Balance of revenue and social welfare in FCC's spectrum allocation

Yanjiao Chen, Lingjie Duan, Jianwei Huang, Qian Zhang

***Abstract*—To accommodate users' ever-increasing traffic in wireless broadband services, the Federal Communications Commission (FCC) in the U.S. is considering allocating additional spectrum to the wireless market. There are two major directions: licensed (e.g. 3G) and unlicensed services (e.g. Wi-Fi). On the one hand, 3G service can realize a high spectrum efficiency and provide ubiquitous connection. On the other hand, the Wi-Fi service (often with limited coverage) can provide users with high-speed local connections, but is subject to uncontrollable interferences. Regarding spectrum allocation, prior studies only focused on revenue maximization. However, one of FCC's missions is to better improve all wireless users' utilities. This motivates us to design a spectrum allocation scheme that jointly considers social welfare and revenue. In this paper, we formulate the interactions among the FCC, typical 3G and Wi-Fi operators, and the end-users as a three-stage dynamic game and derive the equilibrium of the entire game. Compared to the benchmark case where the FCC only maximizes its revenue, the consideration of social welfare will encourage the FCC to allocate more spectrum to the service which lacks spectrum to better serve its users. Such consideration for the social welfare, to our delight, brings limited revenue loss for the FCC.**

I. INTRODUCTION

The number of customers using wireless broadband services has been increasing dramatically during the recent years. Some studies suggested that such demands will surpass the capacities supported by currently available commercial wireless spectrum by as soon as 2013 [1]. To provide more resources for supporting these services, the Federal Communications Commission (FCC) intends to make 500 MHz of new wireless spectrum (e.g., TV Whitespace) available within 10 years (300 MHz for the next 5 years) [2]. In July of 2012, the President's Council of Advisors on Science and Technology (PCAST) of the U.S. [3], further proposed to identify 1,000 MHz of Federal spectrum for shared-use among commercial users. How to allocate and make the best use of the new spectrum resource in different services is not only a technical issue, but also a complicated policy and socio-economic issue.

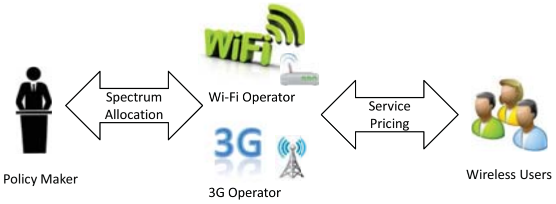


Fig. 1: Illustration of Mobile Service Model.

Cellular technology (represented by 3G[1]) and wireless local area network (WLAN) (represented by Wi-Fi) are two major technology choices of providing high-speed mobile Internet access. Wi-Fi and 3G services are very different. 3G service provides ubiquitous Internet connection and achieves high spectrum efficiency with low interferences, thanks to the exclusive spectrum licensing and centralized control (e.g., traffic scheduling and power control). Wi-Fi service often has limited overall coverage (due to a limited number of access points) and a low spectrum efficiency (due to interference generated by other services operating on the same unlicensed bands). But Wi-Fi is often cheaper and much easier to deploy and maintain than 3G services [4]. Users with different QoS requirements thus have different preferences over these two services.

When additional spectrum is available, the FCC needs to efficiently allocate them to these two existing services to satisfy different types of users. In the past, the FCC mainly conducts spectrum auction to assign license to the 3G operators, aiming at maximizing either economic return [5] or the utility of winning operators [6]–[8][2]. Spectrum auction for licensed use only caters to the need of those cellular users who are willing to pay a higher price for a full network coverage. This will lead to the loss of social welfare of Wi-Fi users, especially many low-end price-sensitive users who prefer affordable Wi-Fi service operating on unlicensed spectrum with limited coverage. The impact of unlicensed spectrum usage on social welfare was studied by [9], but these results only considered the price competition among unlicensed operators, which is a two-tier model involving the unlicensed operator and the

---

This work was supported by SUTD-MIT International Design Center Grant (Project No.: IDSF1200106OH), SUTD Start-up Research Grant (Project No.: SRG ESD 2012 042), the General Research Funds (Project No. CUHK 412710 and CUHK 412511) established under the University Grant Committee of the Hong Kong SAR, China, grants from RGC under the contracts CERG 623209 and 622410, HKUST grant SRFI11FYT01, the grant from Huawei-HKUST joint lab, and the National Natural Science Foundation of China under Grants No. 60933012.

Y. Chen and Q. Zhang are with the Department of Computer Science and Engineering, Hong Kong University of Science and Technology, {chenyanjiao, qianzh}@ust.hk.

L. Duan is with the Engineering Systems and Design Pillar, Singapore University of Technology and Design, lingjie_duan@sutd.edu.sg.

J. Huang is with the Department of Information Engineering, The Chinese University of Hong Kong, jwhuang@ie.cuhk.edu.hk.

[1] 4G is a successor of 3G. The candidate systems for 4G include WiMAX and LTE, yet it is controversial whether they truly meet 4G standards. In this paper, we use 3G as the representative of the existing cellular technology.

[2] It is argued that truthful auction can reveal the operators' true valuation for the spectrum, which equals the social welfare that can be created in the market. However, in the case of competition between 3G and Wi-Fi operators, the market may fail due to the strong market power of the cellular operators and the "tragedy of commons" of Wi-Fi's unlicensed spectrum usage.

978-1-4673-5946-7/13/$31.00 ©2013 IEEE

users. Existing results based on three-tier models involving the interaction of the policy maker, the operators and the end users include [10]–[12], but none of the prior results considered the spectrum allocation issue for the FCC. As hundreds of millions of users worldwide now make regular use of Wi-Fi to access the Internet, their social welfare cannot be ignored. A lack of comprehensive analysis on spectrum allocation between licensed and unlicensed usage motivates us to propose a new allocation scheme that jointly considers the economic return and social welfare of both 3G and Wi-Fi end-users.

In this paper, we formulate the interactions among the FCC, the 3G and the Wi-Fi operators, and the end-users as a three-stage dynamic game as illustrated in Fig. 1. In Stage I, the FCC decides the spectrum allocations to both services by jointly considering revenue and social welfare. In Stage II, two competitive operators optimally price their services to maximize their profits based on their limited spectrum allocations. Finally in Stage III, the users choose between the two services to maximize their own utilities after observing the prices.

Our key results and major contributions are as follows.

- *Multi-stage dynamic game formulation:* We formulate the interactions among the FCC, the 3G and the Wi-Fi operators, and the users as a three-stage dynamic game. Such a tight-knit three-tier structure enables the FCC to adjust the spectrum allocations to different operators to effectively improve end users' utilities. Despite the modeling complexity, we are able to fully characterize the subgame perfect equilibrium of the whole game.
- *Impact of social welfare on FCC's spectrum allocations:* As our theoretical analysis suggests, if social welfare is taken into consideration, when one service has rather a high user demand compared to the capacity determined by its spectrum resource, the FCC has to allocate more spectrum to this service in order to enhance its users' utilities and social welfare.
- *Operators' price competition under limited resources:* Since the Wi-Fi and the 3G operators compete for users in the same market, the price of one service tends to decrease if the price of the other service decreases due to the pressure of retaining customers.
- *Users' utility improvements:* Analysis on the user distribution shows that the number of users for each service will be determined by two factors: 1) *physical factor*: which is determined by the spectrum allocation and network capacity; 2) *economic factor*: which is determined by the price competition between the two services. When one service's opponent service has a low price, and the operator of this service also sets a low price, the physical factor becomes the dominant one to determine its number of users; otherwise, the economic factor becomes the dominant one. Extensive simulation results show that the proposed spectrum allocation scheme can greatly improve users' utilities and social welfare while inducing only limited revenue loss for the FCC, compared with a prior revenue-centric spectrum allocation scheme.

The rest of the paper is organized as follows. We briefly review the related work in Section II. The system model and basic assumptions are given in Section III. We describe the three-stage game framework in Section IV and use backward induction to analyze the game, from the *market response* of Stage III in Section V, to the *service competition game* of Stage II in Section VI, finally to the spectrum allocations of Stage I in Section VII. Simulation results are presented in Section VIII. We summarize our work in Section IX.

II. RELATED WORK

Lehr and McKnight in [4] surveyed the competitive and complementary relationship of Wi-Fi and 3G technology. Niyato and Hossain in [13] built a 2-tier pricing competition model between Wi-Fi and WiMAX operators. However, the paper did not provide any suggestions for the FCC on spectrum allocation between the two operators. Several recent results studied three-tier models that involve the spectrum owner, the operators and the users [10], [11]. But the focus of their work is not FCC's spectrum allocation problem.

Spectrum auction is another way of distributing spectrum to operators [6], [8]. However, the auctioned spectrum is only for exclusive licensed use by the 3G operators. This will impair utility of users who subscribe to unlicensed wireless service. That's why we propose a new spectrum allocation scheme that fulfils both licensed and unlicensed spectrum usage.

Nguyen *et al.* in [9] studied the influence of unlicensed spectrum quantity on social welfare, by using an over-simplified user demand function without suggestion on how spectrum should be allocated between licensed and unlicensed usage. However, the model did not consider the competition from licensed operators, nor the spectrum allocation issue.

III. SYSTEM MODEL

A. Spectrum Allocation

We consider a policy maker (referred to as FCC for the illustration purpose) who possesses S units spectrum chunks, and intends to assign the spectrum to wireless service operators to satisfy their increasing wireless data demands. In this paper, we only consider two wireless service operators: one cellular operator providing 3G services, and one wireless local area network (WLAN) operator providing Wi-Fi services. Also, we focus on the *additional* capacity yielded by the spectrum to be assigned to the two operators by the FCC, without considering their existing capacity.

We assume that the FCC charges the Wi-Fi and the 3G operators according to the amount of spectrum allocated to them. Let S_w and S_g denote the spectrum allocated to Wi-Fi and 3G, respectively. The FCC has no intention to reserve the spectrum for other purposes. Therefore, $S_g + S_w = S$.

We assume that the FCC charges the 3G operator a total amount of $\phi_g(S_g)$, and charges the Wi-Fi operator a total amount of $\phi_w(S_w)$. Both pricing functions are non-decreasing in the spectrum allocation, and they can be different .

As the government regulator, the FCC cares not only about the economic return (revenue) but also the social welfare.

Therefore, when making the spectrum allocation decisions, the FCC needs to properly trade off between the revenue and end users' utilities, by taking users' heterogenous preferences into consideration.

B. Service Model

1) Network Capacity: We assume that each user requires a fixed data rate of δ for applications[3] that require guaranteed QoS[4]. The capacity of 3G network is $f(S_g)$, under an exclusive spectrum license and efficient interference management. The capacity of Wi-Fi network is $f(\eta S_w)$, where $\eta < 1$ is the interference parameter and characterizes the low spectrum efficiency in sharing the unlicensed band with other unlicensed services (e.g., nearby microwaves and medical diathermy machines) [14]. So the maximum number of concurrent in-service users that can be supported by each service is $f(S_g)/\delta$ and $f(\eta S_w)/\delta$, respectively. We consider the requirement of data rate instead of bandwidth, thus circumventing the complicated issue of interference management among users. Therefore, our setup can be easily generalized and applied to different types of technologies, as long as we are able to characterize the relationship between bandwidth and capacity.

2) Network Coverage: A base station in cellular networks can serve cellular users over a relatively large contiguous geographic area. Therefore, one of cellular network's key features is to offer ubiquitous coverage in the new spectrum [4] based on the current network infrastructure without any *additional* investment. We assume that the 3G network's coverage equals 1.

Generally speaking, Wi-Fi access points (APs) are deployed at selected areas with high traffic demand, the so-called "hot-spots" (such as hotel, airport, and campus). A Wi-Fi hot-spot can serve users in a radius around $100 \sim 300$ meters [4].

Even if using the whitespace spectrum with better propagation characteristics, the existing ad hoc deployment of Wi-Fi access points may not be able to achieve a full coverage. Therefore, we assume Wi-Fi network's coverage equals $\theta < 1$, indicating that the Wi-Fi service only provides a partial coverage. According to [15], the coverage does not depend on bandwidth S_w, but mainly depends on the deployment of APs, which is related to the Wi-Fi operator's investment. According to [16], approximately 700 Wi-Fi hot-spots would be needed to cover the same area as one cellular base station under the current technology.

3) Service Price: We assume that the flat-rate subscription fees of Wi-Fi service and 3G service are p_w and p_g per user, respectively, as long as the data rate requirement is fulfilled. We consider a pool of N wireless users within the coverage of the 3G and Wi-Fi network. The numbers of users who finally choose the two services are denoted by N_w and N_g, respectively. When the user demand exceeds the network capacity, the operators need to perform proper admission controls, so we have $N_w \leq f_g(S_g)/\delta$ and $N_g \leq f_w(\eta S_w)/\delta$ at the equilibrium.

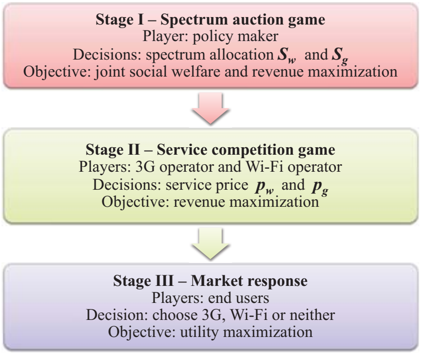


Fig. 2: Three stages of the dynamic game.

C. User Preference

Users are different in their preferences to the coverage of Internet access. We characterize such heterogeneity as by the type parameter $\alpha \in [0, A]$, which represents users' sensitivity due to their mobility [17]. A user with a high mobility prefers high coverage, while a user who always stays at one location is less sensitive to coverage in other places. The cumulative distribution function (cdf) of the user types is Γ. For simplification, we assume that Γ is a uniform distribution in $[0, A]$.

For a type α user, his utility is $u_g^\alpha = \alpha - p_g$ if choosing 3G; and $u_w^\alpha = \alpha\theta - p_w$ if choosing Wi-Fi. Each user also has a reserve utility u_0[5]. A type α user's service choice is as follows:

$$\begin{cases} \text{3G service,} & \text{if } u_g^\alpha \geq \max\{u_w^\alpha, u_0\} \\ \text{Wi-Fi service,} & \text{if } u_w^\alpha > \max\{u_g^\alpha, u_0\} \\ \text{No service,} & \text{otherwise} \end{cases} \tag{1}$$

IV. THREE-STAGE GAME FRAMEWORK

The three-stage game is illustrated in Fig. 2: in Stage I, the FCC allocates spectrum to the two network operators; in Stage II, two network operators determine service prices to maximize their profits; in Stage III, each user chooses the service type to maximize his utility.

A. Stage I - Spectrum Allocation

The FCC decides the spectrum allocation S_w and S_g to maximize its utility, U_f, which is the weighted sum of the revenue and the social welfare SW,

$$U_f = \beta SW + \phi_g(S_g) + \phi_w(S_w). \tag{4}$$

Here the FCC can adjust the weight β to tailor for different spectrum allocation goals. If the FCC cares more about social welfare than revenue, then β can be set larger than 1.

As the charges between the FCC and the operators, as well as between the operators and the users are internal transfers, they do not affect the social welfare.

---

[3] 3G service can satisfy users' fixed δ requirement, as licensed spectrum access can ensure QoS. Thanks to the better propagation characteristics of the whitespace than the ISM band, future Wi-Fi 2.0 network also intends to provide QoS-guaranteed service to satisfy users' fixed δ requirement [10].

[4] In the future work, we plan to consider wireless applications that does not require guaranteed QoS, thus a variant δ.

[5] We assume the same reserve utility for 3G and Wi-Fi when users do not differentiate the Internet access service provided by 3G and Wi-Fi

$$N_w = \begin{cases} \frac{f(\eta S_w)}{\delta}, & \text{if } p_g - (1-\theta)A \le p_w < \theta p_g - (1-\theta)u_0 - \frac{A\theta(1-\theta)f(\eta S_w)}{N\delta}, \\ \overline{N_w}, & \text{if } \theta p_g - (1-\theta)u_0 - \frac{A\theta(1-\theta)f(\eta S_w)}{N\delta} \le p_w < \theta p_g - (1-\theta)u_0, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

$$N_g = \begin{cases} \frac{f(S_g)}{\delta}, & \text{if } \frac{1}{\theta}[p_w + (1-\theta)u_0] \le p_g < p_w + (1-\theta)A - \frac{A(1-\theta)f(S_g)}{N\delta}, \\ \overline{N_g}, & \text{if } p_w + (1-\theta)A - \frac{A(1-\theta)f(S_g)}{N\delta} \le p_g < p_w + (1-\theta)A, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The key factor that impacts social welfare is how wireless users value the network coverage. So we define social welfare as the weighted sum of Wi-Fi users' enjoyed coverage and 3G users' enjoyed coverage:

$$SW = \omega_1 \int_{\text{Wi-Fi users}} \frac{\theta}{A} d\alpha + \omega_2 \int_{\text{3G users}} \frac{1}{A} d\alpha \tag{5}$$

where the first term is Wi-Fi users' aggregated valuation of coverage, and the second term is 3G users' aggregated valuation of coverage. The parameters ω_1 and ω_2 represent the relative importance of Wi-Fi and 3G network from FCC's point of view.

B. Stage II - Service Competition Game

Two operators play a pricing game in Stage II, where they determine the prices to their own users to maximize the profits. The spectrum allocations are assumed to be fixed in this stage.

The utility of the 3G operator is the difference between the revenue and the spectrum payment:

$$U_g = N_g p_g - \phi_g(S_g) \tag{6}$$

The utility of the Wi-Fi operator is the difference between the revenue and the spectrum payment:

$$U_w = N_w p_w - \phi_w(S_w) \tag{7}$$

C. Stage III - Market Response

After observing the prices of 3G and Wi-Fi services, users compare and decide which service to subscribe[6]. We will show in Section VII that the number of users choosing a certain service is influenced by two factors: 1) the *physical factor*: the network capacity determined by spectrum resource, and 2) the *economic factor*: the prices of both services.

Definition 1: Nash Equilibrium: Given a game $\{\mathcal{I}, (S_i)_{i\in\mathcal{I}}, (u_i)_{i\in\mathcal{I}}\}$[7], a (pure) strategy profile $s^* \in S$ is a Nash Equilibrium if the following is true for all $i \in \mathcal{I}$

$$u_i(s_i^*, s_{-i}^*) \ge u_i(s_i, s_{-i}^*), \forall s_i \in S_i. \tag{8}$$

Definition 2: Subgame Perfect Equilibrium (SPE): A strategy profile of the three-stage game is an SPE if the choices of the FCC, the 3G and the Wi-Fi operators, as well as the end users constitute a Nash Equilibrium in each of the subgame[8] of the whole game. In other words, no player at SPE will deviate from his equilibrium strategy.

In Sections V to VII, we will derive the SPE of the three-stage game using backward induction, from Stage III to I.

V. STAGE III- MARKET RESPONSE

In Stage III, users can observe the two prices given in Stage II: p_w and p_g. Recall that the numbers of subscribers of the Wi-Fi and 3G services are N_w and N_g, respectively.

Proposition 1: Given the 3G and Wi-Fi service prices p_g and p_w, and their capacities $f_g(S_g)$ and $f_w(\eta S_w)$, the number of Wi-Fi users N_w depends on the 3G price p_g as follows:

- *Low 3G price* ($p_g \le A - u_0 - A\theta f(\eta S_w)/N\delta$): the subscriber number of Wi-Fi service is given in (2).
- *High 3G price* ($p_g > A - u_0 - A\theta f(\eta S_w)/N\delta$): the subscriber number of Wi-Fi service is

$$N_w = \begin{cases} \overline{N_w}, & \text{if } p_g - (1-\theta)A \le p_w < \theta p_g - (1-\theta)u_0, \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

in which

$$\overline{N_w} = \frac{N}{A\theta(1-\theta)}[\theta p_g - p_w - (1-\theta)u_0]. \tag{10}$$

Proposition 2: Given the 3G and Wi-Fi service prices p_g and p_w, and their capacities $f_g(S_g)$ and $f_w(\eta S_w)$, the number of 3G users N_g depends on the Wi-Fi price p_w as follows:

- *Low Wi-Fi price* ($p_w \le \theta A - u_0 - A\theta f(S_g)/N\delta$): the subscriber number of 3G service is given in (3).
- *High Wi-Fi price* ($p_w > \theta A - u_0 - A\theta f(S_g)/N\delta$): the subscriber number of 3G service is

$$N_g = \begin{cases} \overline{N_g}, & \text{if } \frac{1}{\theta}[p_w + (1-\theta)u_0] \le p_g < p_w + (1-\theta)A, \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

in which

$$\overline{N_g} = \frac{N}{A(1-\theta)}[-p_g + p_w + (1-\theta)A] \tag{12}$$

Propositions 1 and 2 do not include all Wi-Fi and 3G price combinations. Due to the page limitation, we skip discussions on price combinations that are proven to be impossible in the equilibrium of Stage II. The complete user distribution characterization under all possible prices is given in the online technical report [18]. The proofs of Propositions 1 and 2 are also given in the online technical report [18]. The number of Wi-Fi users corresponding to Proposition 1 is shown in Fig. 3 and Fig. 4, in which the red curves denote the Wi-Fi user number. Fig. 3 corresponds to equation (2), and Fig. 4 corresponds to equation (9).

---

[6] Note that a user's subscription to one service may be rejected if the capacity of that service is not enough to accommodate the user's demand.

[7] $\mathcal{I}$ is the set of players, S_i is the strategy space of player i, and u_i is the utility of player i. s_{-i} is the strategy profile of all players other than i. We denote by $S = \Pi_i S_i$ the set of strategy profiles.

[8] There are three subgames in our model: Stage III is a subgame, Stages II and III together is another subgame, and the whole game with three stages is also a subgame.

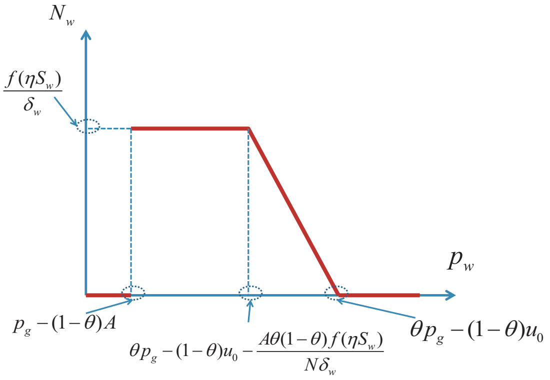


Fig. 3: Number of Wi-Fi users when $p_g \leq A - u_0 - \frac{A\theta f(\eta S_w)}{N\delta}$.

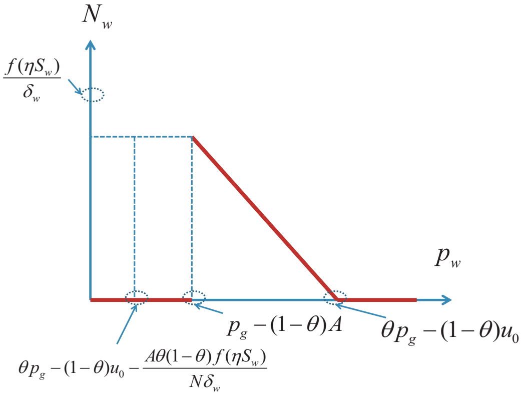


Fig. 4: Number of Wi-Fi users when $p_g \geq A - u_0 - \frac{A\theta f(\eta S_w)}{N\delta}$.

Propositions 1 and 2 show that there are two factors that influence the number of users for each service.

- *Physical factor*: $f(\cdot)/\delta$. When both operators' prices are relatively low, the subscriber number of a service is determined by its spectrum allocation (see the first cases of (2) in low 3G price and (3) in low Wi-Fi price).
- *Economic factors*: p_w, p_g, A, and u_0. (10) and (12) show that the demand for one service decreases in its own price but increases in its opponent service's price. In (10), it is shown that if the users' reserve utility u_0 is low, Wi-Fi may have more users and its highest profitable price (that still yields a non-negative revenue) is higher (as shown in Figures 3 and 4). In (12), it is shown that if the highest user type A increases, 3G may have more users and its highest profitable price is higher.

VI. STAGE II - SERVICE COMPETITION GAME

In Stage II, given the spectrum allocations from the FCC, two operators need to optimize their prices by incorporating the users' behavior in Stage III.

It is clear that one operator's revenue not only depends on his own price but also on his competitor's price (see Propositions 1 and 2). Given an operator's price, the other operator can decide his best pricing choice that maximizes his profit (also called best response). The equilibrium prices correspond to the intersection point of two operators' best response functions. Formally, we define the following concepts.

Definition 3 (Best Response): Given the 3G service price p_g, the Wi-Fi operator's best response price is $p_w^*(p_g)$, satisfying that $U_w(p_w^*(p_g), p_g) \geq U_w(p_w, p_g)$, for any $p_w \geq 0$; Given the Wi-Fi service price p_w, the 3G operator's best response price is $p_g^*(p_w)$, satisfying that $U_g(p_g^*(p_w), p_w) \geq U_g(p_g, p_w)$, for any $p_g \geq 0$.

When each operator employs its best response with regard to the other operator's price, it has no incentive to change. When both are choosing best responses simultaneously, they reach the equilibrium in this Stage II pricing game.

Proposition 3: In Stage II, the equilibrium 3G and Wi-Fi prices satisfy the following conditions:

$$\frac{1}{\theta}[p_w + (1-\theta)u_0] \leq p_g \leq p_w + (1-\theta)A,$$

$$\text{or equivalently, } p_g - (1-\theta)A \leq p_w \leq \theta p_g - (1-\theta)u_0. \tag{13}$$

Proposition 3 suggests that at the equilibrium, the price of one service will be upper bounded and lower bounded depending on the price of the other service. The upper bound ensures that that service is not too expensive so as to have no subscribers and zero revenue. The lower price bound is due to the maximum number of users in the system, as a price too low will only decrease the revenue without attracting more users.

To better understand Proposition 3, we introduce three critical user type.

- *Indifferent to Wi-Fi and no service user type*. There exists a critical user type α_1^{th}, who is indifferent to choosing Wi-Fi service or no service at all. It satisfies $\alpha_1^{th}\theta - p_w = u_0$, which leads to $\alpha_1^{th} = (p_w + u_0)/\theta$. Users of type $\alpha > \alpha_1^{th}$ prefer Wi-Fi service to no service.
- *Indifferent to 3G and no service user type*. There exists a critical user type α_2^{th}, who is indifferent to choosing 3G service or no service at all. It satisfies $\alpha_2^{th} - p_g = u_0$, which leads to $\alpha_2^{th} = p_g + u_0$. Users of type $\alpha > \alpha_2^{th}$ prefer 3G service to no service.
- *Indifferent to Wi-Fi and 3G service user type*. There exists a critical user type α_3^{th}, who is indifferent to choosing Wi-Fi service or 3G service. It satisfies $\alpha_3^{th}\theta - p_w = \alpha_3^{th} - p_g$, which leads to $\alpha_3^{th} = (p_g - p_w)/(1-\theta)$. Here, it is possible that $\alpha_3 \leq 0$. Users of type $\alpha > \alpha_3^{th}$ prefer 3G service to Wi-Fi service.

We can show that $\alpha_2^{th} = \theta\alpha_1^{th} + (1-\theta)\alpha_3^{th}$, so there are only two possibilities: $\alpha_1^{th} \leq \alpha_2^{th} \leq \alpha_3^{th}$ or $\alpha_3^{th} \leq \alpha_2^{th} \leq \alpha_1^{th}$.

Lemma 1: At an equilibrium of the pricing game in Stage II, we can only have $\alpha_1^{th} \leq \alpha_2^{th} \leq \alpha_3^{th} \leq A$.

Proof: First, it is trivial to prove that $\alpha_1^{th} \leq A$. This is because if $\alpha_1^{th} > A$, no users will choose Wi-Fi service, and thus the Wi-Fi operator has an incentive to decrease p_w to achieve a positive revenue. Similarly, we can prove that $\alpha_2^{th} \leq A$.

Now we prove that $\alpha_1^{th} \leq \alpha_2^{th} \leq \alpha_3^{th} \leq A$ by contradiction. Assume that $\alpha_3^{th} \leq \alpha_2^{th} \leq \alpha_1^{th} \leq A$ as shown in the upper

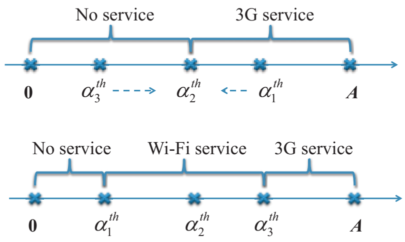


Fig. 5: Users' partitions into three types 1) $\alpha_3^{th} \leq \alpha_2^{th} \leq \alpha_1^{th} \leq A$; 2) $\alpha_1^{th} \leq \alpha_2^{th} \leq \alpha_3^{th} \leq A$.

subfigure in Fig. 5, which means that (i) users of type $\alpha \in [0, \alpha_2^{th}]$ will choose no service, and (ii) users of type $\alpha \in [\alpha_2^{th}, A]$ will choose 3G service. However, the Wi-Fi operator has an incentive to reduce p_w to decrease α_1^{th} (and thus also decrease α_3^{th} under a fixed p_g.) until $\alpha_1^{th} < \alpha_3^{th}$, so that the Wi-Fi operator's revenue increases from zero to positive. This completes the proof.

■

By combining Lemma 1 and expressions of $\alpha_1^{th}, \alpha_2^{th}$, and α_3^{th}, we can prove Proposition 3 (details in [18]). With Proposition 3, we can now analyze the best responses and the equilibrium prices of Wi-Fi and 3G services.

Let's define two spectrum benchmarks, which are the bandwidth thresholds to distinguish whether the bandwidth in Wi-Fi or 3G service is limited or adequate.

$$\begin{aligned} \widetilde{S_w} &= \frac{1}{\eta} f^{-1}\left(\frac{N\delta(\theta A - u_0)}{\theta(2-\theta)A}\right), \\ \widetilde{S_g} &= f^{-1}\left(\frac{N\delta(A - u_0)}{\theta(2-\theta)A}\right). \end{aligned} \tag{14}$$

A. The Wi-Fi Operator's Best Response

Proposition 4: Define

$$\overline{p_w} = \frac{1}{2}[\theta p_g - (1-\theta)u_0]. \tag{15}$$

Given the 3G operator's service price as p_g, the best response of the Wi-Fi operator $p_w^*(p_g)$ is as follows.

- *Limited Wi-Fi Spectrum Resource*. If $S_w \leq \widetilde{S_w}$, the optimal price for the Wi-Fi operator is given in (16).
- *Adequate Wi-Fi Spectrum Resource*. If $S_w \geq \widetilde{S_w}$, the optimal price for the Wi-Fi operator is

$$p_w^* = \begin{cases} \overline{p_w}, & \text{if } p_g \leq \frac{1-\theta}{2-\theta}(2A - u_0) \\ p_g - (1-\theta)A, & \text{otherwise} \end{cases} \tag{17}$$

The proof of Proposition 4 is given in the online technical report [18]. Proposition 4 suggests that in general, the best response of the Wi-Fi price increases with the 3G price, because it is the relative price that decides user's choice. As the 3G meets the spectrum bottleneck (the third term in (16) and the second term in (17)), the increase speed of Wi-Fi price becomes faster (the slope of the best response function becomes steeper). This is because users can no longer switch to 3G service due to 3G capacity limitation even if the Wi-Fi operator raises Wi-Fi price. However, if the spectrum resource for Wi-Fi service is limited itself, hence the increase speed of Wi-Fi price experiences a transition period (the second term in (16)).

B. The 3G Operator's Best Response

Proposition 5: Define

$$\overline{p_g} = \frac{1}{2}[p_w + (1-\theta)A]. \tag{18}$$

Given the Wi-Fi operator's service price as p_w, the best response of the 3G operator $p_g^*(p_w)$ is as follows.

- *Limited 3G Spectrum Resource*. If $S_g \leq \widetilde{S_g}$, the optimal price for the 3G operator is given in (19).
- *Adequate 3G Spectrum Resource*. If $S_g \geq \widetilde{S_g}$, the optimal price for the 3G operator is

$$p_g^* = \begin{cases} \overline{p_g}, & \text{if } p_w \leq \frac{1-\theta}{2-\theta}(\theta A - 2u_0) \\ \frac{1}{\theta}[p_w + (1-\theta)u_0], & \text{otherwise} \end{cases} \tag{20}$$

The proof of Proposition 5 is given in our online technical report [18]. Proposition 5 shows that $\overline{p_g}$ increases in p_w and A. As A increases, users' average valuation increases and operator can charge a higher price.

C. Equilibrium Prices

Let's define

$$\begin{aligned} \widehat{S_w} &= \frac{1}{\eta} f^{-1}\left(\frac{N\delta(\theta A - 2u_0)}{\theta(4-\theta)A}\right), \\ \widehat{S_g} &= f^{-1}\left(\frac{N\delta(2A - u_0)}{(4-\theta)A}\right). \end{aligned} \tag{21}$$

As analyzed in Sections VI-A and VI-B, there are four resource conditions, generating different best response combinations.

Theorem 1: In Stage II, the optimal pricing decisions of the Wi-Fi and 3G operators are summarized in Table I. The four spectrum conditions are characterized as

- *Crowded Wi-Fi and Crowded 3G*. $S_w \leq \widehat{S_w}, S_g \leq \widehat{S_g}$;
- *Idle Wi-Fi versus Crowded 3G*. $S_w \geq \widehat{S_w}, S_g \leq \widehat{S_g}$;
- *Crowded Wi-Fi versus Idle 3G*. $S_w \leq \widehat{S_w}, S_g \geq \widehat{S_g}$;
- *Idle Wi-Fi and Idle 3G*. Spectrum condition other than the above three conditions.

According to Theorem 1 and Table I, when the spectrum resource is abundant for both services, the equilibrium prices do not depend on the spectrum allocation; when the spectrum resource is scarce for 3G service, the 3G price will decrease with S_g, and the Wi-Fi price decreases as well but at a lower speed. We have the similar observation when the spectrum resource is scarce for Wi-Fi service. When the spectrum resource is scarce for both services, their prices decrease in both its own spectrum amount and the competitor's spectrum.

VII. STAGE I - SPECTRUM ALLOCATION

In Stage I, the FCC solves an optimization problem of spectrum allocation based on the equilibrium prices in Stage II and the user choices in Stage III. We consider different situations in accordance with the four spectrum conditions in Stage II.

TABLE I: Equilibrium price in Stage II

	3G service price p_g	**Wi-Fi service price p_w**
Crowded Wi-Fi service & 3G service	$A - u_0 - \frac{A[\theta f(\eta S_w)+f(S_g)]}{N\delta}$	$\theta A - u_0 - \frac{A\theta[f(\eta S_w)+f(S_g)]}{N\delta}$
Idle Wi-Fi service vs Crowded 3G service	$\frac{(1-\theta)[2A-u_0-\frac{2Af(S_g)}{N\delta}]}{2-\theta}$	$\frac{(1-\theta)[\theta A-u_0-\frac{\theta Af(S_g)}{N\delta}]}{2-\theta}$
Crowded Wi-Fi service vs Idle 3G service	$\frac{(1-\theta)[A-u_0-\frac{\theta Af(\eta S_w)}{N\delta}]}{2-\theta}$	$\frac{(1-\theta)[\theta A-2u_0-\frac{2\theta Af(\eta S_w)}{N\delta}]}{2-\theta}$
Idle Wi-Fi service & 3G service	$A - u_0$	$\theta A - u_0$

$$p_w^* = \begin{cases} \overline{p_w}, & \text{if } p_g \le (\frac{1}{\theta} - 1)u_0 + \frac{2A(1-\theta)f(\eta S_w)}{N\delta} \\ 2\overline{p_w} - \frac{A\theta(1-\theta)f(\eta S_w)}{N\delta}, & \text{if } (\frac{1}{\theta} - 1)u_0 + \frac{2A(1-\theta)f(\eta S_w)}{N\delta} < p_g \le A - u_0 - \frac{A\theta f(\eta S_w)}{N\delta} \\ p_g - (1-\theta)A, & \text{otherwise} \end{cases} \tag{16}$$

$$p_g^* = \begin{cases} \overline{p_g}, & \text{if } p_w \le \frac{2A(1-\theta)f(S_g)}{N\delta} - (1-\theta)A \\ 2\overline{p_g} - \frac{A(1-\theta)f(S_g)}{N\delta}, & \text{if } \frac{2A(1-\theta)f(S_g)}{N\delta} - (1-\theta)A < p_w \le \theta A - u_0 - \frac{A\theta f(S_g)}{N\delta} \\ \frac{1}{\theta}[p_w + (1-\theta)u_0], & \text{otherwise} \end{cases} \tag{19}$$

TABLE II: Optimal spectrum allocation in Stage I

	Spectrum Condition $\mathcal{S}$	$S_w^* = \overline{S_w}$**, if** $\overline{S_w} \in \mathcal{S}$	S_w^***, if** $\overline{S_w} >$ **upperbound**	S_w^***, if** $\overline{S_w} <$ **lowerbound**
Spectrum shortage	$S - \widehat{S_g} \le S_w \le \widehat{S_w}$	$\overline{S_w} : -\frac{\beta(1-\theta\eta)f'}{N\delta} - \phi'_g + \phi'_w = 0$	$\widehat{S_w}$	$S - \widehat{S_g}$
3G spectrum shortage	$S_w > \max(\widetilde{S_w}, S - \widehat{S_g})$	$\overline{S_w} : -\frac{2\beta(1-\theta)f'}{(2-\theta)N\delta} - \phi'_g + \phi'_w = 0$	S	$\max(\widetilde{S_w}, S - \widehat{S_g})$
Wi-Fi spectrum shortage	$S_w < \min(S - \widetilde{S_g}, \widehat{S_w})$	$\overline{S_w} : \frac{\beta\theta(1-\theta)\eta f'}{(2-\theta)N\delta} - \phi'_g + \phi'_w = 0$	$\min(S - \widetilde{S_g}, \widehat{S_w})$	0
Adequate spectrum	$\widetilde{S_w} \le S_w \le S - \widetilde{S_g}$	$\overline{S_w} : -\phi'_g + \phi'_w = 0$	$S - \widetilde{S_g}$	$\widetilde{S_w}$

We first analyze the social welfare in (5). As an example, we assign $\omega_1 = \omega_2 = 1$, i.e., we view Wi-Fi and 3G network as equally important[9]. So the social welfare becomes

$$SW = \frac{\theta}{A} \int_{\text{Wi-Fi users}} d\alpha + \frac{1}{A} \int_{\text{3G users}} d\alpha = \frac{1}{N}(\theta N_w + N_g).$$

According to Propositions 4 and 5 in the Stage II, the best responses of the 3G and Wi-Fi operators always result in $N_w = \overline{N_w}$ and $N_g = \overline{N_g}$. (Note that $\overline{N_w}$ and $\overline{N_g}$ are defined in equations (10) and (12), respectively.) Intuitively, when the number of users preferring 3G exceeds 3G's capacity ($N_g = f(S_g)/N\delta > \overline{N_g}$), the 3G operator can slightly raise its price p_g to make $\overline{N_g}$ equal $f(S_g)/N\delta$. In this way, the 3G operator can get a higher revenue in serving the same group of users. Therefore,

$$SW = \frac{1}{N}(\theta\overline{N_w} + \overline{N_g}) = -\frac{1}{A}(p_g - A + u_0). \tag{22}$$

Here, it seems that the social welfare only depends on price p_g; however, operators' interactions make p_w and p_g interdependent at the equilibrium in Stage II, so the social welfare will also be affected indirectly by p_w. In the following analysis, we focus on computing S_w, by keeping in mind that $S_g = S - S_w$. To derive clean insights for FCC's allocations to 3G and Wi-Fi services, we assume that $A = 1$ and $u_0 = 0$.

A. *Revenue-centric Spectrum Allocation*

We first derive the spectrum allocation when the FCC only cares about its revenue. This is the special case with $\beta = 0$, which serves as a benchmark to compare with our proposed spectrum allocation scheme. FCC's revenue-only utility is $U_f^0 = \phi_g(S_g) + \phi_w(S_w)$. The first derivative is $\frac{\partial U_f^0}{\partial S_w} = -\phi'_g + \phi'_w$. By forcing $\partial U_f^0/\partial S_w = 0$, we can get S_w^0. The second derivative is $\frac{\partial^2 U_f^0}{\partial S_w^2} = \phi''_g + \phi''_w$. We assume that $\phi''_g < 0$ and $\phi''_w < 0$, which means that the unit price will decrease as the spectrum increases[10]. In this case, spectrum allocation S_w^0 generates the maximum U_f^0, so it is the optimal choice for the FCC. This spectrum allocation scheme only depends on how the FCC designs the payment function ϕ_g and ϕ_w, without considering end users' utility.

B. *Joint Social Welfare and Revenue Spectrum Allocation*

When jointly considering social welfare and revenue, the FCC maximizes its utility as in (4).

Theorem 2: In Stage I, FCC's optimal spectrum allocation for Wi-Fi service S_w is summarized in Table II, and the optimal spectrum allocation for 3G service is $S_g = S - S_w$.

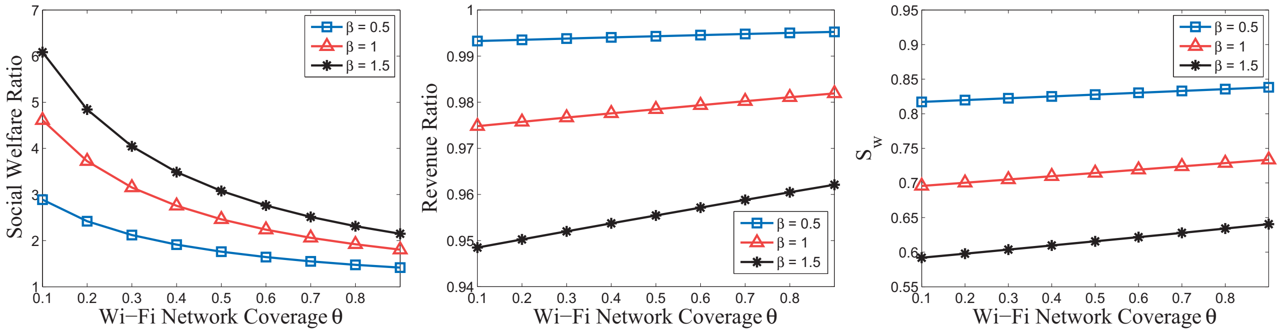
The proof of Theorem 2 is given in online technical report [18]. Theorem 2 and Table II yield the following insights:

- *Spectrum compensation to the service suffering from spectrum shortage.* In the second row of Table II, optimal

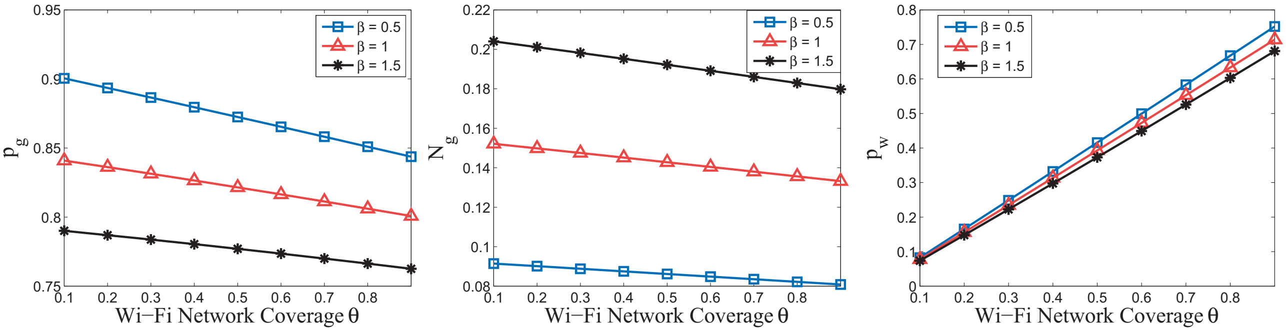
---

[9] Our results can also be extended to different weights of ω_1 and ω_2 when the FCC has different preference towards serving users, e.g., if he intends to focus on high-end users, he would pay more attention to 3G service with $\omega_1 < \omega_2$.

[10] This is reasonable because, according to diminishing marginal returns [19] in economics, the marginal improvement in transmission QoS will decrease as the amount of spectrum increases while keeping other factors constant. So the FCC should charge a smaller unit price as the amount of spectrum increases.



(a) Social Welfare Ratio (b) Revenue Ratio (c) Wi-Fi spectrum allocation



(d) 3G service price (e) 3G user number (f) Wi-Fi service price

Fig. 6: Impact of Wi-Fi network coverage θ and emphasis weight β on social welfare.

value for S_w always satisfies $\phi'_w > \phi'_g$. Since $\phi''_w < 0$, ϕ'_w is a decreasing function. This means $\overline{S_w} < S_w^0$, which indicates when the spectrum for 3G is scarce (under the revenue centric benchmark in Section VII-A), the FCC will reduce the allocation to the Wi-Fi service in order to increase the spectrum allocation to the 3G service to increase social welfare. Similarly, when the spectrum for the Wi-Fi service is scarce, the FCC will allocate more spectrum for the Wi-Fi service.

- *3G-favored spectrum allocation when both service suffer from spectrum shortage.* In the first row of Table II, optimal value for S_w always satisfies $\phi'_w > \phi'_g$, so $\overline{S_w} < S_w^0$. the FCC tends to prioritize alleviating the spectrum shortage of 3G network first, partly due to the 3G service's wider coverage and high spectrum efficiency.
- *No need to consider social welfare when spectrum is abundant for both service.* In the fourth row of Table II, the spectrum allocation results are the same as the revenue-centric spectrum allocation. This indicates that when spectrum resource is abundant, the FCC can safely ignore the influence of social welfare.
- *Optimal spectrum allocation is closely related to Wi-Fi network coverage θ and spectrum efficiency η.* For instance, the first row of Table II shows that, if θ or η increases, ϕ'_w decreases and S_w increases. This means that, if the Wi-Fi operator improves its network coverage or spectrum efficiency, it can obtain more spectrum from the FCC.

VIII. SIMULATION RESULTS

The insights in Section VII show that the Wi-Fi network coverage θ and weight for social welfare β greatly impact the spectrum allocation. In this section, we conduct simulations to evaluate these impacts as well as the performance of the proposed spectrum allocation scheme in terms of social welfare and revenue. Due to the page limit, we focus on the most interesting scenario where the spectrum resource is severely limited for both Wi-Fi and 3G network, i.e., $S_w \leq \widehat{S_w}$ and $S_g \leq \widehat{S_g}$. We set the parameters as follows: $N = 1, S = 1, \delta = 1, f(x) = 0.5x, \phi_g(S_g) = \ln 10(1 + S_g)$, and $\phi_w(S_w) = \ln(1 + 10S_w)$.

We define the **Social Welfare Ratio** as the ratio between the social welfare of our proposed Social-aware Spectrum Allocation Scheme (SSAS) and that of the Revenue-centric Spectrum Allocation Scheme (RSAS). We also define the **Revenue Ratio** in a similar fashion

Fig. 6 comprehensively shows the social welfare ratio, the revenue ratio, the spectrum allocation S_w to Wi-Fi, 3G price p_g, 3G user number N_g, and Wi-Fi price p_w as functions of Wi-Fi coverage θ and the FCC's emphasis on social welfare (i.e., weight β in (4)).

A. Impact of Wi-Fi Network Coverage

When the Wi-Fi coverage θ is small, Fig. 6(a) shows that our proposed allocation scheme brings in significant improvement of social welfare than SSAS, because the RSAS scheme only considers the revenue and tends to assign excessively high amount of spectrum to the 3G operator, sacrificing the benefits of (low-end) Wi-Fi users. Such social welfare gain will decrease as the Wi-Fi coverage θ increases, since the QoS difference between the coverage of Wi-Fi network and 3G network becomes smaller and the Wi-Fi no longer attracts only low-end users. Fig. 6(b) shows that the SSAS sacrifices limited revenue as the revenue ratio are all above 85%. As the network coverage increases, Fig. 6(c) shows that the spectrum allocated to Wi-Fi network S_w is slightly increased, because now the Wi-Fi can provide a higher utility and generate higher social welfare. When the Wi-Fi coverage is small, the Wi-Fi service price is very low compared with 3G service price. Surprisingly, when the Wi-Fi coverage increases, the two service prices head in different directions. When θ increases, Fig. 6(d) shows that p_g decreases because Wi-Fi competes with 3G more intensely and 3G operator has to reduce 3G price to maintain users. When θ increases, the Wi-Fi service price p_w is influenced by two factors: 1) p_w may decrease since its competitor's price p_g decreases; 2) p_w may increase since its coverage improved. Fig. 6(f) shows that the second factor dominates.

B. Impact of Weight of Social Welfare

When the weight of social welfare β increases, the FCC emphasizes more on the social welfare compared with the revenue. Therefore, the utility maximization results yield higher social welfare as shown in Fig. 6(a). This, in turn, decreases the revenue as shown in Fig. 6(b). As we discussed in Section VII-B, when both 3G and Wi-Fi services suffer from spectrum shortage, the FCC tends to favors 3G service by allocating more spectrum to 3G, which decreases Wi-Fi spectrum allocation as shown in Fig. 6(c). The reason is as follows. When the 3G operator gets more spectrum from the FCC, 3G network capacity increases and more users can be supported. Hence, 3G operator reduces 3G price (as shown in Fig. 6(d)) to allow more users access 3G service (as shown in Fig. 6(e)). Meanwhile, though Wi-Fi network can support less users, great price pressure from 3G service forces the Wi-Fi operator to reduce Wi-Fi price (as shown in Fig. 6(f)). Though the number of Wi-Fi users decreases a little, the total number of users served increases[11] and the number of 3G users increases. This means that some of the users have switched from Wi-Fi to 3G, thus having a higher utility. So the overall social welfare is improved.

IX. CONCLUSION

In this paper, we proposed a novel spectrum allocation scheme that enables the FCC to take into account both the revenue and the social welfare of the entire wireless market. We build a comprehensive three-tier game model for the wireless market interactions, which involves the FCC, the Wi-Fi and the 3G operators, and the end users. We used backward induction to analyze the game, by first calculating the user distribution in the Stage III, then deriving the equilibrium prices of both Wi-Fi and 3G services in Stage II, and finally demonstrating the optimal spectrum allocation for the FCC to maximize the weighted sum of revenue and social welfare in the Stage I. The results show that when the spectrum resource is limited for one service, the social welfare consideration will lead to more spectrum allocation to that service. Extensive simulations show that the proposed spectrum allocation scheme can significantly improve social welfare while generating more than $\%85$ revenues compared with the revenue-centric spectrum allocation scheme.

In the future, we plan to further consider best-effort wireless applications, where each end user's demand is no longer a constant δ and will change based on the prices. We will also look at the case where the FCC does not have complete network information, e.g., the coverage of the Wi-Fi provider.

REFERENCES

[1] M. Reardon, "Rethinking the wireless spectrum crisis," *http://news.cnet.com/8301-30686_3-20005831-266.html*, 2010.

[2] ——, "Fcc unveils national broadband plan," *http://news.cnet.com/8301-30686_3-20000453-266.html*, 2010.

[3] E. O. of the President President's Council of Advisors on Science and Technology, "Realizing the full potential of government-held spectrum to spur economic growth."

[4] W. Lehr and L. McKnight, "Wireless internet access: 3g vs. wifi?" *Telecommunications Policy*, vol. 27, no. 5-6, pp. 351–370, 2003.

[5] J. Jia, Q. Zhang, Q. Zhang, and M. Liu, "Revenue generation for truthful spectrum auction in dynamic spectrum access," in *ACM, Mobihoc 2009*.

[6] B. L. Wei WAng and B. Li, "Designing truthtul spectrum double auctions with local markets," *Mobile Computing, IEEE Transactions on*.

[7] M. Dong, G. Sun, X. Wang, and Q. Zhang, "Combinatorial auction with time-frequency flexibility in cognitive radio networks," in *IEEE, INFOCOM 2012*.

[8] Y. Zhu, B. Li, and Z. Li, "Truthful spectrum auction design for secondary networks," in *IEEE, INFOCOM 2012*.

[9] T. Nguyen, H. Zhou, R. Berry, M. Honig, and R. Vohra, "The impact of additional unlicensed spectrum on wireless services competition," in *IEEE, DySPAN 2011*.

[10] H. Kim, J. Choi, and K. Shin, "Wi-fi 2.0: Price and quality competitions of duopoly cognitive radio wireless service providers with time-varying spectrum availability," in *IEEE, INFOCOM 2011*.

[11] L. Duan, J. Huang, and B. Shou, "Duopoly competition in dynamic spectrum leasing and pricing," *IEEE Transactions on Mobile Computing*, no. 99, pp. 1–1, 2011.

[12] J. Jia and Q. Zhang, "Competitions and dynamics of duopoly wireless service providers in dynamic spectrum market," in *ACM, MobiHoc 2008*.

[13] D. Niyato and E. Hossain, "Competitive pricing in heterogeneous wireless access networks: Issues and approaches," *Network, IEEE*, vol. 22, no. 6, pp. 4–11, 2008.

[14] S. Yun, Y. Yi, D. Cho, and J. Mo, "The economic effects of sharing femtocells," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, p. 595, 2012.

[15] R. I.-R. P.1238-1, *Propagation Data and Prediction Methods for the Planning of Indoor Radiocomm*, 1999.

[16] S. Dekleva, J. Shim, U. Varshney, and G. Knoerzer, "Evolution and emerging issues in mobile wireless networks," *Communications of the ACM*, vol. 50, no. 6, pp. 38–43, 2007.

[17] M. Manshaei, J. Freudiger, M. Félegyházi, P. Marbach, and J. Hubaux, "On wireless social community networks," in *IEEE, INFOCOM 2008*.

[18] Y. Chen, L. Duan, J. Huang, and Q. Zhang, "Appendix of spectrum allocation for balancing revenue and social welfare of licensed and unlicensed usage," *http://jianwei.ie.cuhk.edu.hk/publication/Appendix3GWifi.pdf*, 2012.

[19] A. Samuelson and D. Nordhaus, *The economics of climate change: the Stern review*. McGraw Hill, 2001.

---

[11] Due to page limitation, we do not show the figure here.