TAMES: A Truthful Auction Mechanism for Heterogeneous Spectrum Allocation

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***Abstract*—Spectrums are heterogeneous, especially from the aspect of their central frequency. According to signal propagation properties, low-frequency spectrum generally has lower path loss, thus longer transmission range, compared with high-frequency spectrum. Cellular operators with different targeted cell size will have different preferences for spectrums with different frequencies. Furthermore, the transmission range also affects the interference relationships among transmitters. Transmitters who can reuse the same high-frequency spectrum may interfere with each other when reusing the low-frequency spectrum, so it is difficult to decide how to construct the interference graph to exploit spectrum reusability among transmitters. Auction is considered as an efficient way for spectrum allocation. However, most of the previous works only considered homogenous spectrum auction, failing to address the problem of spectrum heterogeneity. In this paper, we propose TAMES, a Truthful Auction Mechanism for hEterogeneous Spectrum allocation, which allows buyers to freely express their different preferences towards different spectrums. Frequency-specific interference graphs are constructed to determine buyer groups. The proposed heterogeneous spectrum auction is theoretically proved to be truthful and individual rational. The simulation results verifies that the proposed auction mechanism outperforms other auction mechanisms with homogenous bid or homogenous interference graph. The proposed auction mechanism is able to yield higher buyers' satisfaction, seller's revenue and spectrum utilization.**

I. Introduction

Spectrum is an essential and limited resource for wireless communication. With the soaring user demand, wireless service providers are facing critical spectrum crunch. In 2010, FCC has opened up a significant amount of TV whitespace to cope with spectrum shortage. In July, 2012, the President's Council of Advisors on Science and Technology (PCAST) of the U.S. [1], proposed to identify 1,000 MHz of Federal spectrum for shared-use among commercial users.

Auction is considered as an efficient way for spectrum allocation. However, most of the existing works only focused on homogenous spectrum auction, where buyers bid the same price for all spectrums, which may result in potential utility loss. The reason is that, although generally a buyer has different preferences for different spectrums, he will only bid the valuation of the least-favored spectrum for the fear of overbid[1]. However, with low bid, the buyer may miss the chance of winning his high-favored spectrum.

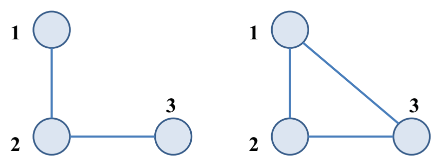

Fig. 1: Heterogeneous interference graph. The left and right figures are based on high-frequency spectrum s_h and low-frequency spectrum s_l respectively.

Spectrum heterogeneity also affects the interference relationships among transmitters. One of the distinctive property of spectrum is spatial reusability. The seller can choose multiple interference-free winners for one channel based on the interference graph. However, the interference graphs for spectrum with different frequencies are no longer the same. If the highest frequency is three times the lowest frequency, the path loss difference can be as large as $10dB$. This will change the interference graph structure and make the spectrum allocation more challenging. Fig.1 shows interference graphs of three buyers in terms of high-frequency spectrum s_h and low-frequency one s_l. If we use the interference graph of s_h and assign s_l simultaneously to buyer 1 and 3, they end up interfering with each other. If we use the interference graph of s_l and assign s_h to only one user, the spectrum utilization becomes low. With numerous frequencies and buyers, the problem is even more complicated.

Apart from spectrum heterogeneity, a well-designed auction should also possess robust economic properties. Truthfulness guarantees that no bidders have incentive to cheat in the auction and the spectrum is allocated to those who value it the most. Individual rationality ensures that buyers pay less than their valuation for the winning-item and positive utility can be achieved so that they are willing to participate in the auction.

Classic auction mechanisms, such as secondary-price or

[1]Overbid refers to the case where the buyer wins a spectrum of which his valuation is lower than his bid.

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VCG mechanisms, have been proved to be untruthful or individual irrational when applied to spectrum auction. In [2], [3], the authors designed auction for spectrum allocation, but spectrum reusability is not considered. In [4]–[7], spectrum reusability is exploited by assigning the same spectrum to non-interfering neighbors, but they don't consider that the set of interfering neighbors is frequency dependent. In [8], the interference graph is assumed to be complete, therefore there is no channel reusability. In [9], the authors exploit spectrum locality but haven't considered how the difference in transmission range would impact the allocation results. The paper [10] considers heterogeneous reserve price and bidding price but not the influence of spectrum frequency heterogeneity on the interference graph structure.

In this paper, we propose TAMES, a Truthful Auction Mechanism for hEterogeneous Spectrum allocation, which resolves above-mentioned problems. We consider a scenario where one spectrum owner wants to sell numerous channels to a pool of buyers. Buyers submit a bidding profile to express different valuations for different spectrums. The seller also has different reserve prices for each spectrum. During the auction, the seller groups buyers who can reuse the same spectrum based on frequency-tailored interference graphs. At the same time, the buyer groups are matched to each spectrum. After the group bids are computed, auction winners can be determined by comparing the group bid with the reserve price of the channel. Winning buyers evenly share the group payment. The proposed auction mechanism is proved to be truthful as well as improve buyers' ex ante utility.

The main contributions are as follows:

- TAMES considers spectrum heterogeneity, makes it possible for customized spectrum trading.
- TAMES is proved to be economic robust in terms of truthfulness and individual rationality.
- The simulation results show that TAMES greatly improves users' satisfaction, seller's revenue and spectrum utilization.

The rest of the paper is organized as follows. We define and formulate the problem of heterogeneous spectrum auction in Section II. We develop the auction mechanism in Section III, explaining the main algorithms and analyzing its properties. In Section IV, we use simulations to evaluate the performance of the proposed heterogeneous auction mechanism. We finally summarize our work in Section V.

II. Preliminaries

We consider a network consisting of one spectrum seller and N buyers. The seller owns K spectrum units, denoted by $S = \{s_1, s_2, ..., s_K\}$; and the reserve prices for them are $R = \{r_1, r_2, ..., r_K\}$, meaning the lowest acceptable price for each spectrum. Each buyer submits a bidding profile $b_i = (b_i^1, b_i^2, ..., b_i^k)$, in which b_i^j is buyer i's bidding price for s_j. The bid of a buyer is based on his true valuation for the spectrum, denoted by $v_i = (v_i^1, v_i^2, ..., v_i^k)$. We use $w_i = (w_i^1, w_i^2, ..., w_i^k)$ to represent buyer i's winning profile and p_i to represent buyer i's payment profile. $w_i^j = 1$ means buyer i wins spectrum s_j; otherwise $w_i^j = 0$. If $\sum_{j=1}^{K} w_i^j = 0$, the buyer does not pay anything, so $p_i = 0$. After the auction, the utility u_i of the buyer i is his true valuation for the procured spectrum minus his payment.

$$u_i = \begin{cases} 0, & \text{if } \sum\limits_{j=1}^{K} w_i^j = 0 \\ \sum\limits_{j=1}^{K} w_i^j * v_i^j - p_i, & \text{otherwise} \end{cases} \quad (1)$$

III. Auction Mechanism

A. Main Algorithm

The algorithm for the proposed heterogeneous spectrum auction is shown in Algorithm 1.

- For each spectrum s_i, we create the spectrum-specific interference graph and find interference-free buyers on that graph. These buyers are grouped together and are matched to s_i. In fact, it is not necessary to construct interference graph from scratch every time. We only have to eliminate 1) vertices of already grouped buyers; 2) corresponding edges; 3) add or delete edges according to transmission range change.
- The group bid is calculated according to equation (2), that is, the group bid is the lowest bid in the group for s_i multiplies the group size minus one. The group members' bids for other spectrums are ignored. The way of calculating the group bid is essential for guaranteeing truthfulness and individual rationality.
- The group bid is compared to the reserve price of the matched spectrum to decide whether the group wins the spectrum. If the group bid is higher than the reserve price of the matched spectrum, the group wins the spectrum.
- All the winning group members except the one with the lowest bid will be assigned the spectrum. The total payment of the group equals the group bid. That means, each winning buyer pays exactly the lowest bid in his group.
- Iteratively process steps $4 \sim 8$ of the algorithm until there are no buyers to be grouped or there are no channels left to be assigned.

B. A Toy Example

Now, we give a simple example to show how the proposed heterogeneous auction mechanism works.

In Fig.2, there are 6 buyers. The seller has 3 channels with reserve price $r_1 = 6, r_2 = 3, r_1 = 2$. The bidding profile of each buyer is shown in the figure.

Interference graph for s_1 is shown in the topmost graph. Buyer a, e form g_1 with group bid 5, the bid of e for s_1. In the second iteration, a, e are eliminated from the graph, along with the edge between (b, d) and (c, d) because these two pairs no longer interfere with each other on spectrum s_2, which has a higher frequency and shorter transmission range. $g_2 = \{b, d, f\}$ with group bid 6. Vertices b, d, f are further eliminated in the third iteration, leaving c forming g_3 with group bid 0 since the group size is 1.

Algorithm 1 Heterogeneous spectrum auction mechanism

$V = \{1, 2, ..., N\}$
for all $s_i, i = 1, 2, ..., K$ **do**
  if V is non-empty **then**
    Find a group of interference-free buyers g_i and match g_i to s_i;
    The group bid of g_i is

$$\Phi_i = \min_{j \in g_i} b_j^i \times (|g_i| - 1), \tag{2}$$

    in which $|g_i|$ is the number of buyers in the group;
    Eliminate members in g_i from V.
    if $\Phi_i > r_i$ **then**
      Each group members in g_i except the lowest-bid one are winners, and each winner pays

$$p_m^i = \frac{\Phi_i}{|g_i| - 1} = \min_{j \in g_i} b_j^i; \tag{3}$$

    end if
  end if
end for

After comparing group bids with the reserve price of their matched spectrum, we know that b and d are both assigned s_2 and each of them pays 3 respectively. Other buyers lose and don't pay anything.

C. Economic Properties

Proposition 1: The proposed heterogeneous auction mechanism is individually rational.

Proof: It can be easily proved that each winner pays no more than its bid because they are charged the price of the lowest bid in their group. ■

Proposition 2: The proposed heterogeneous auction mechanism is truthful.

Proof:

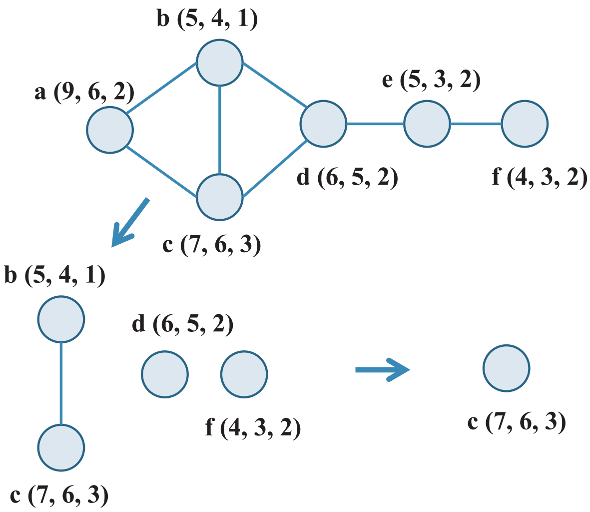
A buyer m's bid b_m^i does not affect the winning result $w_m^j, j \neq i$, because the buyer has no control over the grouping process, and which spectrum it is matched to.

Now we consider a buyer m's bid b_m^i and the winning result w_m^i. When m bids truthfully, the winning result is w_m^i and m's utility is u_m; otherwise, the winning result is $w_m^i{}'$ and m's utility is $u_m{}'$.

If $b_m^i \neq v_m^i$, there are four cases:

(1) $w_m^i = 0, w_m^i{}' = 0$. In this case, $u_m = u_m{}' = 0$.
(2) $w_m^i = 1, w_m^i{}' = 1$. In this case, $u_m = u_m{}' = v_m^i - p_i$, in which p_i is the lowest bid in the group for s_i. Since buyer m does not bid the lowest in the group (otherwise it loses the channel), p_i is independent of b_m^i.
(3) $w_m^i = 1, w_m^i{}' = 0$. In this case, $u_m{}' = 0$, $u_m = v_m^i - p_i \geq 0$ due to individual rationality.
(4) $w_m^i = 0, w_m^i{}' = 1$. In this case, $u_m = 0$. There are three possibilities: 1) The buyer doesn't belong to any group after the grouping process; 2) The buyer belongs to a group whose group bid is lower than the reserve price of the matched spectrum; 3) The buyer has the lowest bid in the group. Bidding untruthfully cannot help a buyer win the channel in situation 1). In situation 2) and 3), if a buyer wants to win the channel, it must bid higher than the second lowest bid in the group. Therefore $p_i{}' > v_m^i$ and $u_m{}' = v_m^i - p_i{}' < 0 = u_m$.

Therefore, $u_m \geq u_m{}'$ always stands. So the buyers don't have incentive to be untruthful. ■



Group	Group bid	Allocation	Reserve price
$g_1 = \{a, e\}$	5	s_1	6
$g_2 = \{b, d, f\}$	6	s_2	3
$g_3 = \{c\}$	0	s_3	2

Fig. 2: A toy example.

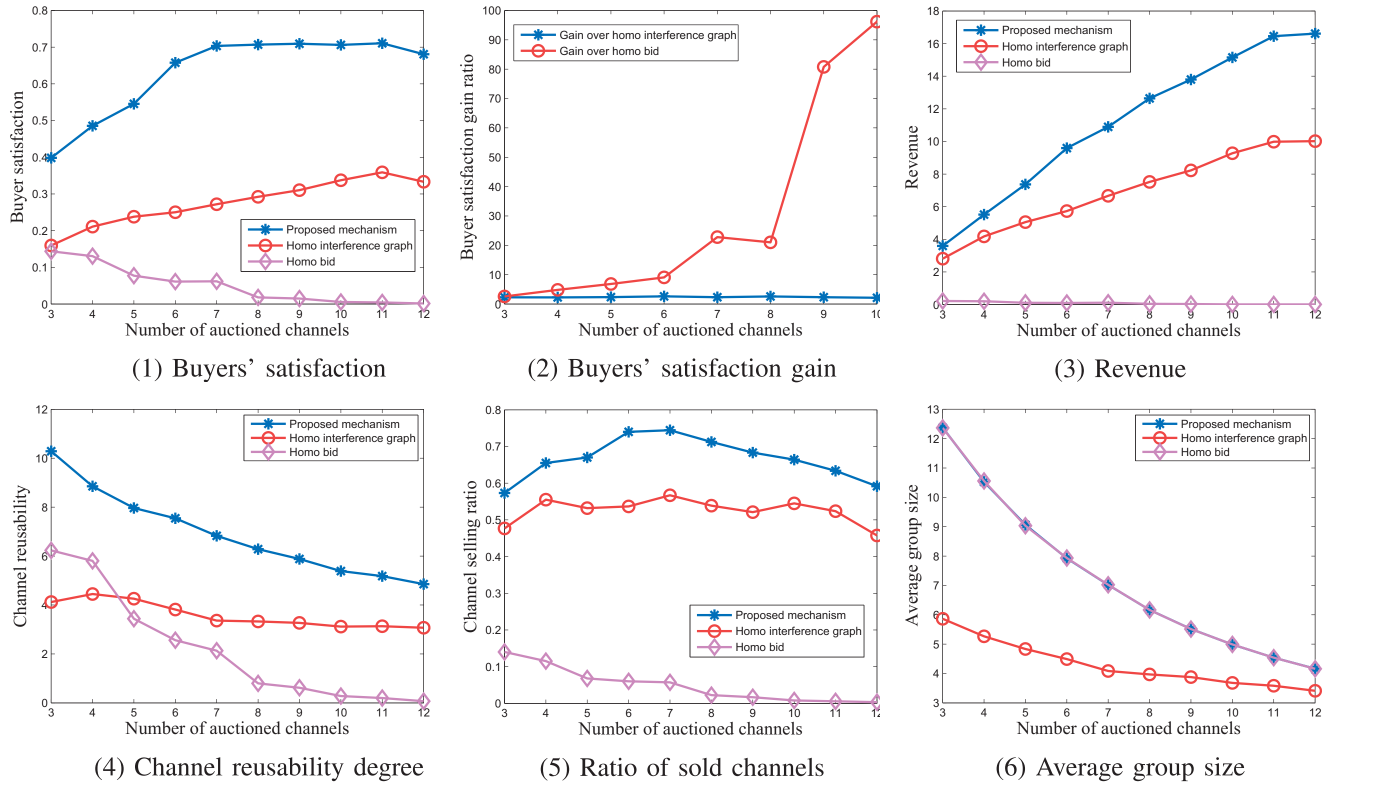
IV. Performance Evaluation

The simulation setup is as follows: the buyers scatter within a 1×1 area. The seller's reserve price is uniformly drawn on the range $[0, 1]$. In order to evaluate the proposed heterogeneous auction mechanism, we compare it with two other benchmarks: 1) Homogenous interference graph; 2) Homogenous bids. We define the following metrics to evaluate the auction results:

- *Buyers' satisfaction*, the ratio of winning bids to the total demands.
- *Buyers' satisfaction gain ratio*, the comparison of the buyers' satisfaction between the proposed auction mechanism and the other two benchmark auction mechanism.
- *Seller's Revenue*, all winning bidders' payment.
- *Channel reusability degree*, the ratio of winning buyer-channels to the number of sold channels, denoted by CR.

$$CR = \frac{\sum_{i=1}^{N} \sum_{j=1}^{K} w_i^j}{|\{s_j \in S | \exists i, w_i^j = 1\}|} \tag{4}$$

in which $|\cdot|$ denotes the size of the set.



(1) Buyers' satisfaction (2) Buyers' satisfaction gain (3) Revenue

(4) Channel reusability degree (5) Ratio of sold channels (6) Average group size

Fig. 3: Proposed auction mechanism vs 1) Homogenous interference graph and 2) Homogenous valuation auctions. $3 \sim 12$ channels are auctioned to 50 buyers.

- *Ratio of sold channels*, the ratio of number of sold channels to total on-sale channels.
- *Average group size* of the sequential grouping.

Fig.3 shows that if there are more units of spectrum to be auctioned, the ratio of winning buyers increases and so does buyers' satisfaction. Accordingly, seller's revenue increases because more spectrum are sold.

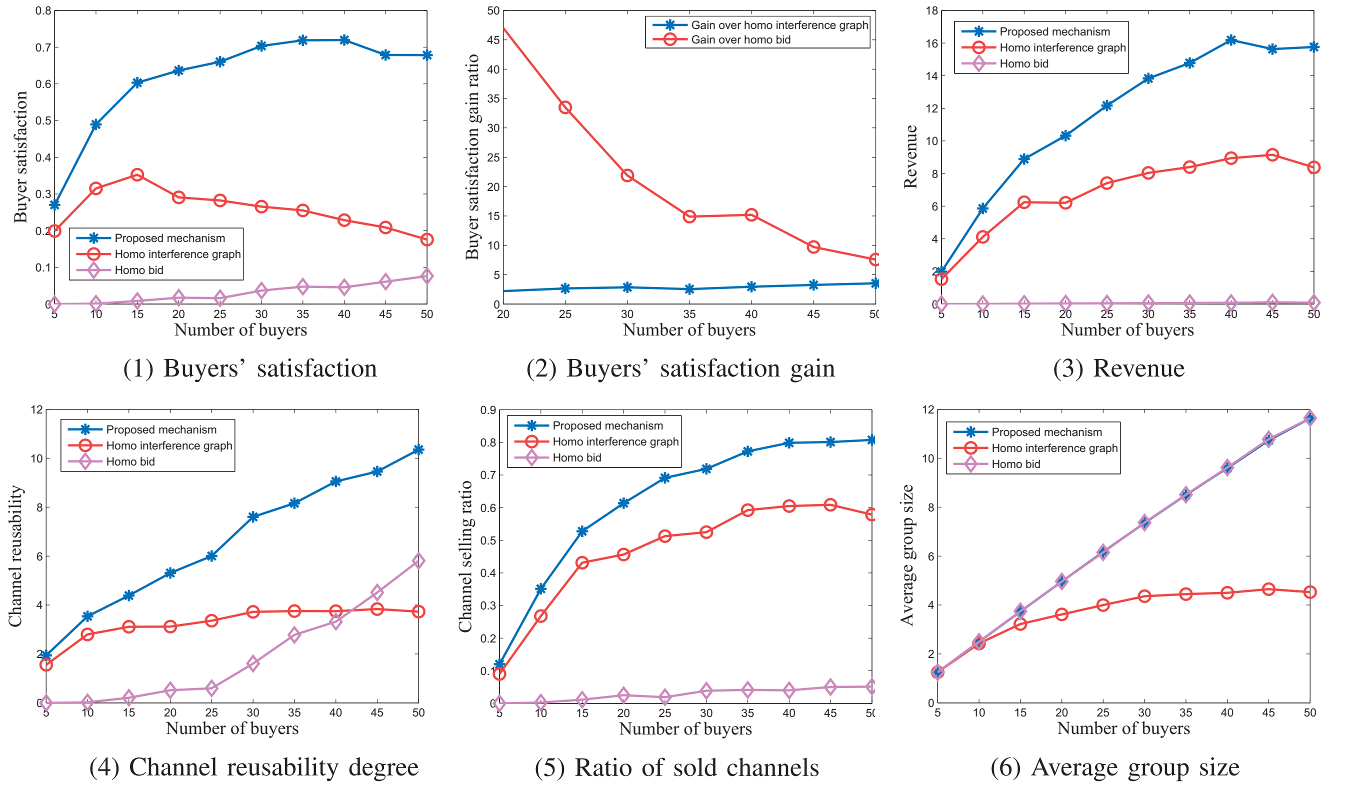
In Fig.4, as the number of buyers increases, the ratio of winners first increases and then decreases. This is because, at first, the channel resource is relatively abundant; when the number of buyers rises, it is easier to form large-size groups, the group bids are higher and more buyers become winners. As the number of buyers increases further, the number of winning groups is bounded by the number of channels. Revenue, average group size and the channel reusability degree increase as shown in Fig.4.

A. Proposed Heterogeneous Auction Mechanism vs Homogenous Interference Graph Auctions

In the homogenous interference graph auction, we have to use the interference graph constructed on the lowest-frequency (longest-transmission-range) channel to group buyers, because if we use interference graph of higher-frequency channel, the buyers that are matched to lower-frequency channel may interfere with each other since the transmission range is longer. Intuitively, the grouping process in this situation has low efficiency because redundant edges exist in the interference graph when forming groups for high-frequency channels. Due to the over-strict interference graph, the group size is small, which reduces the chance for the group to win, and reduces the number of channels sold by the spectrum owner. Both buyers' satisfaction and seller's revenue are low. Thus, it is essential for us to consider heterogeneous interference graph in order to improve spectrum utilization.

B. Proposed Heterogeneous Auction Mechanism vs Homogenous Valuation Auction

In the homogenous valuation auction, buyers are only allowed to bid the same price for any channel, which limit their free expression of diverse valuation for different spectrums. Therefore, buyers will bid the lowest possible bid, because if they bid the valuation for higher-preferred spectrum and end up winning the lower-preferred spectrum, they may achieve negative utility due to overpayment. Though this homogenous valuation does not affect the grouping process of the auction mechanism, the group bid is low due to low individual bid, which leads to low winning chance. The average group size is the same as that of the proposed heterogeneous auction mechanism. Nevertheless, the buyers' satisfaction and ratio of winning buyers are both low. Hence, it is also important for us to consider heterogeneous valuation in order to improve spectrum utilization.



(1) Buyers' satisfaction (2) Buyers' satisfaction gain (3) Revenue

(4) Channel reusability degree (5) Ratio of sold channels (6) Average group size

Fig. 4: Proposed auction mechanism vs 1) Homogenous interference graph and 2) Homogenous valuation auctions. 8 channels are auctioned to $5 \sim 50$ buyers

V. Conclusion

In this paper, we propose TAMES, a heterogeneous spectrum auction mechanism which takes into consideration buyers' different preferences for different spectrums. In the auction, the single seller groups buyers based on the propagation property of spectrums with different central frequencies. We prove in theory that the proposed auction mechanism is economic robust: buyers have no incentive to be untruthful and a buyer's payment is always lower than his bid. We compare the performance of the proposed auction mechanism with the auction mechanism where the buyers are only allowed to bid the same price, or the seller uses the same interference graph for all spectrums. The simulation results show that the proposed heterogeneous spectrum auction improve buyers' satisfaction, yields higher revenue for the seller as well as increases spectrum utilization.

VI. Acknowledgement

The research was support in part by grants from RGC under the contracts CERG 623209 and 622410, HKUST grant SRFI11FYT01, the grant from Huawei-HKUST joint lab, and the National Natural Science Foundation of China under Grants No. 60933012, Guangdong Natural Science Funds for Distinguished Young Scholar No.S20120011468, the grant from Guangzhou Pearl River New Star Technology Training Project No.2012J2200081, Guangdong NSF Grant No.S2012010010427, China NSFC Grant 61202454.

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