Incentivizing Crowdsourcing Systems with Network Effects

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Abstract—In a crowdsourcing system, it is important for the crowdsourcer to engineer extrinsic rewards to incentivize the participants. With mobile social networking, a user enjoys an intrinsic benefit when she aligns her behavior with the behavior of others. Referred to as *network effects*, such an intrinsic benefit becomes more significant as the number of users grows in the crowdsourcing system. But should a crowdsourcer design her extrinsic rewards differently when such network effects are taken into account? In this paper, we, for the first time, consider network effects as a contributing factor to intrinsic rewards, and study its influence on the design of extrinsic rewards. Rather than assuming a fixed participant population, we show that the number of participating users evolves to a steady equilibrium, thanks to subtle interactions between intrinsic rewards due to network effects and extrinsic rewards offered by the crowdsourcer. Taken network effects into consideration, we design progressively more sophisticated extrinsic reward mechanisms, and propose new and optimal strategies for a crowdsourcer to obtain a higher utility. Via extensive simulations, we demonstrate that with our new strategies, a crowdsourcer is able to attract more participants with higher contributed efforts; and participants gain higher utilities from both intrinsic and extrinsic rewards.

I. INTRODUCTION

Crowdsourcing combines the collective efforts of the crowd to accomplish a specific task, such as collecting large volumes of data, which is otherwise extremely costly or even unattainable. Crowdsourcing systems, such as Uber [1], Waze [2], and Amazon Mechanical Turk (MTurk) [3], have become increasingly popular. Another excellent example is ResearchKit, recently launched by Apple as a mobile application framework that supports a crowdsourcing platform for medical research [4].

The success of a new crowdsourcing platform relies on the scale of user participation, as well as the contribution from each individual participant. To recruit and maintain a large number of users, the crowdsourcer usually provides users with monetary compensation, referred to as *extrinsic rewards*. In contrast to such extrinsic rewards offered by the crowdsourcer, a participant often enjoys a reward that is derived from a sense of satisfaction, social status, or honor, and such rewards are inherently *intrinsic*. In ResearchKit, for example, it has been reported that only a few hours after its release, over 7,000 people has voluntarily enrolled in a study on Parkinson's disease without any extrinsic rewards, and the largest study ever was only 1,700 [5].

Such intrinsic rewards, however, do not typically remain unchanged as the size of participant population grows. When a user's behavior aligns with other users, she will obtain higher intrinsic rewards, usually due to social factors. This is known as the *network effects* [6]. In the example of Waze, a mobile crowdsourcing platform for sharing traffic information, a driver can get a better route if more users join and contribute their local traffic data. It is intuitively conceivable that a growing population of the crowdsourcing platform — with more intrinsic rewards due to network effects — would help reduce the amount of extrinsic rewards that a crowdsourcer will need to provide.

Unfortunately, existing works in the literature have not yet considered the influence of intrinsic rewards, as well as the exquisite and subtle interactions between intrinsic and extrinsic rewards. Reverse auctions (*e.g.,* [7]) and Stackelberg games (*e.g.,* [8]) are typically used to model extrinsic rewards in existing works. With reverse auctions, users submit bids with their desired extrinsic rewards, and the crowdsourcer chooses the users based on their bids, with a selection process that is typically NP-hard and impractical. With Stackelberg games, the crowdsourcer first announces her policy on extrinsic rewards, and the users would then make decisions on their contribution levels. In both models, the user population is assumed to be fixed, without considering intrinsic rewards and network effects.

In this paper, we bring intrinsic rewards into the spotlight, with a focus on how network effects affect the mechanism design when a crowdsourcer provides extrinsic rewards to incentivize crowdsourcing systems. In particular, we study the dynamics of participation levels as a result of the interaction between intrinsic rewards incurred by network effects and extrinsic rewards from the crowdsourcer. Based on our analyses, we propose two extrinsic reward mechanisms, taking advantage of the intrinsic rewards to boost user participation and increase the crowdsourcer's utility.

To start with, we first present a simple mechanism with fixed extrinsic rewards (Section III), which is simple to implement. In fact, mechanisms with fixed extrinsic rewards have been adopted by many existing crowdsourcing systems, such as MTurk. Thanks to intrinsic rewards, the participation level is above-zero even without extrinsic rewards. Given a certain extrinsic reward, the participation level will evolve to a stable

equilibrium. Targeting the most profitable participation level, we are able to compute the corresponding optimal fixed extrinsic reward.

Taking it a step further, we proceed to design a flexible extrinsic reward mechanism, where the extrinsic reward of a user is a function of her effort level (Section IV). To specify the extrinsic reward function is challenging since we do not have any prior knowledge about its form (*e.g.*, linear or logarithmic). Moreover, different extrinsic reward functions, via a complex interaction with intrinsic rewards, lead to different equilibrium participation levels. To tackle this problem, we first focus on a certain participation level, and obtain the extrinsic reward function that achieves the highest utility for the crowdsourcer under this participation level. We then choose the best participation level that yields the maximum utility, and derive the corresponding optimal extrinsic reward function.

The superiority of our proposed extrinsic reward mechanisms is verified through extensive simulations. With the help of intrinsic rewards, the crowdsourcer is able to reach a higher participation level and obtain a higher utility. Stronger network effects will contribute to higher intrinsic rewards, thus more beneficial to the crowdsourcer. Compared with the fixed extrinsic reward mechanism, the flexible extrinsic reward mechanism is more efficient in soliciting more user contributions. Interestingly, the flexible extrinsic reward mechanism improves both the crowdsourcer's and the users' utilities, resulting in a win-win situation. This is because users' higher contributions not only earn themselves higher extrinsic rewards, but also become a valuable asset to the crowdsourcer.

II. SYSTEM MODEL

By participating in a crowdsourcing system, a user receives both extrinsic rewards from the crowdsourcer, and intrinsic rewards due to the benefits or social status she obtains. More formally, user i exerts an effort of b_i. $b_i \in [\underline{b}, \overline{b}]$, in which $\underline{b}$ is the minimum effort, for example, a user has to register and fill in the basic information; $\overline{b}$ is the maximum effort due to limitations such as time, battery life and manpower. To contribute an effort of b_i, the user incurs a cost of $c_i b_i$, where c_i is user i's unit cost.

Intrinsic rewards. On one side, a user benefits from her own effort, for example, a healthcare crowdsourcing platform enables a user to get a better understanding of her health condition by keeping track of her diet, exercise and heart rate. On the other side, a user enjoys the social advantage of a large crowd owing to network effects. Let v_i denote the unit value a user gets from her own effort, and $E(n)$ denote the network effects, where $n \in [0, 1]$ represents the normalized participation level and $E(\cdot)$ is a concave function, satisfying $E(0) = 0, E'(\cdot) > 0$ and $E''(\cdot) < 0$. This indicates that network effects monotonically increase with the participation level, but the marginal return decreases. Therefore, a user's intrinsic reward is $v_i b_i + E(n)$, a function of her effort and the participation level.[1]

Extrinsic rewards. The crowdsourcer provides users with an extrinsic reward of $P(b_i)$, satisfying $P(0) = 0$. In the fixed extrinsic reward mechanism, $P(b_i) = p$, irrespective of the users' effort levels; in the flexible extrinsic reward mechanism, on the other hand, $P(\cdot)$ is a function of b_i.

User i's utility u_i is the sum of intrinsic and extrinsic rewards minus the cost:

$$u_i = v_i b_i + E(n) + P(b_i) - c_i b_i. \quad (1)$$

We combine $v_i b_i$ and $c_i b_i$ as they have the common term b_i:

$$u_i = E(n) + P(b_i) - \beta_i b_i, \quad (2)$$

in which $\beta_i = c_i - v_i$ is defined as the *net cost* of user i. For some users, $v_i > c_i$, so the net cost β_i is negative. Even without extrinsic rewards, these self-motivated users have incentives to participate in the crowdsourcing system. These pioneers help attract others via network effects. The net cost is a user's private information. Thus, we assume that $\beta_i \in [\underline{\beta}, \overline{\beta}]$ is a random variable, with a cumulative distribution function $F(\beta)$, and a probability density function $f(\beta) = F'(\beta)$. We have $\underline{\beta} < 0$ and $\overline{\beta} > 0$, as users may have negative or positive net costs.

Aiming at maximizing her utility in (2), a user's optimal effort level b_i^* is a function of her net cost β_i, i.e., $b_i^* = g(\beta_i)$. She will drop out if her utility is always negative whatever the effort level is.

The crowdsourcer makes a profit from the users' contributions, while having to pay extrinsic rewards. Her utility U is the total aggregated contribution from all participants minus the total extrinsic rewards:

$$U = \mu \int_\beta \ln(1 + g(\beta)) \mathrm{d}F(\beta) - \int_\beta P(g(\beta)) \mathrm{d}F(\beta), \quad (3)$$

in which μ is the equivalent monetary worth of users' contributions. Note that $g(\beta)$ is a user's effort. We use the logarithmic function $\ln(\cdot)$ to transform a user's effort to the perceived utility by the crowdsourcer, which features the law of diminishing return: a user's contribution increases with her effort level but the marginal return decreases. If a user does not contribute any effort, the utility received by the crowdsourcer is $\ln(1 + 0) = 0$.

The crowdsourcer's objective is to maximize her utility in (3) by determining the optimal extrinsic rewards. In the fixed extrinsic reward mechanism, the crowdsourcer has to decide the optimal uniform extrinsic reward p^*; in the flexible extrinsic reward mechanism, the crowdsourcer has to design the optimal extrinsic reward function $P^*(\cdot)$.

III. FIXED EXTRINSIC REWARDS

In this section, we first study how the interplay of a fixed extrinsic reward and network effects leads to participation levels at equilibrium, and then we derive the optimal value of the fixed extrinsic reward.

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[1] For simplicity, in this paper, we assume that network effects $E(n)$ are the same for all users. In the future, we will study the case where users obtain heterogeneous rewards from network effects, i.e., $E_i(n)$ for user i.

A. Equilibrium Participation Level

Given a fixed extrinsic reward, a user's utility becomes:

$$u_i = E(n) + p - \beta_i b_i. \tag{4}$$

If $\beta_i < 0$, a user will definitely participate with her maximum effort $\bar{b}$; otherwise, she will participate with her minimum effort $\underline{b}$ if a positive utility can be obtained. Given an expected participation level n^e and the corresponding network effects $E(n^e)$, the marginal user, who is indifferent to the choices of participating or not, has a utility of zero. Let β_n denote the net cost of the marginal user. We have:

$$\beta_n(p) = \frac{1}{\underline{b}}\big(E(n^e) + p\big). \tag{5}$$

$\beta_n(p)$ is upward sloping in p, and the curve will shift up if n^e increases. This implies that the users with higher net costs will participate if either extrinsic or intrinsic rewards go up. Since $F(\beta_n) = n$, we have:

$$n = F\Big(\frac{E(n^e) + p}{\underline{b}}\Big). \tag{6}$$

At equilibrium, the expected participation level equals the real participation level, that is, $n = n^e$. With $p > \underline{b}\bar{\beta} - E(1)$, $n = 1$ will be an equilibrium,[2] but the crowdsourcer will never set a p that is more than enough to achieve full participation. Thus, we stipulate that $p \leq \underline{b}\bar{\beta} - E(1)$.

Proposition 1. *Existence of an Equilibrium Participation Level. For any extrinsic reward p, Equation (6) has at least one root.*

Proof. Define $\Phi(n) = F\big(\frac{E(n)+p}{\underline{b}}\big) - n$. $\Phi(n)$ is continuous in $[0,1]$. $\Phi(0) = F(p/\underline{b}) > 0$ and $\Phi(1) = F((E(1)+p)/\underline{b}) - 1 \leq 0$. By the intermediate value theorem, 0 is a value of $\Phi(n)$ for some $n \in [0,1]$, which is just the equilibrium participation level. □

Fig. 1 shows the value of $\Phi(n)$ under different participation levels, and the condition $\Phi(n) = 0$ pinpoints the equilibria. Given a certain extrinsic reward, there are multiple equilibria, but they have different stability attributes.

(1) *Stable equilibria*, such as n_B and n_D in Fig. 1. Suppose there is a small perturbation Δn upwards at n_B, $\Phi(n_B + \Delta n) < 0$, *i.e.*, $\beta_{n_B + \Delta n} \underline{b} > E(n_B + \Delta n) + p$. The participation level will be pushed downwards back to n, because the net costs of the new participants are greater than their rewards, so they will leave. Similarly, suppose there is a small perturbation Δn downwards at n_B, $\Phi(n_B - \Delta n) > 0$, *i.e.*, $\beta_{n_B - \Delta n} \underline{b} > E(n_B - \Delta n) + p$. The participation level will be pushed upwards back to n, because users whose net costs are smaller than their rewards will rejoin.

(2) *Unstable equilibria*, such as n_A and n_C in Fig. 1. Suppose there is a small perturbation Δn upwards at n_A, $\Phi(n_A + \Delta n) > 0$, which implies that more users will rush in, and the participation level will be pushed further up to n_B.
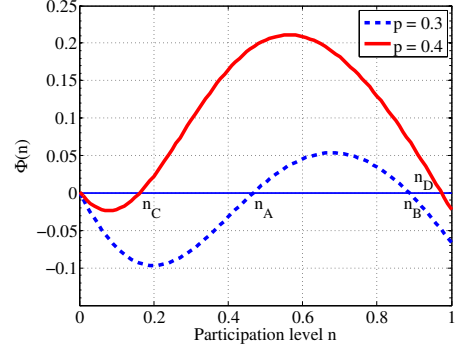

Fig. 1: Multiple equilibria under a certain extrinsic reward. $E(n) = \sqrt{n}, \underline{b} = -1, F(\cdot) \sim N(1, 0.2)$.

Similarly, suppose there is a small perturbation Δn downwards at n_A, $\Phi(n_A - \Delta n) < 0$, therefore, more users will leave, and the participation level will be pushed further down to 0.

In summary, we have the following lemma to characterize the stability of an equilibrium.

Lemma 1. *Stable Equilibrium. An equilibrium participation level is stable if $\Phi'(n) < 0$.*

Proof. Since $\Phi(n) = 0$ and $\Phi'(n) < 0$, it can be easily proved that $\Phi(n + \Delta n) < 0$ and $\Phi(n - \Delta n) > 0$. In this case, the equilibrium is stable according to our preceding analysis. □

Proposition 2. *Existence of a Stable Equilibrium Participation Level. For any extrinsic reward p, there exists at least one stable equilibrium participation level. In particular, the highest equilibrium participation level is stable.*

Proof. Let $n^{\max}$ denote the highest equilibrium participation level. We prove that $\Phi'(n_{e,\max}) < 0$ must be true, then $n^{\max}$ is stable according to Lemma 1.

$$\Phi'(n^{\max}) = \lim_{h \to 0} \frac{\Phi(n^{\max} + h) - \Phi(n^{\max})}{h} = \lim_{h \to 0} \frac{\Phi(n^{\max} + h)}{h}.$$

$\Phi(n^{\max} + h) < 0$ must be true, otherwise there will be another equilibrium between $n_{\max} + h$ and 1, which contradicts the fact that $n^{\max}$ is the highest. □

B. Optimal Fixed Extrinsic Reward

The fixed extrinsic reward p leads to the equilibrium participation level, which in turn, affects the crowdsourcer's utility.[3] Since users with $\beta_i \in [\underline{\beta}, 0]$ make an effort of $\bar{b}$ and users with $\beta \in (0, \beta_n]$ make an effort of $\underline{b}$, the crowdsourcer's utility becomes:

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[2]If $p \leq \underline{b}\underline{\beta}$, $n = 0$ will be an equilibrium. Nevertheless, $\underline{\beta} < 0$, so this will not happen.

[3]For tractability, we only consider the highest stable equilibrium participation level. In the future, we will study the possibilities of other equilibria, and their influence on the design of extrinsic rewards.
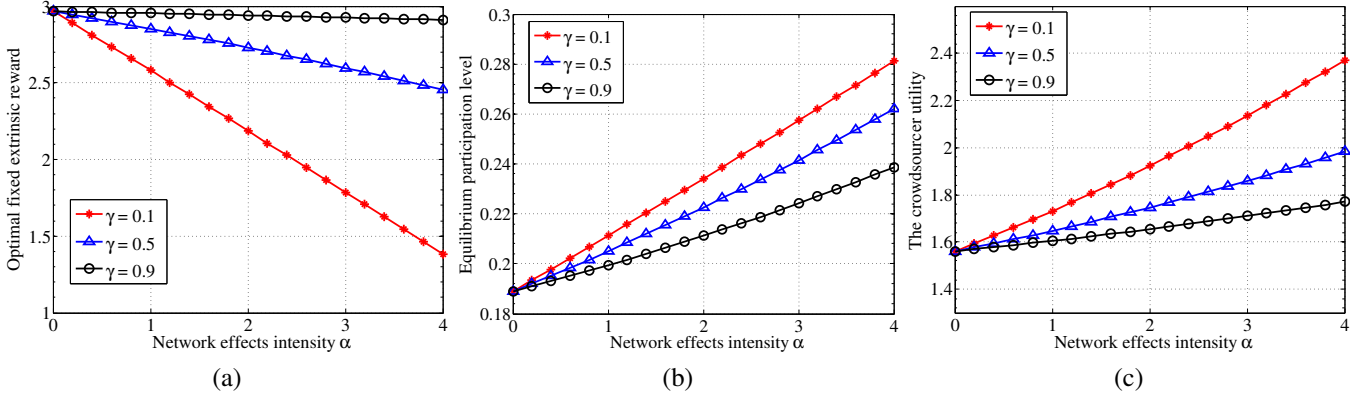
Fig. 2: The fixed extrinsic reward mechanism. $b \in [1, 10], F(\cdot) \sim \text{UNIF}(-1, 15)$.

$$U = \mu \int_{\underline{\beta}}^{0} \ln(1 + \bar{b}) dF(\beta) + \mu \int_{0}^{\beta_n} \ln(1 + \underline{b}) dF(\beta) - np$$

$$= \mu F(0) \ln(1 + \bar{b}) + \mu(n - F(0)) \ln(1 + \underline{b}) - np$$

$$= \mu F(0) \ln \frac{1 + \bar{b}}{1 + \underline{b}} + [\mu \ln(1 + \underline{b}) - p]n. \tag{7}$$

The extrinsic reward p determines the equilibrium participation level n according to Equation (6). Therefore, finding the optimal extrinsic reward p is equivalent to finding the optimal participation level n induced by p:

$$\max_{p} U \Rightarrow \max_{n} \left[\mu \ln(1 + \underline{b}) + E(n) - \underline{b}F^{-1}(n) \right]n. \tag{8}$$

Proposition 3. *Optimal fixed extrinsic reward. Given the optimal equilibrium participation level as the solution of (8), the optimal fixed extrinsic reward is:*

$$p^* = \underline{b}F^{-1}(n^*) - E(n^*). \tag{9}$$

Fig. 2 shows the optimal extrinsic reward, the equilibrium participation level, and the crowdsourcer's utility under the fixed extrinsic reward mechanism. Exponential functions are commonly used to model network effects in the existing literature [6], [9], [10]. Therefore, we assume that the network effects function is $E(n) = \alpha \cdot n^{\gamma}$. Stronger network effects (a larger α and a smaller γ) yield higher intrinsic rewards. The crowdsourcer therefore can take advantage of this to obtain a higher equilibrium participation level (Fig. 2(b)) with a lower fixed extrinsic reward (Fig. 2(a)), and her utility rises as well (Fig. 2(c)). Neglecting network effects ($\alpha = 0$) will potentially cause a significant amount of loss to the crowdsourcer.

An anecdotal real-world example that substantiates the observations above is that, according to its co-founder Elon Musk, when PayPal Inc. was started in 1998, a fixed monetary reward was paid for each user to sign up for the service. As the number of users grew, the amount of the reward was gradually reduced to zero, without affecting the growth of the user population due to network effects [11].

IV. FLEXIBLE EXTRINSIC REWARDS

In this section, we first analyze the users' strategic behavior in response to an extrinsic reward function, which leads to an equilibrium participation level. Given a targeted participation level, we can then derive the extrinsic reward function which helps the crowdsourcer achieve the highest possible utility (referred to as the conditional optimal extrinsic reward function). Finally, we compute the optimal participation level that maximizes the crowdsourcer's utility, and the corresponding global optimal extrinsic reward function.

A. Users' Optimal Effort Level

Given an extrinsic reward function instead of a fixed extrinsic reward, users will strategically choose their effort levels to maximize their utilities, rather than toggling between $\bar{b}$ and $\underline{b}$.

Proposition 4. *Optimal Effort Level. The optimal effort level of user i, based on her net cost β_i, i.e., $b_i = g(\beta_i)$, is implicitly given by the following equation:*

$$E(n) + P(b_i) - \beta_i g(\beta_i) = \int_{\beta_i}^{F^{-1}(n)} g(x) dx. \tag{10}$$

Proof. We prove this by the envelop theory [12]. Take partial derivative of β_i on both sides of Equation (2):

$$\frac{\partial u_i}{\partial \beta_i} = -b_i. \tag{11}$$

Let β_n represent the net cost of the marginal user, who is indifferent towards the options of participating or not. Integrating both sides of Equation (11) from β_i to β_n:

$$u(\beta_n) - u(\beta_i) = -\int_{\beta_i}^{\beta_n} g(x) dx. \tag{12}$$

This yields Equation (10) as $u(\beta_n) = 0$ and $n = F(\beta_n)$. □

Given a complicated extrinsic reward function $P(\cdot)$, it is difficult to solve Equation (10) to obtain the optimal effort level function $g(\beta)$. Fortunately, we show in the following section that, as the crowdsourcer intentionally designs $P(\cdot)$ for utility maximization, $g(\beta)$ has a closed-form expression.

B. Optimal Extrinsic Reward Function

If a user cannot gain positive utility even with the optimal effort level, she will not participate at all. Therefore, different extrinsic reward functions will result in different equilibrium participation levels. Based on this knowledge, the crowdsourcer can design the conditional optimal extrinsic reward function for a targeted participation level.

Proposition 5. *Conditional optimal extrinsic reward function. Given a targeted participation level n and the corresponding marginal user's net cost $\beta_n = F^{-1}(n)$, under conditions that $\widetilde{\beta} > 0$ and $E(n) \leq \underline{b}\beta_n$, the crowdsourcer's conditional optimal extrinsic reward function, and the users' optimal effort level are given as follows, in which $\widehat{\beta}$ satisfies $\widehat{\beta} + \frac{F(\widehat{\beta})}{f(\widehat{\beta})} = \frac{\mu}{1+\underline{b}}$, and $\widetilde{\beta}$ satisfies $\widetilde{\beta} + \frac{F(\widetilde{\beta})}{f(\widetilde{\beta})} = \frac{\mu}{1+\overline{b}}$.*

- *If $\beta_n \in [\underline{\beta}, \widetilde{\beta})$, the conditional optimal extrinsic reward function is:*

$$P(b) = \beta_n b - E(n). \tag{13}$$

The users' optimal effort level is:

$$g(\beta) = \overline{b}, \beta \in [\underline{\beta}, \beta_n]. \tag{14}$$

- *If $\beta_n \in [\widetilde{\beta}, \widehat{\beta})$, the conditional optimal extrinsic reward function is:*

$$P(b) = g^{-1}(b)b - E(n) + \int_b^{g(\beta_n)} x \mathrm{d}g^{-1}(x). \tag{15}$$

The users' optimal effort level is:

$$g(\beta) = \begin{cases} \overline{b}, & \beta \in [\underline{\beta}, \widetilde{\beta}], \\ \frac{\mu f(\beta)}{\beta f(\beta) + F(\beta)} - 1, & \beta \in [\widetilde{\beta}, \beta_n]. \end{cases} \tag{16}$$

In particular, $g^{-1}(\overline{b}) = \widetilde{\beta}$.

- *If $\beta_n \in [\widehat{\beta}, \overline{\beta}]$, the conditional optimal extrinsic reward function is:*

$$P(b) = g^{-1}(b)b - E(n) + \int_b^{g(\beta_n)} x \mathrm{d}g^{-1}(x) + \underline{b}(\beta_n - \widehat{\beta}). \tag{17}$$

The users' optimal effort level is:

$$g(\beta) = \begin{cases} \overline{b}, & \beta \in [\underline{\beta}, \widetilde{\beta}], \\ \frac{\mu f(\beta)}{\beta f(\beta) + F(\beta)} - 1, & \beta \in [\widetilde{\beta}, \widehat{\beta}], \\ \underline{b}, & \beta \in [\widehat{\beta}, \beta_n]. \end{cases} \tag{18}$$

In particular, $g^{-1}(\overline{b}) = \widetilde{\beta}$ and $g^{-1}(\underline{b}) = \widehat{\beta}$.

Proof. The crowdsourcer's utility is:

$$U = \int_{\underline{\beta}}^{\beta_n} \left[\mu \ln(1 + g(\beta)) - P(g(\beta)) \right] \mathrm{d}F(\beta). \tag{19}$$

Substitute $P(b(\beta))$ with (10):

$$U = \int_{\underline{\beta}}^{\beta_n} \left[\mu \ln(1 + g(\beta)) - g(\beta)\beta + E(n) \right. \\ \left. - \int_{\underline{\beta}}^{\beta_n} g(x) dx \right] \mathrm{d}F(\beta). \tag{20}$$

The last term is:

$$\int_{\underline{\beta}}^{\beta_n} \int_{\beta}^{\beta_n} g(x) \mathrm{d}x \mathrm{d}F(\beta) = \left[\int_{\beta}^{\beta_n} g(x) \mathrm{d}x * F(\beta) \right] \Big|_{\underline{\beta}}^{\beta_n} \\ - \int_{\underline{\beta}}^{\beta_n} F(\beta) \frac{\partial \int_{\beta}^{\beta_n} g(x)\mathrm{d}x}{\partial \beta} \mathrm{d}\beta = \int_{\underline{\beta}}^{\beta_n} F(\beta)g(\beta)\mathrm{d}\beta. \tag{21}$$

Therefore, $U = \int_{\underline{\beta}}^{\beta_n} I(\beta)\mathrm{d}F(\beta)$, in which:

$$I(\beta) = \mu \ln(1 + g(\beta)) - g(\beta)\beta + E(n) - \frac{F(\beta)}{f(\beta)}g(\beta). \tag{22}$$

Maximizing the crowdsourcer's utility U via the extrinsic reward function $P(\cdot)$ is equivalent to inducing the optimal effort level $b = g(\beta)$ through $P(\cdot)$. Take the first and second derivatives of I with respect to $g(\beta)$:

$$\frac{\partial I}{\partial g(\beta)} = \frac{\mu}{1 + g(\beta)} - \beta - \frac{F(\beta)}{f(\beta)}, \\ \frac{\partial^2 I}{\partial g^2(\beta)} = -\frac{\mu}{(1 + g(\beta))^2} < 0. \tag{23}$$

If $\beta < \widetilde{\beta}$, $\partial I/\partial g(\beta) > 0$, meaning that I monotonically increases with b, thus $g(\beta) = \overline{b}$. If $\beta > \widehat{\beta}$, $\partial I/\partial g(\beta) < 0$, meaning that I monotonically decreases with b, hence $g(\beta) = \underline{b}$. Otherwise, the optimal effort level b is the solution to $\partial I/\partial b = 0$. In this way, we get (14), (16) and (18). Substituting $g(\beta)$ in (10) with (14), (16) and (18), we can get the optimal extrinsic reward functions (13), (15) and (17) for different targeted participation levels. Special cases of $g^{-1}(\underline{b})$ and $g^{-1}(\overline{b})$ are given in the appendix.

□

Corollary 1. *The conditional optimal extrinsic reward function $P(\cdot)$ is non-negative and monotonically increasing.*

Proof. If $\beta_n \in [\underline{\beta}, \widetilde{\beta})$, $P(b) = \beta_n \overline{b} - E(n) > 0$. If $\beta_n \in [\widetilde{\beta}, \overline{\beta}]$, the first derivative of $P(\cdot)$ with respect to b is:

$$\frac{\partial P(b)}{\partial b} = (g^{-1}(b))'b + g^{-1}(b) - (g^{-1}(b))'b = g^{-1}(b) > 0. \tag{24}$$

Therefore, $P(b)$ monotonically increases with b, and the minimum reward is $P(\underline{b}) = \beta_n \underline{b} - E(n) \geq 0$.

□

In Proposition 5, conditions $\widetilde{\beta} > 0$ and $E(n) \leq \underline{b}\beta_n$ are both reasonable. $\widetilde{\beta}$ is the threshold net cost, below which a user will make the maximum effort $\overline{b}$. Users with negative net costs — and those with low positive net costs as well — will exert the maximum effort, with certain extrinsic rewards. Therefore, $\widetilde{\beta}$ is assumed to be positive. $E(n) \leq \underline{b}\beta_n$ indicates that network effects cannot fully cover the cost of the marginal user who makes the minimum effort; otherwise, the crowdsourcer will not provide any extrinsic rewards to them.

Note that the conditional optimal extrinsic reward functions (13), (15), and (17) are functions of users' effort level b,

which is observable by the crowdsourcer, but not the users' net cost β, which is private information. $P(\cdot)$ also depends on β_n, which is known by the crowdsourcer as the participation level n is set as the target by the crowdsourcer. Furthermore, the crowdsourcer is aware of the users' response to the extrinsic reward function, and can use Proposition 5 to derive function $g(\cdot)$, which is indispensible in determining $P(\cdot)$. In the conditional optimal extrinsic reward function, the term $\beta_n b$ or $g^{-1}(b)b$ can be regarded as the compensation for the users' cost; the term $-E(n)$ shows how network effects help curtail the crowdsourcer's payment to users; the rest of the terms are necessary to realize the targeted participation level. More specifically, it is ensured that $u_i > 0, \forall \beta_i < \beta_n, u_i < 0, \forall \beta_i > \beta_n$ and $\beta_i = 0, \beta_i = \beta_n$. Interestingly, a user's optimal effort level is not affected by network effects, which are counteracted by the second term of extrinsic reward functions.

Corollary 2. *Strict individual rationality. With the extrinsic reward functions given by Proposition 5, every participant receives strictly positive utility, i.e., $u_i > 0, \forall \beta_i \in [\underline{\beta}, \beta_n)$. In particular, the marginal user's utility is zero, i.e., $u_i = 0, \beta_i = \beta_n$.*

The proof of Corollary 2 is given in the appendix.

As the net cost increases, a user's optimal effort level declines, as shown in Fig. 3. μ reflects the crowdsourcer's appreciation for users' contributions. If μ is higher, the crowdsourcer is willing to elicit more user contributions with higher extrinsic rewards.
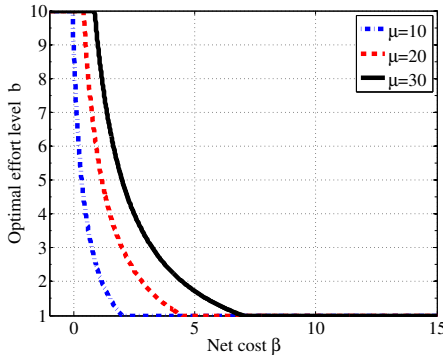

Fig. 3: The user's optimal effort level $g(\beta)$. $b \in [1, 10], F(\cdot) \sim$ UNIF$(-1, 15)$.

C. Optimal Participation Level

Proposition 5 gives the conditional optimal extrinsic reward function for a targeted participation level. By comparing the crowdsourcer's utility under each participation level with the conditional optimal extrinsic reward function, we can find the most lucrative participation level, and the corresponding global optimal extrinsic reward function.

Proposition 6. *Optimal participation level. The optimal participation level is $n^* = \arg\max_n U$, in which:*
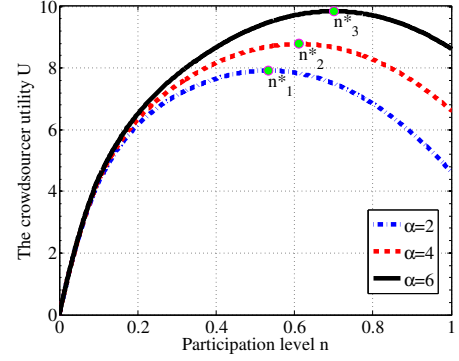

Fig. 4: The crowdsourcer's utility under different participation levels with a conditional optimal extrinsic reward function. $\gamma = 1/2, b \in [1, 10], F(\cdot) \sim$ UNIF$(-1, 15)$.

$$U = \begin{cases} n[\mu\ln(1+\bar{b}) - \bar{b}\beta_n + E(n)], & n \in [0, F(\widetilde{\beta})), \\ U_A + \int_{\widetilde{\beta}}^{\beta_n} [\mu\ln(1+g(\beta)) - \\ (\beta + \frac{F(\beta)}{f(\beta)})g(\beta)]\mathrm{d}F(\beta) + E(n)n, & n \in [F(\widetilde{\beta}), F(\widehat{\beta})), \\ U_B + n[\mu\ln(1+\underline{b}) + E(n) - \underline{b}\beta_n], & n \in [F(\widehat{\beta}), 1]. \end{cases}$$
(25)

in which $U_A = [\mu\ln(1+\bar{b}) - \bar{b}\widetilde{\beta}]F(\widetilde{\beta})$, $U_B = F(\widetilde{\beta})[\mu\ln(1+\bar{b}) - \bar{b}\widetilde{\beta}] - F(\widehat{\beta})[\mu\ln(1+\underline{b}) - \underline{b}\widehat{\beta}] + \int_{\widetilde{\beta}}^{\widehat{\beta}}[\mu\ln(1+b(\beta)) - b(\beta)\beta - \frac{F(\beta)}{f(\beta)}g(\beta)]\mathrm{d}F(\beta)$.

The key idea of Proposition 6 is to achieve the participation level that maximizes the crowdsourcer's utility. Fig. 4 illustrates the attainable utility under each participation level with the conditional optimal extrinsic reward function. The optimal participation level rests at the peak of each curve. The detailed proof of Proposition 6 is in the appendix.

It is difficult to solve the maximization problem in Proposition 6 since the objective function (25) contains an integral. To address this problem, we propose an efficient analytical approach. We first transform the integral in (25) when $n \in [F(\widetilde{\beta}), F(\widehat{\beta}))$ into a quadratic sum as:

$$U_K = U_A + E(n_K)n_K + \sum_{k=0}^{K} f(\beta_k)\Delta\beta\{\mu\ln(1+g(\beta_k)) - g(\beta_k)\beta_k - \frac{F(\beta_k)}{f(\beta_k)}g(\beta_k)\},$$
(26)

in which $\beta_k = \widetilde{\beta} + k\Delta\beta, n_k = F^{-1}(\beta_k)$. U_{K+1} can be easily calculated as:

$$U_{K+1} = U_K - E(n_K)n_K + E(n_{K+1})n_{K+1} + f(\beta_{K+1})\Delta\beta[\mu\ln(1+g(\beta_{K+1})) - g(\beta_{K+1})\beta_{K+1} - \frac{F(\beta_{K+1})}{f(\beta_{K+1})}g(\beta_{K+1})].$$
(27)

Fig. 5 shows the gap between the approximated utility in (26) and the ground truth utility in (25), when the participation
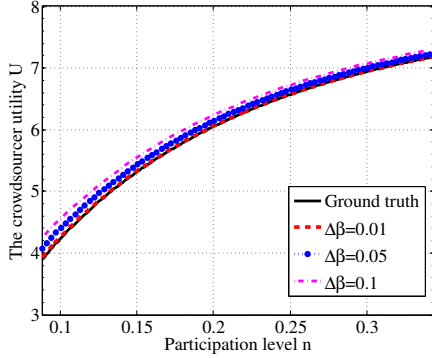
Fig. 5: Algorithm 1 verification. $\gamma = 1/2, b \in [1, 10], F(\cdot) \sim$ UNIF$(-1, 15)$.

level is within the range $[F(\widetilde{\beta}), F(\widehat{\beta}))$. When $\Delta\beta = 0.01$, the quadratic sum can approximate the integral to near perfection. The largest approximation error is 0.72% when $\Delta\beta = 0.05$, and 1.65% when $\Delta\beta = 0.1$. Selecting a proper step size $\Delta\beta$, we can compute the participation level within the range $[F(\widetilde{\beta}), F(\widehat{\beta}))$ that yields the highest utility, then we can find the optimal participation level over the entire range $[0, 1]$. Algorithm 1 summarizes the detailed process.

Algorithm 1 The Optimal Participation Level

Input: Network effects $E(\cdot)$, probability density function $f(\cdot)$, the users' optimal effort level $g(\cdot)$, equivalent monetary worth of users' contributions μ, and the step size $\Delta\beta$.

Output: The optimal participation level n^*.

1: $\beta_K = \widetilde{\beta}, U_{\max} = 0$.
2: Calculate U_K according to (26).
3: **while** $\beta_K \leq \widehat{\beta}$ **do**
4:    **if** $U_K > U_{\max}$ **then**
5:       $U_{\max} = U_K$.
6:       $n_{\max} = F(\beta_K)$.
7:    **end if**
8:    $\beta_K = \beta_K + \Delta\beta$.
9:    Update U_K according to (27).
10: **end while**
11: Find the best participation level $n'_{\max}$ in the range $[0, F(\widetilde{\beta}))$ and range $[F(\widehat{\beta}), 1]$ using `fminbnd` in MATLAB, with the corresponding utility $U'_{\max}$.
12: **if** $U_{\max} > U'_{\max}$ **then**
13:    $n^* = n_{\max}$.
14: **else**
15:    $n^* = n'_{\max}$.
16: **end if**

Fig. 6 shows the optimal extrinsic reward function, the equilibrium participation level, and the crowdsourcer's utility under the flexible extrinsic reward mechanism. The flexible extrinsic reward mechanism remunerates users for different levels of contributions, as shown in Fig. 6(a). It can be observed that the extrinsic reward function is concave, that

is, a user's extrinsic reward increases with her effort level, but the marginal return decreases. Similar to Fig. 2, network effects boost the equilibrium participation level and the crowdsourcer's utility, as shown in Fig. 6(b) and (c).

D. Fixed vs. Flexible Extrinsic Reward Mechanisms

The flexible extrinsic reward mechanism is more efficient than the fixed extrinsic reward mechanism, as verified by Fig. 7. Although the flexible mechanism requires more disbursement from the crowdsourcer than the fixed mechanism in order to provide the extrinsic rewards to incentivize users (Fig. 7(a)), it induces a higher user contribution level in return. With the fixed mechanism, most participants will only provide a minimum level of effort; while with the flexible mechanism, participants are stimulated to work harder in exchange for higher extrinsic rewards. As a result, the crowdsourcer has a higher overall utility with the flexible mechanism, as shown in Fig. 7(b). Interestingly, users also have a higher aggregate utility with the flexible mechanism, since the flexible mechanism gives a higher payment and motivates more users to participate. Both extrinsic rewards and intrinsic rewards induced by network effects are augmented. This suggests that the interests of users and the crowdsourcer are not necessarily in conflict with each other. Quite the contrary, a thriving crowdsourcing system with a higher participation level and a higher user contribution level will be valuable to both users and the crowdsourcer.

V. RELATED WORK

Extrinsic rewards in crowdsourcing. Existing works have used reverse auctions and Stackelberg games to model extrinsic rewards. With reverse auctions, the crowdsourcer selects users based on the bids, which reflect users' anticipated extrinsic rewards [7], [13], [14]. The complexity of reverse auctions is high, making them impractical in real-world implementations. With Stackelberg games, the crowdsourcer determines the optimal extrinsic rewards, while users compete for these rewards by making strategic decisions on their contribution levels [15], [16]. Different from these two multi-winner mechanisms, in [8], [17], the authors proposed a winner-take-all mechanism, where a single best or designated user gets all the extrinsic rewards. In contrast, we focused on the interplay between extrinsic and intrinsic rewards in this paper, with a focus on the impact of network effects.

Network effects. Network effects have been extensively discussed in telecommunication networks [9], the open-source software community [10], and social networks [18]. As the crowdsourcing systems connect a large number of participants, network effects can be observed [19]. Due to network effects, users obtain higher intrinsic rewards when the total number of participants increases, thus requiring less extrinsic rewards to compensate their costs. However, there is a lack of existing works that take advantage of network effects for more efficient extrinsic rewards design.

Empirical studies on intrinsic and extrinsic rewards. User behavior under the influence of extrinsic and intrinsic rewards
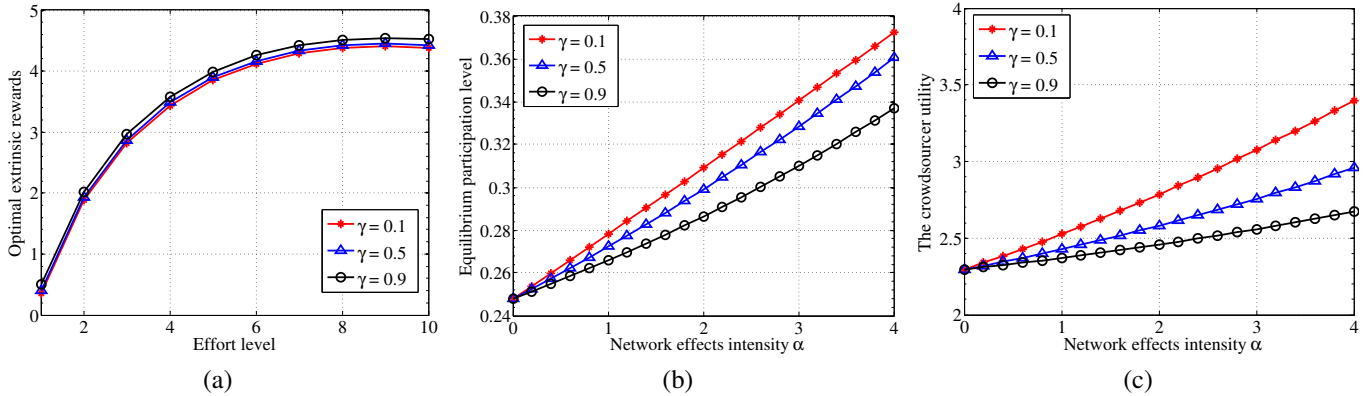
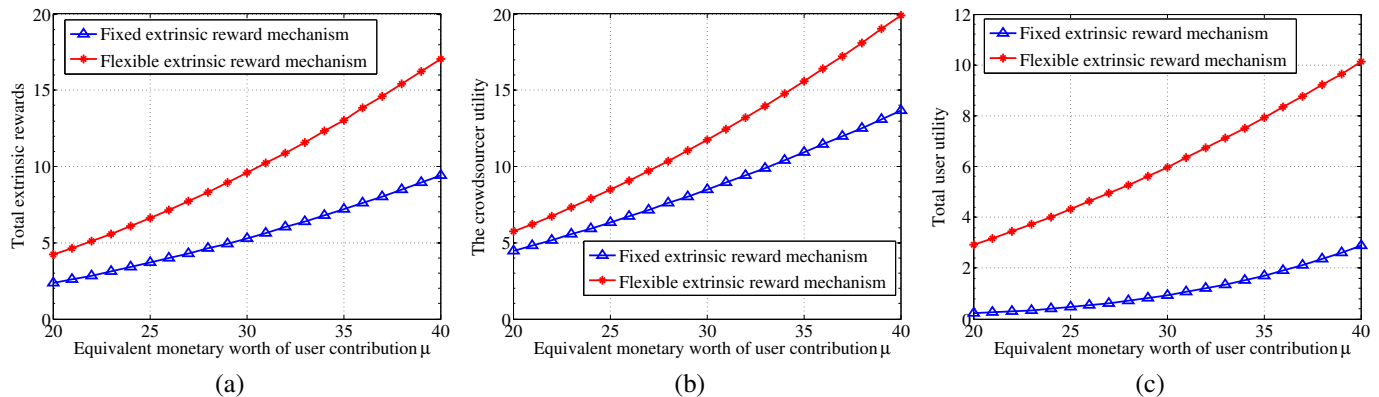Fig. 6: Flexible extrinsic reward mechanism. $b \in [1, 10]$, $F(\cdot) \sim \text{UNIF}(-1, 15)$.

Fig. 7: Fixed VS. flexible extrinsic reward mechanisms. $\alpha = 1, \gamma = 1/2$, $F(\cdot) \sim \text{UNIF}(-1, 20)$.

have been explored by some empirical studies. Schweizer *et al.* [20] have used the feedback for crowdsourcing tasks as a potential intrinsic reward. Anawar *et al.* [21] have adopted self-determination theory to explain intrinsic motivations in a weight loss crowdsourcing system. Competitive extrinsic rewards are found to be more efficient than fixed extrinsic rewards in [22]–[24]. However, intrinsic rewards incurred by network effects, as well as how intrinsic and extrinsic rewards interact with each other, have not been examined in existing empirical studies.

VI. CONCLUSION

In this paper, we have proposed a new framework for extrinsic reward design in crowdsourcing systems, which, for the first time, exploits network effects and intrinsic rewards. Instead of assuming a fixed participant population, we have shown how user participation levels evolve as a result of the interactions between extrinsic rewards and network effects. Based on our analyses, we have proposed a *fixed* and a *flexible* extrinsic reward mechanism, designed to help a crowdsourcer to enlist more users and attain a higher payoff by considering network effects. In particular, our flexible mechanism adjusts the value of extrinsic rewards according to the contributions from the users, and improves the utilities of both users and

the crowdsourcer. We have presented an optimal extrinsic reward function in closed form, which can be easily used by the crowdsourcer to determine the extrinsic reward to an individual user. Extensive simulation results have verified that the proposed extrinsic reward mechanisms have outperformed existing ones that did not take network effects and the corresponding intrinsic rewards into consideration.

VII. ACKNOWLEDGEMENT

VIII. APPENDIX

A. Proof of Proposition 5

1) If $\beta_n \in [\underline{\beta}, \widetilde{\beta}]$, Equation (10) becomes:

$$E(n) + P(\bar{b}) - \beta\bar{b} = \int_{\underline{\beta}}^{\beta_n} \bar{b}\mathrm{d}y,$$

which yields $P(\bar{b}) = \bar{b}\beta_n - E(n)$.

2) If $\beta_n \in [\widetilde{\beta}, \widehat{\beta}]$, when $b \in (\underline{b}, \bar{b})$, Equation (10) becomes:

$$P(b) = \beta b - E(n) + \int_{\beta}^{\beta_n} g(y)\mathrm{d}y$$

$$= g^{-1}(b)b - E(n) + \int_b^{g(\beta_n)} x\mathrm{d}g^{-1}(x).$$

When $b = \bar{b}$, combine (10) and (2):

$$E(n) + P(\bar{b}) - \beta\bar{b} = \int_{\beta}^{\widetilde{\beta}} \bar{b}\mathrm{d}y + \int_{\widetilde{\beta}}^{\beta_n} g(y)\mathrm{d}y$$

$$\Rightarrow g^{-1}(\bar{b}) = \widetilde{\beta}.$$

3) If $\beta_n \in [\widehat{\beta}, \overline{\beta}]$, when $b \in (\underline{b}, \bar{b})$, Equation (10) becomes:

$$P(b) = \beta b - E(n) + \int_{\beta}^{\widehat{\beta}} g(y)\mathrm{d}y + \int_{\widehat{\beta}}^{\beta_n} \underline{b}\mathrm{d}y$$

$$= g^{-1}(b)b - E(n) + \int_{\underline{b}}^{b} x\mathrm{d}g^{-1}(x) + (\beta_n - \widehat{\beta})\underline{b}.$$

It can be easily proved that $g^{-1}(\bar{b}) = \widetilde{\beta}$ and $g^{-1}(\underline{b}) = \widehat{\beta}$.

B. Proof of Corollary 2

- If $\beta_n \in [\underline{\beta}, \widetilde{\beta})$, $u(\bar{b}) = E(n) + \beta_n\bar{b} - E(n) - \beta\bar{b} > 0$.
- If $\beta_n \in [\widetilde{\beta}, \widehat{\beta})$, we have:

$$u(\bar{b}) = E(n) + \widetilde{\beta}\bar{b} - E(n) + \int_{\beta}^{\beta_n} g(y)\mathrm{d}y - \beta\bar{b} > 0.$$

$$u(b) = E(n) + g^{-1}(b)b - E(n) + \int_{\beta}^{\beta_n} g(y)\mathrm{d}y - \beta b > 0.$$

- If $\beta_n \in [\widehat{\beta}, \overline{\beta}]$, we can similarly prove that $u(\bar{b}) > 0$ and $u(b) > 0, \forall b \neq \bar{b}, b \neq \underline{b}$.

$$u(\underline{b}) = E(n) + \beta_n\underline{b} - E(n) - \beta\underline{b} = (\beta_n - \beta)\underline{b} > 0.$$

C. Proof of Proposition 6

- If $\beta_n \in [\underline{\beta}, \widetilde{\beta})$, the crowdsourcer's utility is:

$$U = \int_{\underline{\beta}}^{\beta_n} \left\{ \mu\ln(1 + \bar{b}) - \bar{b}\beta_n + E(n) \right\}\mathrm{d}F(\beta)$$

$$= F(\beta_n)\left[\mu\ln(1 + \bar{b}) - \bar{b}\beta_n + E(n) \right].$$

- If $\beta_n \in [\widetilde{\beta}, \widehat{\beta})$, the crowdsourcer's utility is:

$$U = \int_{\underline{\beta}}^{\widetilde{\beta}} \left\{ \mu\ln(1 + \bar{b}) - \bar{b}\widetilde{\beta} + E(n) - \int_{\widetilde{\beta}}^{\beta_n} b(x)\mathrm{d}x \right\}\mathrm{d}F(\beta)$$

$$+ \int_{\widetilde{\beta}}^{\beta_n} \left\{ \mu\ln(1 + b(\beta)) - b(\beta)\beta + E(n) - \int_{\beta}^{\beta_n} b(x)\mathrm{d}x \right\}\mathrm{d}F(\beta)$$

$$= U_A + E(n)F(\beta_n) + \int_{\widetilde{\beta}}^{\beta_n} \left\{ \mu\ln(1 + b(\beta)) \right.$$

$$\left. - b(\beta)\beta + \frac{F(\beta)}{f(\beta)}b(\beta) \right\}\mathrm{d}F(\beta).$$

- If $\beta_n \notin [\underline{\beta}, \overline{\beta}]$, the crowdsourcer's utility is:

$$U = \int_{\underline{\beta}}^{\widetilde{\beta}} \left\{ \mu\ln(1 + \bar{b}) - \bar{b}\widetilde{\beta} + E(n) - \int_{\widetilde{\beta}}^{\widehat{\beta}} b(x)\mathrm{d}x \right.$$

$$\left. - \underline{b}(\beta_n - \widehat{\beta}) \right\}\mathrm{d}F(\beta) + \int_{\widetilde{\beta}}^{\widehat{\beta}} \left\{ \mu\ln(1 + b(\beta)) \right.$$

$$\left. - b(\beta)\beta + E(n) - \int_{\beta}^{\widehat{\beta}} b(x)\mathrm{d}x - \underline{b}(\beta_n - \widehat{\beta}) \right\}\mathrm{d}F(\beta)$$

$$+ \int_{\widehat{\beta}}^{\beta_n} \left\{ \mu\ln(1 + \underline{b}) - \underline{b}\widehat{\beta} + E(n) - \underline{b}(\beta_n - \widehat{\beta}) \right\}\mathrm{d}F(\beta)$$

$$= U_B + (\mu\ln(1 + \underline{b}) + E(n) - \underline{b}\beta_n)F(\beta_n).$$

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