LOTUS: Location-Aware Online Truthful Double Auction for Dynamic Spectrum Access

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Abstract—In the spectrum auction, if a buyer locates in a "critical" place, interfering with a lot of other buyers, his occupancy of the spectrum may deprive many other transmission opportunities. In this paper, we propose a Location-aware Online Truthful doUble auction Scheme (LOTUS), which incorporates the buyers' location information into auction mechanism design. In the online auction, the biggest challenge is how to allocate the spectrums based on the knowledge in the current time slot, without knowing the spectrum requests that may come afterwards. To solve this problem, we propose to consider the opportunity cost of allocating the spectrum to a buyer based on his local interference conditions. We introduce the "interference discount" to markdown a buyer's bid if he induces a wide range of interference. Furthermore, we take into account the spectrum heterogeneity and design mechanisms that guarantee the economic-robustness of the auction. The simulation results show that LOTUS outperforms both existing online auction and static auction mechanisms, significantly improving buyers' and sellers' utility.

Index Terms—Online Spectrum Auction, Location-Aware, Truthfulness, Interference Discount, Opportunity Cost

1 Introduction

To increase the spectrum efficiency, under-utilized spectrums are being explored with the technical support of dynamic spectrum access [1]. The FCC has opened up a significant amount of TV whitespace for unlicensed usage in 2010. Through online auction, the whitespace database operator can dynamically allocate spectrums based on buyers' spectrum requests, and exploit temporal and spatial reusability to improve spectrum utilization [1].

To exploit the spectrum reusability can greatly enhance spectrum utilization. Static spectrum auction partly addresses the problem by grouping the interference-free buyers together. However, using existing static double auction mechanisms for dynamic spectrum allocation will cause potential utility loss. For example, in Fig.1, suppose that buyer A arrives at time slot 1, requesting for 3 time slots, and his true valuation is 1 per time slots. Later at time slot 2, buyer B and buyer C arrive, each requesting for 2 time slots, and their true valuations are 1 per time slots. If the auctioneer allocates the spectrum to buyer A at time slot 1, buyer A gets utility of 3. However, if the auctioneer rejects buyer A's, leaving the spectrum idle at time slot 1; then allocates the spectrum to buyer B and C simultaneously at time slots 2, they get a total utility of 4. This example shows the necessity of considering the spectrum request dynamics in spectrum allocation.

Most of the previous works only focused on static auction [2] [3], which will incur utility loss in case of dynamic spectrum allocation. Jonathan Bredin [4] *et al.* proposed online auction mechanism for dynamic resource and task allocation. However, spectrum is different from general goods as spectrum can be reused by multiple buyers. To the best of our knowledge, only one existing paper considered online spectrum auction [5]. However, the paper makes the simplified assumption that the interference graph is complete. While such an assumption makes the auction design easier, it does not capture the practical scenarios where some buyers are more interference harmful, and should be rejected or charged a higher price.

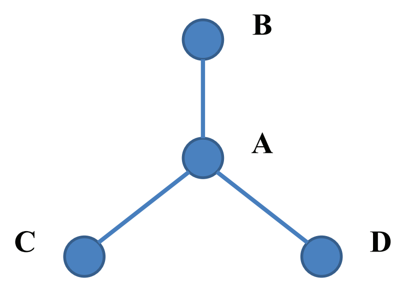

Fig. 1. A motivated example

This motivates us to design an online double auction mechanism that indeed takes into consideration the sporadic nature of spectrum request and buyers' geographic feature. The challenges facing us are two-folds:

- *How to characterize the interference harmfulness of a certain buyer?*
  This is difficult because, although a buyer may interfere with many other buyers, it is possible that one of his interfering neighbors interferes with even more buyers.
- *How to decide whether to allocate the spectrum to a certain buyer based on the knowledge of the spectrum requests in the current time slot?*
  This is difficult because spectrum request is random and it is hard to predict whether a buyer's interfering neighbors will request spectrum in later time slots.

Apart from the above challenges, the proposed double auction mechanism should also be economic-robust, that is, truthful, individual rational and budget balanced for the auctioneer.

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To address these challenges, in this paper, we propose LOTUS, a **L**ocation-aware **O**nline **T**ruthful do**U**ble auction **S**cheme, which aims at exploiting location heterogeneity to improve spectrum utilization while guaranteeing economic robustness. In LOTUS, multiple sellers and multiple buyers participate in the auction that lasts for a time period of T time slots. The buyers can request multiple spectrums at any time slot, and bid different prices for different spectrums. In Section 4, we answer the first question of how to characterize interference harmfulness, by introducing the concept of *interference discount*. If a buyer is considered to be severely interference harmful, his bid will be discounted heavily to compensate the potential utility loss he may cause. In Section 5, we answer the second question of how to deal with spectrum request dynamics, by introducing the concept of *opportunity cost*. Opportunity cost is the utility difference of allocating spectrum to a buyer or not, considering potential spectrum requests by his interfering neighbors in the forthcoming time slots.

The major contributions are as follows:

- To the best of our knowledge, we are the first to design a location-aware online spectrum auction mechanism which incorporates complicated interference relationship among buyers and can deal with dynamic spectrum requests.
- The proposed auction mechanism LOTUS has nice economic properties, including truthfulness, individual rationality and budget balanced for the auctioneer.
- Simulation results show that LOTUS can significantly improve buyers' and sellers' utility, making it desirable for implementation in dynamic spectrum access scenarios.

The rest of the paper is organized as follows. We briefly review the related work in Section 2. We give system model and basic assumptions in Section 3. We explain the concept of interference discount in Section 4. We give a detailed description of LOTUS in Section 5 and analyze its economic properties in Section 6. We present the simulation results in Section 7, and finally summarize our work in Section 8.

2 Literature Review

Spectrum auction has been widely studied for efficient spectrum allocation. Most of the previous works only focused on static auction, in which the spectrum request (either for the present time slot or for future time slots) is determined and submitted all at the same time. In early works of [6], [7], the authors designed auction mechanisms for spectrum allocation, but spectrum reusability is not considered. In [2], [8]–[10], non-double-auction mechanisms are proposed, and spectrum reusability is exploited by allocating the same spectrum to non-interfering buyers. A truthful double spectrum auction is proposed in [11], but it is restricted that each seller or buyer trades only one spectrum. The paper [12] proposes an auction mechanism for heterogeneous spectrum. However, it is restricted that each buyer can only demand one spectrum. In fact, the auction in [12] will become untruthful when extended to the scenario where one buyer demands multiple spectrums. The paper [13] [14] considered spectrum allocation in different locations but the allocation is static. The paper [3] considered buyers' requests for multiple time slots. However, the allocation is still static since all buyers submit their request at the first time slot, and the allocation is decided at the first time slot. As we discussed before, such static auction mechanisms will incur utility loss because it fails to consider the dynamic nature of spectrum request and the influence of early spectrum allocation on the later allocation decisions.

Online auction is considered by Jonathan Bredin [4] *et al.* for general goods. However, spectrum is different from other goods since it can be assigned to multiple buyers simultaneously. To the best of our knowledge, only one existing paper considered online spectrum auction. In [5], the authors proposed a truthful online double auction mechanism. However, the paper makes the simplified assumption that the interference graph is complete, thus avoid the spectrum reusability problem. While such an assumption makes the auction design easier, it does not capture the practical scenarios where some buyers are more interference harmful, and should be rejected or charged higher price.

Instead of truthfulness, some works targets at revenue maximization [15]–[17]. Some works used interference temperature instead of interference graph [18], [19]. Major drawback is that spectrum information between any transmitter-receiver pair is needed, which is difficult to obtain.

3 Preliminaries

In this section, we first introduce the auction participants: the spectrum sellers, the spectrum buyers and the auctioneer. we give the definitions of the economic properties to be achieved via the auction mechanism design.

3.1 Auction Participants

3.1.1 Spectrum Sellers

We assume that there are M potential spectrum sellers in the network. Each seller has one unit bandwidth of spectrum available to be leased to the buyers during the entire time slots $1 \sim T$. If some spectrums are unavailable in certain time slots, the problem can be easily fixed by removing the spectrum from all buyers' available spectrum sets. Before the start of the auction, the ith seller submit his per time slot bid A_i to the auctioneer, and his true valuation for his spectrum is $\widetilde{A}_i$. A seller aims at maximizing individual utility, so A_i is not necessarily equal to $\widetilde{A}_i$. We assume that sellers do not change their asking prices during $[1, T]$[1]. In the following part of the paper, we refer to sellers' prices as "ask".

3.1.2 Spectrum Buyers

We assume that there are N potential spectrum buyers. The buyers' spectrum request is dynamic. Before the start of the auction, the jth buyer submits his per time slot bid $B_j = (b_{j,1}, b_{j,2}, ..., b_{j,M})$ to the auctioneer. $b_{j,i}$ is the buyer's bid for s_i, the spectrum owned by the ith seller. The buyer's

1. It is possible that sellers may change their asking prices according to the demand, e.g., charge higher prices when the demand is high. But in this paper, we only focus on the temporal and spatial features of buyers' spectrum request, and leaves the sellers' price dynamics as a future direction.

true valuation is $\widetilde{B}_j = (\widetilde{b}_{j,1}, \widetilde{b}_{j,2}, ..., \widetilde{b}_{j,M})$. A buyer aims at maximizing individual utility, so B_j may not be equal to $\widetilde{B}_j$. In the following part of the paper, we refer to buyers' prices as "bid". After the online auction begins, at time slot t, the jth buyer will submit an instant spectrum request, specifying the number of requested spectrums d_j and the number of desired time slots t_j to the auctioneer. The number of desired time slots t_j means that the buyer wants to occupy the spectrum during $t \sim t + t_j$[2]. We assume that the buyers do not accept partial fulfillment of time slots fewer than t_j but may accept partial fulfillment of spectrums fewer than d_j[3]. If a buyer does not want any spectrum at time slot t, he does not submit any bids, which will not affect the auction.

One unique feature of the spectrum is reusability, that is, two buyers can reuse the same spectrum if they are far away from each other. However, if two nearby buyers use the same spectrum, they end up interfering with each other. By leveraging the spectrum reusability, we can improve the spectrum utilization, which is essential to solve the seemingly spectrum scarcity problem. Based on the location information of the buyers, the auctioneer is able to construct an interference graph to denote the interference relationship among buyers. Let $G = (V, E)$ denote the interference graph, in which V is the set of all buyers. If two buyers are potential interferers[4], there is an edge between them, otherwise, there is no edge between them. We define the interfering neighbors and interference degree as follows.

Definition 1: Interfering neighbors. The interfering neighbors of the jth buyer is the set of buyers who interfere with the buyer, i.e., sharing an edge in E with the buyer.

Definition 2: Interference degree. The interference degree of the jth buyer is the number of his interfering neighbors, that is, the degree of his representing vertex in the interference graph G.

We use $N(j)$ to denote the set of interfering neighbors of the jth buyer and θ_j to denote his interference degree.

3.1.3 Auctioneer

There is a third-party auctioneer who hosts the auction. At the very start of the auction, the auctioneer should collect spectrum information and the asks from the sellers, and the bids from the buyers. The auctioneer has to know the spatial availability of each spectrum to decide available spectrum set for each buyer. The buyers' should give the auctioneer their location information, which is essential for the auctioneer to compute the interference discount. In this paper, we assume that the objective of the auctioneer is to maintain a balanced budget[5] at each time slot, that is, $U_t^{auctioneer} \geq 0, t = 1, 2, \cdots, T$.

2. We make the assumption that if the buyer wants multiple spectrums, the number of desired time slots is the same for each spectrum. In the future work, we may consider different numbers of desired time slots for different spectrums.

3. Other conditions of partial fulfillment or full fulfillment of desired time slot and requested spectrums will be a future direction.

4. The interference relationship can be derived based on the radio propagation models [20]. In this paper, we assume that the interference relationship is the same for all spectrums.

5. An alternative assumption is that the auctioneer aims at maximizing its revenue. Revenue maximization in forward auction is considered in [15]–[17], but is not the focus of this paper.

At each time slot t, all the buyers submit their spectrum request to the auctioneer, then the auctioneer decides the spectrum allocation and the corresponding payments. The auction results is represented by a 3-dimensional matrix $\mathcal{X}_{i,j,t}, i \in [1, M], j \in [1, N], t \in [1, T]$. At time slot t, if the auctioneer decides to allocate the ith seller's spectrum to the jth buyer for t_j time slots, then $x_{i,j,t} = 1, x_{i,j,t+1} = 1, ..., x_{i,j,t+t_j} = 1$; otherwise, $x_{i,j,t} = 0$. At time slot t, The auctioneer will pay the ith seller $p_{i,t}^{seller}$, and charges the jth buyer $p_{j,i,t}^{buyer}$ for s_i.

At time slot t, the utility of the ith seller is his payment from the auctioneer minus his true valuation:

$$U_{i,t}^{seller} = \begin{cases} p_{i,t}^{seller} - \widetilde{A}_i & \text{if} \sum_j x_{i,j,t} > 0 \\ p_{i,t}^{seller} & \text{otherwise} \end{cases} \tag{1}$$

At time slot t, the utility of the jth buyer is his true valuation minus his payment to the auctioneer:

$$U_{j,t}^{buyer} = \sum_i [(\widetilde{b}_{j,i} - p_{j,i,t}^{buyer}) \times x_{i,j,t}]. \tag{2}$$

The auctioneer's budget at time t is the payment obtained from all the buyers minus the payment to all the sellers:

$$U_t^{auctioneer} = \sum_{j,i} p_{j,i,t}^{buyer} - \sum_i p_{i,t}^{buyer} \tag{3}$$

We make the following assumptions throughout the paper.

Assumption 1: Buyers' spectrum request is Independent and Identically Distributed random variable (IID), more specifically, Poisson distribution with arrival rate λ.

Assumption 2: Buyers' bids are IID, and the distribution is known by the auctioneer.

Assumption 3: The number of requested time slot of each buyer is IID, and the distribution is known by the auctioneer.

Assumption 1 is a common assumption for traffic in wireless network ([5] [21]). Assumption 2 is reasonable as the auctioneer can estimate the trend of buyers' bids either from the bidding history or by evaluating the value of the spectrum in an open market. Assumption 3 is reasonable as the time for performing common tasks in wireless communication can be estimated based on historical data.

3.2 Economic Properties

In this paper, we focus on the following three economic properties in designing our auction mechanism:

Truthfulness. Truthfulness is the most fundamental property of an auction mechanism. The buyers and sellers are selfish and rational players, who will manipulate their asks and bids to maximize their own utilities. The property of truthfulness ensures that neither the sellers nor the buyers can get higher payoff by misreporting their true valuation, thus avoiding market manipulation. For the online spectrum auction, we have to consider the truthfulness at each time slot. Let $\widetilde{U}_{i,t}^{seller}$ and $U_{i,t}^{seller}$ denote the utility of the ith seller when being truthful and untruthful respectively. In a truthful auction, $U_{i,t}^{seller} \leq \widetilde{U}_{i,t}^{seller}$ always holds. As for the buyers, the truthfulness includes three aspects: 1) buyers bid their true valuation; 2) buyers submit their true number of requested spectrums; 3) buyers submit their true number of desired time slots. We assume that the buyers do not lie about their number of requested spectrums and desired time slots. The reason is: if a buyer submits a fewer number of spectrums or time slots, he cannot finish his task, and will get negative utility; if a buyer submits a larger number of spectrums or time slots, he has to make extra payments, and will get negative utility. Therefore, we only consider the truthfulness regarding buyers' bidding price. Assume that the jth buyer gets $\widetilde{U}_{j,t}^{buyer}$ and $U_{j,t}^{buyer}$ when being truthful and untruthful respectively. In a truthful auction, $U_{j,t}^{buyer} \leq \widetilde{U}_{j,t}^{buyer}$ always holds.

Individual rationality. Individual rationality ensures that all sellers and buyers achieve non-negative utility so that they have incentive to participate in the auction. In an individual rational auction, the seller is paid more than his ask, i.e., $p_{i,t}^{seller} \geq A_i$; the buyer pays less than his bid, i.e., $p_{j,i,t}^{buyer} \leq b_{j,i}$.

Budget balance. Budget balance means that the auctioneer maintains non-negative budget. If the auctioneer is a profit institution, it is important to keep a balanced budget to avoid bankruptcy[6].

In this paper, the objective of the proposed online spectrum auction mechanism is: ***Given the temporal and spatial features of the buyers' spectrum request, how to find a truthful, individually rational and budget balanced double auction mechanism to dynamically allocate the spectrum at each time slot?***

4 Interference Discount

In this section, we first introduce two factors that contribute to the interference harmfulness of a certain buyer. Then we combine the two factors to compute the interference discount.

We have shown in Fig.1 that, if we simply run an existing single-round double auction mechanism for each time slot where the auctioneer considers only the spectrum request at that time slot, it will incur utility loss. The reason is that, if the jth buyer wins a certain spectrum at time t, he will occupy the spectrum from $t \sim t+t_j$. His interfering neighbors cannot use the same spectrum when they request at a later time during $t+1 \sim t+t_j$ even if they bid higher prices, or can reuse the spectrum more efficiently. An important observation is that, granting the spectrum request of a buyer who is located in a "critical" place and interferes with a lot of other buyers, may generate high potential utility loss. Therefore, we have to take into consideration buyers' spatial feature, especially their interference relationship, in order to improve spectrum utilization.

We analyze the interference harmfulness of a buyer from two aspects. First, we compare the interference degree of a buyer with the average interference degree of his interfering neighbors. Second, we consider the reusability efficiency of a buyer's interfering neighbors. We combine the two aspects together to decide the opportunity cost of allocating the spectrum to a certain buyer.

6. It is possible that the governmental bodies or non-profit groups may assume the responsibility of the spectrum allocation, and even subsidize the auction to promote efficient spectrum usage.

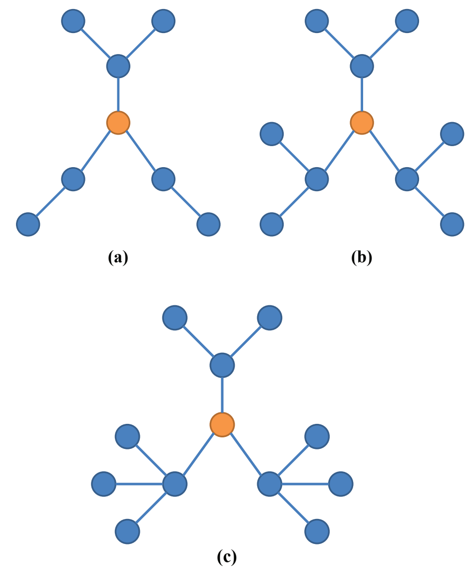


Fig. 2. Comparison of interference degree

4.1 Comparison of Interference Degree

For the jth buyer, there exists two spectrum allocation options: allocate the spectrum to the jth buyer; or allocate the spectrum to one of his interfering neighbors[7]. In the former case, none of the buyers in $N(j)$ can use the same spectrum during the jth buyer's occupancy. In the later case, none of the buyers who are the interfering neighbors of the buyers in $N(j)$ can use the same spectrum. Such usage exclusivity has a ripple effect, which may extend to the entire interference graph. To make the problem tractable, we only consider 3 layers, from the jth buyer to the interfering neighbors of the buyers in $N(j)$. Let r_j denote the ratio of the interference degree of the jth buyer and the average interference degree of his interfering neighbors.

$$r_j = \frac{\theta_j}{(\sum_{k \in N(j)} \theta_k)/\theta_j} = \frac{\theta_j^2}{\sum_{k \in N(j)} \theta_k} \tag{4}$$

If r_j is large, to allocate the spectrum to the jth buyer will have negative effect on more buyers than to allocate the spectrum to the buyers in $N(j)$, vice versa. Hence, r_j shows the harmfulness of a buyer in terms of his interference degree.

The r_j in different cases is shown in Fig.2. The central node in orange color denotes the jth buyer. In the first subgraph, the buyer's interfering neighbors have smaller average interference degree, so $r_j > 1$, which means that the jth buyer is quite interference harmful. In the second subgraph, the buyer's interfering neighbors have exactly the same average interference degree, so $r_j = 1$. In the third subgraph, the buyer's interfering neighbors have larger average interference degree, so $r_j < 1$, which means that the jth buyer may be a good candidate for spectrum allocation than his interfering neighbors.

7. The third choice is not to assign the spectrum to either the ith buyer or any of his interfering neighbors, but it is unreasonable since we can always improve spectrum utilization by allocating the spectrum to either party without violating the interference constraint.

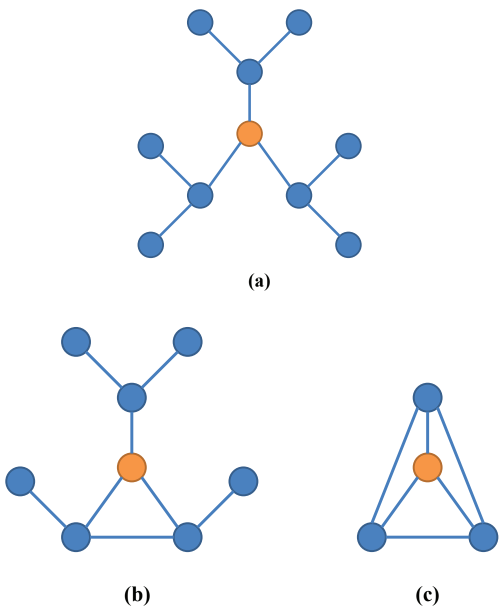


Fig. 3. Reuse degree of interference neighbors

4.2 Reusability Efficiency of Interfering Neighbors

It is not enough to only consider the interference degree. As shown in Fig.3, the central node in orange color denotes the jth buyer. By definition, $r_j = 1$ holds for all three cases. However, in the first subgraph, since the interfering neighbors can reuse the same spectrum, to allocate the spectrum to the jth buyer causes more utility loss than the case in the last subgraph, where none of the interfering neighbors can reuse the same spectrum. Therefore, we have to consider the spectrum reusability efficiency of a buyer's interfering neighbors.

To determine the reusability efficiency among a set of buyers is not easy. For instance, in Fig.3, some of the interfering neighbors of the jth buyer can reuse the spectrum, but some cannot. One way is to compute the average reusability efficiency by considering all possible subset of the 3 buyers. However, when the number of buyers increases, the number of subsets increases exponentially. For simplification, we only consider the maximum reusability efficiency (denoted by m_i), that is, the maximum number of buyers in $N(j)$ that can reuse the spectrum[8].

In Fig.3, the first subgraph shows the case in which all the interfering neighbors are interference free, so $m_j = 3$. In the second subgraph, the maximum number of interfering neighbors who can reuse the spectrum is 2, so $m_j = 2$. In the last subgraph, which is a complete graph, $m_j = 1$. As we discussed before, to allocate the spectrum to the jth buyer causes the most utility loss in the first subgraph as his interfering neighbors can reuse the spectrum most efficiently.

4.3 Interference Discount

We define the *Interference Discount* as follows.

8. To derive a more precise calculation for average reusability efficiency is a future direction.

Definition 3: Interference Discount. Interference discount is the discount factor that marks down a buyer's bid by the comparison of interference degree and the reusability efficiency of his interfering neighbors. The interference discount of the jth buyer is

$$I_i = \frac{1}{r_i m_i} \tag{5}$$

In other words, if the jth buyer bids $b_{j,i}$ for s_i, the auctioneer discounts the bid as $I_j * b_{j,i}$ when screening the buyers for valid candidate of the auction. Since the interference relationship among the buyers is relatively static, the interference discount of each buyer can be calculated offline, and maintained as a lookup table by the auctioneer.

5 LOTUS

In this section, we give a detailed description of the proposed auction mechanism LOTUS. At the start of time slot t, the auctioneer collects spectrum requests from all buyers, including their bids, number of requested spectrums and number of desired time slots. Then the auction proceeds in two phases:

- *Candidate screening*. In a static auction, since all buyers' spectrum requests arrive at the same time, it is possible for the auctioneer to perform the spectrum allocation in an "optimal" way to maximize the aggregated utility or maximize the spectrum utilization. However, in the online auction, the spectrum requests happen sporadically. The spectrum requests in the early time slots influence those in the later time slots. It is necessary for the auctioneer to screen the requests from the buyers according to their 1) discounted bids, and; 2) opportunity cost. We will further explain the opportunity cost in the following part.
- *Spectrum allocation determination*. After screening phase, the valid candidate buyers become potential auction winners. Then the auctioneer runs the auction mechanism to determine the spectrum allocation and the payments. The auction mechanism should be truthful, individual rational and budget balanced.

5.1 Candidate Screening

Now we explain how to calculate the opportunity cost of allocating the spectrum s_i to the jth buyer at time slot t. Before the auction starts, the auctioneer will compute the expected valuation matrix $\mathcal{V}_{N\times M\times T}$. Entry $v_{j,i,t}, j \in [1,N], i \in [1,M], t \in [1,T]$ denotes the expected value of allocating s_i to the interfering neighbors of the jth buyer from time slot t on. Let $\mathbf{Pr}(\tau)$ denote the probability that the number of desired time slots of a typical buyer is τ, $\bar{b}_{j,i} = \sum_{k\in N(j)} b_{k,i} I_k/|N(j)|$ denote the average discounted bids of the interfering neighbors in $N(j)$ for spectrum s_i. Then $v_{j,i,t}$ is calculated as follows:

$$v_{j,i,t} = \sum_{\tau=1}^{T-t} [\mathbf{Pr}(\tau)(\bar{b}_{j,i}\tau + v_{j,i,t+\tau})] \tag{6}$$

$\mathcal{V}$ is an $N \times M \times T$ matrix and can be calculated using dynamic programming.

The opportunity cost of allocating s_i to the jth buyer at time t for t_j time slots is

$$C_{j,i}(t, t_i) = v_{j,i,t} - v_{j,i,t+t_j}. \quad (7)$$

At time slot t, the auctioneer compares the opportunity cost $C_{j,i}(t, t_i)$ with the discounted bid $b_{j,i} * I_i$ to decide whether the jth buyer is a valid candidate for s_i.

- If $C_{j,i}(t, t_i) > b_{j,i} * I_i * t_i$, meaning that the jth buyer does not bid high enough to compensate the potential spectrum utilization loss. Therefore, the jth buyer will not be considered as a valid candidate for winning s_i.
- If $C_{j,i}(t, t_i) \leq b_{j,i} * I_i * t_i$, meaning the jth buyer's bid exceeds the potential spectrum utilization loss, the auctioneer considers him as a valid candidate for winning s_i.

Based on the above spectrum candidate screening, the auctioneer can construct the available spectrum set for each buyer who submits the spectrum request at time slot t, as shown in Algorithm 1. Let $\mathcal{S}_{j,t}$ denote the available spectrum set of the jth buyer. Firstly, all the spectrums are put into $\mathcal{S}_{j,t}$. Then, the spectrums that are occupied by the interfering neighbors of the jth buyer are removed from $\mathcal{S}_{j,t}$. Finally, the spectrums for which the jth buyer is not a valid candidate are removed from $\mathcal{S}_{j,t}$.

Algorithm 1 Available Spectrum Set Construction

At time slot t
for all Buyer $j, j = 1, 2, ..., N$ **do**
  Put all the spectrums in $\mathcal{S}_{j,t}$.
  for all $s_i, i = 1, 2, ..., M$ **do**
    if s_i is being occupied by any buyer in $N(j)$ **then**
      Remove s_i from $\mathcal{S}_{j,t}$.
    end if
    if Buyer j is not a valid candidate for s_i **then**
      Remove s_i from $\mathcal{S}_{j,t}$.
    end if
  end for
end for

After the candidate screening phase, each buyergets an available spectrum set. Then the auctioneer moves on to the spectrum allocation determination phase.

5.2 Spectrum Allocation Determination

The spectrum allocation includes two parts: grouping, and winner and payment determination.

5.2.1 Grouping Process

Buyers who can reuse the same spectrums can be grouped together, and allocated the same spectrum in order to improve spectrum utilization. In the previous double auction design [11], [13], [14], it is assumed that all the spectrums are available to each buyer, so the grouping process can be easily done by partitioning the interference graph into multiple independent sets. However, this no longer works in our settings where different buyers have different available spectrum sets. Two buyers who do not have any common available spectrums cannot be grouped together even if they do not interfere with each other. Another problem is how to deal with multiple spectrum requests. If a buyer requests more than one spectrum, his request can never be satisfied if we simply put the buyer in one of the independent sets. To solve these problem, we propose the Algorithm 2 for grouping.

Algorithm 2 Grouping Process

At time slot t
$G_t = \{V_t, E_t\}$ is the subgraph consisting of buyers who submit the spectrum request.
Add edges between any two buyers who do not have a common spectrum in their available spectrum sets.
for all Buyer $j, j \in V_t$ **do**
  Create d_j dummies for buyer j if $d_j > 1$.
  Delete the original buyer j in G_t and insert the dummies in G_t.
end for
for all $s_i, i = 1, 2, ..., M$ **do**
  if V_t is non-empty **then**
    $V_{t,i}$ is the set of buyers for which s_i is available
    Find one independent set g_i on $V_{t,i}$;
    Match g_i to s_i;
    Let k denote the buyer with minimum $b_{k,i}t_k$ in g_i. The group bid of g_i is

$$\Phi_i = \frac{b_{k,i}t_k}{\max_{j \in g_i \setminus k} t_j} \times (|g_i| - 1)$$

    in which $|g_i|$ is the size of g_i.
    Remove g_i from V_t
  end if
end for

- We add virtual edges in the interference graph to make sure that buyers with no common available spectrum will not be grouped together.
- We create dummies to replace the original buyer. If the buyer requests d_j spectrums, then we create d_j dummies. In this way, the buyer has d_j chances to be included in an independent set, and to win up to d_j spectrums.
- When computing the group bid, we make sure that no buyer will pay more than his bid, thus guaranteeing individual rationality. We sacrifice the buyer who bids the minimum bid (by reducing the group size by 1) in order to guarantee truthfulness.

5.2.2 Winner Determination

we propose the Algorithm 3 for determining winning sellers and buyers as well as their payment.

6 THEORETICAL ANALYSIS

In this section, we will prove that the proposed auction mechanism is economic robust in terms of 1) truthfulness; 2) individual rationality; and 3) balanced budget.

Algorithm 3 Winner and Payment Determination

for all $g_i, i = 1, 2, ..., M$ **do**
  if $\phi_i > A_i$ **then**
    All buyers in g_i except k win s_i.
    Group g_i pays a total amount of $\Phi_i \times \max_{j \in g_i \setminus k} t_j$, which is equally shared by the winning buyers (except k).
    Seller of s_i is paid Φ_i per time slot till $t + \max_{j \in g_i \setminus k} t_j$;
  else
    All buyers in group g_i lose and pay nothing.
    Seller of s_i is paid nothing.
  end if
end for

6.1 Individual Rationality

Theorem 1: LOTUS is individual rational for both the sellers and the buyers.

Proof: Let's consider the winning seller i and the winning group of buyers g_i.

- For the seller i, the per time slot payment Φ_i is greater than A_i, and the spectrum occupancy is no more than $\max_{j \in g_i \setminus k} t_j$. Therefore, the seller i gets positive utility as calculated in (1).
- For a winning buyer j in g_i, the total payment is

$$\frac{\Phi_i \times \max_{j \in g_i \setminus k} t_j}{(|g_i| - 1)} = b_{k,i} t_k \leq b_{j,i} t_j$$

  Therefore, the total payment is less than the total bids of the buyer j, and he can get positive utility as calculated in (2).

□

6.2 Balanced Budget

Theorem 2: LOTUS is budget balanced for the auctioneer.

Proof: Let's consider the payment to the winning seller i and the payment from the winning group of buyers g_i. The auctioneer's budget is

$$\Phi_i \times \max_{j \in g_i \setminus k} t_j - \Phi_i \times \max_{j \in g_i \setminus k} t_j = 0$$

Therefore, the auctioneer's budget is always zero, which is a non-negative budget. □

6.3 Truthfulness

In this paper, we only consider the truthfulness after the screening phase[9]. We first prove the truthfulness on the buyers' side, then prove the truthfulness on the sellers' side. Since the buyers can bid for multiple spectrums, and can bid different prices for different spectrums, we have to consider intra-spectrum truthfulness as well as inter-spectrum truthfulness.

9. In fact, a buyer's bid will affect his available spectrum set because the bid affects the comparison between the discounted bid and the opportunity cost. To consider the truthfulness in the screening phase is a future direction.

Lemma 1: Inter-spectrum truthfulness. A buyer cannot manipulate the bidding price for one spectrum to affect the winning result of another spectrum.

Proof: The buyers have no control over the grouping process. After group formation, the group members are matched to a specific spectrum. It is not sure whether the matched spectrum will be allocated to them or not but it is definitely sure that other spectrums will *not* be allocated to them.

The group bid only relies on the bid of each group member for the matched spectrum, independent of their bids for other spectrums. Therefore, a buyer cannot manipulate the bidding price for one spectrum to affect the winning result of another spectrum. □

Lemma 2: If the jth buyer wins spectrum s_i, he pays the same price regardless of his bid for s_i, that is, $b_{j,i}$.

Proof: If the jth buyer wins in group g_i, he is not the one with the minimum bid and he will pay $b_{k,i} t_k t_j / \max_{j \in g_i \setminus k} t_j$, which is unaffected by $b_{j,i}$. □

Lemma 3: If the jth buyer wins spectrum s_i by bidding $b_{j,i}$, he will also win by bidding $b'_{j,i} > b_{j,i}$.

Proof: Since the jth buyer is a winning buyer in group g_i, $b_{j,i} t_j$ is not the minimum in group g_i. Therefore, the group bid will not be affected if $b_{j,i}$ increases to $b'_{j,i}$. Group g_i will still win spectrum s_i, and the jth buyer will still be a winning buyer in group g_i. □

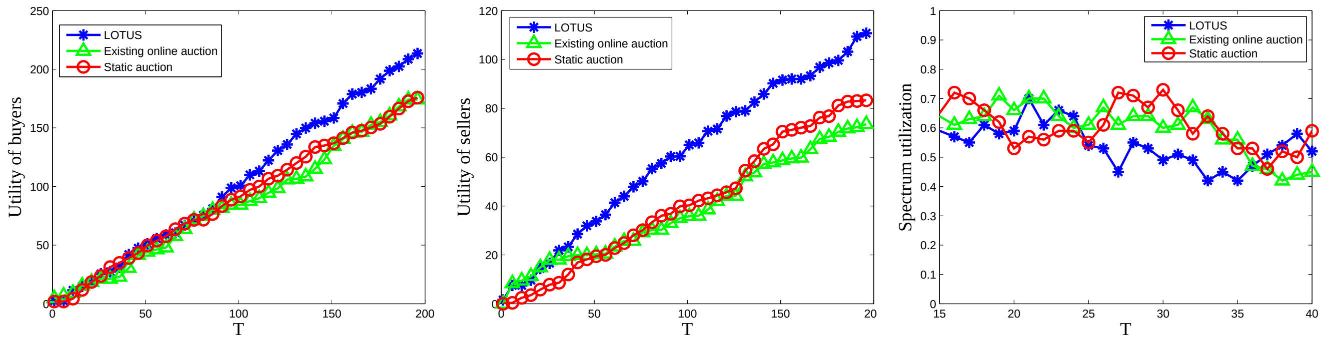
TABLE 1
Possible Auction Results

Case	I	II	III	IV
The seller/buyer is truthful	Lose	Win	Win	Lose
The seller/buyer is untruthful	Lose	Lose	Win	Win

Lemma 4: Intra-spectrum truthfulness. A buyer cannot manipulate the bidding price for one spectrum to affect the winning result of the spectrum.

Proof: Let's consider the jth buyer and the spectrum s_i. The jth buyer bids truthfully as $b_{j,i} = \widetilde{b}_{j,i}$, and untruthfully as $b'_{j,i}$. There are four possible auction results as shown in Table 1.

- **Case I**. The jth buyer loses the spectrum s_i when he bids truthfully and untruthfully, and his utility is both zero. Therefore, the buyer does not gain higher utility by being untruthful.
- **Case II**. The jth buyer wins the spectrum s_i when he bids truthfully and loses the spectrum s_i when he bids untruthfully. In the former case, he achieves non-negative utility due to individual rationality. In the later case, he achieves zero utility. Therefore, the buyer has higher utility when being truthful.
- **Case III**. The jth buyer wins the spectrum s_i when he bids truthfully and untruthfully, and he pays the same price according to Lemma 2. Since the buyer also gains the same valuation for using spectrum s_i, his utility is the same when he bids truthfully and untruthfully.
- **Case IV**. The jth buyer loses the spectrum s_i when he bids truthfully and wins the spectrum s_i when he bids untruthfully. In the former case, he achieves zero utility.



(a) Utility of buyers (b) Utility of sellers (c) Spectrum utilization

Fig. 4. Performance comparison of 1) LOTUS; 2) Online auction without interference discount (referred to as Non-ID online auction in the figure) and; 3) Static auction

Now we have to analyze his utility in the later case. For Case IV to happen, $b_{j,i}t_j$ has to be the minimum in group g_i, and $b'_{j,i} > b_{j,i}$. When the jth buyer increases his bid, the group bid Φ'_i will be higher than Φ_i, and group g_i can win the spectrum s_i. Also, $b'_{j,i}t_j$ will no longer be the minimum, otherwise, the buyer j will still lose the spectrum s_i. Let buyer j' be the new minimum bids, and we have $b_{j,i}t_j < b_{j',i}t_{j'} < b'_{j,i}t_j$. The jth buyer's utility becomes

$$\widetilde{b}_{j,i}t_j - b_{j',i}t_{j'} \leq \widetilde{b}_{j,i}t_j - b_{j,i}t_j = 0$$

The jth buyer achieves negative utility when being untruthful. Therefore, he has no incentive to lie.

In summary, the jth buyer has no incentive to bid untruthfully as his utility cannot be improved. □

Theorem 3: LOTUS is truthful for the buyers.

Proof: Theorem 3 can be easily proved by combining the inter-spectrum truthfulness of Lemma 1 and the intra-spectrum truthfulness of Lemma 4. □

Now we consider the truthfulness on the sellers' side.

Lemma 5: If the ith seller wins, he gets paid the same price regardless of his ask A_i.

Proof: If the ith seller wins, the received payment is always determined by ϕ_i and the maximum number of desired time slots in g_i, both of which are not affected by A_i. □

Lemma 6: If the ith seller wins by A_i, it will also win by asking $A'_i < A_i$.

Proof: If the ith seller asks for a lower price, $A'_i < A_i < \phi_i$. Therefore, the seller will still be able to sell his spectrum to group g_i. □

Theorem 4: LOTUS is truthful for the sellers.

Proof: Let's consider the ith seller and the group g_i. The ith seller asks truthfully as $A_i = \widetilde{A}_i$, and untruthfully as A'_i. We again consider the four possible cases in Table 1.

- **Case I**. The ith seller loses when he bids truthfully and untruthfully, and his utility is both zero. Therefore, the seller does not gain higher utility by being untruthful.
- **Case II**. The ith seller wins when he bids truthfully and loses when he bids untruthfully. In the former case, he achieves non-negative utility due to individual rationality. In the later case, he achieves zero utility. Therefore, the seller has higher utility when being truthful.
- **Case III**. The ith seller wins when he bids truthfully and untruthfully, and he is paid the same price according to Lemma 5. Since the seller also loses the same valuation for leasing spectrum s_i, his utility is the same when he bids truthfully and untruthfully.
- **Case IV**. The ith seller loses when he bids truthfully and wins when he bids untruthfully. In the former case, he achieves zero utility. Now we have to analyze his utility in the later case. For Case IV to happen, $A_i > \Phi_i$ must be true, and seller i must lower his bid so that $A'_i < \Phi_i$. The ith seller's utility becomes $\Phi_i - \widetilde{A}_i < A_i - \widetilde{A}_i = 0$. The ith seller achieves negative utility when being untruthful. Therefore, he has no incentive to lie.

In summary, the ith seller has no incentive to bid untruthfully as his utility cannot be improved. □

In conclusion, LOTUS is truthful, individual rational and budget-balanced.

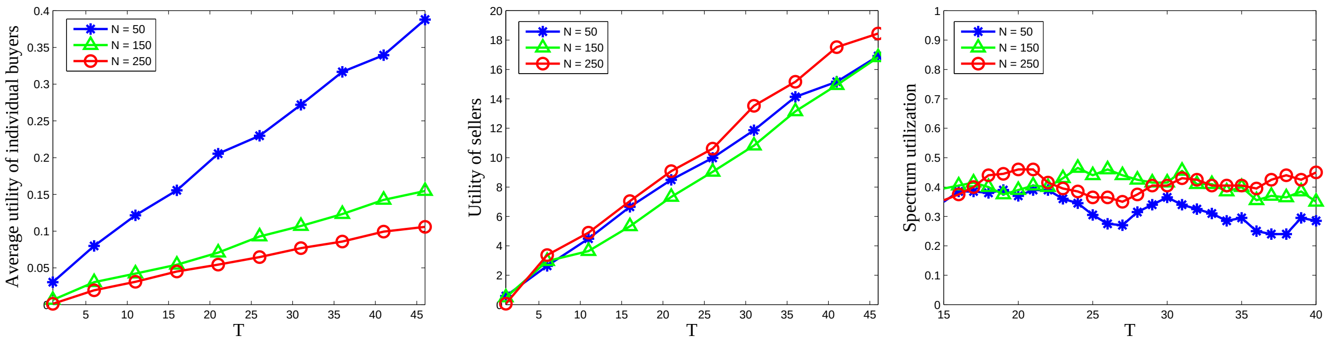
7 Performance Evaluation

7.1 Simulation Setup

We consider the network where buyers scatter randomly within a square area layout of size $100m \times 100m$ area. The transmission range of secondary devices is $50m$. The number of buyers is 100 at default. The probability that an buyer requests the spectrum is 0.5. The average number of desired time slots is 3 at default, and the average number of requested spectrums is 2 at default. The number of sellers (and correspondingly their contributed spectrums) is fixed as 5. Sellers' asks and buyers' bids both follow independent uniform distribution in the range (0,1].

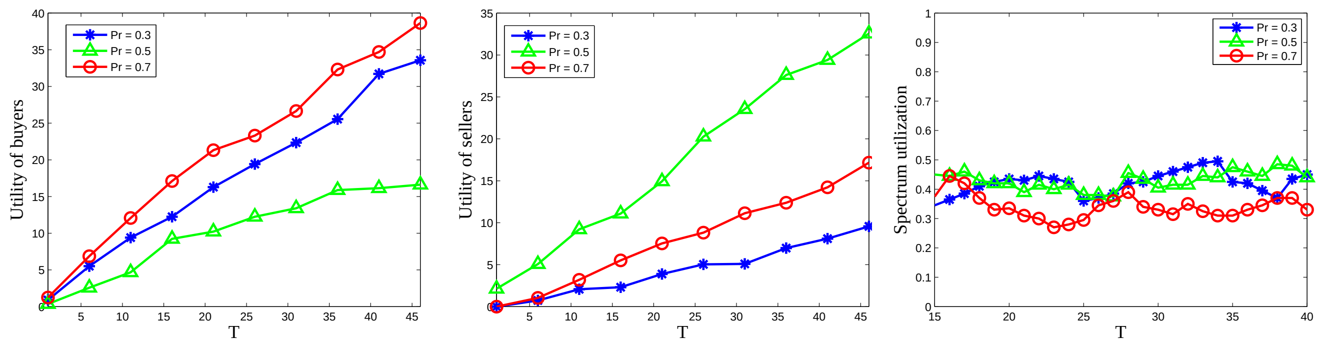
Firstly, we compare the performance of LOTUS with the following cases:

- *Static auction without opportunity cost consideration*. In this case, the influence of the spectrum allocation at an early time slot on the spectrum requests at later time slots is totally ignored. The auctioneer does not calculate the opportunity cost. Thus there is no screening process



(a) Average utility of individual buyer (b) Utility of sellers (c) Spectrum utilization

Fig. 5. Impact of buyer density.



(a) Utility of buyers (b) Utility of sellers (c) Spectrum utilization

Fig. 6. Impact of buyer request frequency.

based on the comparison of the discounted bid and the opportunity cost.

- *Online auction without interference discount consideration*. In this case, the influence of the spectrum allocation at an early time slot on the spectrum requests at later time slots is considered. But when calculating the opportunity cost, no interference discount is applied. Actually, this can be viewed as a special case where the interference graph is a complete graph, thus each buyer's interference discount equals 1.

Secondly, we study the key factors that affect the performance of LOTUS, including

- *The density of buyers*, which has a great influence on the interference relationship among buyers;
- *The average frequency of buyers' spectrum request*, which is represented by the probability that a buyer raises spectrum request.
- *The average number of desired time slots*, which affects how long a winning buyer will occupy the spectrum, thus depriving his interfering neighbors the access opportunity.
- *The average number of requested spectrums*. We compare the case where the buyers are restricted to request one spectrum and the case where the buyers are allowed to request multiple spectrums.

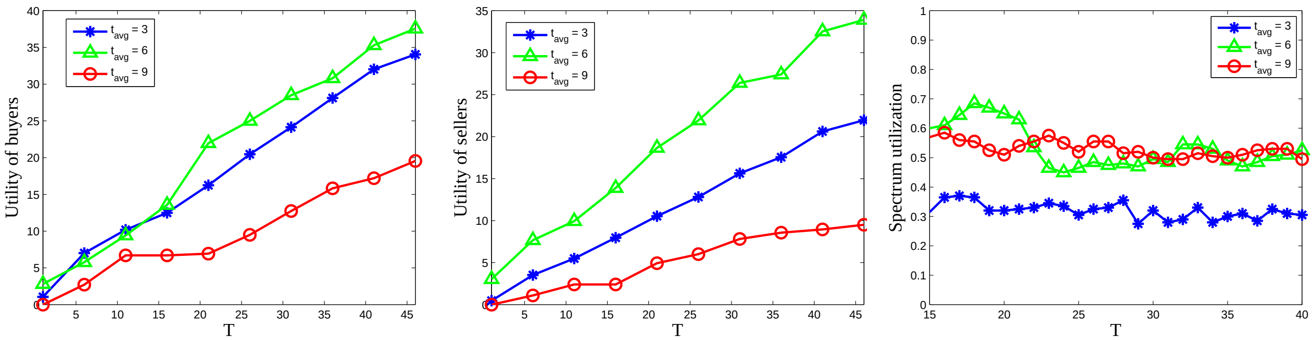
7.2 Comparison of Different Auction Mechanisms

Fig. 4 shows the results of the comparison of different auction mechanisms. It is verified that LOTUS outperforms both existing online auction and static auction mechanisms. Subfigure (a) shows that the total utility of all buyers can be improved by as high as 25.7% over the existing online auction and the static auction; and subfigure (b) shows that the total utility of all sellers can be improved by as high as 52.8% over the existing online auction, 37.5% over the static auction. Subfigure (c) shows the spectrum utilization of the LOTUS is more or less the same as the spectrum utilization of the existing online auction and the static auction, because LOTUS aims at improving the utilities of the buyers and the sellers at the cost of leaving some time slots idle for later buyers who may have higher bids.

7.3 Factors Affecting LOTUS Performance

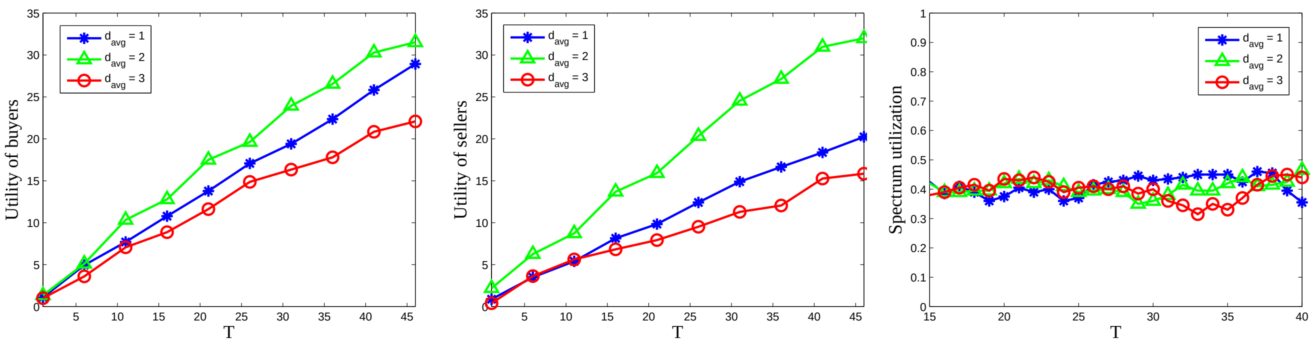
7.3.1 *Impact of Buyer Density*

Fig.5 shows the impact of buyer density. In subfigure (b), it is very interesting that when the number of buyers increases from 50 to 150, the utility of sellers decreases; but when the number of buyers increases from 150 to 250, the utility of sellers increases, and becomes even the highest one. The possible reason is that, when there are more buyers, the network becomes crowded with many buyers interfering with each other. The opportunity cost of allocating a spectrum to a buyer becomes larger and the interference discount becomes larger.



(a) Utility of buyers (b) Utility of sellers (c) Spectrum utilization

Fig. 7. Impact of average number of desired time slots.



(a) Utility of buyers (b) Utility of sellers (c) Spectrum utilization

Fig. 8. Impact of average number of requested spectrums.

Therefore, many buyers are eliminated in the screening phase, resulting in utility decrease for the sellers. However, when the number of buyers further increases, even though many buyers are screened out, the number of the rest of the buyers is still high. Therefore, there are more successful auction results, which contributes to higher seller utility. Subfigure (a) is the total utility of buyer divided by the number of buyers. We can see that when the number of buyers increases, the average utility of individual buyers keeps decreasing, because of more intense price competition and tighter interference relationship. The spectrum utilization does not have clear trend with the increase in number of buyers, because on the one hand, more buyers lead to more potential spectrum requests and higher group bids, but on the other hand, more buyers lead to more severe interference problems.

7.3.2 *Impact of Buyer Request Frequency*

Fig.6 shows the impact of buyer request frequency. When the spectrum request happens quite infrequently ($Pr = 0.3$), which means that the traffic is very light, the total utility of sellers is low because few buyers participates in the auction. The increase in the request frequency ($Pr = 0.5$) increases sellers' utility due to increased demand but a further increase ($Pr = 0.7$) will degrade the utility since the network is too crowded and many buyers' request cannot be satisfied. Also, the opportunity cost for an buyer increases since the chance of his interfering neighbor requesting the spectrum in later time becomes higher. Therefore, more buyers will be rejected in the screening phase. The trend of the utility of buyers just goes the opposite direction of the utility of sellers. The possible explanation is that when sellers obtain more utility from the auction, the buyers are left with less utility. The spectrum utilization change has similar trend to that of the sellers' utility, due to the same reasons.

7.3.3 *Impact of Average Number of Desired Time Slots*

Fig.7 shows the impact of average number of desired time slots. Spectrum utilization is almost not affected since the number of arriving buyers in each time slot is nearly the same. So the number of buyers participating in the auction and the number of final winners are nearly the same, too. When the average requested time slots increases from 3 to 6, the utilities of both buyers and sellers increase. The reason is that more available time slots are requested and assigned to buyers, making them better off and generating higher revenue for the sellers. On the contrary, when the average requested time slots increases from 6 to 9, the utilities of both buyers and sellers decrease. This is because the buyers arrive at early time slots occupy the spectrum for a long period, during which the unpredicability of later spectrum requests increases, making the calculation of opportunity cost less reliable. There is higher probability that an buyer with higher valuation cannot access the spectrum because its interfering neighbors still occupy the spectrum. This reduces not only buyers' utility but also buyers' utility.

7.3.4 Impact of Average Number of Requested Spectrums

Fig.8 shows the impact of average number of requested spectrums. When the buyers are restricted to request only one spectrum, the spectrum demand is low, and the possible number of successful spectrum trade is small. Therefore, the utilities of both buyers and sellers are low. When the buyers are allowed to bid for multiple spectrums, the spectrum demand increases, leading to utility increase for both buyers and sellers. However, if the spectrum demand further decreases, the utilities of both sellers and buyers will decrease due to similar reasons for the impact of average number of desired time slots. The spectrum utilization is generally higher when with the increase of spectrum demand.

8 Conclusion

In this paper, we proposed a location-aware online truthful double auction framework for spectrum allocation. We determine the interference harmfulness of a buyer by marking down his bid by a factor defined as *interference discount*. The interference discount depends on a buyer's geographic location and his interference relationship with other buyers. If a buyer has a larger interference degree than his interfering neighbors, or if many of his interfering neighbors can reuse the same spectrum, then the interference discount will greatly reduce this buyer's bid. A buyer will be considered as a valid candidate for winning a spectrum if his discounted bid is greater than his opportunity cost. The auctioneer executes the auction mechanism LOTUS, which is proved to be truthful, individual rational and budget balanced. The simulation results show that LOTUS is effective in improving the utility of sellers and buyers, as well as the spectrum utilization.

For the future work, there are many interesting directions since online auction is an important but rather untapped area. First, the truthfulness in the screening phase in LOTUS should be considered, as the buyers may bid untruthfully in order to become a valid candidate for a certain spectrum. Furthermore, collusion in online spectrum auction is another interesting topic. Buyers who arrive at the same time slot may collude, and buyers who arrive at different time slots may also collude. It has to be defined how buyers may collude in the later case and how to avoid it.

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