Financial Analysis of 4G Network Deployment

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Abstract—**Major cellular operators are planning to upgrade to high-speed 4G networks, but due to budget constraints, they have to dynamically plan and deploy the 4G networks through multiple stages of time. By considering one-time deployment cost, daily operational cost and 3G network congestion, this paper studies how an operator financially manages the cash flow and plans the 4G deployment in a finite time horizon to maximize his final-stage profit. The operator provides both the traditional 3G service and the new 4G service, and we show that users will start to use the 4G service only when it reaches a sizable coverage. At each time stage, the operator first decides an additional 4G deployment size, by predicting users' responses in choosing between the 3G and 4G services. We formulate this problem as a dynamic programming problem, and propose an optimal threshold-based 4G deployment policy. We show that the operator will not deploy to a full 4G coverage in an area with low user density or high deployment/operational cost. Perhaps surprisingly, during the 4G deployment process, we show that the 4G subscriber number first increases and then decreases, as the 4G service helps mitigate 3G network congestion and increases its QoS.**

I. Introduction

As the market penetration of smartphones increases and more wireless users get used to data services, the existing 3G networks become more and more congested. As the successor of 3G, a 4G technology (e.g., LTE, featuring OFDM and MIMO schemes) better utilizes the wireless resources, and provides a much higher date rate to enable latest mobile applications and resolve the network congestion [1]. Besides this technological improvement, 4G service with a higher data rate is more profitable than 3G in the wireless market. According to [2], 4G service is usually priced significantly higher than 3G even when 4G users keep similar data usage. Hence, major cellular operators are planning to deploy the new 4G service. However, they have to carefully plan their 4G deployment roadmap due to budget constraints. If not well planned beforehand, they may experience an unexpected financial cut-off. For example, as the 4G pioneer from 2008, Sprint is now short of money to expand its 4G coverage and has to request financial help from Softbank in Japan [3]. In Europe, many operators are also budget-limited and are still discussing when to deploy 4G [4].

The deployment of 4G network is not an overnight establishment but a long-term process requiring careful cash-flow management. An operator's budget at a time is determined by the operator's initial funding, ongoing revenue collection, and total cost till now. Revenue collected in the current stage, from both 3G and 4G services, helps relax the budget constraint for the next-stage deployment. The collected revenue is affected by two major factors in the wireless market:

- *User density* is the number of all potential users in a unit area who can choose and pay for cellular services. We observe that in rural areas with a low user density, it may not be profitable to deploy advanced and costly 4G networks. For example, Verizon densely deployed 4G networks in populous states like New York, but only sparsely deployed 4G networks in underpopulated states like Utah [5].
- *Market response* is about all potential users' service choices, and such a response depends on service qualities (including network coverage, data rate, and network congestion) and service prices. Users with high QoS requirements prefer 4G service, while those price-sensitive ones prefer 3G service.

Accompanied with revenue collection, total cost of the 4G network includes the deployment cost to physically enlarge the 4G coverage and the operational cost to maintain the 4G network all the time. More specifically,

- *Deployment cost* is the one-time expenditure to purchase and build 4G network infrastructure (e.g., 4G cell towers). According to [6], the cost to deploy a 4G LTE tower ranges from US $\$75,000$ to US $\$200,000$, depending on the leasing site. The 4G coverage is approximately proportional to the number of 4G towers.
- *Operational cost* is the daily expenditure related to management and maintenance of 4G network (e.g., energy and manpower costs), which is approximately proportional to the network coverage according to [7].

To our best knowledge, this paper is the first work that financially studies the dynamic deployment planning for a large-scale wireless network, by taking various costs and market response into account. We consider a finite and time-slotted horizon.[1] At each time stage, the operator first decides his investment amount to expand the 4G coverage based on his current budget, and users observing the network update then choose between the 3G and 4G services. By predicting users' dynamic responses to the services qualities, the operator wants to optimize his 4G deployment planning over time and maximize the final-stage profit.

Our main results and key contributions are summarized as follows:

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[1] A finite time horizon is reasonable as 4G has its own advantageous cycle and may be replaced by another (e.g., 5G) in the future.

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- *Financial modeling of cash flow for 4G deployment*. Very few studies have studied financial management in wireless industry, and they assume the deployment of 4G is an overnight effort and solve the static deployment problem without considering the cash flow (e.g., [8] [9]). Our financial model investigates the cash flow driven by an operator's initial budget, ongoing revenue collection (depending on users' dynamic subscriptions), and the total cost for 4G network.
- *Dynamic programming formulation for deployment planning*. Since the operator's current investment decision affects next-stage budget and revenue collection, we formulate the cash flow management problem as a dynamic program. The operator wants to maximize his final-stage budget (cash level) by trading off deployment progress and budget saving over time.
- *Optimal threshold-based policy for 4G deployment*. By solving the dynamic program, we show that it is optimal for the operator to accumulate the collected revenue till a threshold for initial deployment. After starting with a sizable deployment, the operator will gradually enlarge the 4G coverage conforming to the time-varying budget constraint. We also show that the operator should not deploy to a full coverage in an under-populated area, or if the deployment/operational cost is high.
- *Impact of 3G network congestion*. We show that the expansion of 4G coverage helps resolve the 3G network congestion. As the 4G coverage increases, the 4G service's subscriber number can first increase but then decrease as fewer subscribers churn from the improved 3G service to the 4G service.

The rest of the paper is organized as follows. We briefly review the related work in Section II, and present the system model in details in Section III. We analyze the market response in Section IV, which is useful for the operator to predict and decide optimal deployment policy in Section V. We study the impact of operational cost in Section VI and the impact of 3G congestion effect in Section VII. We present the simulation results in Section VIII, and finally summarize the work in Section IX. **Due to page limitation, we give all the proofs in our online technical report [10].**

II. Related Work

People just started to study network deployment or upgrade problems recently. In [11], the Internet service providers choose between "Upgrade" and "Not-Upgrade", and pay a one-time fee for infrastructure upgrade. In [12], Internet providers choose an investment level to minimize their long-term security risk in a one-shot static model. In [13], the user adoption of a new technology and an incumbent technology is studied without considering network deployment process or any economic return. In [8], wireless operators' 4G network upgrade is assumed to be an overnight establishment, without considering the time duration and cash flow for deployment. [9] analyzes the financial impact of pico-cellular base station deployment by focusing on a static model without considering the deployment time or any dynamics in user subscription. Task scheduling with energy constraints in cloud computing has been studied in [14]–[16], but their optimizations do not consider the change in payoff due to network update.

All prior works focus on a simplified one-time (static) network upgrade, and our focus is on how an operator should dynamically manage the cash flow for the 4G network deployment. Furthermore, we comprehensively incorporate users' dynamic responses and various costs in the financial analysis.

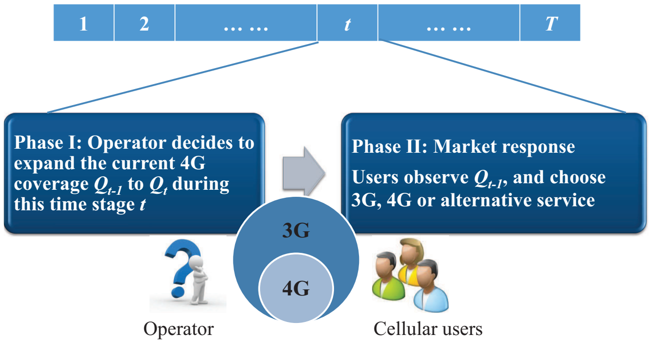


Fig. 1: System model.

III. System Model

We consider a cellular operator, who plans to deploy a new 4G network in a wireless market[2]. We consider that the operator has already deployed a ubiquitous 3G network, and the area of this market region is normalized to 1. We denote the user density as ρ in this region, and all ρ users are potential subscribers to choose the 3G or 4G service provided by the operator. The operator needs to make 4G deployment decisions in a time-slotted horizon including a finite T stages, and the decision-making in each stage $t \in \{1, ..., T\}$ is further divided into two phases as illustrated in Fig. 1:

- *Phase I (Operator planning)*: The 4G network coverage at the beginning of time stage t is denoted as Q_{t-1}, depending on last time stage's effort. Then the operator will decide the enlarged 4G coverage by the end of time stage t, Q_t. Therefore, the 4G coverage expansion after time stage t is $Q_t - Q_{t-1} \geq 0$.
- *Phase II (Market response)*: After Phase I, users will choose their preferred service (or nothing), depending on the current 4G coverage Q_{t-1}. Such a dependency on Q_{t-1} rather than Q_t is because that deploying a larger Q_t is still ongoing till the end of time stage t. After deciding service choices for this time stage, users make payments once and these payments can be used directly for ongoing deployment in time stage t.

In the following, we will first introduce the models of 3G and 4G services including their QoS requirements and prices, and then specify users' utility models in choosing different services.

[2]We plan to extend this monopoly model to oligopoly model in the future, where the revenue collections among competitive operators are inter-dependent and operators have more incentive to deploy earlier as in [8].

3G service: The subscription fee of 3G service per time stage is P^{3G}.[3] Subscribers can access the mature 3G network everywhere at a data rate R^{3G} but may experience network congestion.

4G service: The subscription fee of 4G service per time stage is P^{4G}. Subscribers can connect to 4G network at a high data rate R^{4G} whenever they are within the 4G coverage ($Q_t \leq 1$ after time stage t). According to the ITU standards, 3G and 4G data rates should be larger than 2 Mbit/s and 1 Gbit/s, respectively according to [18] [19], and the 4G service price is globally 20% higher than 3G according to [2]. Thus we reasonably assume that the quasi-unit-price of 4G is lower than that of 3G ($P^{4G}/R^{4G} < P^{3G}/R^{3G}$) [4].

A user's utility depends on his valuation of the service choice and the price he pays. We adopt the widely-used multi-attribute linear-weighted utility function (e.g., [20], [21]), and define a subscriber's utility as the difference between his valuation of the chosen service and the service price. Since different users have different sensitivity levels towards the QoS in the valuation, we denote user-specific sensitivity level as α. We follow a widely-used assumption that α follows uniform distribution in a normalized range [0,1]. In Phase II of time stage $t+1$, given the 4G coverage Q_t, a user with sensitivity α has valuations αR^{3G} and αR^{4G} when choosing the two services, and his utility when choosing 3G, 4G or none is:

$$\begin{cases} u_\alpha^{3G}(Q_t) = \alpha R^{3G} - P^{3G}, & \text{if 3G} \\ u_\alpha^{4G}(Q_t) = \alpha[Q_t R^{4G} + (1-Q_t)R^{3G}] - P^{4G}, & \text{if 4G} \\ u_0. & \text{o.w.} \end{cases} \tag{1}$$

where $Q_t R^{4G} + (1-Q_t)R^{3G}$ is the 4G service's expected data rate by considering the actual rate R^{4G} in 4G coverage Q_t and rate R^{3G} in uncovered area $1-Q_t$. For ease of analysis, the congestion effect in the 3G network is not presented here and will be modeled later in Section VII for clear comparison purpose.

As shown in Fig. 1, in each time stage, the operator in Phase I needs to decide deployment after predicting users' responses in Phase II for revenue collection, and users' responses are determined by their utilities. By using backward induction, we will first analyze the market response in phase II in different time stages in Section IV, then analyze the operator's optimal 4G deployment in phase I in different time stages in Section V.

IV. Market Response Analysis for Revenue Estimation

In this section, we analyze the market response in Phase II of time stage $t+1, t \in [0,T]$ given the 4G coverage Q_t. The choice of a user with QoS sensitivity α is:

$$\begin{cases} \text{3G service}, & \text{if } u_\alpha^{3G}(t) \geq \max\{u_\alpha^{4G}(Q_t), u_0\}, \\ \text{4G service}, & \text{if } u_\alpha^{4G}(Q_t) \geq \max\{u_\alpha^{3G}(Q_t), u_0\}, \\ \text{No subscription}, & \text{otherwise}. \end{cases}$$

For ease of reading, we denote the price difference and QoS difference as $\Delta P = P^{4G} - P^{3G}$, and $\Delta R = R^{4G} - R^{3G}$. By comparing all users' utilities of different choices, we can partition users' choices according to three α thresholds:

- $\alpha_{(3,0)} = P^{3G}/R^{3G}$ partitions users in choosing between 3G service and no subscription. A user with $\alpha \geq \alpha_{(3,0)}$ prefers 3G service to no subscription.
- $\alpha_{(4,0)} = P^{4G}/(QR^{4G} + (1-Q)R^{3G})$ partitions users in choosing between 4G service and no subscription. A user with $\alpha \geq \alpha_{(4,0)}$ prefers 4G service to no subscription.
- $\alpha_{(4,3)} = \Delta P/(Q\Delta R)$ partitions users in choosing between 3G and 4G services. A user with $\alpha \geq \alpha_{(4,3)}$ prefers 4G service to 3G service.

Note that $\alpha_{(4,0)}$ and $\alpha_{(4,3)}$ increase with Q_t, since the expansion of 4G network will attract more subscribers to 4G service.

Based on the above analysis, we can derive the users' equilibrium subscription in Phase II and estimate the resultant revenue $R(Q_t)$ for the operator in Phase I of each time stage. The operator's revenue $R(Q_t)$ depends on Q_t and is the sum of subscription fee collected from both services.

Proposition 1. Depending on the 4G coverage Q_t, the operator's revenue in time stage $t+1$ is:

- *Low 4G coverage regime.* When $Q_t < \Delta P/\Delta R$, no users choose 4G service and users with $\alpha \in [\alpha_{(3,0)}, 1]$ choose 3G. The operator's current revenue is:
$$R(Q_t) = \rho P^{3G}(1 - P^{3G}/R^{3G}). \tag{2}$$
- *Medium 4G coverage regime.* When $\Delta P/\Delta R \leq Q_t < \Delta P R^{3G}/(P^{3G}\Delta R)$, users with $\alpha \in [\alpha_{(3,0)}, \alpha_{(4,3)}]$ choose 3G, and $\alpha \in [\alpha_{(4,3)}, 1]$ choose 4G. The operator's current revenue is:
$$R(Q_t) = \rho\left(P^{4G} - \frac{\Delta P^2}{Q_t \Delta R} - \frac{(P^{3G})^2}{R^{3G}}\right). \tag{3}$$
- *High 4G coverage regime.* When $Q_t \geq \Delta P R^{3G}/(P^{3G}\Delta R)$,[5] no users choose 3G service and users with $\alpha \in [\alpha_{(4,0)}, 1]$ choose 4G. The operator's current revenue is:
$$R(Q_t) = P^{4G}\left(1 - \frac{P^{4G}}{Q_t \Delta R + R^{3G}}\right). \tag{4}$$

In low 4G coverage regime, compared with the existing 3G service, 4G service is not attractive to any user, and one can imagine that the operator should initially deploy beyond this regime directly, if budget allows. It should be noted that as Q_t increases, the operator's revenue in both medium and high coverage regimes will increase. The revenue increase in

[3] The subscription fee can be monthly based. We assume a flat-rate fee model here, which is a common practice in the existing data market [17].

[4] We consider arbitrary price values and assume that the 3G and 4G prices are static. This is reasonable as the time scale of each stage is about one month, but the price is not flexible to change from time to time as users in practice do not welcome price change.

[5] Due to the fact $P^{3G}/R^{3G} > P^{4G}/R^{4G}$ in Section III, we can prove $\Delta P R^{3G}/(P^{3G}\Delta R) < 1$ and the high coverage regime exists.

the high coverage regime is faster, as 4G further attracts the original users out of 3G service and the operator's market penetration increases.

V. Dynamic Programming Formulation for Optimal 4G Deployment

In this section, we formulate the operator's deployment problem as a dynamic program to maximize the final-stage cash level and analytically solve the optimization problem. We want to first characterize the deployment policy with the deployment cost only, which serves as a benchmark to compare with the additions of operational cost in Section VI and congestion effect in Section VII[6].

A. Dynamic Programming Formulation

Let S_t denote the operator's cash level at the end of time stage t. S_t depends on three parts: 1) S_{t-1}, the cash level at the end of time stage $t-1$, 2) $R(Q_{t-1})$ given in Proposition 1, operator's revenue collected during time stage t based on Q_{t-1}, and 3) the deployment cost from Q_{t-1} to Q_t. S_0 is the initial budget before time stage $t=1$. The state transition function of cash flow is:

$$S_t = S_{t-1} + R(Q_{t-1}) - f(Q_t, Q_{t-1}), t \in \{1, ..., T\}, \quad (5)$$

in which $f(Q_t, Q_{t-1})$ is the deployment cost function during time stage t. We need to make sure that the budget constraint holds under current deployment investment, that is, S_t should be non-negative for any time stage t. We do not allow capital raise to fill in the cash flow gap.

To manage the cash flow, the operator makes decisions from time stages 1 to T, that is, to decide $Q_1, ..., Q_T$. The objective is to maximize the final profit (cash level), denoted as S_{T+1}, which consists of 1) S_T, the cash level at the end of time stage T, and 2) $R(Q_T)$, operator's revenue based on market response to Q_T. The operator will not further deploy 4G in stage T. That is,

$$S_{T+1} = S_T + R(Q_T). \quad (6)$$

By deciding deployment plan Q_t for each stage, the operator wants to maximize S_{T+1}, subject to the budget constraint and cash level transition equation. The dynamic programming problem is:

$$\begin{aligned} \max_{Q_1,...,Q_T} \quad & S_{T+1} \\ \text{s.t.} \quad & S_t \geq 0, \forall t = 1, \cdots, T, \\ & S_t = S_{t-1} + R(Q_{t-1}) - f(Q_t, Q_{t-1}), \\ & 0 \leq Q_1 \leq Q_2 \leq \cdots \leq Q_T \leq 1. \end{aligned} \quad (7)$$

The deployment cost of enlarging the 4G coverage is mainly the cost to purchase and set up 4G base stations, and this cost can be approximated as a linear function of the number of 4G base stations according to [9]. In our online technical report [10], we also investigate Verizon's real data to show this linear relationship. Thus, we reasonably assume that the deployment cost function is $f(Q_{t+1}, Q_t) = C_d(Q_{t+1} - Q_t)$, in which C_d is the unit deployment cost to purchase and install 4G base stations per unit coverage. By substituting the state transition function of the cash flow in (5) and the final-stage cash level in (6) into the objective of problem (7), we can simplify the problem as:

$$\begin{aligned} \max_{Q_1,\cdots,Q_T} \quad & \underbrace{S_0}_{initial\ budget} + \underbrace{\sum_{t=0}^{T} R(Q_t)}_{revenue\ flow} - \underbrace{C_d Q_T}_{total\ deployment\ cost} \\ \text{s.t.} \quad & S_0 + \sum_{\tau=1}^{t-1} R(Q_\tau) - C_d Q_t \geq 0, t = 2, ..., T \\ & S_0 - C_d Q_1 \geq 0, \\ & 0 \leq Q_1 \leq ... \leq Q_T \leq 1. \end{aligned} \quad (8)$$

where the first constraint is the budget constraint to ensure that the cash level at each time stage is non-negative.

B. Optimal deployment policy

The dynamic programming problem in (8) is difficult to solve. This is because $R(Q_t)$ in the objective and budget constraints is not concave, and the optimization problem is not convex.

Lemma 1. *$R(Q_t)$ is concave within two separate Q_t ranges $[0, \Delta P/\Delta R]$ and $(\Delta P/\Delta R, 1]$, respectively[7], but is just quasi-concave (not concave) in the entire range $Q_t \in [0, 1]$.*

To make the problem (8) solvable, we decompose the problem into two convex subproblems by focusing on the Q_t ranges $[0, \Delta P/\Delta R]$ and $(\Delta P/\Delta R, 1]$, respectively. First, we define $T_{th} \in \{1, ..., T\}$ as the threshold time stage, such that $Q_t \leq \Delta P/\Delta R, \forall t \leq T_{th} - 1$, and $Q_t > \Delta P/\Delta R, \forall t \geq T_{th}$. Due to the budget constraint in (8), the feasible regime of T_{th} is:

$$T_{th} \geq \left\lceil (C_d \Delta P/\Delta R - S_0)/(\rho P^{3G}(1 - P^{3G}/R^{3G})) \right\rceil. \quad (9)$$

Then, we solve the optimization problem by the following three steps:

1) *Optimal deployment policy before time stage T_{th}.* Given any T_{th} value, we find the optimal 4G deployment policy for the convex optimization problem (8) by optimizing over Q_1 to $Q_{T_{th}-1}$ and replacing the last constraint with $Q_1 \leq \cdots \leq Q_{T_{th}-1} \leq \Delta P/\Delta R$.
2) *Optimal deployment policy after time stage T_{th}.* Given any T_{th} value, we find the optimal 4G deployment policy for the convex optimization problem by optimizing over $Q_{T_{th}}$ to Q_T and replacing the last constraint with $\Delta P/\Delta R \leq Q_{T_{th}} \leq \cdots \leq Q_T$.
3) *Search of the optimal T_{th}^*.* By changing T_{th} and comparing all S_{T+1} values given by the two subproblems,

[6]We assume that the 4G spectrum license has already been obtained before the deployment, so we do not consider spectrum cost in this paper.

[7]Recall that $\Delta P/\Delta R < 1$ as we assume $P^{3G}/R^{3G} > P^{4G}/R^{4G}$.

we can find the optimal T_{th}^* yielding the largest value S_{T+1}^*.

By following the first step before time stage T_{th}, we have the following result.

Proposition 2. *(Optimal deployment policy before time stage T_{th})* It is the best for the operator not to deploy any 4G network before T_{th}. That is, $Q_t^* = 0, 1 \leq t \leq T_{th} - 1$.

Based on Proposition 2, we can rewrite the optimization problem (8) as

$$\max_{Q_{T_{th}},\cdots,Q_T} \underbrace{T_{th}\rho P^{3G}(1-\frac{P^{3G}}{R^{3G}})}_{revenue\ before\ T_{th}} + \sum_{t=T_{th}}^{T} R(Q_t) - C_d Q_T$$
$$\text{s.t. } S_0 + T_{th}\rho P^{3G}(1-\frac{P^{3G}}{R^{3G}}) + \sum_{\tau=T_{th}}^{t-1} R(Q_\tau) - C_d Q_t \geq 0,$$
$$\forall t = T_{th}, ..., T$$
$$\Delta P/\Delta R \leq Q_{T_{th}} \leq ... \leq Q_T \leq 1 \tag{10}$$

Since the objective function and the constraints in (10) are convex, KKT conditions, the first order necessary conditions, are also sufficient for a solution to be optimal [22]. We show that the optimal idea is to find the "stopping time stage" $\bar{T}_{th} \geq T_{th} - 1$, such that before the time stage $\bar{T}_{th}$, the operator will use all current budget to deploy 4G in each time stage (with tight budget constraint in (8)); after the time stage $\bar{T}_{th}$, no more new 4G network is deployed. The only problem is how to find the final 4G coverage at time stage $\bar{T}_{th}$, which depends on the deployment cost C_d, time length T, and revenue function $R(Q_t)$. Let Q_t^* denote the optimal 4G coverage at time stage t with $T_{th} \leq t \leq T$. By solving the subproblem we have the following proposition.

Proposition 3. *(Optimal deployment policy after time stage T_{th})* Given any T_{th} value, there exists a mature deployment stage $\bar{T}_{th}$, before and after which the operator has different deployment strategies. The value of $\bar{T}_{th}$ is determined by Algorithm 1. The special case of $\bar{T}_{th}$=$T_{th} - 1$ leads to no further deployment after T_{th}, i.e., $Q_t^* = Q_{T_{th}-1}$ for any time stages $t \in \{T_{th}, ..., T\}$. More generally, when $\bar{T}_{th} > T_{th} - 1$, we have:

- *Aggressive deployment period:* In the time stages $t \in [T_{th}, \bar{T}_{th}]$, the operator will use up all his current budget at each time stage t for 4G deployment, i.e., $Q_t^* = Q_t^m$ in (11), the maximum achievable coverage that can be supported by the budget at time stage t.
- *Conservative deployment period:* When $t = \bar{T}_{th}$, the operator will conservatively upgrade according to:

$$\begin{cases} Q_{t-1}^*, & \text{if } C_d > (T+T_{th}-t)\max\{R'(Q_{t-1}^*), R'(Q_t^m)\} \\ q_t^*, & \text{if } C_d \in [(T+T_{th}-t)R'(Q_{t-1}^*), \\ & \quad (T+T_{th}-t)R'(Q_t^m)] \\ Q_t^m, & \text{if } C_d < (T+T_{th}-t)\min\{R'(Q_{t-1}^*), R'(Q_t^m)\} \end{cases}$$

in which q_t^* is the unique solution to the equation $C_d = (T + T_{th} - t)R'(q_t^*)$.

Algorithm 1 Calculation of mature deployment stage $\bar{T}_{th}$

1: Set $t = T_{th} - 1$, $Q_t^m = \Delta P/\Delta R$.
2: **if** $C_d > (T + T_{th} - t)R'(Q_t^m)$ **then**
3:     Set $\bar{T}_{th} = T_{th} - 1$; STOP;
4: **else**
5:     Set $Q_t^* = Q_t^m$, $t = t + 1$
6:
$$Q_t^m = \min\{1, (S_0 + \sum_{\tau=0}^{t-1} R(Q_\tau^*))/C_d\} \tag{11}$$
7:     **if**
    1) $C_d > \max\{(T + T_{th} - t)R'(Q_{t-1}^*), (T + T_{th} - t)R'(Q_t^m)\}$, or;
    2) $C_d \in [(T+T_{th}-t)R'(Q_{t-1}^*), (T+T_{th}-t)R'(Q_t^m)]$, or;
    3) $C_d < \min\{(T + T_{th} - t)R'(Q_{t-1}^*), (T + T_{th} - t)R'(Q_t^m)\}$, but $Q_t^m = 1$ or $t = T$.
    then
8:         Set $\bar{T}_{th} = t$; STOP;
9:     **else**
10:         GOTO step 5.
11:     **end if**
12: **end if**

- *No deployment period.* When $\bar{T}_{th} + 1 \leq t \leq T$, $Q_t^* = Q_{t-1}^*$.

Intuitively, the final optimal 4G coverage requires that the marginal cost C_d equals the marginal revenue $(T + T_{th} - t)R'(q_t^*)$. If $\bar{T}_{th} = T_{th}$, *aggressive deployment period* does not exist. If $\bar{T}_{th} = T$, *conservative coverage period* does not exist.

After solving the two subproblems in Propositions 2 and 3, now we are ready to combine their solutions for the global optimum, by searching through all possible T_{th} values.

Lemma 2. The optimal T_{th} is chosen from the following two candidates:

- $T_{th} = T + 1$, that is, the operator will not deploy 4G network to a coverage level $\Delta P/\Delta R$;
- $T_{th} = \left\lceil (C_d\Delta P/\Delta R - S_0)/[\rho P^{3G}(1 - P^{3G}/R^{3G})] \right\rceil$, that is, the operator deploys the 4G network to the coverage level $\Delta P/\Delta R$ as soon as possible.

Lemma 2 helps us limit our attention to two candidates only, without widely searching all T_{th} values. Finally, to determine which T_{th} candidate is better, we compare their corresponding final-stage cash levels S_{T+1}: the one that yields a higher final-stage cash level is the optimal T_{th}^*.

By following the above decompose-and-compare approach, we summarize the optimal 4G deployment policy.

Theorem 1. The optimal 4G deployment policy is one of the following two options.

- *No deployment scheme:* the operator never deploys any 4G network, i.e., $Q_t^* = 0, t = 1, ..., T$.

- *Threshold-based deployment scheme:* Set threshold $T_{th}^* = \lceil (C_d \Delta P/\Delta R - S_0)/[\rho P^{3G}(1 - P^{3G}/R^{3G})] \rceil$.
  - When $t \in [1, T_{th}^* - 1]$, the operator does not deploy any 4G network, i.e., $Q_t^* = 0$;
  - When $t \in [T_{th}^*, T]$, the operator deploys 4G network according to Proposition 3.

The special case is that, if the deployment cost satisfies

$$C_d > \left(T + 1 - \left\lceil \frac{C_d \Delta P/\Delta R - S_0}{\rho P^{3G}(1 - P^{3G}/R^{3G})} \right\rceil \right) \rho \Delta R, \quad (12)$$

the *No deployment scheme* is optimal.

Notice that the right-hand side term in (12) is increasing in user density ρ, therefore (12) is more likely to hold as ρ decreases. This tells us that when user density is low, the operator is unwilling to deploy 4G network. Moreover, if the time length T is long enough, any one-time deployment cost C_d is negligible compared to the infinitely long 4G benefit, and initial 4G deployment will be started as soon as possible.

Proposition 4. *Final 4G coverage level Q_T^* increases with time length T and user density ρ, but decreases with the deployment cost C_d. As $T \to \infty$, $Q_T^* = 1$ (full 4G coverage).*

As T increases, the operator has more time to collect the 4G revenue. As ρ increases, a higher revenue can be collected from more users, encouraging the operator to increase the 4G coverage. Finally, if C_d is large, the operator wants to avoid high deployment cost by deploying a smaller 4G coverage.

VI. Impact of Operational Cost

In this section, we model and analyze how the operational cost affects the operator's 4G deployment policy, by comparing with Section V under deployment cost only. The operational cost (OPEX) is related to daily management and maintenance of 4G network, which can be approximated as a linear function of the network coverage with unit cost C_o according to [7] [23]. In time stage $t+1$, the total deployment and operational cost is

$$f(Q_{t+1}, Q_t) = C_d(Q_{t+1} - Q_t) + C_o Q_t. \quad (13)$$

By adding the operational cost $C_o Q_t$ in each time t, the optimization problem in (8) becomes

$$\max_{Q_1, \cdots, Q_T} S_0 + \sum_{t=1}^{T} R(Q_t) - C_d Q_T - \underbrace{C_o \sum_{t=1}^{T} Q_t}_{\text{total operational cost}}$$

$$\begin{aligned} \text{s.t. } & S_0 + \sum_{\tau=1}^{t-1} (R(Q_\tau) - C_o Q_\tau) - C_d Q_t \geq 0, t = 2, ..., T \\ & S_0 - C_d Q_1 \geq 0, \\ & 0 \leq Q_1 \leq ... \leq Q_T \leq 1 \end{aligned} \quad (14)$$

By following a similar 3-step solution approach as in Section V with a threshold stage T_{th}, we can first propose the *optimal deployment policy before time stage T_{th}* and the *optimal deployment policy after T_{th}*, and finally compare and find the optimal T_{th}^*.

Due to page limit, we skip the detailed analysis here. The *Optimal deployment policy before time stage T_{th}* is the same as Proposition 2 in Section V. And the *Optimal deployment policy after time stage T_{th}* is similar to Proposition 3, by changing (3) to:

$$\begin{cases} Q_{t-1}^*, & \text{if } C_d > (T + T_{th} - t) \max\{(R'(Q_{t-1}^*) - C_o), \\ & (R'(Q_t^m) - C_o)\} \\ q_t^*, & \text{if } C_d \in [(T + T_{th} - t)(R'(Q_{t-1}^*) - C_o), \\ & (T + T_{th} - t)(R'(Q_t^m) - C_o)] \\ Q_t^m, & \text{if } C_d < (T + T_{th} - t) \min\{(R'(Q_{t-1}^*) - C_o), \\ & (R'(Q_t^m) - C_o)\} \end{cases} \quad (15)$$

in which q_t^* is the unique solution to the equation $C_d = (T + T_{th} - t)(R'(q_t^*) - C_o)$.

Based on these results, we can search for the optimal T_{th}. Recall that in Lemma 2, if there is no operational cost, one of the two possible choices of T_{th} suggests the operator deploy the 4G coverage above $\Delta P/\Delta R$ once collecting enough budget. However, with operational cost, this choice may not be optimal as the marginal revenue at coverage $\Delta P/\Delta R$ may be less than the marginal revenue at the initial coverage level 0.

Lemma 3. *The optimal T_{th}^* with consideration of the operational cost, is chosen from the following two candidates:*

- $T_{th} = T + 1$, *that is, the operator will not deploy the 4G coverage to $\Delta P/\Delta R$;*
- $T_{th} = \widetilde{T}$, *which satisfies*

$$R(Q_{\widetilde{T}}) - C_o Q_{\widetilde{T}} \geq R(0) \quad (16)$$

$$R(Q_{\widetilde{T}-1}) - C_o Q_{\widetilde{T}-1} \leq R(0) \quad (17)$$

Lemma 3 shows that it is the best for the operator to wait until he can immediately reach a sizable and profitable coverage level satisfying (16) and (17), where the marginal revenue (depending on the coverage level) is larger than the marginal revenue $R(0)$ without 4G deployment.

According to the discussion above, by using the decompose-and-compare approach, we have the following theorem for the optimal 4G deployment policy with operational cost.

Theorem 2. *(Optimal 4G deployment policy with operational cost).* The optimal 4G deployment policy is one of the following two options.

- *No deployment scheme:* the operator never deploys any 4G network, i.e., $Q_t^* = 0, t = 1, ..., T$.
- *Threshold-based deployment scheme:*
  - When $t \in [1, T_{th}^* - 1]$, the operator does not deploy any 4G network, i.e., $Q_t^* = 0$;
  - When $t \in [T_{th}^*, T]$, the operator deploys 4G network according to Proposition 3 by replacing (3) with (15);
  - T_{th}^* is determined by Lemma 3.

By comparing the final-stage cash levels of the above two options, the one that yields higher final-stage cash level is the optimal choice. The operational cost changes the optimal 4G deployment in two aspects.

Proposition 5. *The operational cost reduces the final 4G coverage level, and delays the time to deploy the 4G network.*

Without operational cost, Proposition 4 tells that $Q_T^* = 1$ as $T \to \infty$. However, when there exists an operational cost, and the cost is high enough ($R'(Q_t) < C_o$), we can show that no matter how large T is, the operator will never deploy 4G network. If the operational cost is not that high, to compensate for the operational cost, the operator will first accumulate enough budget during a longer time for a sizable initial deployment coverage.

VII. Impact of Network Congestion

In this section, we model and analyze the congestion effect in the 3G network by comparing to the congestion-free scenario in Section V.[8] Note that the network congestion in 4G network is incomparable compared to 3G network. Actually, 4G technology is proposed to resolve the network congestion.

The congestion affects the QoS and users' responses in Phase II, which should be taken into account into the operator's deployment planning in Phase I in each time stage. More specifically, the revenue function $R(Q_t)$ in time stage t under congestion effect is now different from that in Proposition 1. Both optimal 4G deployment policies in Theorem 1 and 2 can be applied by replacing congestion-free revenue function with revenue function with congestion effect. Due to page limit, in the following, we only look at the modeling and calculation of the revenue function $R(\cdot)$.

As 3G users' traffic increases, the network congestion increases and reduces the 3G QoS. Let $g(\cdot)$ denote the 3G congestion cost, which is an increasing function of the number of the users in 3G network. Note that these users include not only 3G subscribers but also 4G subscribers roaming outside 4G coverage. Let D_t denote the number of total users connecting to 3G network. By incorporating the congestion effect, given the 4G coverage Q_t, users' utility function changes from (1) to:

$$u_\alpha^{3G}(Q_t) = \alpha R^{3G} - g(D_t) - P^{3G} \tag{18}$$

$$u_\alpha^{4G}(Q_t) = \alpha Q_t R^{4G} + (1-Q_t)[\alpha R^{3G} - g(D_t)] - P^{4G} \tag{19}$$

We assume that $g(\cdot)$ is linear in user demand, i.e., $g(x) = \gamma x$, which is a common approximation (e.g., [21]) to make the problem tractable and deliver clean engineering insight. When $\gamma = 0$, the results will degenerate to be the same as those in Section IV. Similar to Section IV, we can partition user's choices according to three thresholds:

- $\alpha_{(3,0)} = (P^{3G} + \gamma D_t)/R^{3G}$ partitions users in choosing between 3G service and no subscription. A user with $\alpha > \alpha_{(3,0)}$ prefers 3G service to no subscription.
- $\alpha_{(4,0)} = (P^{4G} + (1-Q_t)\gamma D_t)/(Q_t \Delta R + R^{3G})$ partitions users in choosing between 4G service and no subscription. A user with $\alpha > \alpha_{(4,0)}$ prefers 4G service to no subscription.
- $\alpha_{(4,3)} = (\Delta P - Q_t \gamma D_t)/(Q_t \Delta R)$ partitions users in choosing between 3G and 4G services. A user with $\alpha > \alpha_{(4,3)}$ prefers the 4G service to 3G service.

When there is no congestion effect in Section IV, the three thresholds are independent of the equilibrium number of users who choose 3G and 4G service, i.e., D_t. However, when there is congestion effect, the three thresholds are functions of D_t, making it difficult to obtain the equilibrium market response. Despite complexity, we present the operator's revenue under congestion effect in Proposition 6, which is quite different from Proposition 1.

Proposition 6. Depending on the 4G coverage Q_t, the operator's revenue is:

- *Low 4G coverage regime.* No users choose 4G service and users with $\alpha \in [\alpha_{(3,0)}, 1]$ choose 3G. The operator's revenue is:
$$R(Q_t) = P^{3G} \frac{R^{3G} - P^{3G}}{R^{3G} + \gamma}.$$
- *Medium 4G coverage regime.* Users with $\alpha \in [\alpha_{(3,0)}, \alpha_{(4,3)}]$ choose 3G, and $\alpha \in [\alpha_{(4,3)}, 1]$ choose 4G. The operator's revenue is:

$$R(Q_t) = P^{3G} \frac{\frac{\Delta P R^{3G}}{Q_t} - \Delta R P^{3G} + \gamma(1-Q_t)(P^{3G} - R^{4G} + \frac{\Delta P}{Q_t})}{\Delta R R^{3G} + \gamma(R^{3G} Q_t + \Delta R)} + P^{4G} \frac{\Delta R R^{3G} - \Delta P R^{3G}/Q_t + \gamma(R^{4G} - P^{3G} - \Delta P/Q_t)}{\Delta R R^{3G} + \gamma(R^{3G} Q_t + \Delta R)}$$

- *High 4G coverage regime.* No users choose 3G service and users with $\alpha \in [\alpha_{(4,0)}, 1]$ choose 4G. The operator's revenue is:
$$R(Q_t) = P^{4G} \frac{Q_t \Delta R + R^{3G} - P^{4G}}{Q_t \Delta R + R^{3G} + (1-Q_t)^2 \gamma}$$

The cutting point between the Low and Medium 4G coverage regime is lower when 3G congestion is considered, because the 3G congestion encourages users to switch to 4G service even when 4G coverage is small. We can see that $R(Q_t)$ is greatly influenced by the congestion factor γ. We will analyze by numerical results the influence of congestion effect on 4G deployment in Section VIII.

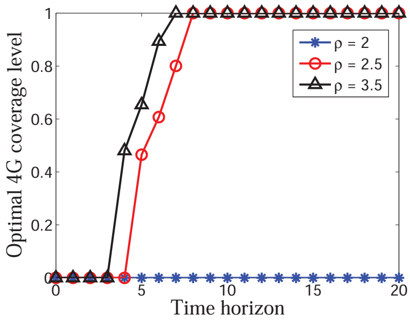
VIII. Numerical Results

In this section, we use numerical results to illustrate and highlight some interesting engineering insights.

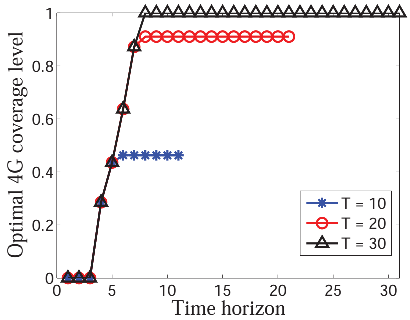
A. Impact of User Density, Service Prices, and Time Span

1) The user density: Fig. 2(a) shows that a higher user density ρ boosts the 4G deployment because there are more potential subscribers and the total 4G revenue is higher. When the user density is below a certain threshold, it is optimal for

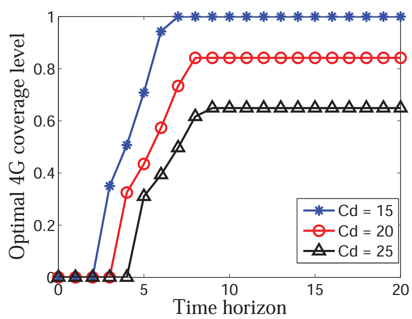
[8] Note that we do not directly compare to Section VI as the comparison is not clean to tell the impact of congestion effect.



(a) Impact of user density ρ

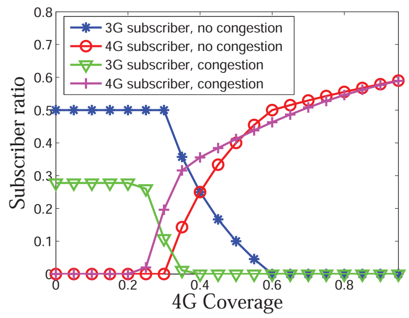


(b) Impact of T

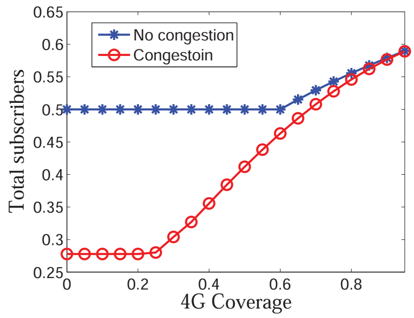


(c) Impact of C_d

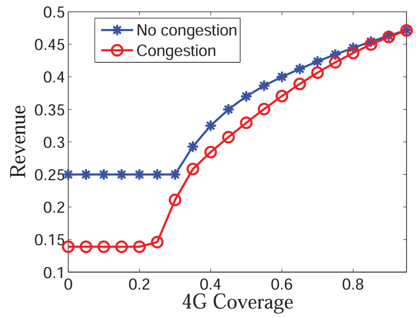
Fig. 2: Impact of the user density, time span and deployment cost.



(a) 3G and 4G subscriber numbers



(b) Total subscriber numbers



(c) Total revenue at a time stage

Fig. 3: Impact of the 3G congestion effect on the revenue and market response.

the operator not to deploy 4G network (See the $\rho = 2$ case). This explains why in some under-populated rural area, there is no 4G development.

2) Time span: Fig. 2(b) shows that the time span T does not change the optimal 4G deployment progress in the aggressive deployment stage ($T_{th} \leq t \leq \bar{T}_{th}$), but it does change the final 4G coverage level Q_T^*. Shorter T decreases $\bar{T}_{th}$ in Lemma 2 of Section V, the time stage when there is no more deployment. Larger T increases the final 4G coverage. Intuitively, as the time span T decreases, the operator has fewer time stages to collect the revenue from the newly deployed 4G network. Therefore, the operator stops at an earlier time stage for deployment and saves the deployment cost. We can also observe from Fig. 2(c) that, if T is long enough, the operator deploys a full coverage 4G network as the long-term 4G benefits outweigh the one-time deployment cost.

B. Impact of Deployment and Operational Costs

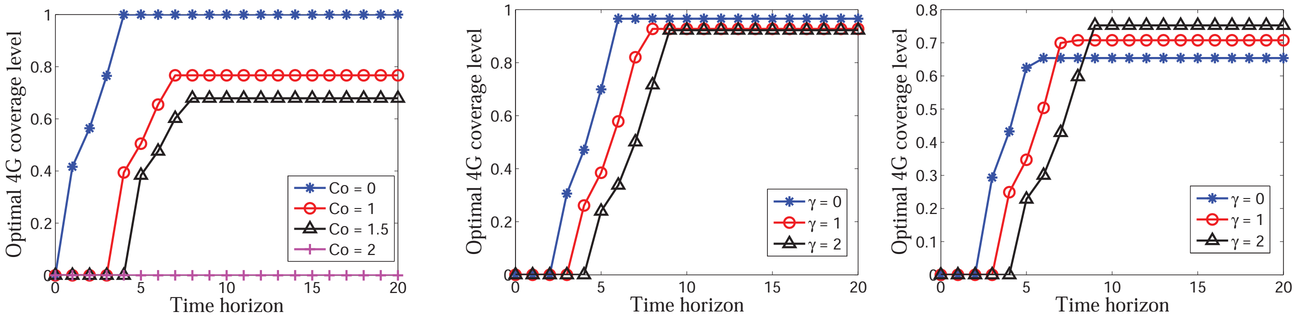
1) Deployment cost: Fig. 2(c) shows the 4G deployment roadmap as a function of time and deployment cost C_d. A larger deployment cost will reduce the final 4G deployment coverage, and delay the initial time stage when the operator starts to deploy. Actually, it can be proved theoretically that Q_T decreases with C_d (see [10]). The operator starts to deploy 4G network later because he needs more time to collect enough budget to reach sizable coverage $\Delta P/\Delta R$.

2) Operational Cost: Fig. 4(a) shows that a larger operational cost delays the deployment timing and reduces the final 4G coverage level. The difference between operational cost and deployment cost is that, there is no deployment cost once the 4G coverage becomes stable, but the operational cost always exists. If the time horizon T is large enough, with only deployment cost, the 4G network will always reach full coverage. However, with operational cost, if the difference between the marginal revenue $R'(Q_T^* = 1)$ is smaller than the operational cost C_o, the 4G network will never reach full coverage no matter how large T is, according to Theorem 2. This is illustrated in the case of $C_o = 3$ in Fig. 4(a).

C. Impact of Congestion Effect

Fig. 3 shows the impact of the congestion effect with coefficient γ on the market response and revenue, depending on different 4G coverage levels. Fig. 3(a) shows that, when the 4G coverage is small but still attractive to users (between 0.2 and 0.5), there are more 4G subscribers when there is 3G congestion, but when the 4G coverage is large (more than 0.5), there are fewer 4G subscribers. This is because, the increase in 4G coverage initially eases the 3G congestion, but later aggravates the 3G congestion since more 4G subscribers connect to 3G network. Fig. 3(b)(c) further show that congestion effect reduces the total subscriber number and the operator's revenue.

Figs. 4(b)(c) show that the congestion effect delays the deployment timing, because the revenue collected over time is smaller and the operator needs more time to collect enough budget to reach the initial coverage threshold $\Delta P/\Delta R$. If the operational cost C_o is low, the final 4G coverage decreases as the congestion factor γ increases, as shown in Fig. 4(b). But if the operational cost C_o is high, the final 4G coverage increases with γ, as shown in Fig. 4(c). One possible explanation for



(a) Impact of C_o (b) Impact of γ, $C_o = 0.1$ (c) Impact of γ, $C_o = 1$

Fig. 4: Impact of operational cost and congestion effect.

these two different directions of final 4G coverage versus congestion factor γ is as follows. When the operational cost is high (Fig.4(c)), the final 4G coverage is relatively low (around 0.7), thus the 4G subscribers are relatively low. The 3G congestion mostly results from 3G subscribers. So the increase in γ encourages the operator to deploy more 4G coverage to ease 3G congestion effect. When the operational cost is low (Fig.4(b)), the final 4G coverage is relatively high (around 0.9), thus the 4G subscribers are relatively high. The 3G congestion mostly results from 4G subscribers who access 3G network. So the operator instead decreases 4G coverage when 3G congestion factor γ increases.

IX. Conclusion

In this paper, we conduct financial analysis for 4G network deployment. We model the operator's cash flow management as a dynamic process through a limited time horizon: at each time stage, the operator decides the 4G deployment level with the budget constraint. The goal is to maximize the final cash level, which can be formulated as an optimization problem, and we solve the problem using dynamic programming. A practical and easy-to-implement optimal 4G deployment policy is proposed. In the first phase, the operator always exhausts the capital for deployment. In the second phase, the operator strategically sets the coverage level, which will remain unchanged in the third phase. We find that the operator delays the 4G deployment timing and reduces the final 4G coverage level if there is operational cost because the marginal revenue becomes smaller. We further consider the congestion effect in the 3G network and show that it results in a lower total subscription level, a lower revenue, and a smaller final 4G coverage.

There are several future directions. First, we may consider the discounted cash flow, and the reduction in deployment and operational costs due to technology advancement. Second, we may also consider uncertainties in user response, as some users may not be rational enough to make optimal decisions. Finally, we may consider the bandwidth reallocation between the growing 4G service and traditional 3G service.

References

[1] "4g LTE market upgrade," *Actiontec Corporation, White paper*, 2010.

[2] ABIresearch, "4G data is priced 20 percent higher than 3G," 2012. [Online]. Available: https://www.abiresearch.com/press/4g-data-is-priced-20-higher-than-3g

[3] "Sprint still bleeding customers, money; looks to softbank to finance LTE upgrade," 2013. [Online]. Available: http://www.dailytech.com

[4] "Teliasonera layoffs a sign of trouble for mobile operators," *TECH EUROPE, http://blogs.wsj.com/tech-europe*, 2012.

[5] "4G LTE coverage." [Online]. Available: http://www.verizonwireless.com/wcms/consumer/4g-lte.html

[6] "How much do cell tower cost," 2011. [Online]. Available: www.deadzones.com

[7] "Construction, not capacity, is the real lte challenge in U.S." 2012. [Online]. Available: http://www.telecomengine.com

[8] L. Duan, J. Huang, and J. Walrand, "Economic analysis of 4G network upgrade," in *IEEE, INFOCOM 2013*.

[9] H. Claussen, L. T. Ho, and L. G. Samuel, "Financial analysis of a pico-cellular home network deployment," in *IEEE International Conference on Communications (ICC), 2007*, pp. 5604–5609.

[10] "Technical report of financial analysis of 4G network deployment," https://www.dropbox.com/s/zx74gtphrm3chnb/TechnicalReport.pdf.

[11] J. Musacchio, J. Walrand, and S. Wu, "A game theoretic model for network upgrade decisions," in *Proceedings of the 44th Annual Allerton Conference on Communication, Control, and Computing, Monticello, IL*, 2006, pp. 191–200.

[12] L. Jiang, V. Anantharam, and J. Walrand, "How bad are selfish investments in network security?" *IEEE/ACM Transactions on Networking (TON)*, vol. 19, no. 2, pp. 549–560, 2011.

[13] S. Sen, Y. Jin, R. Guérin, and K. Hosanagar, "Modeling the dynamics of network technology adoption and the role of converters," *IEEE/ACM Transactions on Networking (TON)*, vol. 18, no. 6, pp. 1793–1805, 2010.

[14] W. Zhang, Y. Wen, and D. O. Wu, "Energy-efficient scheduling policy for collaborative execution in mobile cloud computing," in *IEEE INFOCOM, 2013*, pp. 190–194.

[15] Y. Jin, Y. Wen, Q. Chen, and Z. Zhu, "An empirical investigation of the impact of server virtualization on energy efficiency for green data center," *The Computer Journal*, vol. 56, no. 8, pp. 977–990, 2013.

[16] Y. Jin, Y. Wen, and Q. Chen, "Energy efficiency and server virtualization in data centers: An empirical investigation," in *IEEE INFOCOM Computer Communications Workshops, 2012*, pp. 133–138.

[17] R. W. Costas Courcoubetis, *Pricing Communication Networks: Economics, Technology and Modelling*. WILEY, 2003.

[18] ITU, "Cellular standards for the third generation," 2005. [Online]. Available: http://www.itu.int/osg/spu/imt-2000/technology.html

[19] ——, "ITU global standard for international mobile telecommunications 'IMT-advanced'," 2008. [Online]. Available: http://www.itu.int/ITU-R/information/promotion/e-flash/2/article4.html

[20] R. L. Keeney, *Decisions with multiple objectives: preferences and value trade-offs*. Cambridge University Press, 1993.

[21] C. Joe-Wong, S. Sen, and S. Ha, "Offering supplementary wireless technologies: Adoption behavior and offloading benefits," in *IEEE, INFOCOM 2013*.

[22] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[23] L. Duan, J. Huang, and B. Shou, "Economics of femtocell service provision," *IEEE Transactions on Mobile Computing*, vol. 12, no. 11, pp. 2261–2273, 2013.