Technical Report of Financial Analysis of 4G Network Deployment

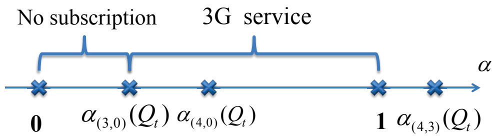
I. Proof of Proposition 1

Proposition 1. Depending on the 4G coverage Q_t, the operator's revenue in time stage $t+1$ is:

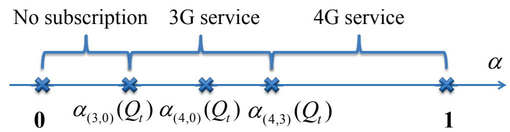
- *Low 4G coverage regime.* When $Q_t < \Delta P/\Delta R$, no users choose 4G service and users with $\alpha \in [\alpha_{(3,0)}, 1]$ choose 3G. The operator's current revenue is:
$$R(Q_t) = \rho P^{3G}(1 - P^{3G}/R^{3G}), \tag{1}$$
- *Medium 4G coverage regime.* When $\Delta P/\Delta R \leq Q_t < \Delta P R^{3G}/(P^{3G}\Delta R)$, users with $\alpha \in [\alpha_{(3,0)}, \alpha_{(4,3)}]$ choose 3G, and $\alpha \in [\alpha_{(4,3)}, 1]$ choose 4G. The operator's current revenue is:
$$R(Q_t) = \rho\left(P^{4G} - \frac{\Delta P^2}{Q_t\Delta R} - \frac{(P^{3G})^2}{R^{3G}}\right), \tag{2}$$
- *High 4G coverage regime.* When $Q_t \geq \Delta P R^{3G}/(P^{3G}\Delta R)$, no users choose 3G service and users with $\alpha \in [\alpha_{(4,0)}, 1]$ choose 4G. The operator's current revenue is:
$$R(Q_t) = P^{4G}\left(1 - \frac{P^{4G}}{Q_t\Delta R + R^{3G}}\right), \tag{3}$$

Proof. Let $D^{3G}(Q_t)$ and $D^{4G}(Q_t)$ denote the proportion of users who choose 3G or 4G service respectively. There are three cases of users' service partition as shown in Fig. 1.

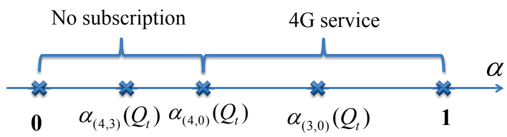
- *Low 4G coverage regime*, as shown in Fig. 1(a). When $Q_t < \Delta P/\Delta R$, we can derive that
$$D^{4G}(Q_t) = 0$$
$$D^{3G}(Q_t) = 1 - \alpha_{(3,0)}(Q_t) = \frac{R^{3G} - P^{3G}}{R^{3G}}$$
- *Medium 4G coverage regime*, as shown in Fig. 1(b). When $\Delta P/\Delta R \leq Q_t < \Delta P R^{3G}/(P^{3G}\Delta R)$, we can derive that
$$D^{3G}(Q_t) = \alpha_{(4,3)}(Q_t) - \alpha_{(3,0)}(Q_t) = \frac{\Delta P R^{3G} - Q_t\Delta R P^{3G}}{Q_t\Delta R R^{3G}}$$
$$D^{4G}(Q_t) = 1 - \alpha_{(4,3)}(Q_t) = \frac{Q_t\Delta R - \Delta P}{Q_t\Delta R}$$



(a) Low 4G coverage regime



(b) Medium 4G coverage regime



(c) High 4G coverage regime

Fig. 1: Users' service partitions according to 4G coverages.

- *High 4G coverage regime*, as shown in Fig. 1(c). When $\Delta P R^{3G}/(P^{3G}\Delta R) \le Q_t \le 1$, we can derive that

$$
\begin{aligned}
D^{3G}(Q_t) &= 0 \\
D^{4G}(Q_t) &= 1 - \alpha_{(4,0)} = \frac{Q_t R^{4G} + (1-Q_t)R^{3G} - P^{4G}}{Q_t R^{4G} + (1-Q_t)R^{3G}}
\end{aligned}
$$

In summary,

$$
D^{3G}(Q_t) = \begin{cases} 1 - \frac{P^{3G}}{R^{3G}}, & \text{if } Q_t < \frac{\Delta P}{\Delta R} \\ \frac{\Delta P}{Q_t \Delta R} - \frac{P^{3G}}{R^{3G}}, & \text{if } \frac{\Delta P}{\Delta R} \le Q_t < \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \\ 0, & \text{if } \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \le Q_t \le 1 \end{cases}
$$

$$
D^{4G}(Q_t) = \begin{cases} 0, & \text{if } Q_t < \frac{\Delta P}{\Delta R} \\ 1 - \frac{\Delta P}{Q_t \Delta R}, & \text{if } \frac{\Delta P}{\Delta R} \le Q_t < \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \\ 1 - \frac{P^{4G}}{Q_t R^{4G} + (1-Q_t)R^{3G}}, & \text{if } \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \le Q_t \le 1 \end{cases}
$$

Given that the 4G coverage is Q_t, the total revenue from the two services is

$$
R(Q_t) = P^{4G}\rho D^{4G}(Q_t) + P^{3G}\rho D^{3G}(Q_t)
$$

$$R(Q_t) = \begin{cases} \rho P^{3G}(1 - \frac{P^{3G}}{R^{3G}}), & \text{if } Q_t \le \frac{\Delta P}{\Delta R} \\ \rho(P^{4G} - \frac{\Delta P^2}{Q_t \Delta R} - \frac{(P^{3G})^2}{R^{3G}}), & \text{if } \frac{\Delta P}{\Delta R} \le Q_t < \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \\ \rho(P^{4G} - \frac{(P^{4G})^2}{Q_t \Delta R + R^{3G}}), & \text{if } \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \le Q_t \le 1 \end{cases}$$

□

II. Proof of Lemma 1

Lemma 1. *$R(Q_t)$ is concave within two separate Q_t ranges $[0, \Delta P/\Delta R]$ and $(\Delta P/\Delta R, 1]$, respectively*[1]*, but is just quasi-concave (not concave) in the entire range $Q_t \in [0, 1]$.*

Proof. The first derivative of $R(Q)$ with regard to Q is (Here we use Q instead of Q_t for clarity of expression):

$$R'(Q) = \begin{cases} 0, & \text{if } Q \le \frac{\Delta P}{\Delta R} \\ \frac{\rho \Delta P^2}{Q^2 \Delta R}, & \text{if } \frac{\Delta P}{\Delta R} \le Q < \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \\ \frac{\rho \Delta R (P^{4G})^2}{(Q\Delta R + R^{3G})^2}, & \text{if } \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \le Q \le 1 \end{cases}$$

Note that $R'(Q)$ is continuous at $Q = \frac{\Delta P R^{3G}}{P^{3G}\Delta R}$.

The second derivative of $R(Q)$ with regard to Q is:

$$R''(Q) = \begin{cases} 0, & \text{if } Q \le \frac{\Delta P}{\Delta R} \\ -\frac{2\rho \Delta P^2}{Q^3 \Delta R}, & \text{if } \frac{\Delta P}{\Delta R} \le Q < \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \\ -\frac{2\rho \Delta R^2 (P^{4G})^2}{(Q\Delta R + R^{3G})^3}, & \text{if } \frac{\Delta P R^{3G}}{P^{3G}\Delta R} \le Q \le 1 \end{cases}$$

$R'(Q)$ is decreasing in the range $[\Delta P/\Delta R, \Delta P R^{3G}/(P^{3G}\Delta R)]$, and range $[\Delta P R^{3G}/(P^{3G}\Delta R), 1]$, and $R'(Q)$ is continuous at $Q = \Delta P R^{3G}/(P^{3G}\Delta R)$. Therefore, $R'(Q)$ is decreasing in the range $[\Delta P/\Delta R, 1]$, and $R(Q)$ is concave in the same range.

However, $R'(Q)$ is 0 in the range $[0, \Delta P/\Delta R]$, but is positive in the range $[\Delta P/\Delta R, \Delta P R^{3G}/(P^{3G}\Delta R)]$. Therefore, $R(Q)$ is not concave in the entire range $[0, 1]$. $R(Q)$ is quasi-concave, because $R(Q)$ is monotonic increasing function, i.e., $f(\lambda Q_1 + (1-\lambda)Q_2) \le \max(f(Q_1), f(Q_2)), \forall \lambda \in [0, 1]$ is always true, because $\lambda Q_1 + (1-\lambda)Q_2 \le \max(Q_1, Q_2), \forall \lambda \in [0, 1]$.

□

[1]Recall that $\Delta P/\Delta R < 1$ as we assume $P^{3G}/R^{3G} > P^{4G}/R^{4G}$.

III. Proof of Proposition 2

Proposition 2. *(Optimal deployment policy before time stage T_{th})* It is the best for the operator not to deploy any 4G network before T_{th}. That is, $Q_t^* = 0, 1 \leq t \leq T_{th} - 1$.

Proof. First, it is easy to prove that $Q_t = 0, t = 1, ..., T_{th} - 1$ is feasible. The objective function is

$$
\begin{aligned}
& S_0 + \sum_{t=0}^{T_{th}-1} R(Q_t) + \sum_{t=T_{th}}^{T} R(Q_t) - kQ_T \\
& = S_0 + T_{th}\rho P^{3G}(1 - P^{3G}/R^{3G}) + \sum_{t=T_{th}}^{T} R(Q_t) - kQ_T
\end{aligned}
$$

We can see that the objective function is irrelevant of $Q_t, t = 1, ..., T_{th} - 1$. We can also prove that the constraints are irrelevant of $Q_t, t = 1, ..., T_{th} - 1$. Therefore, any feasible $Q_t, t = 1, ..., T_{th} - 1$ will generate the same objective function value. □

IV. Proof of Proposition 3

Proposition 3. *(Optimal deployment policy after time stage T_{th})* Given any T_{th} value, there exists a mature deployment stage $\bar{T}_{th}$, before and after which the operator has different deployment strategies. The value of $\bar{T}_{th}$ is determined by Algorithm 1. The special case of $\bar{T}_{th}$=$T_{th} - 1$ leads to no further deployment after T_{th}, i.e., $Q_t^* = \Delta P/\Delta R$ for any time $t \in \{T_{th}, , T\}$. More generally, when $\bar{T}_{th} < T_{th} - 1$, we have:

- *Aggressive deployment period:* In the time period $t \in [T_{th}, \bar{T}_{th}]$, the operator will use up all his current budget at each time stage t for 4G deployment, i.e., $Q_t^* = Q_t^m$ in (10), the maximum achievable coverage that can be supported by the budget at time stage t.
- *Conservative deployment period:* When $t = \bar{T}_{th}$, the operator will conservatively upgrade according to:

$$
\begin{cases}
Q_{t-1}^*, & \textit{if } C_d > (T + T_{th} - t)\max\{R'(Q_{t-1}^*), R'(Q_t^m)\} \\
q_t^*, & \textit{if } C_d \in [(T + T_{th} - t)R'(Q_{t-1}^*), \\
& \quad (T + T_{th} - t)R'(Q_t^m)] \\
Q_t^m, & \textit{if } C_d < (T + T_{th} - t)\min\{R'(Q_{t-1}^*), R'(Q_t^m)\}
\end{cases}
$$

  in which q_t^* is the unique solution to the equation $C_d = (T + T_{th} - t)R'(q_t^*)$.
- *No deployment period.* When $\bar{T}_{th} + 1 \leq t \leq T$, $Q_t^* = Q_{t-1}^*$.

Proof. We first give the solution to the following problem, then use the solution to address the problem in Proposition 3.

$$\max_{Q_1,\cdots,Q_T} \quad S_0 + C_d Q_0 + \sum_{t=0}^{T} R(Q_t) - C_d Q_T$$
$$\text{s.t. } C_d Q_t \le S_0 + C_d Q_0 + \sum_{\tau=0}^{t-1} R(Q_\tau), t = 1, ..., T$$
$$Q_0 \le Q_1 \le Q_2 \le \cdots \le Q_T \le 1$$

in which $Q_0 = \Delta P / \Delta R$.

It is easy to see that the problem in Proposition 3 is equivalent to the above optimization problem, with initial coverage $\Delta P/\Delta R$, initial capital $(T_{th}-1)\rho P^{3G}(1-P^{3G}/R^{3G})-C_d\Delta P/\Delta R$, and time horizon $T+1-T_{th}$. The above optimization problem is convex optimization, so we can use KKT conditions to derive the optimal solution.

We rewrite the constraints as

$$Q_t - Q_{t+1} \le 0, t = 0, \cdots, T-1$$
$$Q_T - 1 \le 0$$
$$C_d Q_t - S_0 - C_d Q_0 - \sum_{\tau=0}^{t-1} R(Q_\tau) \le 0, t = 1, ..., T$$

We define the Lagrangian L associated with the problem as

$$\begin{aligned} L =& S_0 + C_d Q_0 + \sum_{t=0}^{T} R(Q_t) - C_d Q_T \\ & - \sum_{t=0}^{T-1} \lambda_t (Q_t - Q_{t+1}) - \lambda_T (Q_T - 1) \\ & - \sum_{t=1}^{T} \mu_t (C_d Q_t - S_0 - C_d Q_0 - \sum_{\tau=0}^{t-1} R(Q_\tau)) \end{aligned}$$

The KKT conditions are:

$$\lambda_t \ge 0, t = 0, \cdots, T$$
$$\mu_t \ge 0, t = 1, \cdots, T$$

$$Q_t - Q_{t+1} \leq 0, t = 0, \cdots, T-1$$

$$Q_T - 1 \leq 0$$

$$C_d Q_t - S_0 - C_d Q_0 - \sum_{\tau=0}^{t-1} R(Q_\tau) \leq 0, t = 1, \cdots, T$$

$$\lambda_t (Q_t - Q_{t+1}) = 0, t = 0, \cdots, T-1$$

$$\lambda_T (Q_T - 1) = 0$$

$$\mu_t (C_d Q_t - S_0 - C_d Q_0 - \sum_{\tau=0}^{t-1} R(Q_\tau)) = 0, t = 1, \cdots, T$$

$$\frac{\partial L}{\partial Q_t} = (1 + \sum_{\tau=t+1}^{T} \mu_\tau) R'(Q_t) + \lambda_{t-1} - \lambda_t - C_d \mu_t = 0, t \in [1, T-1]$$

$$\frac{\partial L}{\partial Q_T} = R'(Q_T) - C_d + \lambda_{T-1} - \lambda_T - C_d \mu_T = 0$$

We give the solution to the KKT conditions as follows.

There exists a solution $(Q_1^*, \cdots, Q_T^*, \lambda_0, \cdots, \lambda_T, \mu_1, \cdots, \mu_T)$, in which $Q_t^*, t = 1, \cdots, T$ is given defined in Proposition 3 , and $\bar{T}_{th}$ in the Algorithm 1.

If $\bar{T}_{th} = 0$,

$$\lambda_t = \begin{cases} C_d - (T-t) R'(Q_0), & t = 0, \cdots, T-1 \\ 0, & t = T \end{cases}$$

$$\mu_t = 0, t = 1, \cdots, T$$

If $1 \leq \bar{T}_{th} \leq T$,

- If $C_d \in [(T + T_{th} - t) R'(Q_{t-1}^*), (T + T_{th} - t) R'(Q_t^m)]$:

$$\lambda_t = \begin{cases} 0, & t = 0, \cdots, \bar{T}_{th} - 1 \\ C_d - (T-t) R'(Q_{\bar{T}_{th}}^*), & t = \bar{T}_{th}, \cdots, T-1 \\ 0, & t = T \end{cases}$$

  Note that if $\bar{T}_{th} = T, \lambda_t = 0, t = 0, \cdots, T$.

$$\mu_t = \begin{cases} (1 + \sum_{\tau=\min\{T,t+1\}}^{T} \mu_\tau) R'(Q_t^*)/C_d, & t = 1, \cdots, \bar{T}_{th} - 1 \\ 0, & t = \bar{T}_{th}, \cdots, T \end{cases}$$

  Note that if $\bar{T}_{th} = 1, \mu_t = 0, t = 1, \cdots, T$.

- If $C_d > \max\{(T+T_{th}-t)R'(Q^*_{t-1}), (T+T_{th}-t)R'(Q^m_t)\}$:

$$\lambda_t = \begin{cases} 0, & t = 0, \cdots, \bar{T}_{th} - 2 \\ C_d - (T-t)R'(Q^*_{\bar{T}_{th}-1}), & t = \bar{T}_{th} - 1, \cdots, T-1 \\ 0, & t = T \end{cases}$$

Note that $\bar{T}_{th} \geq 2$ because $C_d - TR'(Q_0) \leq 0$.

$$\mu_t = \begin{cases} (1 + \sum_{\tau=\min\{T,t+1\}}^{T} \mu_\tau) R'(Q^*_t)/C_d, & t = 1, \cdots, \bar{T}_{th} - 2 \\ (T - (\bar{T}_{th} - 2))R'(Q^*_{\bar{T}_{th}-1})/C_d - 1, & t = \bar{T}_{th} - 1 \\ 0, & t = \bar{T}_{th}, \cdots, T \end{cases}$$

Note that if $\bar{T}_{th} = 2, \mu_1 = TR'(Q^*_1)/k - 1, \mu_t = 0, t = 2, \cdots, T$.

- If $C_d < \min\{(T+T_{th}-t)R'(Q^*_{t-1}), (T+T_{th}-t)R'(Q^m_t)\}$, but $Q^m_t = 1$:

$$\lambda_t = \begin{cases} 0, & t = 0, \cdots, \bar{T}_{th} - 1 \\ (t + 1 - \bar{T}_{th})R'(1), & t = \bar{T}_{th}, \cdots, T-1 \\ (T - (\bar{T}_{th} - 1))R'(1) - C_d, & t = T \end{cases}$$

Note that if $\bar{T}_{th} = T, \lambda_t = 0, t = 0, \cdots, T-1$ and $\lambda_T = R'(1) - C_d$.

$$\mu_t = \begin{cases} (1 + \sum_{\tau=\min\{T,t+1\}}^{T} \mu_\tau) R'(Q^*_t)/C_d, & t \in [1, \bar{T}_{th} - 1] \\ 0, & t = \bar{T}_{th}, \cdots, T \end{cases}$$

Note that if $\bar{T}_{th} = 1, \mu_t = 0, t = 1, \cdots, T$.

- If $C_d < \min\{(T+T_{th}-t)R'(Q^*_{t-1}), (T+T_{th}-t)R'(Q^m_t)\}$, but $t = T$:

$$\lambda_t = 0, t = 0, \cdots, T$$

$$\mu_t = \begin{cases} (1 + \sum_{\tau=t+1}^{T} \mu_\tau) R'(Q^*_t)/C_d, & t = 1, \cdots, T-1 \\ (R'(Q^*_T) - C_d)/C_d, & t = T \end{cases}$$

Since we find that the solution to the KKT conditions, the solution is optimal. □

V. Proof of Lemma 2

Lemma 2. The optimal T_{th} is chosen from the following two candidates:

- $T_{th} = T + 1$, that is, the operator will never deploy the 4G network to a coverage level $\Delta P/\Delta R$;

- $T_{th} = \left\lceil (C_d \Delta P/\Delta R - S_0)/[\rho P^{3G}(1 - P^{3G}/R^{3G})] \right\rceil$, that is, the operator deploys the 4G network to the coverage level $\Delta P/\Delta R$ as soon as possible.

Proof. We only have to prove that if $T_{th} \in [\left\lceil (C_d \Delta P/\Delta R - S_0)/[\rho P^{3G}(1 - P^{3G}/R^{3G})] \right\rceil, T]$, the optimal one is $T_{th}^* = \left\lceil (C_d \Delta P/\Delta R - S_0)/[\rho P^{3G}(1 - P^{3G}/R^{3G})] \right\rceil$.

The original optimization problem is equivalent to

$$\begin{aligned} \max_{Q_1, \cdots, Q_{T+1-T_{th}}} \quad & (T_{th} - 1)\rho P^{3G}(1 - \frac{P^{3G}}{R^{3G}}) + \sum_{t=0}^{T+1-T_{th}} R(Q_t) - C_d Q_{T+1-T_{th}} \\ \text{s.t. } & C_d Q_t \leq (T_{th} - 1)\rho P^{3G}(1 - \frac{P^{3G}}{R^{3G}}) + \sum_{\tau=0}^{t-1} R(Q_\tau), t = 1, ..., T + 1 - T_{th} \\ & Q_0 \leq Q_1 \leq Q_2 \leq \cdots \leq Q_{T+1-T_{th}} \leq 1 \end{aligned}$$

in which $Q_0 = \Delta P/\Delta R$ and $R(Q_0) = \rho P^{3G}(1 - P^{3G}/R^{3G})$. Let $OPT(T_{th})$ denote the optimal value of the above objective function. We only have to prove that $OPT(T_{th}) - OPT(T_{th}) \geq 0, T_{th} \leq T - 1$.

We define another optimization problem to help us compare $OPT(T_{th})$ and $OPT(T_{th} + 1)$.

$$\begin{aligned} \max_{Q_1, \cdots, Q_{T+1-T_{th}}} \quad & \sum_{t=1}^{T+1-T_{th}} R(Q_t) - C_d Q_{T+1-T_{th}} \\ \text{s.t.} \quad & Q_0 \leq Q_1 \leq \cdots \leq Q_{T+1-T_{th}} \leq 1 \\ & C_d Q_t \leq (T_{th} - 1)\rho P^{3G}(1 - \frac{P^{3G}}{R^{3G}}) + \sum_{\tau=0}^{t-1} R(Q_\tau), t = 1, ..., T + 1 - T_{th} \end{aligned} \quad (4)$$

Let $\Phi^*(T_{th})$ denote the optimal value of the above objective function. Therefore, $OPT(T_{th}) - OPT(T_{th} + 1) = \Phi^*(T_{th}) - \Phi^*(T_{th} + 1) - \rho P^{3G}(1 - P^{3G}/R^{3G}), T_{th} \leq T - 1$. Let $Q_t^*(T_{th})$ denote the optimal coverage at time stage t when the time horizon is $T + 1 - T_{th}$.

(1) If $Q_1^*(T_{th}) \leq Q_1^*(T_{th} + 1)$, then $Q_1^*(T_{th}), Q_1^*(T_{th} + 1), Q_2^*(T_{th} + 1), \cdots, Q_{T-T_{th}}^*(T_{th} + 1)$ is a feasible solution to the problem (4). We have

$$\Phi^*(Q_1^*(T_{th}), Q_1^*(T_{th}+1), \cdots, Q_{T-T_{th}}^*(T_{th}+1)) \leq \Phi^*(Q_1^*(T_{th}), Q_2^*(T_{th}), \cdots, Q_{T-T_{th}+1}^*(T_{th}))$$

$$\Rightarrow \sum_{t=1}^{T-T_{th}} R(Q_t^*(T_{th}+1)) - C_d Q_{T-T_{th}}^*(T_{th}+1) + R(Q_1^*(T_{th})) = \Phi^*(T_{th}+1) + R(Q_1^*(T_{th})) \leq \Phi^*(T_{th})$$

$$\Rightarrow \Phi^*(T_{th}) - \Phi^*(T_{th}+1) \geq R(Q_1^*(T_{th})) \geq R(Q_0) = \rho P^{3G}(1 - P^{3G}/R^{3G})$$

$$\Rightarrow OPT(T_{th}) - OPT(T_{th}+1) \geq 0.$$

(2) If $Q_1^*(T_{th}) > Q_1^*(T_{th}+1)$, then $Q_1^*(T_{th}+1), \cdots, Q_{T-T_{th}}^*(T_{th}+1), Q_{T-T_{th}}^*(T_{th}+1)$ is a feasible solution to $\Phi(T_{th})$. We have

$$\Phi^*(Q_1^*(T_{th}+1), Q_2^*(T_{th}+1), \cdots, Q_{T-T_{th}}^*(T_{th}+1), Q_{T-T_{th}}^*(T_{th}+1))$$
$$\leq \Phi^*(Q_1^*(T_{th}), Q_2^*(T_{th}), \cdots, Q_{T-T_{th}+1}^*(T_{th}))$$
$$\Rightarrow \sum_{t=1}^{T-T_{th}} R(Q_t^*(T_{th}+1)) - C_d Q_{T-T_{th}}^*(T_{th}+1) + R(Q_{T-T_{th}}^*(T_{th}+1))$$
$$= \Phi^*(T_{th}+1) + R(Q_{T-T_{th}}^*(T_{th}+1)) \leq \Phi^*(T_{th})$$
$$\Rightarrow \Phi^*(T_{th}) - \Phi^*(T_{th}+1) \geq R(Q_{T-T_{th}}^*(T_{th}+1)) \geq R(Q_0) = \rho P^{3G}(1 - P^{3G}/R^{3G})$$
$$\Rightarrow OPT(T_{th}) - OPT(T_{th}+1) \geq 0.$$

□

VI. Proof of Theorem 1

Theorem 1. The optimal 4G deployment policy is one of the following two options.

- *No deployment scheme:* the operator never deploys any 4G network, i.e., $Q_t^* = 0, t = 1, ..., T$.
- *Threshold-based deployment scheme:* Set threshold $T_{th}^* = \lceil (C_d \Delta P/\Delta R - S_0)/[\rho P^{3G}(1 - P^{3G}/R^{3G})] \rceil$.
  - When $t \in [1, T_{th}^* - 1]$, the operator does not deploy any 4G network, i.e., $Q_t^* = 0$;
  - When $t \in [T_{th}^*, T]$, the operator deploys 4G network according to Proposition 3.

The special case is that, if the deployment cost satisfies

$$C_d > \left(T + 1 - \left\lceil \frac{C_d \Delta P/\Delta R - S_0}{\rho P^{3G}(1 - P^{3G}/R^{3G})} \right\rceil \right) \rho \Delta R, \tag{5}$$

then 4G network will not be deployed.

Proof. No deployment scheme corresponds to $T_{th} = T + 1$ in Lemma 2. Threshold-based deployment scheme corresponds to $T_{th} = \lceil (C_d \Delta P/\Delta R - S_0)/[\rho P^{3G}(1 - P^{3G}/R^{3G})] \rceil$ in Lemma 2. The proof of the special case is as follows.

Let $T_1 = \lceil (C_d \Delta P/\Delta R - S_0)/[\rho P^{3G}(1 - P^{3G}/R^{3G})] \rceil$ If $C_d \geq (T+1-T_1)R'(\Delta P/\Delta R)$, then $Q_t^*(T_1) = \Delta P/\Delta R, t = T_{th}^1, \cdots, T$, then $OPT(T_1) - OPT(T+1) = (T+1-T_1)[R(\Delta P/\Delta R) - \rho P^{3G}(1 - P^{3G}/R^{3G})] - C_d \Delta P/\Delta R = -C_d \Delta P/\Delta < 0$. So we have $T_{th}^* = T + 1$. □

VII. Proof of Proposition 2 when there is operational cost

Proof. First, it is easy to prove that $Q_t = 0, t = 1, ..., T_{th} - 1$ is feasible. The objective function is

$$S_0 + \sum_{t=0}^{T_{th}-1} (R(Q_t) - C_o Q_t) + \sum_{t=T_{th}}^{T} (R(Q_t) - C_o Q_t) - C_d Q_T$$
$$= S_0 + T_{th} \rho P^{3G}(1 - P^{3G}/R^{3G}) - \sum_{t=0}^{T_{th}-1} C_o Q_t + \sum_{t=T_{th}}^{T} (R(Q_t) - C_o Q_t) - C_d Q_T$$

We can see that the objective function monotonically decreases with $Q_t, t = 1, ..., T_{th} - 1$. And the budget also monotonically decreases with $Q_t, t = 1, ..., T_{th} - 1$. Therefore, $Q_t = 0, t = 1, ..., T_{th} - 1$ is the optimal solution. □

VIII. Proof of Proposition 3 when there is operational cost

Proof. We first give the solution to the following problem, then use the solution to address the optimization problem when there is operational cost.

$$\begin{aligned}
\max_{Q_1, \cdots, Q_T} \quad & S_0 + C_d Q_0 + \sum_{t=0}^{T} (R(Q_t) - C_o Q_t) - C_d Q_T \\
\text{s.t.} \quad & Q_t - Q_{t+1} \leq 0, t = 0, \cdots, T-1 \\
& Q_T - 1 \leq 0 \\
& C_d Q_t - S_0 - k Q_0 - \sum_{\tau=0}^{t-1} (R(Q_\tau) - C_o Q_\tau) \leq 0, t = 1, ..., T
\end{aligned}$$

in which $Q_0 = \Delta P/\Delta R$.

It is easy to see that the optimization problem in Proposition 3 when there is operational cost is equivalent to the above optimization problem, with initial coverage $\Delta P/\Delta R$, initial capital

$(T_{th}-1)\rho P^{3G}(1-P^{3G}/R^{3G})-(C_d-C_o)\Delta P/\Delta R$, and time horizon $T+1-T_{th}$. The above optimization problem is convex optimization. So we only have to find the solution that satisfies the KKT conditions, which is guaranteed to be optimal.

The Lagrangian L associated with the problem is

$$\begin{aligned} L =& S_0 + C_d Q_0 + \sum_{t=0}^{T} R(Q_t) - C_d Q_T - C_o \sum_{t=0}^{T} Q_t \\ & - \sum_{t=0}^{T-1} \lambda_t (Q_t - Q_{t+1}) - \lambda_T (Q_T - 1) \\ & - \sum_{t=1}^{T} \mu_t [C_d Q_t - S_0 - C_d Q_0 - \sum_{\tau=0}^{t-1} (R(Q_\tau) - C_o Q_\tau)] \end{aligned}$$

The KKT conditions are:

$$\lambda_t \geq 0, t = 0, \cdots, T$$

$$\mu_t \geq 0, t = 1, \cdots, T$$

$$Q_t - Q_{t+1} \leq 0, t = 0, \cdots, T-1$$

$$Q_T - 1 \leq 0$$

$$C_d Q_t - S_0 - C_d Q_0 - \sum_{\tau=0}^{t-1} [R(Q_\tau) - C_o Q_\tau] \leq 0, t = 1, \cdots, T$$

$$\lambda_t (Q_t - Q_{t+1}) = 0, t = 0, \cdots, T-1$$

$$\lambda_T (Q_T - 1) = 0$$

$$\mu_t (C_d Q_t - S_0 - C_d Q_0 - \sum_{\tau=0}^{t-1} (R(Q_\tau) - C_o Q_\tau)) = 0, t = 1, \cdots, T$$

$$\frac{\partial L}{\partial Q_t} = (1 + \sum_{\tau=t+1}^{T} \mu_\tau)(R'(Q_t) - C_o) + (\lambda_{t-1} - \lambda_t) - C_d \mu_t = 0, t = 1, \cdots, T-1$$

$$\frac{\partial L}{\partial Q_T} = R'(Q_T) - C_o - C_d + \lambda_{T-1} - \lambda_T - C_d \mu_T = 0$$

There exists a solution $(Q_1^*, \cdots, Q_T^*, \lambda_0, \cdots, \lambda_T, \mu_1, \cdots, \mu_T)$, in which $Q_t^*, t = 1, \cdots, T$ is the optimal coverage level defined by the adaptation of Proposition 2 when there is operational cost, and

If $\bar{T}_{th} = 0$,

$$\lambda_t = \begin{cases} C_d - (T-t)(R'(Q_0) - C_o), & t = 0, \cdots, T-1 \\ 0, & t = T \end{cases}$$

$$\mu_t = 0, t = 1, \cdots, T$$

If $1 \leq \bar{T}_{th} \leq T$,

- If $C_d \in [(T + T_{th} - t)(R'(Q^*_{t-1}) - C_o), (T + T_{th} - t)(R'(Q^m_t) - C_o)]$:

$$\lambda_t = \begin{cases} 0, & t = 0, \cdots, \bar{T}_{th} - 1 \\ C_d - (T-t)(R'(Q^*_{\bar{T}_{th}}) - C_o), & t = \bar{T}_{th}, \cdots, T-1 \\ 0, & t = T \end{cases}$$

  Note that if $\bar{T}_{th} = T, \lambda_t = 0, t = 0, \cdots, T$.

$$\mu_t = \begin{cases} [(1 + \sum_{\tau=\min\{T,t+1\}}^{T} \mu_\tau)(R'(Q^*_t) - C_o)]/C_d, & t = 1, \cdots, \bar{T}_{th} - 1 \\ 0, & t = \bar{T}_{th}, \cdots, T \end{cases}$$

  Note that if $\bar{T}_{th} = 1, \mu_t = 0, t = 1, \cdots, T$.

- If $C_d > \max\{(T + T_{th} - t)(R'(Q^*_{t-1}) - C_o), (T + T_{th} - t)(R'(Q^m_t) - C_o)\}$:

$$\lambda_t = \begin{cases} 0, & t = 0, \cdots, \bar{T}_{th} - 2 \\ C_d - (T-t)(R'(Q^*_{\bar{T}_{th}-1}) - C_o), & t = \bar{T}_{th} - 1, \cdots, T-1 \\ 0, & t = T \end{cases}$$

  Note that $\bar{T}_{th} \geq 2$ because $C_d - TR'(Q_0) \leq 0$.

$$\mu_t = \begin{cases} [(1 + \sum_{\tau=\min\{T,t+1\}}^{T} \mu_\tau)(R'(Q^*_t) - C_o)]/C_d, & t = 1, \cdots, \bar{T}_{th} - 2 \\ (T - (\bar{T}_{th} - 2))(R'(Q^*_{\bar{T}_{th}-1}) - C_o)/C_d - 1, & t = \bar{T}_{th} - 1 \\ 0, & t = \bar{T}_{th}, \cdots, T \end{cases}$$

  Note that if $\bar{T}_{th} = 2, \mu_1 = T(R'(Q^*_1) - C_o)/C_d - 1, \mu_t = 0, t = 2, \cdots, T$.

- If $C_d < \min\{(T + T_{th} - t)(R'(Q^*_{t-1}) - C_o), (T + T_{th} - t)(R'(Q^m_t) - C_o)\}$, but $Q^m_t = 1$:

$$\lambda_t = \begin{cases} 0, & t = 0, \cdots, \bar{T}_{th} - 1 \\ (t + 1 - \bar{T}_{th})(R'(1) - C_o), & t = \bar{T}_{th}, \cdots, T-1 \\ (T - (\bar{T}_{th} - 1))(R'(1) - C_o) - C_d, & t = T \end{cases}$$

Note that if $\bar{T}_{th} = T, \lambda_t = 0, t = 0, \cdots, T-1$ and $\lambda_T = R'(1) - C_d$.

$$\mu_t = \begin{cases} [(1+\sum_{\tau=\min\{T,t+1\}}^{T} \mu_\tau)(R'(Q_t^*) - C_o)]/C_d, & t = 1, \cdots, \bar{T}_{th} - 1 \\ 0, & t = \bar{T}_{th}, \cdots, T \end{cases}$$

Note that if $\bar{T}_{th} = 1, \mu_t = 0, t = 1, \cdots, T$.

- $C_d < \min\{(T + T_{th} - t)(R'(Q_{t-1}^*) - C_o), (T + T_{th} - t)(R'(Q_t^m) - C_o)\}$, but $\bar{T}_{th} = T$:

$$\lambda_t = 0, t = 0, \cdots, T$$

$$\mu_t = \begin{cases} [(1+\sum_{\tau=\min\{T,t+1\}}^{T} \mu_\tau)(R'(Q_t^*) - C_o)]/C_d, & t = 1, \cdots, T - 1 \\ (R'(Q_T^*) - C_o - C_d)/C_d, & t = T \end{cases}$$

Since the solution satisfies the KKT conditions, it is the optimal solution. □

IX. Proof of Proposition 4

Proposition 4. *Final 4G coverage level Q_T^* increases with time length T and user density ρ, but decreases with the deployment cost C_d. As $T \to \infty$, $Q_T^* = 1$ (full 4G coverage).*

Proof. It can be easily proved that Q_T^* increases with T and ρ, but decreases with C_d. So we only prove that as $T \to \infty$, $Q_T^* = 1$ (full 4G coverage). If the total time horizon T is long enough, $Q_t^*(T_1) = 1, t = \theta, \cdots, T$. Hence $OPT(T_1) - OPT(T+1) \geq (T+1-T_{th}-\theta)(R(1) - \rho P^{3G}(1 - P^{3G}/R^{3G})) - C_d > 0$, because $R(1) - \rho P^{3G}(1 - P^{3G}/R^{3G}) > 0, T \to \infty$. So we have $T_{th}^* = T_1$. □

X. Proof of Lemma 3

Lemma 3. *The optimal T_{th}^* with consideration of the operational cost, is chosen from the following two candidates:*

- $T_{th} = T + 1$*, that is, the operator will not deploy the 4G coverage to* $\Delta P/\Delta R$*;*
- $T_{th} = \widetilde{T}$*, which satisfies*

$$R(Q_{\widetilde{T}}) - C_o Q_{\widetilde{T}} \geq R(0) \tag{6}$$

$$R(Q_{\widetilde{T}-1}) - C_o Q_{\widetilde{T}-1} \leq R(0) \tag{7}$$

Proof. We just have to prove that $\widetilde{T}$ is optimal for $T_{th} \leq T$.

Given that the coverage before T_{th} is zero, the original optimization problem is equivalent to the following optimization problem:

$$\begin{aligned} \max_{Q_1,\cdots,Q_{T+1-T_{th}}} \quad & (T_{th}-1)\rho P^{3G}(1-\frac{P^{3G}}{R^{3G}}) + \sum_{t=0}^{T+1-T_{th}} (R(Q_t) - C_o Q_t) - C_d Q_{T+1-T_{th}} \\ \text{s.t.} \quad & Q_0 \le Q_1 \le \cdots \le Q_{T+1-T_{th}} \le 1 \\ & C_d Q_t \le (T_{th}-1)\rho P^{3G}(1-\frac{P^{3G}}{R^{3G}}) + \sum_{\tau=0}^{t-1}(R(Q_\tau) - C_o Q_\tau), t = 1, ..., T+1-T_{th} \end{aligned}$$

in which $Q_0 = \Delta P/\Delta R$ and $R(Q_0) = \rho P^{3G}(1 - P^{3G}/R^{3G})$. Let $OPT(T_{th})$ denote the optimal value of the above objective function.

We define another optimization problem to help us compare $OPT(T_{th})$ and $OPT(T_{th}+1)$.

$$\begin{aligned} \max_{Q_1,\cdots,Q_{T+1-T_{th}}} \quad & \sum_{t=1}^{T+1-T_{th}} (R(Q_t) - C_o Q_t) - C_d Q_{T+1-T_{th}} \\ \text{s.t.} \quad & Q_0 \le Q_1 \le \cdots \le Q_{T+1-T_{th}} \le 1 \\ & C_d Q_t \le (T_{th}-1)\rho P^{3G}(1-\frac{P^{3G}}{R^{3G}}) + \sum_{\tau=0}^{t-1}(R(Q_\tau) - C_o Q_\tau), t = 1, ..., T+1-T_{th} \end{aligned}$$

Let $\Phi^*(T_{th})$ denote the optimal value of the above objective function. Therefore, $OPT(T_{th}) - OPT(T_{th}+1) = \Phi^*(T_{th}) - \Phi^*(T_{th}+1) - \rho P^{3G}(1 - P^{3G}/R^{3G}), T_{th} \le T$. Let $Q_t^*(T_{th})$ denote the optimal coverage at time stage t, given T_{th}.

We first prove the following argument.

Argument 1. *For $T_{th} \le T$, the value of $R(Q_1^*(T_{th})) - C_o Q_1^*(T_{th})$ has the trend of one of the following two cases:*

1) *first increases then decreases with T_{th}, or*
2) *monotonously increases with T_{th}.*

Proof. When T_{th} increases, the stopping time $\bar{T}_{th}$ decreases, because the total time horizon $T+1-T_{th}$ decreases and the upperbound Q_1^m increases.

- When $\bar{T}_{th}(T_{th}) > 1$, $Q_1^*(T_{th}) = Q_1^m$ increases along with T_{th}. $R(Q_1^*(T_{th})) - C_o Q_1^*(T_{th})$ also increases because $R'(Q_1^*(T_{th})) - C_o > k/(T+1-T_{th}) > 0$.
- When $\bar{T}_{th}(T_{th}) = 1$,

- if $C_d \in [(T+T_{th}-\bar{T}_{th})(R'(Q^*_{\bar{T}_{th}-1})-C_o), (T+T_{th}-\bar{T}_{th})(R'(Q^m_{\bar{T}_{th}}))-C_o)]$, $Q^*_1(T_{th})$ decreases along with T_{th}, $R(Q^*_1(T_{th}))-C_oQ^*_1(T_{th})$ also decreases because $R'(Q^*_1(T_{th}))-C_o = k/(T+1-T_{th}) > 0$;
- if $C_d > \max\{(T+T_{th}-\bar{T}_{th})(R'(Q^*_{\bar{T}_{th}-1})-C_o), (T+T_{th}-\bar{T}_{th})(R'(Q^m_{\bar{T}_{th}})-C_o)\}$, $Q^*_1(T_{th}) = Q_0$, and $R(Q^*_1(T_{th}))-C_oQ^*_1(T_{th})$ remains unchanged;
- if $C_d < \min\{(T+T_{th}-\bar{T}_{th})(R'(Q^*_{\bar{T}_{th}-1})-C_o), (T+T_{th}-\bar{T}_{th})(R'(Q^m_{\bar{T}_{th}})-C_o)\}$ and $Q^m_1 = 1$, $Q^*_1(T_{th}) = 1$, and $R(Q^*_1(T_{th}))-C_oQ^*_1(T_{th})$ remains unchanged;
- if $C_d < \min\{(T+T_{th}-\bar{T}_{th})(R'(Q^*_{\bar{T}_{th}-1})-C_o), (T+T_{th}-\bar{T}_{th})(R'(Q^m_{\bar{T}_{th}})-C_o)\}$ and $\bar{T}_{th} = T$, we know that for $T_{th} < T$, $\bar{T}_{th}(T_{th}) > 1$, and for all $T_{th} \leq T$, $Q^*_1(T_{th}) = Q^m_1$ increases along with T_{th}. $R(Q^*_1(T_{th}))-C_oQ^*_1(T_{th})$ also increases because $R'(Q^*_1(T_{th}))-C_o > k/(T+1-T_{th}) > 0$.

In summary, $R(Q^*_1(T_{th}))-C_oQ^*_1(T_{th})$ first increases then decreases with T_{th}. □

Based on Argument 1, we define $T_1 \leq T_2 \leq T$, such that 1) when $T_{th} < T_1$, $R(Q^*_1(T_{th}))-C_oQ^*_1(T_{th}) < \rho P^{3G}(1-P^{3G}/R^{3G})$; 2) when $T_1 \leq T_{th} < T_2$, $R(Q^*_1(T_{th}))-C_oQ^*_1(T_{th}) \geq \rho P^{3G}(1-P^{3G}/R^{3G})$; 3) $T_2 \leq T_{th} \leq T$, $R(Q^*_1(T_{th}))-C_oQ^*_1(T_{th}) < \rho P^{3G}(1-P^{3G}/R^{3G})$. Note that $\widetilde{T} = T_1$, so we just have to prove that T_1 is optimal for $T_{th} \leq T$.

Argument 2. *When $T_1 \leq T_{th} \leq T_2$, the maximum objective function value is achieved when $T_{th} = T_1$.*

Proof. We know that $OPT(T_{th}) - OPT(T_{th}+1) = \Phi^*(T_{th}) - \Phi^*(T_{th}+1) - \rho P^{3G}(1-P^{3G}/R^{3G}), T_{th} \in [T_1, T_2-1]$. We only have to prove that $OPT(T_{th}) - OPT(T_{th}+1) \geq 0, T_1 \leq T_{th} \leq T_2$.

(1) If $Q^*_1(T_{th}) \leq Q^*_1(T_{th}+1)$, then $Q^*_1(T_{th}), Q^*_1(T_{th}+1), Q^*_2(T_{th}+1), \cdots, Q^*_{T-T_{th}}(T_{th}+1)$ is a feasible solution to $\Phi(T_{th})$. The proof is as follows. We have

$$\Phi^*(Q_1^*(T_{th}), Q_1^*(T_{th}+1), \cdots, Q_{T-T_{th}}^*(T_{th}+1)) \le \Phi^*(Q_1^*(T_{th}), Q_2^*(T_{th}), \cdots, Q_{T-T_{th}+1}^*(T_{th}))$$
$$\Rightarrow \sum_{t=1}^{T-T_{th}} (R(Q_t^*(T_{th}+1)) - C_o Q_t^*(T_{th}+1)) - C_d Q_{T-T_{th}}^*(T_{th}+1) + R(Q_1^*(T_{th})) - C_o Q_1^*(T_{th})$$
$$= \Phi^*(T_{th}+1) + R(Q_1^*(T_{th})) - C_o Q_1^*(T_{th}) \le \Phi^*(T_{th})$$
$$\Rightarrow \Phi^*(T_{th}) - \Phi^*(T_{th}+1) \ge R(Q_1^*(T_{th})) - C_o Q_1^*(T_{th}) \ge \rho P^{3G}(1 - P^{3G}/R^{3G})$$
$$\Rightarrow OPT(T_{th}) - OPT(T_{th}+1) \ge 0.$$

(2) If $Q_1^*(T_{th}) > Q_1^*(T_{th}+1)$, then $Q_1^*(T_{th}+1), \cdots, Q_{T-T_{th}}^*(T_{th}+1), Q_{T-T_{th}}^*(T_{th}+1)$ is a feasible solution to $\Phi(T_{th})$. We have

$$\Phi^*(Q_1^*(T_{th}+1), Q_2^*(T_{th}+1), \cdots, Q_{T-T_{th}}^*(T_{th}+1), Q_{T-T_{th}}^*(T_{th}+1))$$
$$\le \Phi^*(Q_1^*(T_{th}), Q_2^*(T_{th}), \cdots, Q_{T-T_{th}+1}^*(T_{th}))$$
$$\Rightarrow \sum_{t=1}^{T-T_{th}} (R(Q_t^*(T_{th}+1)) - C_o Q_t^*(T_{th}+1)) - C_d Q_{T-T_{th}}^*(T_{th}+1)$$
$$+ R(Q_{T-T_{th}}^*(T_{th}+1)) - C_o Q_{T-T_{th}}^*(T_{th}+1)$$
$$= \Phi^*(T_{th}+1) + R(Q_{T-T_{th}}^*(T_{th}+1)) - C_o Q_{T-T_{th}}^*(T_{th}+1) \le \Phi^*(T_{th})$$
$$\Rightarrow \Phi^*(T_{th}) - \Phi^*(T_{th}+1) \ge R(Q_1^*(T_{th}+1)) - Q_1^*(T_{th}+1) \ge \rho P^{3G}(1 - P^{3G}/R^{3G})$$
$$\Rightarrow OPT(T_{th}) - OPT(T_{th}+1) \ge 0.$$

In summary, $OPT(T_{th}), T_{th} \in [T_1, T_2]$ is a decreasing function of T_{th}, so the maximum $OPT(T_{th})$ is achieved when $T_{th} = T_1$. □

Argument 3. *For any $T_{th} \le T_1$, the maximum objective function value is achieved when $T_{th} = T_1$.*

Proof. We know that $OPT(T_{th}) - OPT(T_{th}+1) = \Phi^*(T_{th}) - \Phi^*(T_{th}+1) - \rho P^{3G}(1 - P^{3G}/R^{3G}), T_{th} \le T_1$. We only have to prove that $OPT(T_{th}) - OPT(T_{th}+1) \le 0, T_{th} \le T_1$. $Q_2^*(T_{th}), Q_3^*(T_{th}), \cdots, Q_{T+1-T_{th}}^*(T_{th})$ is a feasible solution to $\Phi(T_{th}+1)$. We have

$$\Phi^*(Q_2^*(T_{th}), Q_3^*(T_{th}), \cdots, Q_{T+1-T_{th}}^*(T_{th})) \leq \Phi^*(Q_1^*(T_{th}+1), Q_2^*(T_{th}+1), \cdots, Q_{T-T_{th}}^*(T_{th}+1))$$

$$\Rightarrow \sum_{t=1}^{T+1-T_{th}} (R(Q_t^*(T_{th})) - C_o Q^* t(T_{th})) - C_d Q_{T+1-T_{th}}^*(T_{th}) - (R(Q_1^*(T_{th})) - C_o Q_1^*(T_{th}))$$

$$= \Phi^*(T_{th}) - (R(Q_1^*(T_{th})) - C_o Q_1^*(T_{th})) \leq \Phi^*(T_{th}+1)$$

$$\Rightarrow \Phi^*(T_{th}) - \Phi^*(T_{th}+1) \leq R(Q_1^*(T_{th})) - C_o Q_1^*(T_{th}) < \rho P^{3G}(1 - P^{3G}/R^{3G})$$

$$\Rightarrow OPT(T_{th}) - OPT(T_{th}+1) < 0.$$

Similarly, we have the following argument.

Argument 4. *For any $T_{th} \in [T_2+1, T+1]$, the maximum objective function value is achieved when $T_{th} = T+1$.*

Proof. It is easy to see that for any $T_{th} \in [T_2+1, T]$, the maximum objective function value is achieved when $T_{th} = T$. The proof is similar to the above argument. Then we compare $OPT(T)$ and $OPT(T+1)$. $OPT(T) = S_0 + T\rho P^{3G}(1 - P^{3G}/R^{3G}) + R(Q_1^*(T)) - (C_o + C_d)Q_1^*(T) < S_0 + (T+1)\rho P^{3G}(1 - P^{3G}/R^{3G}) = OPT(T+1)$. □

In summary, we have proved that $\widetilde{T} = T_1$ is optimal for $T_{th} \leq T$.

□

□

XI. PROOF OF THEOREM 2

Theorem 2. *(Optimal 4G deployment policy with operational cost).* The optimal 4G deployment policy is one of the following two options.

- *No deployment scheme:* the operator never deploys any 4G network, i.e., $Q_t^* = 0, t = 1, ..., T$.
- *Threshold-based deployment scheme:* when $t = 1, ..., T_{th}^* - 1$, the operator does not deploy any 4G network, i.e., $Q_t^* = 0$; when $t = T_{th}^*, ..., T$, the operator deploy 4G network according to Proposition 3 by replacing (3) by (14); and T_{th}^* is determined by Lemma 3.

Proof. No deployment scheme corresponds to $T_{th} = T+1$ in Lemma 3. Threshold-based deployment scheme corresponds to $T_{th} = \widetilde{T}$ in Lemma 3. □

XII. Proof of Proposition 5

Proposition 5. *The operational cost reduces the final 4G coverage level, and delays the time to deploy the 4G network.*

Proof. Without loss of generality, we assume that for both case with/without operational case, $Q_0 = \Delta P/\Delta R$, so $T_{th} = 0$. Let Q_T^{op} and Q_T^{nop} denote the final coverage level when there is operational cost and when there is no operational cost respectively, $\bar{T}_{th}^{op}$ and $\bar{T}_{th}^{nop}$ denote the stopping time when there is operational cost and when there is no operational cost respectively. When there is no operational cost, Q_T^{op} can be reached at time τ, i.e., $Q_{\tau-1}^{nop} \leq Q_T^{op} \leq Q_\tau^{m,nop}$, and $\tau \leq \bar{T}_{th}^{op}$ because the operator has more capital for investment when there is no operational cost. Now we consider at the stopping time $t = \bar{T}_{th}^{op}$, when there is operational cost.

If $C_d \in [(T-t)(R'(Q_{t-1}^*) - C_o), (T-t)(R'(Q_t^m) - C_o)]$, then $C_d - [T-(\tau-1)]R'(Q_{\tau-1}^{nop}) < C_d - [T-(\tau-1)]R'(Q_T^{op}) = (\tau - \bar{T}_{th}^{op})R'(Q_T^{op}) - [T - (\bar{T}_{th}^{op} - 1)]C_o < 0$. Hence $\bar{T}_{th}^{nop} \geq \tau$. If $\bar{T}_{th}^{nop} > \tau$, it is obviously that $Q_T^{nop} \geq Q_\tau^{m,nop} \geq Q_T^{op}$. If $\bar{T}_{th}^{nop} = \tau$, 1) if $C_d < \min\{(T - \bar{T}_{th}^{nop})R'(Q^*_{\bar{T}_{th}^{nop}-1}), (T - \bar{T}_{th}^{nop})R'(Q^m_{\bar{T}_{th}^{nop}})\}$, then $Q_T^{nop} = Q_\tau^{m,nop} \geq Q_T^{op}$; 2) if $C_d \in [(T - \bar{T}_{th}^{nop})R'(Q^*_{\bar{T}_{th}^{nop}-1}), (T - \bar{T}_{th}^{nop})R'(Q^m_{\bar{T}_{th}^{nop}})]$, then it can be easily derived that $R'(Q_T^{nop}) < R'(Q_T^{op})$, so $Q_T^{nop} > Q_T^{op}$.

If $C_d > \max\{[(T-t)(R'(Q_{t-1}^*) - C_o), (T-t)(R'(Q_t^m) - C_o)\}$, $\tau \leq \bar{T}_{th}^{op} - 1$ must be true. Because if $\tau = \bar{T}_{th}^{op}$, then $Q_{\tau-1}^{nop} \leq Q_T^{op} = Q^{op}_{\bar{T}_{th}^{op}-1} = Q^{op}_{\tau-1}$, but $Q_{\tau-1}^{nop} > Q_{\tau-1}^{op}$ since at the same time period, the achievable coverage when there is no operational cost is strictly higher than that when there is operational cost. $C_d - [T-(\tau-1)]R'(Q_{\tau-1}^{nop}) < C_d - [T - (\bar{T}_{th}^{op} - 2)](R'(Q_T^{op}) - C_o) < 0$, so $\bar{T}_{th}^{nop} \geq \tau$. If $\bar{T}_{th}^{nop} > \tau$, it is obviously that $Q_T^{nop} \geq Q_\tau^{m,nop} \geq Q_T^{op}$. If $\bar{T}_{th}^{nop} = \tau$, 1) if $C_d < \min\{(T - \bar{T}_{th}^{nop})R'(Q^*_{\bar{T}_{th}^{nop}-1}), (T - \bar{T}_{th}^{nop})R'(Q^m_{\bar{T}_{th}^{nop}})\}$, then $Q_T^{nop} = Q_\tau^{m,nop} \geq Q_T^{op}$; 2) if $C_d \in [(T - \bar{T}_{th}^{nop})R'(Q^*_{\bar{T}_{th}^{nop}-1}), (T - \bar{T}_{th}^{nop})R'(Q^m_{\bar{T}_{th}^{nop}})]$, then it can be easily derived that $R'(Q_T^{nop}) < R'(Q_T^{op})$, so $Q_T^{nop} > Q_T^{op}$.

If $C_d < \min\{[(T-t)(R'(Q_{t-1}^*) - C_o), (T-t)(R'(Q_t^m) - C_o)\}$, it is easy to prove that $C_d - [T-(\tau-1)]R'(Q_{\tau-1}^{nop}) < 0$ and $\bar{T}_{th}^{nop} \geq \tau$. If $\bar{T}_{th}^{nop} > \tau$, it is obviously that $Q_T^{nop} \geq Q_\tau^{m,nop} \geq Q_T^{op}$. If $\bar{T}_{th}^{nop} = \tau$, 1) if $C_d < \min\{(T - \bar{T}_{th}^{nop})R'(Q^*_{\bar{T}_{th}^{nop}-1}), (T - \bar{T}_{th}^{nop})R'(Q^m_{\bar{T}_{th}^{nop}})\}$, then $Q_T^{nop} = Q_\tau^{m,nop} \geq Q_T^{op}$; 2) if $C_d \in [(T - \bar{T}_{th}^{nop})R'(Q^*_{\bar{T}_{th}^{nop}-1}), (T - \bar{T}_{th}^{nop})R'(Q^m_{\bar{T}_{th}^{nop}})]$, then $R'(Q_T^{nop}) < R'(Q_T^{op})$, so $Q_T^{nop} > Q_T^{op}$. $\square$

XIII. Proof of Proposition 6

Proposition 6. Depending on the 4G coverage Q_t, the operator's revenue is:

- *Low 4G coverage regime.* No users choose 4G service and users with $\alpha \in [\alpha_{(3,0)}, 1]$ choose 3G. The operator's revenue is:

$$R(Q_t) = P^{3G} \frac{R^{3G} - P^{3G}}{R^{3G} + \gamma}.$$

- *Medium 4G coverage regime.* Users with $\alpha \in [\alpha_{(3,0)}, \alpha_{(4,3)}]$ choose 3G, and $\alpha \in [\alpha_{(4,3)}, 1]$ choose 4G. The operator's revenue is:

$$\begin{aligned} R(Q_t) = \\ P^{3G} \frac{\frac{\Delta P R^{3G}}{Q_t} - \Delta R P^{3G} + \gamma(1 - Q_t)(P^{3G} - R^{4G} + \frac{\Delta P}{Q_t})}{\Delta R R^{3G} + \gamma(R^{3G} Q_t + \Delta R)} \\ + P^{4G} \frac{\Delta R R^{3G} - \Delta P R^{3G}/Q_t + \gamma(R^{4G} - P^{3G} - \Delta P/Q_t)}{\Delta R R^{3G} + \gamma(R^{3G} Q_t + \Delta R)} \end{aligned}$$

- *High 4G coverage regime.* No users choose 3G service and users with $\alpha \in [\alpha_{(4,0)}, 1]$ choose 4G. The operator's revenue is:

$$R(Q_t) = P^{4G} \frac{Q_t \Delta R + R^{3G} - P^{4G}}{Q_t \Delta R + R^{3G} + (1 - Q_t)^2 \gamma}$$

Proof. Without confusion, here to ignore the subscript t for clarity of expression. Let D^{3G} and D^{4G} denote the number of users who choose 3G or 4G service respectively. Let $D = \gamma(D^{3G} + (1 - Q)D^{4G})$.

- *Low 4G coverage regime*, as shown in Fig.1(a). We see that $\alpha_{(4,3)} > 1$, so $Q < \frac{\Delta P R^{3G} + \gamma \Delta P}{\Delta R R^{3G} + \gamma R^{4G} - \gamma P^{3G}}$, we have

$$\begin{aligned} D^{4G} &= 0 \\ D^{3G} &= \frac{R^{3G} - P^{3G}}{R^{3G} + \gamma} \end{aligned}$$

- *Medium 4G coverage regime*, as shown in Fig. 1(b). The condition is $\alpha_{(3,0)} < \alpha_{(4,0)} < \alpha_{(4,3)}$, that is, $Q < \Delta P R^{3G} / (P^{3G} \Delta R + R^{4G} D)$. We can derive that

$$D = \frac{\gamma(\Delta P R^{3G} - \Delta R P^{3G} + (1-Q)R^{3G}\Delta R)}{\Delta R R^{3G} + \gamma(R^{3G}Q + \Delta R)}$$

$$D^{3G} = \frac{\Delta P R^{3G} + \gamma(1-Q)\Delta P + Q\gamma(1-Q)P^{3G} - \gamma(1-Q)QR^{4G} - Q\Delta R P^{3G}}{Q[\Delta R R^{3G} + \gamma(R^{3G}Q + \Delta R)]}$$

$$D^{4G} = \frac{Q(\Delta R R^{3G} + \gamma R^{4G} - \gamma P^{3G}) - \Delta P R^{3G} - \gamma \Delta P}{Q[\Delta R R^{3G} + \gamma(R^{3G}Q + \Delta R)]}$$

Now we can get the close form expression of the condition $Q < \Delta P R^{3G}/(P^{3G}\Delta R + R^{4G}D)$. We get the following inequality.

$$\gamma(R^{4G} - P^{3G})Q^2 - [\Delta R P^{3G} + \gamma(\Delta P - P^{3G} + R^{4G})]Q + \Delta P(R^{3G} + \gamma) > 0$$

Define

$$M_1 = \gamma(R^{4G} - P^{3G})$$

$$M_2 = \Delta P(R^{3G} + \gamma)$$

$$M_3 = \Delta R P^{3G} - \Delta P R^{3G} > 0$$

So we have

$$M_1 Q^2 - (M_1 + M_2 + M_3)Q + M_2 > 0$$

$$Q > Q_1 = \frac{M_1 + M_2 + M_3 + \sqrt{(M_1 - M_2 + M_3)^2 + 4M_2M_3}}{2M_1}$$

$$\text{or } Q < Q_2 = \frac{M_1 + M_2 + M_3 - \sqrt{(M_1 - M_2 + M_3)^2 + 4M_2M_3}}{2M_1}$$

We can easily prove that $Q_1 > 1$. So the reasonable one is $\frac{\Delta P R^{3G} + \gamma \Delta P}{\Delta R R^{3G} + \gamma R^{4G} - \gamma P^{3G}} \leq Q < Q_2$.

- *High 4G coverage regime*, as shown in Fig. 1(c). The condition is $\alpha_{(3,0)} > \alpha_{(4,0)} > \alpha_{(4,3)}$, that is $Q_2 \leq Q \leq 1$. We can derive the user distribution as follows:

$$D^{3G} = 0$$

$$D^{4G} = \frac{QR^{4G} + (1-Q)R^{3G} - P^{4G}}{(QR^{4G} + (1-Q)R^{3G}) + (1-Q)^2\gamma}$$

$\square$