing the trade and making a positive profit: suppose his bid to the seller is some number \( b \), and his ask to the buyer is a number \( a > b \). Since T2 is not performing the trade, he currently has a payoff of zero. But then it must be that T2's current strategy is not a best response to what T1 is doing: T2 could instead offer a bid slightly above \( b \) and an ask slightly below \( a \), thereby taking the trade away from T1 and receiving a positive payoff instead of zero.

So it follows that whichever trader is performing the trade at equilibrium must have a payoff of 0: he must be offering the same value \( x \) as his bid and ask. Suppose that trader T1 is performing the trade. Notice that this equilibrium involves indifference on his part: he is indifferent between performing the trade at zero profit and not performing the trade. As in the earlier case of indifference by sellers and buyers, we assume that we (the modelers) can choose an outcome in this case, and we will assume that the transaction is performed. Here too, we could handle indifference by assuming a minimum increment of money (e.g., 0.01) and having the transaction take place with S1, T1, T2, B1, 0, 1, xx.
Perfect competition

T1 at equilibrium has a common bid and ask of \( x \)

If T1 performs the trade, T2 must have a bid and ask of \( x \) since \( b \leq x \) and \( a \geq x \)

But if \( a > b \), then T1 could lower the bid or raise the ask and make a positive profit.
Back to our example

We now consider a further example that illustrates how the network structure can also produce more complex effects that are not explained by these two principles.

Implicit Perfect Competition.

In our examples so far, when a trader makes no profit from a transaction, it is always because there is another trader who can precisely replicate the transaction – a trader who is connected to the same seller and buyer. However, it turns out that traders can make zero profit for reasons based more on the global structure of the network, rather than on direct competition with any one trader.

The network in Figure 11.9 illustrates how this can arise. In this trading network there is no direct competition for any one "trade route" from a seller to a buyer.

Figure 11.9. A form of implicit perfect competition: all bid/ask spreads will be zero in equilibrium, even though no trader directly "competes" with any other trader for the same buyer–seller pair.
Implicit perfect competition

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Modelling single-item second-price auctions

Figure 11.10. (a) A single-item auction can be represented using a trading network. (b) Equilibrium prices and flow of goods. The resulting equilibrium implements the second-price rule from Chapter 9.

However, in any equilibrium, all bid and ask prices take on some common value between 0 and 1, and the goods flow from the sellers to the buyers. So all traders again make zero profit.

It is easy to see that this is an equilibrium: we can simply check that each trader is using a best response to all the other traders' strategies. It takes a bit more work to verify that in every equilibrium all bid and ask prices are the same value: this is most easily done by checking alternatives in which some trader posts a bid that is less than the corresponding ask, and by identifying a deviation that arises.

11.4 Further Equilibrium Phenomena: Auctions and Ripple Effects

The network model we've been considering is expressive enough that it can represent adverses to the phenomena. Here we consider two distinct examples: the first shows how the second-price auction for a single item arises from a trading network equilibrium, and the second explores how small changes to a network can produce effects that ripple to other nodes.

Second-Price Auctions.

Figure 11.10 shows how we can represent the structure of a single-item auction using a trading network. Suppose there is a single individual S1 with an item to sell, and four potential buyers who value the item at values $w$, $x$, $y$, and $z$, listed in descending order: $w > x > y > z$. We use four buyers in this example, but our analysis would work for an arbitrary number of buyers.

In keeping with our model in which trading happens through intermediaries, we assume that each buyer is represented by a distinct trader — essentially, someone who serves as the buyer’s “proxy” for the transaction. This gives us a trading network as depicted in Figure 11.10(a).

$w > x > y > z$
Modelling single-item second-price auctions

The second-price rule is not built into the formulation of the auction; it emerges naturally as an equilibrium.
Social welfare in trading networks

- Are the solutions at equilibrium socially optimal? — maximizing the sum of payoffs of all the players

- Each good that moves from seller i to buyer j contribute $v_j - v_i$ to the social welfare

\[(b_{ti} - v_i) + (a_{tj} - b_{ti}) + (v_j - a_{tj}) = v_j - v_i\]

- As a result, more richly connected networks can potentially allow a flow of goods achieving a higher social welfare
Equilibria and social welfare: example

\[ 0 \leq x \leq 2 \]

\[ 1 \leq y \leq 2 \]
\[ 1 \leq z \leq 3 \]

**Figure 11.11.** (a) Equilibrium before the new S2-T2 link is added. (b) Equilibrium after the S2-T2 edge is added. A number of changes take place in the equilibrium. Among these changes is the fact that buyer B1 no longer gets a copy of the good, and B3 gets one instead.

In Figure 11.11(b), once the edge from S2 to T2 has been added, we need to work out the equilibrium bids and asks charged to S2 and B2, and the flow of goods. Reasoning about these bids and asks requires a bit more work than we've seen in previous examples, so we build up to it in a sequence of steps.

1. The two bids to S2 must be the same as each other; otherwise the trader getting the good could slightly lower his bid. For a similar reason, the two asks to B2 must also be the same as each other. Let's call the common bid \( z \) and the common ask \( y \).
2. We can next determine how the seller–trader transactions work out in equilibrium. Seller S2 will sell to T2 rather than T1 in equilibrium: if S2 were selling to T1, and T1 were receiving a nonnegative payoff from this transaction, then S2 would be selling for at most 2. In this case, T2 could slightly outbid T1 and sell S2's copy of the good to B3. So in equilibrium, T2 buys two copies of the good, while T1 buys only one.
3. Now let's figure out the possible values for the ask \( y \). The ask \( y \) must be at least 1: otherwise, one of the traders is selling to B2 for a low price, and the trader performing this sale has an alternate trader whom he monopolizes and from whom he would get a higher payoff. This can't happen in equilibrium, so \( y \) is at least 1.
   Also, the ask \( y \) cannot be greater than 2 in equilibrium: in this case, B2 would not buy, and so T1 could perform a payoff-improving change in strategy by lowering his ask to B2, thereby getting B2 to buy from T1 for a price between 1 and 2.
4. Next, we determine how the trader–buyer transactions work out in equilibrium. We've already concluded that T2 is buying two copies of the good, and so he maximizes his payoff by selling them to B3 and B4. Therefore, T2 is not selling.

**Social Welfare**

\[ a \]
\[ 1+2+4 = 7 \]

\[ b \]
\[ 2+3+4 = 9 \]
In every trader network, there is always at least one equilibrium, and every equilibrium produces a flow of goods that is socially optimal.
Chapter 11.1 — 11.5