Balancing Income and User Utility in Spectrum Allocation

Yanjiao Chen, *Student Member, IEEE*, Lingjie Duan, *Member, IEEE*, Jianwei Huang, *Senior Member, IEEE*, and Qian Zhang, *Fellow, IEEE*

Abstract—To match wireless users' soaring traffic demand, spectrum regulators are considering allocating additional spectrum to the wireless market. There are two major directions for the spectrum allocation: licensed (e.g., 4G cellular service) and unlicensed services (e.g., Super Wi-Fi service). The 4G service provides a ubiquitous coverage, has a higher spectrum efficiency, and often charges users a high service price. The Super Wi-Fi service has a limited coverage, a lower spectrum efficiency, but often charges users a low service price. The spectrum regulator now simply allocates the spectrum to maximize its income, but such an income-centric allocation does not ensure the best spectrum utilization by the users. This motivates us to design a new spectrum allocation scheme which jointly considers the spectrum regulator's income and the users' aggregate utility by investigating three market tiers: the spectrum regulator, 4G and Super Wi-Fi operator coalitions, and all the wireless users. We formulate it as a three-stage game and derive the unique subgame perfect equilibrium. Compared with the traditional income-centric allocation, we prove that the proposed scheme significantly improves users' aggregate utility with a limited spectrum regulator's income loss.

Index Terms—Spectrum allocation, user utility improvement, three-tier dynamic game

1 Introduction

The number of customers using wireless broadband services has been increasing dramatically during recent years. Such a demand will surpass the capacity of allocated wireless spectrum for mobile broadband services by as soon as 2013 [1]. To provide more spectrum resources to support mobile broadband services, the Federal Communications Commission (FCC) has decided to make 500 MHz of new wireless spectrum available within 10 years for licensed and unlicensed use [2]. In July 2012, the President's Council of Advisors on Science and Technology (PCAST) of the U.S. [3] further proposed to identify 1,000 MHz of Federal spectrum for shared-use among commercial users. The remaining key question is: how to allocate these spectrum bands to different operators and how to make the best use of these spectrum to improve end users' utility? It is not only a technical issue, but also a complicated policy and socio-economic issue [4].

There are two mainstream systems of providing mobile Internet access: licensed cellular networks and unlicensed wireless local area networks. The representative next generation technologies of these two systems are 4G and Super Wi-Fi. However, these two representative technologies are quite different.

- Y. Chen and Q. Zhang are with the Department of Computer Science and Engineering, Hong Kong University of Science and Technology, Hong Kong. E-mail: {chenyanjiao, qianzh}@ust.hk.
- L. Duan is with the Engineering Systems and Design Pillar, Singapore University of Technology and Design, Singapore. E-mail: lingjie_duan@sutd.edu.sg.
- J. Huang is with the Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong. E-mail: jwhuang@ie.cuhk.edu.hk.

Manuscript received 3 Nov. 2013; revised 6 Sept. 2014; accepted 6 Jan. 2015. Date of publication 5 Feb. 2015; date of current version 30 Oct. 2015.
For information on obtaining reprints of this article, please send e-mail to: reprints@ieee.org, and reference the Digital Object Identifier below.
Digital Object Identifier no. 10.1109/TMC.2015.2393309

- The 4G cellular system is based on OFDM and MIMO technologies, and can achieve a high spectrum efficiency due to careful network planning and efficient interference mitigation. Moreover, as a cellular network technology, 4G can provide ubiquitous Internet access. However, a 4G cellular operator usually charges a high price to the end users to compensate the high deployment and operational cost.
- Super Wi-Fi has a relatively low spectrum efficiency, as it operates in the unlicensed mode and different networks or systems may share the same spectrum without a centralized interference management [5]. Furthermore, it only has limited overall coverage due to a limited number of access points and regulatory power limitation[1]. But Super Wi-Fi can be easier to deploy and cheaper to maintain than 4G [6], resulting in a low service price. For example, China Mobile Hong Kong charges a monthly fee of HK$278 for 2 GBytes of 4G data, but only a monthly fee of HK$38 for unlimited Wi-Fi Service [7].

Users with different coverage requirements and price sensitivities thus have different preferences over these two services.

The FCC's current practice is to reserve certain amount of unlicensed spectrum for Wi-Fi service, while auctioning the remaining spectrum to cellular operators who bid the highest price [8]. However, it is not clear what is the optimal amount of spectrum that should be reserved for unlicensed use. Recent study (e.g., [9]) has shown that an inappropriate allocation of unlicensed spectrum may hurt the total surplus

1. Even with the Super Wi-Fi operating on the TV white space (which has a better propagation property than the traditional 2.4GHz spectrum bands), the ad hoc deployment of access points may still make it hard to achieve the full coverage.

Fig. 1. Illustration of three-stage wireless allocation and service model.

of both operators and users. On the other hand, the spectrum auction mechanisms [10], [11], [12], [13], though aiming at maximizing the total utility of winning operators, do not guarantee that these operators will use the spectrum to provide desirable and affordable services for all wireless users. Hence from the social persecutive, the FCC should not only interact with the operators, but also consider the impact on the users. Such multi-level interactions can be modeled by a three-tier model. Existing works on three-tier models (the regulator, the operators, and the end users, e.g., [14], [15], [16]) mainly concerned about how the operators should lease spectrum to maximize their profits in the market, without concerning about how the regulator should allocate the spectrum to maximize a weighted sum of its income and the end users' utility. The main purpose of this paper is to fill in this important policy gap.

In this paper, we formulate the interactions among the FCC (the regulator), the 4G and the Super Wi-Fi operators, and the wireless users as a three-stage dynamic game as illustrated in Fig. 1. We assume that all licensed 4G operators form a coalition when interacting with the FCC; similarly, the Super Wi-Fi coalition represents all the Super Wi-Fi operators, who need spectrum for unlicensed use. In the rest of the paper, we will use the "4G operator" to refer to the 4G coalition, and the "Wi-Fi operator" to refer to the Super Wi-Fi coalition. [2] In Stage I, the FCC decides the spectrum allocations to both operators. In Stage II, two operators optimally price their services to maximize their profits based on their limited spectrum amounts. Finally in Stage III, the users choose between the two services to maximize their own utilities based on the service prices.

Our key results and major contributions are as follows.

- *Spectrum allocation with consideration of end-user utility.* The proposed three-tier dynamic game model, which practically characterizes all wireless users' utility and service choices, is helpful for the FCC's supervision of spectrum utilization in end-market.
- *Significant user utility improvement with limited income loss*. The proposed spectrum allocation scheme, shown by simulation results, significantly improves user utility; while, by theoretical analysis, have bounded income loss (compared with the income-centric spectrum allocation).
- *Price competition under different spectrum conditions.* We comprehensively analyze the price competition between different operators, given the spectrum allocation. We find that an operator will selfishly reserve some capacity unused and charge a high price if assigned relatively large amount of spectrum.
- *Users' service choices:* We analyze how the prices (decided by the operators) and capacity (decided by the FCC's spectrum allocation) jointly influence the users' service choice. The proposed spectrum allocation scheme will lead to lower service prices and serve more wireless users.

The rest of the paper is organized as follows. We briefly review the related work in Section 2. The system model and key assumptions are given in Section 3. We describe the three-stage game framework in Section 4. Then we use backward induction to analyze the game, starting from the market response of Stage III in Section 5, to the service competition game of Stage II in Section 6, finally to the spectrum allocations of Stage I in Section 7. In Section 8, we derive the bound of the regulator's income loss and analyze the impacts of critical parameters. Finally, we summarize our work in Section 9. Due to page limitations, all proofs are given in our online technical report [17].

2. We focus on the competition and interaction between licensed and unlicensed services. We will further study the competition within each service in a future work.

2 Related Work

Multi-tier game models for wireless markets have been widely studied. Lehr and McKnight in [6] surveyed the competitive and complementary relationship of Wi-Fi and 3G technology. Niyato and Hossain in [18] built a 2-tier pricing competition model between Wi-Fi and WiMAX operators. However, they did not consider the regulator's spectrum allocation in a higher tier. Several recent results studied three-tier models that involve the spectrum owner, the operators, and the users [14], [15]. These prior results focused on how much spectrum each operator should lease or buy from the spectrum owners to maximize their profits, while our work focuses on how much spectrum the regulator should allocate to Wi-Fi (for unlicensed use) and 4G (for licensed use) considering the utility of end users.

Spectrum auction is a way of distributing spectrum to different operators for licensed access [11], [13], [19]. Usually the spectrum is auctioned for exclusive licensed use by the cellular operators. This will not help users who want to use unlicensed wireless services. In addition, spectrum auction usually considers the utility of operators rather than the users' aggregate utility. However, the operators who value the spectrum the most may not choose to maximize the aggregate utility of all users, especially when the operators have enough market power and the market entry barrier is significant.

Spectrum is also reserved for unlicensed access. Nguyen *et al.* in [9] studied the influence of additional unlicensed spectrum on social welfare, but the user demand function is a bit over-simplified. Furthermore, they did not make suggestions regarding how spectrum should be allocated between licensed and unlicensed ones. The follow-up work [20] established a pricing model among unlicensed operators. However, the model did not consider the competition from licensed operators, nor the spectrum allocation issue of the regulator in a higher tier.

Our paper is different from the previous results: 1) we aim at addressing the spectrum allocation issue for the FCC, considering allocating additional spectrum available for licensed and unlicensed use, respectively; 2) we build a three-tier model, in which the FCC not only cares about its own income, but also the users' aggregate utility.

3 System Model

3.1 Spectrum Allocation

We consider a regulator (referred to as the FCC for the illustration purpose), who possesses S units of spectrum and intends to assign the spectrum to wireless operators to satisfy their users' increasing wireless data demands. In this paper, we only consider two wireless operator coalitions: one cellular coalition providing 4G service, and one wireless local area network (WLAN) coalition providing Wi-Fi service.[3] Also, we focus on the *additional* capacity yielded by the spectrum to be allocated to the two operators by the FCC, without considering their existing capacity.[4] Let S_w and S_g denote the spectrum allocated to Wi-Fi and 4G, respectively. The FCC has no intention to reserve the spectrum for other purposes. Therefore, $S_g + S_w = S$.

We assume that the FCC charges the Wi-Fi and the 4G operators according to the quantity of spectrum allocated to them. The FCC charges the 4G operator a total amount of $\phi_g(S_g)$, and charges the Wi-Fi operator a total amount of $\phi_w(S_w)$. Both pricing functions are non-decreasing in the spectrum allocation. The FCC can design different pricing schemes based on the estimation for the profitability of 4G and Wi-Fi services.[5]

3.2 Service Model

3.2.1 Network Capacity

We assume that the expected data rate required by a user is δ.[6,7] The capacity of 4G network is $f(S_g)$, under an exclusive spectrum license and an efficient interference management, in which $f(\cdot)$ is a non-decreasing function. The capacity of Wi-Fi network is $f(\eta S_w)$, where $\eta < 1$ is the interference parameter and characterizes the low spectrum efficiency in sharing the unlicensed band with other Wi-Fi networks and other unlicensed services (e.g., ZigBee devices, Bluetooth devices, and cordless phones) [21]. So the maximum number of concurrent in-service users that can be supported by 4G service and Wi-Fi service is $f(S_g)/\delta$ and $f(\eta S_w)/\delta$, respectively. We consider the requirement of data rate instead of bandwidth, circumventing the complicated issue of interference management. Thus, our setup can be easily generalized and applied to different types of technologies, as long as we are able to characterize the relationship between bandwidth and capacity.

3.2.2 Network Coverage

The transmission range of a cellular base station is in the order of kilometers, being able to serve users over a large contiguous area. 4G operators generally deploy enough base stations to realize full coverage. Hence, we assume that the future 4G network's coverage is 1 (full coverage) in the new spectrum.[8]

Though improved greatly on existing Wi-Fi, the Super Wi-Fi is still limited in coverage. For example, the Super Wi-Fi base station launched by Altai has a signal coverage of 500 m in radius [22]. According to [23], approximately 700 traditional Wi-Fi access points would be needed to cover the same area as one cellular base station does under the current technology, but such a dense deployment to achieve full coverage will have a formidably high cost. Even if using the whitespace spectrum with better propagation characteristics (due to low frequency), the existing ad hoc deployment of Wi-Fi may not be able to achieve a full coverage. Hence, we assume Wi-Fi network's coverage is $\theta < 1$.

3.2.3 Service Price

We assume that the Wi-Fi and 4G operators charge p_w and p_g per subscriber, respectively, as long as the data rate requirement is fulfilled [24]. We consider N wireless users who are potential Wi-Fi or 4G subscribers. The numbers of users who eventually choose the two services (called subscribers) are denoted by N_w and N_g, respectively. Naturally we have $N_w + N_g \leq N$. For each network, the operator needs to perform admission control if its subscribers' total demand exceeds the network capacity, hence we have $N_w \leq f(S_g)/\delta$ and $N_g \leq f(\eta S_w)/\delta$ at the equilibrium.

3.3 User Preference

Users are different in their preferences for the coverage of Internet access. We characterize such heterogeneity by a type parameter $\alpha \in [0, A]$, which represents users' sensitivity due to their mobility [25]. A user with a higher mobility prefers a higher network coverage, while a user who always stays at one location is less sensitive to network coverage in other places. Therefore, a higher mobility user will have a higher preference for coverage than for price. For simplification, we assume that α follows a uniform distribution in $[0, A]$ in the following analysis.[9]

For a type α user, he achieves a utility $u_g^\alpha = \alpha - p_g$ if choosing 4G,[10] and a utility of $u_w^\alpha = \alpha\theta - p_w$ if choosing Wi-Fi. All users have the same reservation utility $u_0 \geq 0$, which means that a type α user will choose neither 4G nor Wi-Fi if

3. In the following texts, we use Wi-Fi to represent Super Wi-Fi, for simplicity.

4. We assume that the additional spectrum to be distributed to the two operators will not affect how they manage their existing spectrum and capacity. We will consider the impact of new spectrum allocation on old services in the future work.

5. In this paper, we consider the case that $\phi_g(\cdot)$ and $\phi_w(\cdot)$ are determined by the FCC, unlike the spectrum auction where the operators' bids determine the price. In the future work, we will study the situation where the operators have the right to (partly) decide the spectrum fee.

6. The main results will still hold if we allow a user to have different data rate requirements under the Wi-Fi and 4G services.

7. The 4G service can satisfy a user's fixed δ requirement, as licensed spectrum access can ensure QoS. Thanks to the better propagation characteristics of the White Space than the ISM band, future Super Wi-Fi network also intends to provide QoS-guaranteed service to satisfy users' fixed δ requirement [14]. In the future work, we plan to consider the case where users have heterogeneous demands.

8. At the regions where 4G base stations haven't been deployed yet, subscribers can still enjoy cellular service through the existing 3G network [7].

9. Uniform distribution of the QoS sensitivity is commonly used for tractable analysis and changing to another continuous distribution is unlikely to change the key results (see [24] and [26]).

10. If choosing 4G, the user's utility is $u_g^\alpha = \alpha \cdot 1 - p_g$, as the 4G coverage is 1.

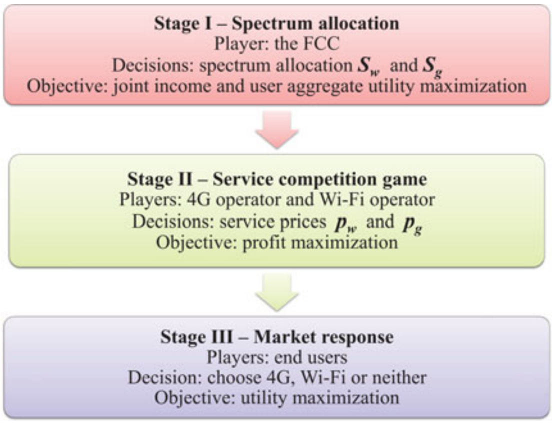


Fig. 2. Three stages of the dynamic game.

$\max\{u_g^\alpha, u_w^\alpha\} < u_0$. To summarize, a type α user's service choice is as follows[11]:

$$\begin{cases} \text{4G service,} & \text{if } u_g^\alpha \geq u_w^\alpha \text{ and } u_g^\alpha \geq u_0 \\ \text{Wi-Fi service,} & \text{if } u_w^\alpha > u_g^\alpha \text{ and } u_w^\alpha \geq u_0 \\ \text{No service,} & \text{otherwise.} \end{cases} \tag{1}$$

4 THREE-STAGE GAME FRAMEWORK

The three-stage game model is illustrated in Fig. 2, showing the game players, what decisions they make and their objectives.

4.1 Stage I-Spectrum Allocation

The FCC decides the spectrum allocation S_w and S_g to maximize its utility, U_f, which is the weighted sum of the end users' aggregate utility U_{user} and the FCC's income,

$$U_f^\beta = \beta U_{user} + \phi_g(S_g) + \phi_w(S_w). \tag{2}$$

Here the weight $\beta \geq 0$, represents how much the FCC values the user utility over its income. The FCC can adjust β to tailor for different spectrum allocation purposes. In the benchmark case, where the FCC only cares about its income, $\beta = 0$. If the FCC cares more about user utility than its income, then β can be set larger than 1. We define end users' aggregate utility as the weighted sum of Wi-Fi subscribers' enjoyed coverage and 4G subscribers' enjoyed coverage:

$$\begin{aligned} U_{user} &= \omega_1 \int_{\text{Wi-Fi subscribers}} \frac{\theta}{A} d\alpha + \omega_2 \int_{\text{4G subscribers}} \frac{1}{A} d\alpha \\ &= \frac{\omega_1 \theta N_w}{N} + \frac{\omega_2 N_g}{N}. \end{aligned} \tag{3}$$

where the first term is Wi-Fi subscribers' aggregated valuation of coverage, and the second term is 4G subscribers' aggregated valuation of coverage. The parameters ω_1 and ω_2 represent the relative importance of Wi-Fi and 4G network from the FCC's point of view. End users' aggregate utility is determined by the numbers of subscribers to the Wi-Fi service and 4G service (N_w and N_g), which are directly affected by the service prices p_w and p_g decided by the two operators, and indirectly affected by the spectrum allocation S_w and S_g decided by the FCC.

11. In this paper, we assume that each user can choose at most one service. In future work, we will study the case when a user may choose both Wi-Fi and 4G services at the same time.

4.2 Stage II-Service Competition Game

After observing the spectrum allocation in Stage I, two operators play a pricing game in Stage II, where they determine the prices of their own services to maximize the profits. The profit of the 4G operator is the difference between the revenue and the spectrum payment:

$$U_g = N_g p_g - \phi_g(S_g). \tag{4}$$

The profit of the Wi-Fi operator is the difference between the revenue and the spectrum payment:

$$U_w = N_w p_w - \phi_w(S_w). \tag{5}$$

Apart from the cost of paying for spectrum, the cost for the Wi-Fi and 4G operators will also include the operating expense (OPEX) and the capital expense (CAPEX). If the OPEX and CAPEX are fixed and do not depend on the subscriber numbers or the amount of spectrum, our analytical results are directly applicable as long as those costs can be compensated by the profits made by the two operators. On the other hand, if the OPEX and CAPEX are functions of either subscriber number or the amount of spectrum, they will affect the computation of the equilibrium. For example, if the OPEX is an increasing function of the subscriber number, then the service prices of both Wi-Fi and 4G in Stage II will be higher. The influence on the spectrum allocation is more complicated, since we have to compare the OPEX and CAPEX of Wi-Fi and 4G. We will leave the detailed study of the impact of OPEX and CAPEX in a future work.

4.3 Stage III-Market Response

After observing the prices of 4G and Wi-Fi services, each user compares and decides which service to subscribe (or not to subscribe to any service at all). A user may be rejected by a network if the users' demand exceeds the network's capacity.

4.4 Nash Equilibrium (NE) and Subgame Perfect Equilibrium (SPE)

We introduce the concept of Nash Equilibrium and Subgame Perfect Equilibrium as follows [27]:

Definition 1 (Nash Equilibrium). *Consider a game* $\{\mathcal{I}, (S_i)_{i\in\mathcal{I}}, (u_i)_{i\in\mathcal{I}}\}$, *where* $\mathcal{I}$ *is the set of players,* S_i *is the strategy set of player* $i \in \mathcal{I}$, *and* u_i *is the utility of player* $i \in \mathcal{I}$. *Let* $\mathbf{s} = (s_i, s_{-i})$ *denote the strategy profile of all users, where* s_{-i} *includes the strategy choices of all players other than* i. *Denote* $S = \Pi_i S_i$ *as the set of all strategy profiles. A strategy profile* $s^* \in S$ *is a (pure) Nash Equilibrium if and only if* $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$ *is true for all* $i \in \mathcal{I}$.

The game that we consider is a dynamic game, where players act sequentially in multiple stages. A subgame is part of a dynamic game, and we have three subgames here. Stage III is a subgame, Stages II and III together is another subgame, and the whole game with three stages is also a subgame.

Definition 2 (Subgame Perfect Equilibrium). *A strategy profile of the three-stage game is an SPE if the choices of the FCC, the 4G and the Wi-Fi operators, and the end users constitute a Nash Equilibrium in each of the subgame of the whole game. In other words, no player at SPE will deviate unilaterally from his equilibrium strategy.*

In Sections 5 to 7, we will derive the SPE of the three-stage game using backward induction, that is, first to find the SPE of Stage III, then to find the SPE of Stage II and III together, finally to find the SPE of the whole game with three stages.

5 Stage III: Market Response

In Stage III, users can observe the service prices given in Stage II: p_w and p_g. Recall that the subscriber numbers of the Wi-Fi and 4G services are N_w and N_g, respectively. The following Proposition 1 gives the N_w in response to p_w; and Proposition 2 gives the N_g in response to p_g.

Proposition 1. *Given Wi-Fi capacity $f(\eta S_w)$, Wi-Fi subscriber number N_w in response to Wi-Fi price p_w and 4G price p_g is as follows*[12]*:*

$$N_w = \min\left(\overline{N}_w, \frac{f(\eta S_w)}{\delta}\right), \tag{6}$$

where

$$\overline{N_w} = \begin{cases} \frac{N}{A\theta(1-\theta)}[\theta p_g - p_w - (1-\theta)u_0], & if\ p_w > p_g - (1-\theta)A, \\ \frac{N}{A\theta}[-p_w + \theta A - u_0], & otherwise. \end{cases} \tag{7}$$

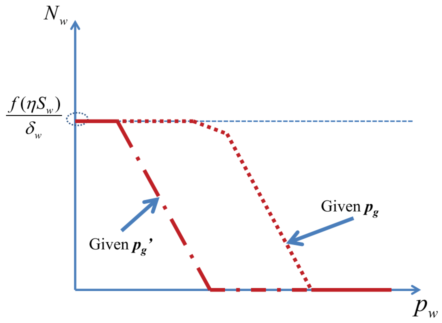
Proposition 2. *Given the 4G capacity $f(S_g)$, 4G subscriber number N_g in response to 4G price p_g and Wi-Fi price p_w is as follows:*

$$N_g = \min\left(\overline{N}_g, \frac{f(S_g)}{\delta}\right), \tag{8}$$

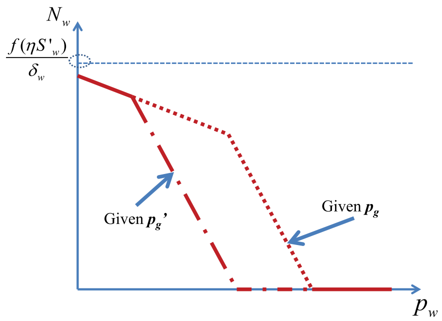
where

$$\overline{N}_g = \begin{cases} \frac{N}{A(1-\theta)}[-p_g + p_w + (1-\theta)A], & if\ p_g > \frac{1}{\theta}[p_w + (1-\theta)u_0], \\ \frac{N}{A}[-p_g + A - u_0], & otherwise. \end{cases} \tag{9}$$

The number of Wi-Fi subscribers corresponding to Proposition 1 is shown in Fig. 3, which shows how the number of Wi-Fi subscribers N_w changes with the Wi-Fi price p_w, given different capacity constraints and competitor's service price.[13] We show two different capacity constraints $f(\eta S_w')/\delta$ and $f(\eta S_w)/\delta$, in which $S_w' > S_w$ (see the two subfigures (a) and (b)); and two curves (solid line shows the overlapped area; dotted line and dot dash line show the difference) under two different competitor's prices $p_g' < p_g$. We have the following observations.



(a) Low capacity constraint



(b) High capacity constraint

Fig. 3. Number of Wi-Fi subscribers in stage III, under two different cellular prices $p_g' < p_g$.

- *Influence of the service's own price*. For all curves, the higher the Wi-Fi service price, the smaller the Wi-Fi subscriber number. The slope of each curve changes at the point when $p_w = p_g - (1-\theta)A$ (or p_g' if the 4G price is p_g'). The reason is that, when $p_w < p_g - (1-\theta)A$, we can prove that users of all types prefer Wi-Fi service (to 4G service) because of the Wi-Fi's low price. Without competition from 4G, the increase in the Wi-Fi price induces a mild decline of the Wi-Fi subscriber number. In contrast, when $p_w > p_g - (1-\theta)A$, Wi-Fi faces competition from 4G, therefore Wi-Fi loses subscribers faster with the increase of the Wi-Fi price p_w.
- *Influence of the competitor's price*. The 4G price mainly horizontally "shifts" the curves in Fig. 3 without changing the slopes. When 4G has a low price (p_g'), more users will be attracted to the 4G service, leading to fewer Wi-Fi subscribers.
- *Influence of the capacity constraint*. When the spectrum level for Wi-Fi is high (e.g., S_w' in Fig. 3b), the capacity constraint $f(\eta S_w')/\delta_w$ does not affect the subscriber number, because the W-Fi network has enough capacity even if all users subscribe to Wi-Fi; However, when the spectrum level for Wi-Fi is low (e.g., S_w in Fig. 3a), the capacity limits the number of subscribers that the Wi-Fi can support when the Wi-Fi price is low. In this case, the further decrease in Wi-Fi price cannot further increase the Wi-Fi subscriber number.

12. The numbers of subscribers to both services are expected values, since the user type α follows a uniform distribution. Therefore, the resultant utility of the two operators and the FCC are also expected values. To have a clean presentation, we omit notation $E[\cdot]$ on these variables.

13. The analysis of Proposition 2 is similar to that of Proposition 1, therefore we ignore it here due to page limitation.

6 Stage II-Service Competition Game

In Stage II, given the spectrum allocations from the FCC, two operators need to optimize their prices based on the analysis of user behavior in Stage III.

Definition 3 (Best Response). *Given the 4G service price p_g, the Wi-Fi operator's best response price is $p_w^*(p_g)$, such that Wi-Fi operator's profit $U_w(p_w^*(p_g), p_g) \geq U_w(p_w, p_g)$ for any $p_w \geq 0$. The 4G service's best response $p_g^*(p_w)$ can be defined similarly.*

According to the analysis in Section 5, the subscriber number of a service is affected by the spectrum allocation of that service, in particular, whether the spectrum level is *low* (that constrains the subscriber number, i.e., $\min(\overline{N}_w, \frac{f(\eta S_w)}{\delta}) = \frac{f(\eta S_w)}{\delta}$ in (6) or $\min(\overline{N}_w, \frac{f(S_g)}{\delta}) = \frac{f(S_g)}{\delta}$ in (8)) or *high* (that has no influence on the subscriber number). Intuitively, the spectrum allocations will also affect the two service prices. Let's define two thresholds[14]:

$$\widetilde{S_w} = \frac{1}{\eta} f^{-1}\left(\frac{N\delta(\theta A - u_0)}{\theta(2-\theta)A}\right), \qquad \widetilde{S}_g = f^{-1}\left(\frac{N\delta(A - u_0)}{\theta(2-\theta)A}\right), \tag{10}$$

which distinguish whether the spectrum level of Wi-Fi or 4G service is low or high. If $S_w \geq \widetilde{S_w}$, the capacity of Wi-Fi is able to support all the potential users who want to subscribe to Wi-Fi service; if $S_w < \widetilde{S_w}$, the number of Wi-Fi subscribers will be limited by the Wi-Fi capacity, and some of the users who want to subscribe to Wi-Fi service have to be rejected. The meaning of $\widetilde{S}_g$ can be understood similarly for the 4G service. If the total user number N or the user requirement δ is high, then both thresholds are high. For the Wi-Fi service, if the spectrum efficiency η is low, the threshold becomes high, since more spectrum is needed to achieve the same capacity.

In the following, we will first exploit the best responses of the Wi-Fi operator and the 4G operator, respectively. Then, we will examine the fixed point of the two operators' best response functions in order to determine the Nash Equilibrium.

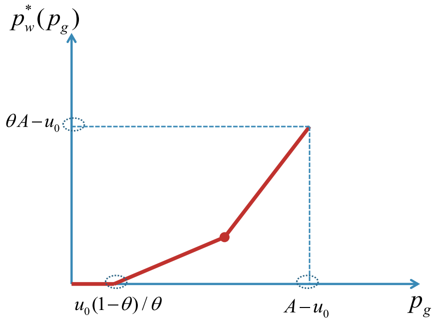
6.1 The Wi-Fi Operator's Best Response

Proposition 3. *Define $\overline{p_w} = [\theta p_g - (1-\theta)u_0]/2$. Given the 4G operator's service price as p_g, the best response of the Wi-Fi operator $p_w^*(p_g)$ is as follows.*

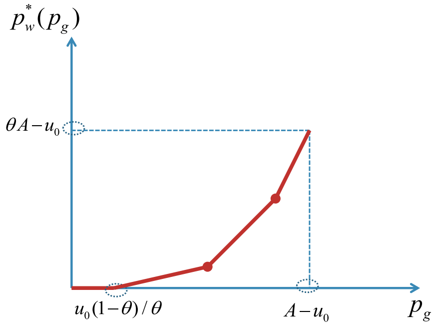
- Low Wi-Fi Spectrum Level. *If $S_w \leq \widetilde{S_w}$, the best response price for the Wi-Fi operator is*

$$p_w^*(p_g) = \begin{cases} \overline{p_w}, & \text{if } (\frac{1}{\theta}-1)u_0 \leq p_g \leq (\frac{1}{\theta}-1)u_0 + \frac{2A(1-\theta)f(\eta S_w)}{N\delta} \\ 2\overline{p_w} - \frac{A\theta(1-\theta)f(\eta S_w)}{N\delta}, & \text{if } (\frac{1}{\theta}-1)u_0 + \frac{2A(1-\theta)f(\eta S_w)}{N\delta} < p_g \leq A - u_0 - \frac{A\theta f(\eta S_w)}{N\delta} \\ p_g - (1-\theta)A, & \text{if } A - u_0 - \frac{A\theta f(\eta S_w)}{N\delta} < p_g \leq A - u_0. \end{cases} \tag{11}$$

14. Refer to the online technical reports [17] for the derivation of the two thresholds.



(a) High Wi-Fi spectrum level



(b) Low Wi-Fi spectrum level

Fig. 4. Wi-Fi operator's best response $p_w^*(p_g)$.

- High Wi-Fi Spectrum Level. *If $S_w > \widetilde{S_w}$, the best response price for the Wi-Fi operator is*

$$p_w^* = \begin{cases} \overline{p_w}, & \text{if } (\frac{1}{\theta}-1)u_0 \leq p_g \leq \frac{1-\theta}{2-\theta}(2A-u_0) \\ p_g - (1-\theta)A, & \text{if } \frac{1-\theta}{2-\theta}(2A-u_0) < p_g \leq A - u_0. \end{cases} \tag{12}$$

Fig. 4 shows Wi-Fi operator's best response in case of high and low Wi-Fi spectrum levels. When p_g is extremely low, $p_w^*(p_g)$ has to remain zero in order to attract users (the left-most flat line at each subfigure). Then, $p_w^*(p_g)$ increases with p_g. When the Wi-Fi spectrum level is high (subfigure (a)), the subscriber number is not affected by the capacity. The slope of $p_w^*(p_g)$ is first small, then becomes large when p_g becomes too high and no users choose 4G service. Hence the Wi-Fi operator can increase its price more aggressively in the high 4G price regime without losing many potential subscribers. However, when Wi-Fi spectrum level is low (subfigure (b)), the slope of $p_w^*(p_g)$ has a medium value between the small and large values, where the subscriber number is bounded by the capacity, and the Wi-Fi operator increases its price at a rate that balances the user demand and the capacity constraint (see the online technical report [17] for more details).

6.2 The 4G Operator's Best Response

Proposition 4. *Define $\overline{p_g} = [p_w + (1-\theta)A]/2$. Given the Wi-Fi operator's service price as p_w, the best response of the 4G operator $p_g^*(p_w)$ is as follows.*

- Low 4G Spectrum Level. *If $S_g \le \widetilde{S}_g$, the best response price for the 4G operator is*

$$p_g^*(p_w) = \begin{cases} \overline{p_g}, & \text{if } 0 \le p_w \le \frac{2A(1-\theta)f(S_g)}{N\delta} - (1-\theta)A \\ 2\overline{p_g} - \frac{A(1-\theta)f(S_g)}{N\delta}, & \text{if } \frac{2A(1-\theta)f(S_g)}{N\delta} - (1-\theta)A < p_w \le \theta A - u_0 - \frac{A\theta f(S_g)}{N\delta} \\ \frac{1}{\theta}[p_w + (1-\theta)u_0], & \text{if } \theta A - u_0 - \frac{A\theta f(S_g)}{N\delta} < p_w \le \theta A - u_0. \end{cases} \tag{13}$$

- High 4G Spectrum Level. *If $S_g \ge \widetilde{S}_g$, the best response price for the 4G operator is*

$$p_g^* = \begin{cases} \overline{p_g}, & \text{if } 0 \le p_w \le \frac{1-\theta}{2-\theta}(\theta A - 2u_0) \\ \frac{1}{\theta}[p_w + (1-\theta)u_0], & \text{if } \frac{1-\theta}{2-\theta}(\theta A - 2u_0) < p_w \le \theta A - u_0. \end{cases} \tag{14}$$

6.3 Equilibrium Prices

To derive the equilibrium Wi-Fi and 4G prices, we have to jointly consider the best responses of Wi-Fi and 4G operators. As analyzed in Sections 6.1 and 6.2, when the spectrum levels for Wi-Fi and 4G operators are low or high (compared with the thresholds $\widetilde{S}_w$ and $\widetilde{S}_g$ defined in (10)), the best responses of Wi-Fi and 4G operators are different. However, to characterize the spectrum conditions for the equilibrium prices, we need two different thresholds $\widehat{S}_w$ and $\widehat{S}_g$. These two thresholds decide the high and low regimes of spectrum allocation for Wi-Fi and 4G services. When the FCC's spectrum allocation at Stage I falls into different regimes, the two operators' best response functions will be determined by $\widetilde{S}_w$ and $\widetilde{S}_g$ (Eqs. (11) or (12) for Wi-Fi operator and Eqs. (13) or (14) for 4G operator), then the intersection points of the two piecewise best response functions will be determined by $\widehat{S}_w$ and $\widehat{S}_g$

$$\widehat{S_w} = \frac{1}{\eta} f^{-1}\left(\frac{N\delta(\theta A - 2u_0)}{\theta(4-\theta)A}\right), \quad \widehat{S}_g = f^{-1}\left(\frac{N\delta(2A-u_0)}{(4-\theta)A}\right). \tag{15}$$

Theorem 1. *In Stage II, a unique equilibrium exists. The equilibrium prices of the Wi-Fi and 4G services are summarized in Table 1. The four spectrum conditions are characterized as*

- Case L_wL_g *(low Wi-Fi Spectrum, low 4G Spectrum):* $S_w \le \widehat{S_w}, S_g \le \widehat{S}_g$;
- Case H_wL_g *(high Wi-Fi Spectrum, low 4G Spectrum):* $S_w > \widehat{S_w}, S_g \le \widehat{S}_g$;
- Case L_wH_g *(low Wi-Fi Spectrum, high 4G Spectrum):* $S_w \le \widehat{S_w}, S_g > \widehat{S}_g$;
- Case H_wH_g *(high Wi-Fi Spectrum, high 4G Spectrum):* $S_w > \widehat{S}_w, S_g > \widehat{S}_g$.

TABLE 1
Equilibrium Price in Stage II

	4G service price p_g	Wi-Fi service price p_w
L_wL_g	$A - u_0 - \frac{A[\theta f(\eta S_w)+f(S_g)]}{N\delta}$	$\theta A - u_0 - \frac{A\theta[f(\eta S_w)+f(S_g)]}{N\delta}$
H_wL_g	$\frac{(1-\theta)\left[2A-u_0-\frac{2Af(S_g)}{N\delta}\right]}{2-\theta}$	$\frac{(1-\theta)\left[\theta A-u_0-\frac{\theta Af(S_g)}{N\delta}\right]}{2-\theta}$
L_wH_g	$\frac{(1-\theta)\left[A-u_0-\frac{\theta Af(\eta S_w)}{N\delta}\right]}{2-\theta}$	$\frac{(1-\theta)\left[\theta A-2u_0-\frac{2\theta Af(\eta S_w)}{N\delta}\right]}{2-\theta}$
H_wH_g	$\frac{1-\theta}{4-\theta}(2A-u_0)$	$\frac{1-\theta}{4-\theta}(\theta A-2u_0)$

There exists a unique equilibrium because the best responses of two operators intersect only once (interested readers can check Figs. 4, 5, 6, 7, 8, 9, 10, 11, and 12 in the technical report [17]) for more details. Note that we have different thresholds for low or high spectrum levels in Theorem 1 and Propositions 3 and 4. We can easily prove that $\widehat{S_w} < \widetilde{S_w}$ and $\widehat{S}_g < \widetilde{S}_g$. One of the reasons that we have different spectrum thresholds is that, the capacity constraint may affect the best response of Wi-Fi operator as shown in Fig. 4, but may not affect the equilibrium price if the intersection of the two best responses does not fall into the transition phase in Fig. 4. In order for the intersection of the two best responses (and the resulting equilibrium prices) to be within the transition phase (which reflects the impact of capacity constraint), the spectrum thresholds $\widehat{S}_w$ and $\widehat{S}_g$ need to be even smaller. Detailed derivation of $\widehat{S}_w$ and $\widehat{S}_g$ is in the online technical report [17].

An interesting observation is that the equilibrium prices are only affected by the spectrum level of the service that has low spectrum level. More specifically: 1) when both the Wi-Fi and 4G spectrum levels are high (the H_wH_g case), the equilibrium prices do not rely on the spectrum allocation; 2) when the 4G spectrum level is low but the Wi-Fi spectrum level is high (the H_wL_g case), both the Wi-Fi and 4G price will decrease with S_g; we have the similar observation when the Wi-Fi spectrum level is low but the 4G spectrum level is high (the L_wH_g case); 3) when both the Wi-Fi and 4G spectrum levels are low (the L_wL_g case), the equilibrium prices decrease in both S_g and S_w. This is because the spectrum level only affects a service's best response price when it is low, in which case it will further affect the equilibrium prices.

6.4 Subgame Perfect Equilibrium

Once we have the equilibrium prices in Stage II, we can determine the user subscriber numbers in Stage III at the SPE according to the analysis in Section 5.

Corollary 1. *The equilibrium prices in Stage II result in such subscriber numers in Stage III that $N_w = \overline{N_w}$ and $N_g = \overline{N_g}$, in which $\overline{N_w}$ and $\overline{N_g}$ are defined in (7) and (9), respectively.*

The subscriber numbers at the SPE (Stages II and III together) under different spectrum allocations are summarized in Table 2. We can see that if one service has low spectrum level, the subscriber number of that service equals its capacity. However, if one service has a high spectrum level, the subscriber number of that service is smaller than its capacity. This means that the network will reserve some

TABLE 2
SPE Subscriber Number in Stage III

	Wi-Fi subscriber number	4G subscriber number
$L_w L_g$	$\frac{f(S_g)}{\delta}$	$\frac{f(\eta S_w)}{\delta}$
$H_w L_g$	$\frac{N(\theta A-u_0)}{(2-\theta)A\theta} - \frac{1}{2-\theta}\frac{f(S_g)}{\delta}$	$\frac{f(S_g)}{\delta}$
$L_w H_g$	$\frac{f(\eta S_w)}{\delta}$	$\frac{N(A-u_0)}{(2-\theta)A} - \frac{\theta}{2-\theta}\frac{f(\eta S_w)}{\delta}$
$H_w H_g$	$\frac{N(\theta A-2u_0)}{(4-\theta)A\theta}$	$\frac{N(2A-u_0)}{(4-\theta)A}$

capacity in this regime in order to maximize the profit. Understanding the influence of spectrum allocation on the subscriber number can help the FCC to achieve a better balance between user benefits and income.

7 Stage I-Spectrum Allocation

In Stage I, the FCC optimizes its spectrum allocation based on the prediction of the equilibrium prices in Stage II (Table 1) and the subscriber numbers in Stage III (Proposition 1 and 2).

We first analyze the end users' aggregate utility U_{user} in (3). As an example, we assign $\omega_1 = \omega_2 = 1$, i.e., we view Wi-Fi and 4G networks as equally important.[15] According to Corollary 1, we have $N_w = \overline{N}_w, N_g = \overline{N}_g$, in which $\overline{N_w}$ and $\overline{N_g}$ are defined in (7) and (9), respectively. Thus we have

$$U_{user} = \frac{1}{N}(\theta\overline{N_w} + \overline{N_g}) = -\frac{1}{A}(p_g - A + u_0), \quad (16)$$

which depends on 4G price p_g, which further depends on the spectrum allocations S_w and S_g according to Table 1. In the following analysis, we focus on the derivation of S_w, by keeping in mind that $S_g = S - S_w$. To derive clean insights for FCC's spectrum allocation to 4G and Wi-Fi services, we assume that $A = 1$ and $u_0 = 0$.

7.1 Income-Centric Spectrum Allocation Benchmark

We first derive the spectrum allocation when the FCC only cares about its income, a special case with $\beta = 0$ that serves as a benchmark to compare with our proposed spectrum allocation with a positive β.

FCC's income-centric utility is $U_f^0 = \phi_g(S_g) + \phi_w(S_w) = \phi_g(S - S_w) + \phi_w(S_w)$. The first order partial derivative with respect to S_w is $\frac{\partial U_f^0}{\partial S_w} = -\phi_g'(S - S_w) + \phi_w'(S_w)$. The second order partial derivative is $\frac{\partial^2 U_f^0}{\partial S_w^2} = \phi_g''(S - S_w) + \phi_w''(S_w)$. We assume that $\phi_g'' < 0$ and $\phi_w'' < 0$, which means that the unit price (charged by the FCC to the network operators) will decrease as the spectrum increases.[16] This means that U_f^0 is a strictly concave function of S_w, and we can obtain the income maximizing S_w^{0*} by solving $\partial U_f^0/\partial S_w = 0$.[17] The spectrum allocation to the 4G network is then $S - S_w^{0*}$. Such a spectrum allocation scheme only depends on how the FCC designs the payment function ϕ_g and ϕ_w, without considering end users' aggregate utility. The analysis in Section 7 is applicable to general forms of ϕ_g and ϕ_w. In Section 8, when we perform numerical study of the bound of income loss and aggregate user utility, we will consider polynomial functions of ϕ_g and ϕ_w. The study of the optimal design of ϕ_g and ϕ_w as well as their influences on the spectrum allocation will be a future work.

7.2 Proposed Spectrum Allocation

When jointly considering end users' utility and the FCC's income, the FCC maximizes its utility as in (2). We assume that the second derivative of capacity function $f''(\cdot) < 0$, which means that capacity per unit spectrum will decrease due to limitations such as interferences. Therefore, (2) is still a concave function of S_w.

Theorem 2. *In Stage I, equilibrium spectrum allocation for Wi-Fi service is*

$$S_w^* = \begin{cases} S_w^{lower}, & if\ S_w^{opt} < S_w^{lower} \\ S_w^{opt}, & if\ S_w^{lower} \le S_w^{opt} \le S_w^{upper} \\ S_w^{upper}, & if\ S_w^{opt} > S_w^{upper} \end{cases} \quad (17)$$

in which $S_w^{opt}, S_w^{upper}, S_w^{lower}$ are summarized in Table 3, and the equilibrium spectrum allocation for 4G service is $S_g^* = S - S_w^*$.

Corollary 2. More spectrum allocation to the service with a low spectrum level. *If one operator receives a low spectrum allocation and the other operator receives a high spectrum allocation in the income-centric benchmark (with $\beta = 0$), then our proposed scheme (with $\beta > 0$) will allocate more spectrum to the operator with the low spectrum allocation (hence less spectrum to the other operator).*

Corollary 2 shows that if the income-centric approach leads to a very imbalanced spectrum allocation across two operators (one low and one high), making the allocation more balance can improve the users' aggregate utility.

Corollary 3. More spectrum allocation to 4G service when both service have low spectrum level. *If both operators receive low spectrum allocations (based on their own thresholds, respectively) in the income-centric benchmark (with $\beta = 0$), then our proposed scheme (with $\beta > 0$) will allocate more spectrum to the 4G operator (hence less spectrum to the Wi-Fi operator).*

Corollary 3 shows that the FCC tends to prioritize alleviating the spectrum shortage of 4G network first, partly due to the 4G service's wider coverage and higher spectrum efficiency.

According to Table 3, we also have the following insights:

15. Our results can also be extended to different weights of ω_1 and ω_2 when the FCC has different preferences towards serving users, e.g., if it intends to focus on high-end users, it would pay more attention to 4G service with $\omega_1 < \omega_2$.

16. This is reasonable because, according to diminishing marginal returns [28] in economics, the marginal improvement in transmission QoS will decrease as the quantity of spectrum increases while keeping other factors constant. So the FCC should charge a smaller unit price as the quantity of spectrum increases.

17. Let $S_w^{0,opt}$ denote the root of $\partial U_f^0/\partial S_w = 0$: if $S_w^{0,opt} < 0, S_w^{0*} = 0$; if $S_w^{0,opt} > S, S_w^{0*} = S$; otherwise, $S_w^{0*} = S_w^{0,opt}$.

TABLE 3
Equilibrium Spectrum Allocation in Stage I

Spectrum allocation	S_w^{opt} satisfies	S_w^{upper}	S_w^{lower}
L_wL_g	$\frac{\beta\left[\theta\eta f'\left(\eta S_w^{opt}\right)-f'\left(S-S_w^{opt}\right)\right]}{N\delta}-\phi_g'(S-S_w^{opt})+\phi_w'\left(S_w^{opt}\right)=0$	$\widehat{S}_w$	$S-\widehat{S}_g$
H_wL_g	$-\frac{2\beta(1-\theta)f'\left(S-S_w^{opt}\right)}{(2-\theta)N\delta}-\phi_g'\left(S-S_w^{opt}\right)+\phi_w'(S_w^{opt})=0$	S	$\max(\widehat{S}_w, S-\widehat{S}_g)$
L_wH_g	$\frac{\beta\theta(1-\theta)\eta f'\left(\eta S_w^{opt}\right)}{(2-\theta)N\delta}-\phi_g'\left(S-S_w^{opt}\right)+\phi_w'\left(S_w^{opt}\right)=0$	$\min(S-\widehat{S}_g, \widehat{S}_w)$	0
H_wH_g	$-\phi_g'\left(S-S_w^{opt}\right)+\phi_w'\left(S_w^{opt}\right)=0$	$S-\widehat{S}_g$	$\widehat{S}_w$

- When both services have a high spectrum level in the income-centric benchmark, the further consideration of users' utility does not change the spectrum allocation. In the case of H_wH_g in Table 3, the spectrum allocation results are the same as the income-centric spectrum allocation. This shows that when spectrum resource is abundant, maximizing the FCC's income does not hurt users' utility.
- The equilibrium spectrum allocation is closely related to the Wi-Fi network coverage θ and spectrum efficiency η. For example, in the case of L_wL_g in Table 3, if θ or η increases, the spectrum allocation to Wi-Fi S_w increases (since $\phi_w'(S_w)-\phi_g'(S-S_w)$ decreases). This means that the FCC will allocate more spectrum to Wi-Fi if the Wi-Fi's network coverage or spectrum efficiency are improved (hence become similar as the 4G network).

7.3 Subgame Perfect Equilibrium of the Entire Game

By substituting S_w and S_g in Tables 1 and 2 with S_w^* and $S-S_w^*$, we get the SPE service prices and subscriber numbers. Now we analyze how the consideration of user utility affects the equilibrium prices in Stage II and subscriber number in Stage III, compared with the income-centric spectrum allocation. We focus on the first three cases, as our scheme is the same as the income-centric benchmark in the H_wH_g case.

7.3.1 Case L_wL_g

In the SPE, the FCC's utility becomes

$$U_f=\underbrace{\phi_g(S_g^*)+\phi_w(S_w^*)}_{FCC's\ income}+\beta\underbrace{\frac{\theta f(\eta S_w^*)+f(S_g^*)}{\delta N}}_{user\ aggregate\ utility}. \tag{18}$$

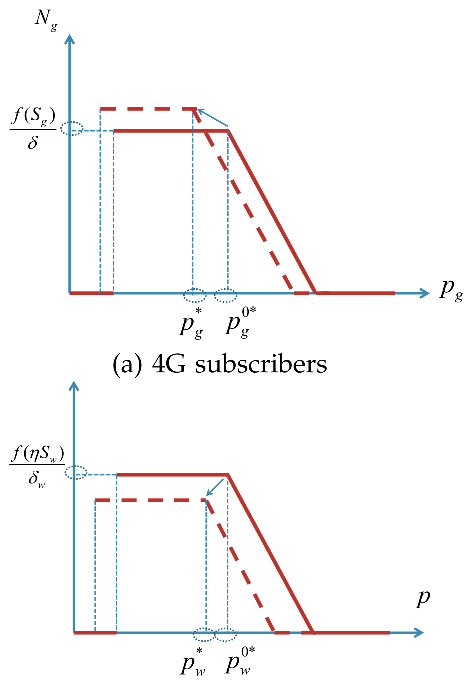
As some spectrum is moved from Wi-Fi to 4G (compared with the income-centric benchmark), the value of $f(\eta S_w)+f(S_g)$ will increase. According to Table 1, both service prices p_g and p_w decrease. It is easy to understand for the 4G operator, which reduces price p_g to attract more users because the 4G capacity increases. Counter-intuitively, the Wi-Fi operator also reduces price p_w, even though the Wi-Fi capacity decreases. The reason is that the 4G service, with a higher spectrum efficiency, is able to support more users with the same amount of spectrum compared with the Wi-Fi. Therefore, the subscriber churn from Wi-Fi to 4G reduces the need for Wi-Fi spectrum more than the amount of spectrum that has been moved to 4G. Hence the Wi-Fi operator needs to reduce price p_w to attract more low-end subscribers, to utilize the available Wi-Fi spectrum due to subscriber churn.

The change of subscriber numbers is shown in Fig. 5. The solid lines are the income-centric benchmark (with the corresponding equilibrium prices p_g^{0*} and p_w^{0*}). The dashed lines are the allocation under our proposed scheme (with the corresponding equilibrium prices p_g^* and p_w^*). The 4G subscriber number N_g increases due to the following reasons: 1) The 4G capacity increases, and can serve more users; 2) The 4G price p_g decreases more than the Wi-Fi price p_w, making some previous price sensitive Wi-Fi subscribers churn to 4G service.

The Wi-Fi subscriber number N_w decreases due to the decrease of Wi-Fi capacity (according to Table 2, in the case of L_wL_g, the Wi-Fi subscriber number is determined by the Wi-Fi capacity).

If we consider two operators' subscribers together, then the total number of 4G and Wi-Fi subscribers increases, as the decrease in Wi-Fi service price attracts lower-end nonsubscribers who previously did not choose any services.

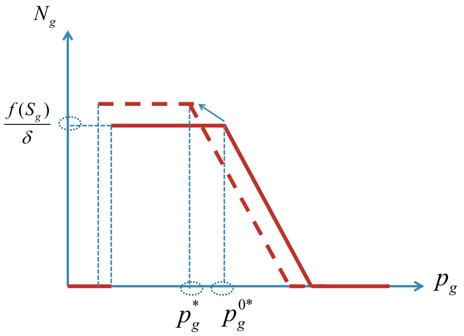
The users' aggregate utility increases because: 1) Some Wi-Fi subscribers churn to 4G service, enjoying a better network



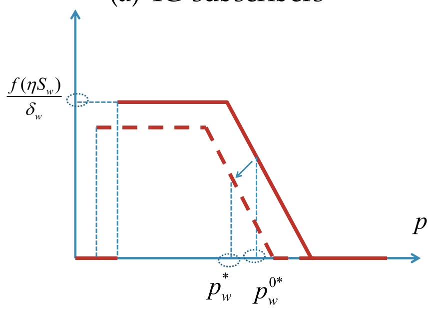
(a) 4G subscribers

(b) Wi-Fi subscribers

Fig. 5. Change of subscriber number in case of low Wi-Fi spectrum and low 4G spectrum.



(a) 4G subscribers



(b) Wi-Fi subscribers

Fig. 6. Change of subscriber number in case of high Wi-Fi spectrum and low 4G spectrum.

coverage; 2) Some previous non-subscribers now choose the Wi-Fi service, enjoying a better network coverage.

7.3.2 Case H_wL_g

In the SPE, the FCC's utility becomes

$$U_f = \underbrace{\phi_g(S_g^*) + \phi_w(S_w^*)}_{FCC's\ income} + \beta \underbrace{\left(\frac{(1-\theta)}{(2-\theta)N} \frac{f(S_g^*)}{\delta} + \frac{\theta}{2-\theta} \right)}_{user\ aggregate\ utility}. \quad (19)$$

Compared with the income-centric benchmark, the proposed spectrum allocation has a higher optimal 4G spectrum allocation S_g^* (than S_g^{0*}), so both prices p_g and p_w are lower according to Table 1. The 4G operator reduces price p_g for a similar reason as in the case of L_wL_g. The Wi-Fi operator reduces price p_w also due to the subscriber churn from Wi-Fi to 4G. Notice that in the case of H_wL_g, the decrease in Wi-Fi capacity no longer affects the Wi-Fi operator's decision, since the Wi-Fi capacity is excessive compared with its user demand.

The change of subscriber numbers is shown in Fig. 6. The major difference between Figs. 5 and 6 is: Although N_w decreases in both cases (as the spectrum is reallocated from Wi-Fi to 4G), N_w equals to the Wi-Fi capacity in the L_wL_g case, but is always lower than the Wi-Fi capacity in the H_wL_g case.

7.3.3 Case L_wH_g

In the SPE, the FCC's utility becomes

$$U_f = \underbrace{\phi_g(S_g^*) + \phi_w(S_w^*)}_{FCC's\ income} + \beta \underbrace{\left(\frac{(1-\theta)\theta}{(2-\theta)N} \frac{f(\eta S_w^*)}{\delta} + \frac{1}{2-\theta} \right)}_{user\ aggregate\ utility}. \quad (20)$$

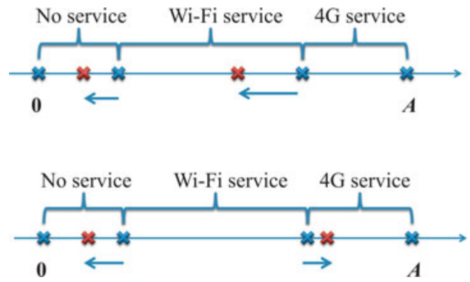


Fig. 7. User flow in the case of H_wL_g (upper plot) and L_wH_g (lower plot).

Compared with the income-centric benchmark, S_w^* is higher than S_w^{0*}, so p_g and p_w both decrease according to Table 1.

Intuitively, H_wL_g and L_wH_g should have symmetrical interpretations. However, they are different in terms of user flow as shown in Fig. 7. In the case of H_wL_g, the increase of 4G users comes from previous Wi-Fi subscribers, and Wi-Fi attracts non-subscribers who previous choose no wireless service. The users' aggregate utility increases, because each of the churning subscribers (from Wi-Fi to 4G) and the new subscribers (from nothing to Wi-Fi) enjoy a higher utility. In the case of L_wH_g, the increase of Wi-Fi subscribers comes from both previous 4G users and new users who previously choose nothing. Although the churning subscribers' service quality (from 4G to Wi-Fi) decreases, such loss is compensated by the increased utility of new subscribers (from choosing nothing to Wi-Fi).

In summary, in all of the above three cases, when user utility is considered (with $\beta > 0$), the proposed spectrum allocation will reduce the service prices of both services (compared with income-centric benchmark with $\beta = 0$). Some lower-end non-subscribers who previously did not choose any services will now choose Wi-Fi service. The total number of 4G and Wi-Fi subscribers and their aggregate utility increase.

8 Bounded Allocation Income Loss

In this section, we analyze the loss of FCC's income (compared with the income-centric benchmark), assuming that the spectrum charge functions ($\phi_g(\cdot)$ and $\phi_w(\cdot)$) and the capacity function ($f(\cdot)$) are polynomial.[18] We show that the income loss can be analytically upper bounded, and we examine how such loss depends on several critical parameters including the weight for user utility β, the Wi-Fi spectrum efficiency η, and the Wi-Fi network coverage θ. We further characterize the increase of the users' aggregate utility that accompanies the loss of FCC's income. We again focus on the three cases of L_wL_g, H_wL_g, and L_wH_g.

8.1 Analysis of Income Loss and Aggregate User Utility Bounds

We define the *user utility ratio* as the ratio between the users' aggregate utility of our proposed spectrum allocation and that of the income-centric benchmark, denoted by $R_{utility}$. We also define the FCC's *income ratio* in a

18. Here, we study polynomial functions for simplicity. In the future, we will consider more complex functions and their influence on the FCC's income and users' utility.

similar fashion, denoted by R_{income}. Intuitively, $R_{utility} \geq 1$ and $0 < R_{income} \leq 1$. We are interested in characterizing the upper-bound of $R_{utility}$ and the lower-bound of R_{income}.

The FCC charges the Wi-Fi and 4G operators based on functions $\phi_w(\cdot)$ and $\phi_g(\cdot)$ respectively. We assume that 1) the operators are willing to pay more for more spectrum, i.e., $\phi_w(\cdot)$ and $\phi_g(\cdot)$ are increasing functions; 2) the operators' willingness to pay for each additional spectrum decreases with the total spectrum allocation ("diminishing returns"), i.e., $\phi'_w(\cdot)$ and $\phi'_g(\cdot)$ (the first order derivatives) are decreasing functions. As a concrete example, we assume that ϕ_w and ϕ_g are negative quadratic functions which satisfy the above two properties. For simplicity, we assume that the network capacity function $f(\cdot)$ is a linear function. In particular, we have

$$\begin{aligned} &\phi_g(S_g) = -aS_g^2 + bS_g, \phi_w(S_w) = -cS_w^2 + dS_w, \\ &f(S_g) = eS_g, f(\eta S_w) = e\eta S_w, \end{aligned} \tag{21}$$

where a, b, c, d, e are positive parameters. We further assume that $S < \min\{b/(2a), d/(2c)\}$, which guarantees that $\phi'_g > 0$, $\phi''_g < 0$, $\phi'_w > 0$, and $\phi''_w < 0$.

Under the income-centric benchmark, we have

$$\begin{aligned} &\text{Wi-Fi Spectrum allocation}: S_w^{0*} = \tfrac{2aS+d-b}{2(a+c)} \\ &\text{FCC's Income}: -aS^2 + bS + \tfrac{(2aS+d-b)^2}{4(a+c)} \end{aligned}$$

8.1.1 The Case of L_wL_g

According to Table 3, we have

$$S_w^* = \frac{1}{2(a+c)}\left[2aS + d - b - (1-\theta\eta)\frac{e\beta}{\delta N}\right]. \tag{22}$$

As β increases, the FCC cares more about user utility. According to Corollary 3, the FCC reallocates more spectrum from Wi-Fi to 4G, so S_w^* decreases. We can prove that the minimum FCC's income is $-aS^2 + bS > 0$ when $S_w^* = 0$ (with a large enough β). Therefore, the income ratio is lower-bounded by

$$R_{income} \geq \frac{-aS^2 + bS}{-aS^2 + bS + \frac{(2aS+d-b)^2}{4(a+c)}}. \tag{23}$$

Correspondingly, the user utility ratio is the highest when $S_w^* = 0$ with the following upper-bound:

$$R_{utility} = \frac{S - (1-\theta\eta)S_w^*}{S - (1-\theta\eta)S_w^{0*}} \leq \frac{S}{S - (1-\theta\eta)S_w^{0*}} \tag{24}$$

The basic reason for the existence of such an upper-bound is the limited spectrum that FCC can allocate. If the Wi-Fi spectrum efficiency η increases, the upper-bound of $R_{utility}$ decreases, because the spectrum efficiency difference between Wi-Fi and 4G is smaller, and reallocating spectrum to 4G no longer significantly increases the overall network capacity (hence the users' utility). If the Wi-Fi coverage θ increases, the upper-bound of $R_{utility}$ also decreases, because the coverage difference between Wi-Fi and 4G gets smaller, and the Wi-Fi subscribers who churn to 4G obtain a smaller utility improvement.

8.1.2 The Case of H_wL_g

According to Table 3, we have

$$S_w^* = \frac{1}{2(a+c)}\left[2aS + d - b - \frac{e\beta(1-\theta)}{\delta N(2-\theta)}\right].$$

The income ratio is lower-bounded by the same value as in (23). However, the user utility ratio has a different upper-bound as follows:

$$R_{utility} \leq \frac{(1-\theta)eS + N\delta\theta}{(1-\theta)e(S - S_w^{0*}) + N\delta\theta}. \tag{25}$$

The upper-bound of $R_{utility}$ decreases in the Wi-Fi coverage θ for the same reason as in the case of L_wL_g. However, the upper-bound of $R_{utility}$ is independent of the Wi-Fi spectrum efficiency η. This is because in this case of H_wL_g, the Wi-Fi subscriber number is lower than the Wi-Fi capacity (determined by η) according to Table 2, so neither the Wi-Fi subscriber number change nor the aggregate utility improvement depend on η. The upper-bound of $R_{utility}$ decreases in the total number of users N, because the equilibrium price gap $p_g - p_w$ increases with N according to Table 1. And the increasing price gap deters Wi-Fi subscribers from churning to 4G for better QoS, so the utility improvement decreases, that is, the upper-bound of $R_{utility}$ decreases.

8.1.3 The Case of L_wH_g

According to Table 3, we have

$$S_w^* = \frac{1}{2(a+c)}\left[2aS + d - b + \frac{e\beta(1-\theta)\theta\eta}{\delta N(2-\theta)}\right].$$

As β increases, according to Corollary 2, the FCC allocates more spectrum to Wi-Fi and S_w^* increases. We can prove that the minimum FCC's income is $-cS^2 + dS$ when $S_w^* = S$.

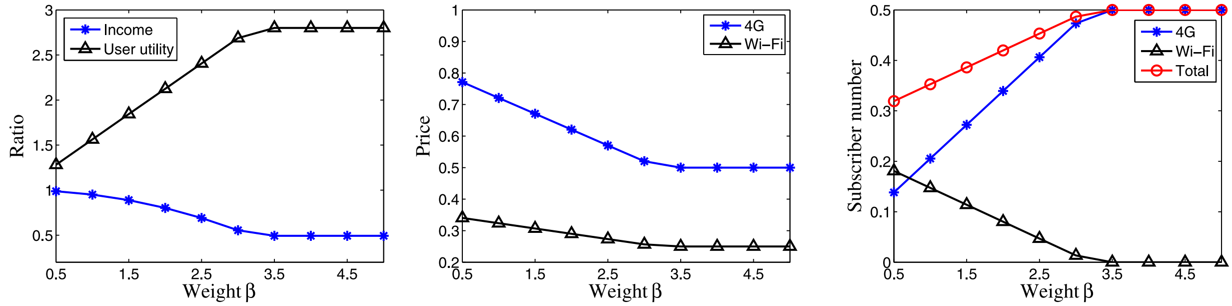
Therefore, the income ratio is lower-bounded by

$$R_{income} \geq \frac{-cS^2 + dS}{-aS^2 + bS + \frac{(2aS+d-b)^2}{4(a+c)}}. \tag{26}$$

The user utility ratio is the highest when $S_w^* = S$

$$R_{utility} = \frac{(1-\theta)\theta\eta e S_w^* + N\delta}{(1-\theta)\theta\eta e S_w^{0*} + N\delta} \leq \frac{(1-\theta)\theta\eta e S + N\delta}{(1-\theta)\theta\eta e S_w^{0*} + N\delta}. \tag{27}$$

If the Wi-Fi spectrum efficiency η increases, the upper-bound of $R_{utility}$ increases, because the spectrum efficiency gap between Wi-Fi and 4G is smaller, and reallocating spectrum to Wi-Fi does not decrease much of the capacity. The influence of a higher Wi-Fi coverage θ is two-fold. On one hand, more non-subscribers will become Wi-Fi subscribers, enjoying a higher network coverage. On the other hand, more 4G subscribers will become Wi-Fi subscribers, hence having a lower network coverage. Which of the two factors dominates depends on the value of θ. When



(a) Income and user utility ratio (b) Service prices (c) User distribution

Fig. 8. Impact of weight β.

$\theta > 1/2$, the upper-bound of $R_{utility}$ decreases in θ, because the subscriber churn from 4G to Wi-Fi is the dominating factor. When $\theta < 1/2$, the upper-bound of $R_{utility}$ increases in θ, because non-subscribers switching to Wi-Fi subscribers becomes the dominant factor. The upper-bound of $R_{utility}$ decreases in N, because the equilibrium price p_w increases with N according to Table 1. Non-subscribers are less likely to choose Wi-Fi service, so the utility improvement decreases.

8.2 Impact of User Utility Weight β

Fig. 8 shows the impact of weight β in the case of L_wL_g. When β increases, the FCC emphasizes more on the user utility compared with the income-centric benchmark. Therefore, the user utility ratio increases and the income ratio decreases with β. When β becomes very large, S_w^* becomes zero according to (22) and $S_g^* = S$, hence the two curves in Fig. 8a become flat (when $\beta > 3.5$). As we have proved, the user utility ratio is upper-bounded and the income ratio is lower-bounded as shown in Fig. 8a. As summarized in Section 7.3, the consideration of user utility will drive down prices of both services. The higher the β, the lower the service prices will be, as shown in Fig. 8b. No matter how the subscriber number (N_w and N_g) changes, the total Wi-Fi and 4G subscriber number increases using the proposed spectrum allocation, and the higher β is, the higher the increase will be, as shown in Fig. 8c.

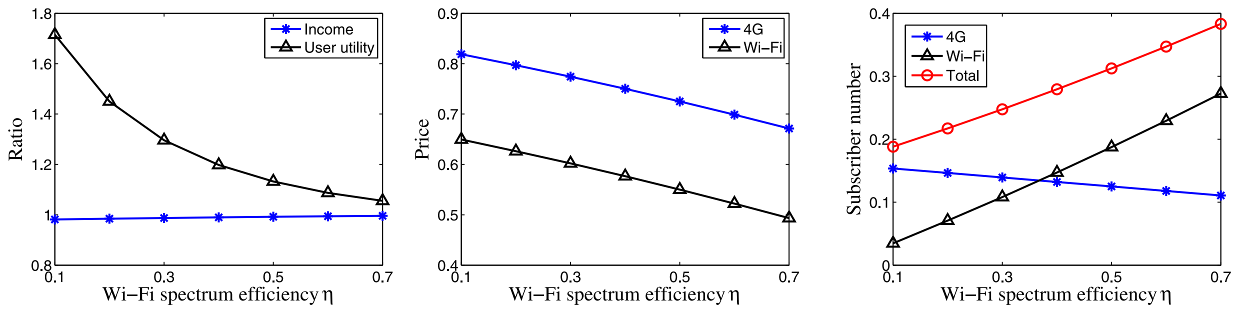
In the case of H_wL_g, the influences of β on user utility ratio, income ratio, service prices, and subscriber numbers are similar as Fig. 8, but the slopes of the curves are steeper. This is because the reallocation of spectrum from Wi-Fi to 4G is more significant as Wi-Fi has a high spectrum level.

In the case of L_wH_g, the influences of β on user utility ratio, income ratio, service prices are similar as Figs. 8a and 8b, but the change of Wi-Fi and 4G subscriber numbers are just opposite to Fig. 8c due to different directions of spectrum reallocation.

In all three cases, the total subscriber number increases in β, since the proposed spectrum allocation improves the aggregate user utility by covering more low-end users (with a small α) with the Wi-Fi service.

8.3 Impact of Wi-Fi Spectrum Efficiency η

Fig. 9 shows the impact of Wi-Fi spectrum efficiency η in the case of L_wL_g. Interestingly, when η increases (hence Wi-Fi can better utilize the spectrum), the user utility ratio decreases, as shown in Fig. 9a. The reason is that if η is already high, reallocating spectrum from Wi-Fi to 4G does not significantly improve the spectrum utilization, thus does not improve the users' aggregate utility much. Therefore, the FCC will reallocate less spectrum from Wi-Fi to 4G, resulting in a smaller utility ratio and a higher income ratio. As η increases, the Wi-Fi capacity increases, so the Wi-Fi operator decreases p_w to attract more users, as shown in Fig. 9b. This increases the market competition, hence forces 4G operator to decrease p_g to try to keep (most of) its subscribers. As shown in Fig. 9c, the Wi-Fi subscriber number increases due to an increased Wi-Fi capacity; the 4G subscriber number decreases due to competition from Wi-Fi; the entire user number increases thanks to the improved spectrum efficiency of Wi-Fi network. In the case of H_wL_g, the change of η does not affect anything. The reason is that



(a) Income and user utility ratio (b) Service prices (c) User distribution

Fig. 9. Impact of Wi-Fi spectrum efficiency η.

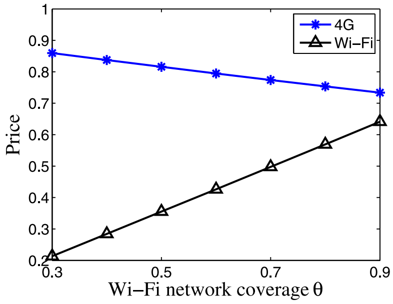


Fig. 10. Impact of Wi-Fi network coverage on service prices.

the Wi-Fi spectrum level is high, so the influence of capacity constraint $(f(\eta S_w)/\delta)$ does not exist. In the case of $L_w H_g$, the influences of η on user utility ratio, income ratio, service prices, and subscriber number are similar to those in Fig. 9.

8.4 Impact of Wi-Fi Network Coverage θ

The influence of Wi-Fi network coverage θ is similar to the influence of Wi-Fi spectrum efficiency η, as they both reflect the performance of Wi-Fi network. The major difference is the impacts of these two factors on the service prices. Fig. 10 shows the case of $L_w H_g$: when the Wi-Fi coverage increases, the two service prices change in different directions. The 4G price p_g decreases because Wi-Fi competes with 4G more intensely, hence the 4G operator has to reduce the 4G price to maintain its subscribers. The Wi-Fi service price p_w is influenced by two factors: (i) the need to decrease to match the decreased cellular price p_g due to competition, and (ii) the need to increase to reflect a better Wi-Fi coverage. Fig. 10 shows that the second factor dominates and p_w increases with its coverage.

9 Conclusion

In this paper, we propose a novel spectrum allocation scheme that enables the FCC to take into account both the income and the users' utility. We model the wireless market interactions as a 3-stage game, which involves the FCC, the Wi-Fi and the 4G operators, and all wireless users. We use backward induction to first calculate users' subscription choice in Stage III, then derive the equilibrium prices of both Wi-Fi and 4G services in Stage II, and finally obtain the equilibrium spectrum allocation for the FCC to maximize the weighted sum of the income and the users' aggregate utility in Stage I. Comparing with the income-centric benchmark, we show that the consideration of users' aggregate utility will make FCC balance the spectrum allocation between two operators. We further characterize the lower-bound of the income loss ratio of the proposed spectrum allocation to the income-centric benchmark, and provide detailed discussions regarding the impacts of weight β, Wi-Fi spectrum efficiency η, and Wi-Fi network coverage θ on the spectrum allocation, service prices, and user subscription.

Acknowledgments

This work was supported by grants from 973 project 2013CB329006, China NSFC under Grant 61173156, RGC under the contracts CERG 622613, 16212714, HKUST6/CRF/12R, and M-HKUST609/13, the grant from Huawei-HKUST joint lab, the General Research Funds (Project Number CUHK 412713 and CUHK 14202814) established under the University Grant Committee of the Hong Kong Special Administrative Region, China, the Competitive Earmarked Research Grants, and the SUTD-MIT International Design Centre (IDC) Grant (Project No.: IDSF1200106OH). Qian Zhang is the corresponding author.

References

[1] M. Reardon. (2010). Rethinking the wireless spectrum crisis [Online]. Available: http://news.cnet.com/8301-30686_3-20005831-266.html
[2] M. Reardon. (2010). Fcc unveils national broadband plan [Online]. Available: http://news.cnet.com/8301-30686_3-20000453-266.html
[3] President's Council of Advisors on Science and Technology (PCAST), "Report to the president realizing the full potential of government-held spectrum to spur economic growth," [Online]. Available: http://www.whitehouse.gov/sites/default/files/microsites/ostp/gorenberg_ppt.pdf, 2012.
[4] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.
[5] A. Min, X. Zhang, J. Choi, and K. Shin, "Exploiting spectrum heterogeneity in dynamic spectrum market," *IEEE Trans. Mobile Comput.*, vol. 11, no. 12, pp. 2020–2032, Dec. 2012.
[6] W. Lehr and L. McKnight, "Wireless internet access: 3g vs. wifi?" *Telecommun. Policy*, vol. 27, nos. 5/6, pp. 351–370, 2003.
[7] Data plan, "three", hong kong [Online]. Available: http://www.three.com.hk, 2014.
[8] Frequency allocation table [Online]. Available: http://www.fcc.gov/, 2014.
[9] T. Nguyen, H. Zhou, R. Berry, M. Honig, and R. Vohra, "The impact of additional unlicensed spectrum on wireless services competition," in *Proc. IEEE Symp. New Frontiers Dynamic Spectrum Access Netw.*, 2011, pp. 146–155.
[10] J. Jia, Q. Zhang, Q. Zhang, and M. Liu, "Revenue generation for truthful spectrum auction in dynamic spectrum access," in *Proc. 10th ACM Int. Symp. Mobile Ad Hoc Netw. Comput.*, 2009, pp. 3–12.
[11] B. Liang, W. Wang, and B. Li, "Designing truthtul spectrum double auctions with local markets," *IEEE Trans. Mobile Comput.*, vol. 13, no. 1, pp. 75–88, Jan. 2014.
[12] M. Dong, G. Sun, X. Wang, and Q. Zhang, "Combinatorial auction with time-frequency flexibility in cognitive radio networks," in *Proc. IEEE Conf. Comput. Commun.*, 2012, pp. 2282–2290.
[13] Y. Zhu, B. Li, and Z. Li, "Truthful spectrum auction design for secondary networks," in *Proc. IEEE Conf. Comput. Commun.*, 2012, pp. 873–881.
[14] H. Kim, J. Choi, and K. Shin, "Wi-fi 2.0: Price and quality competitions of duopoly cognitive radio wireless service providers with time-varying spectrum availability," in *Proc. IEEE Conf. Comput. Commun.*, 2011, pp. 2453–2461.
[15] L. Duan, J. Huang, and B. Shou, "Duopoly competition in dynamic spectrum leasing and pricing," *IEEE Trans. Mobile Comput.*, vol. 11, no. 11, pp. 1706–1719, Nov. 2012.
[16] J. Jia and Q. Zhang, "Competitions and dynamics of duopoly wireless service providers in dynamic spectrum market," in *Proc. 9th ACM Int. Symp. Mobile Ad Hoc Netw. Comput.*, 2008, pp. 313–322.
[17] Y. Chen, L. Duan, J. Huang, and Q. Zhang. Technical report: Balancing income and user utility in spectrum allocation [Online]. Available: http://people.sutd.edu.sg/~lingjie_duan/wp-content/uploads/2014/08/Tech ReportTMC2014.pdf
[18] D. Niyato and E. Hossain, "Competitive pricing in heterogeneous wireless access networks: Issues and approaches," *IEEE Netw.*, vol. 22, no. 6, pp. 4–11, Nov./Dec. 2008.
[19] M. Parzy and H. Bogucka, "Non-identical objects auction for spectrum sharing in tv white spaces the perspective of service providers as secondary users," in *Proc. IEEE Symp. New Frontiers Dynamic Spectrum Access Netw.*, 2011, pp. 389–398.
[20] H. Zhou, R. Berry, M. Honig, and R. Vohra, "Investment and competition in unlicensed spectrum," in *Proc. 46th Annu. Conf. Inf. Sci. Syst.*, 2012, pp. 1–6.

[21] S. Yun, Y. Yi, D. Cho, and J. Mo, "The economic effects of sharing femtocells," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 595–606, Apr. 2012.

[22] Altai super wi-fi products overview [Online]. Available: http://www.altaitechnologies.com/product_overview.php

[23] S. Dekleva, J. Shim, U. Varshney, and G. Knoerzer, "Evolution and emerging issues in mobile wireless networks," *Commun. ACM*, vol. 50, no. 6, pp. 38–43, 2007.

[24] C. Joe-Wong, S. Sen, and S. Ha, "Offering supplementary wireless technologies: Adoption behavior and offloading benefits," in *Proc. IEEE Conf. Comput. Commun.*, 2013, pp. 1061–1069.

[25] M. Manshaei, J. Freudiger, M. Félegyházi, P. Marbach, and J. Hubaux, "On wireless social community networks," in *Proc. IEEE Conf. Comput. Commun.*, 2008, pp. 2225–2233.

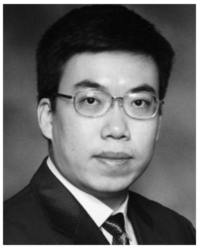
[26] L. Duan, J. Huang, and B. Shou, "Economics of femtocell service provision," *IEEE Trans. Mobile Comput.*, vol. 12, no. 11, pp. 2261–2273, Nov. 2013.

[27] N. Nisan, T. Roughgarden, E. Tardos, and V. V. Vazirani, *Algorithmic Game Theory*. Cambridge, U.K.: Cambridge Univ. Press, 2007.

[28] A. Samuelson and D. Nordhaus, *The Economics of Climate Change: The Stern Review*. New York, NY, USA: McGraw-Hill, 2001.

Yanjiao Chen received the BE degree in electronic engineering from Tsinghua University in 2010. She is currently working toward the PhD degree at the Hong Kong University of Science and Technology. Her research interests include spectrum management for Femtocell networks, network economics and quality of experience (QoE) of multimedia in wireless networks. She is a student member of the IEEE.

Lingjie Duan (S'09-M'12) received the PhD degree from the Chinese University of Hong Kong in 2012. He is an assistant professor of engineering systems and design pillar, Singapore University of Technology and Design. During 2011, he was a visiting scholar in the Department of EECS, University of California at Berkeley. His research interests include network economics and game theory, network optimization, and energy harvesting. He is the TPC cochair of INFOCOM 2014 Workshop on GCCCN and the cochair of the ICCS 2014 Special Issue on "Network Economics and Communication Theory". He is a member of the IEEE.



Jianwei Huang (S'01-M'06-SM'11) received the PhD degree from Northwestern University in 2005. He is an associate professor and the director of the Network Communications and Economics Lab in the Department of Information Engineering, Chinese University of Hong Kong. He is the co-recipient of seven Best Paper Awards. He has served as the editor of *IEEE Journal on Selected Areas in Communications-Cognitive Radio Series* and *IEEE Transactions on Wireless Communications*, and guest editor of *IEEE Journal on Selected Areas in Communications and IEEE Communications Magazine*. He is a senior member of the IEEE.

Qian Zhang is a full professor in the Department of Computer Science and Engineering, Hong Kong University of Science and Technology. She has published about 300 refereed papers in international leading journals and key conferences in the areas of wireless/Internet multimedia networking, wireless communications and networking, and wireless sensor networks. She is a fellow of the IEEE for "contribution to the mobility and spectrum management of wireless networks and mobile communications". She has received MIT TR100 (MIT Technology Review) World's Top Young Innovator Award. She also received the Best Asia Pacific Young Researcher Award elected by IEEE Communication Society in year 2004.