TAHES: Truthful Double Auction for Heterogeneous Spectrums

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***Abstract*—Auction is widely applied in wireless communication for spectrum allocation. Most of prior works have assumed that spectrums are identical. In reality, however, spectrums provided by different owners have distinctive characteristics in both spacial and frequency domains. Spectrum availability also varies in different geo-locations. Furthermore, frequency diversity may cause non-identical conflicts among spectrum buyers since different frequencies have distinct communication ranges. Under such realistic scenario, existing spectrum auction schemes cannot provide truthfulness or efficiency. In this paper, we propose a Truthful double Auction for HEterogeneous Spectrum, called TAHES. TAHES allows buyers to explicitly express their personalized preferences for heterogeneous spectrums and also addresses the problem of interference graph variation. We prove that TAHES has nice economic properties including truthfulness, individual rationality and budget balance.**

I. Introduction

Spectrum is valuable resource for wireless communication. Measurement results have shown that spectrum utilization is highly dynamic in different geo-locations[13]. In some places, spectrums may be under-utilized and are usually referred to as "white spaces". More efficient utilization of white spaces of different frequencies is considered as a potential solution for the next generation of wireless networking.

A promising way to better utilize white spaces is to enable spectrum owners to lease their spectrums to secondary service providers in the geo-locations where the primary users will not be interfered. In return, the spectrum owners can get paid from secondary service providers. Company such as SpectrumBridge[1] has already launched an online platform called SpecEx for spectrum owners to sell their unused spectrums to potential buyers. In such a new business model, spectrums from multiple sellers reside in different frequency bands and have various availability in different locations. Also, spectrum buyers may express different preferences for different spectrums. Another interesting aspect in this scenario is the reusability of spectrum. Two buyers that are far enough from each other may reuse the same spectrum concurrently.

The problem of spectrum redistribution between multiple spectrum owners and multiple secondary service providers can be modeled as a *single round multi-item double auction*. In our case, the spectrum owners are sellers; secondary service providers are buyers; spectrums are goods; and the auctioneer can be a third party providing the auction platform such as SpectrumBridge or regulators such as FCC.

Although auction has been widely applied to spectrum allocation in wireless communication, existing double auction schemes[6]-[9] cannot be directly applied to our scenario. There are three major challenges in our problem. The first challenge is the spatial heterogeneity of spectrum. Spectrums offered by different spectrum owners are available to different buyers. However, existing works[6]-[9] consider only the scenario where all spectrums are available to all buyers. The second challenge comes together with frequency heterogeneity and spectrum reusability. The spectrums may reside in various frequency bands. Lower-frequency bands have larger interference ranges than higher-frequency bands do. In this case, different buyers may have different interference relationships in different frequencies. Nevertheless, existing works considering spectrum reusability[8][9] usually assume the same conflict relationship among all buyers throughout all frequencies. The third challenge comes from the auction mechanism design. A well designed auction scheme should preserve the most critical property: *Truthfulness* (or strategy-proofness). A truthful auction incites all bidders to voluntarily reveal their true valuation for the items they are bidding. The auctioned items are finally assigned to bidders who value it the most. Unfortunately, with heterogenous items, the truthfulness cannot be guaranteed when simply applying existing schemes.

In this paper, we propose TAHES, a **T**ruthful double **A**uction scheme for **HE**terogeneous **S**pectrum, to address the above-mentioned challenges. TAHES groups spectrum buyers considering their non-identical conflict relationships in heterogeneous spectrums to explore spectrum reusability. To guarantee truthfulness, TAHES employs a matching procedure between buyer groups and sellers. In summary, the main contributions of the paper are as follows:

- To the best of our knowledge, TAHES is the first double auction mechanism for heterogeneous spectrum transaction, allowing buyers to express diversified preferences for spectrums of difference frequencies.
- TAHES improves spectrum utilization by taking into consideration of the spectrum reusability. As far as we know, TAHES is the first auction mechanism that can deal with the problem of interference graph variation caused by spectrum frequency heterogeneity.
- TAHES ensures that bidding truthfully is the dominant strategy for all bidders. Despite this, TAHES also con-

978-1-4673-0775-8/12/$31.00 ©2012 IEEE

forms with individual rationality and budget balance.

The rest of the paper is organized as follows. Section II presents the auction model of heterogeneous spectrum trading and then sets the design objectives for the proposed auction mechanism. In section III, challenges in heterogeneous auction design are further explained. We give detailed description of TAHES in section IV. We briefly discuss related works in Section V and summarize the whole work in Section VI.

II. Model Description and Design Targets

In this section, we first formulate the problem of heterogeneous spectrum exchange between spectrum owners and service providers as a double auction. Then, we overview ideal economic properties of an auction mechanism and state our auction design target.

A. Problem Formulation

We consider the scenario where N secondary service providers need to purchase spectrum resource from M spectrum owners. A single-round double auction consists of M seller and N buyers is held to serve this purpose. Let $S = \{s_1, s_2, \cdots, s_M\}$ denotes the set of sellers and $W = \{w_1, w_2, \cdots, w_N\}$ denotes the set of buyers. A third-party acts as the auctioneer and decides the winning bidders and the payment. Each seller contributes one distinct channel. Each buyer would like to purchase one channel, but they have different valuations for the channels. Therefore, the buyers' bids are channel specific. We assume that the auction is sealed-bid, private and collusion-free. In other words, in the auction, all bidders simultaneously submit sealed bids so that no bidder knows the bid of any other participants. In addition, bidders do not collude with each other to improve the utility of the coalitional group.

We use C_i to denote the bid of s_i. $C = (C_1, C_2, \cdots, C_M)$ is the bid matrix of all sellers and C_{-i} denotes the bid matrix with s_i's bid removed. We use b_i^j to denote the bid of w_i for seller s_j's channel. $B_i = (b_i^1, b_i^2, \cdots, b_i^M)$ is the bid vector of w_i. $B = (B_1, B_2, \cdots, B_N)$ is the bid matrix of all buyers. Let B_{-i} denote the bid matrix with buyer w_i's bid B_i excluded. The true valuation of s_i for its channel is V_i^s and the true valuation of w_i for seller s_j's channel is v_i^j. $V_i^w = (v_i^1, v_i^2, \cdots, v_i^M)$ is the valuation vector of buyer w_i. If s_j's channel is unavailable to w_i, v_i^j is zero. The true valuation of both buyers and sellers may or may not equal to their bids. In the auction, the auctioneer determines the payment P_i^s for seller s_i and the price p_i^w that buyer w_i should pay. P_i^s and p_i^w are not necessarily equal.

Therefore, the utility of the seller s_i is defined as:

$$U_i^s = \begin{cases} P_i^s - V_i^s, & \text{if } s_i \text{ wins} \\ 0, & \text{Otherwise} \end{cases} \tag{1}$$

Similarly, the utility of buyer w_i is defined as:

$$u_i^w = \begin{cases} V_i^{\theta(i)} - p_i^w, & \text{if } w_i \text{ wins} \\ 0, & \text{Otherwise} \end{cases} \tag{2}$$

in which $\theta(i)$ is the channel that w_i wins.

B. Design Target

Truthfulness, budget balance and system efficiency cannot be achieved in any double auction at the same time, even without the consideration of individual rationality[5][19]. In our scenario, we need to warrant that spectrum owners have incentives to lease their spectrum and also the third party (spectrum transaction platform, government) is willing to participate as an auctioneer. Therefore, in this paper, we set our design target as achieving truthfulness, individual rationality and budget balance, which are also selected by many existing double auctions[9][19]:

- *Truthfulness*. Neither buyers nor sellers can get higher utility by misreporting their true valuation, i.e., $C_j \neq V_j^s$ or $B_i \neq V_i^w$.
- *Individual rationality*. A winning seller is paid more than its bid and a winning buyer pays less than its bid.
- *Budget balance*. The auctioneer's profit is non-negative. This profit equals to the price paid by the buyers minus the payment to the sellers.

III. Challenges of Heterogeneous Spectrum Auction Design

In this section, we briefly illustrate the challenges of designing a truthful auction mechanism for heterogeneous spectrum. We will first introduce the heterogeneous nature of spectrum. Then we show that existing mechanisms are unsuccessful in meeting our design targets when directly applied to heterogeneous spectrum auction.

A. Spatial Heterogeneity

Spatial heterogeneity means that spectrum availability varies in different locations. For example, one TV channel is available only if there are no nearby TV stations or wireless microphones occupying the same channel.

Traditional auction design involving spectrum reusability usually groups buyers by finding independent sets on their interference graphs [8][9], and then set the group bid according to the minimum bid in the group. However, if two buyers with no common available channels are grouped together, the group bid should be 0 for all channels and this group can never win in the auction.

B. Frequency Heterogeneity

Frequency heterogeneity means that different frequencies have different transmission ranges. According to the propagation model recommended by ITU[10], the center frequency of one spectrum band can impact the path lose between two nodes:

$$L = 10 \log f^2 + \gamma \log d + P_f(n) - 28 \tag{3}$$

where L is the total pass loss in decibel(dB), f is the frequency of transmission in megahertz(MHz), d is the distance in meter(m), γ is the distance power loss coefficient and $P_f(n)$ is the floor loss penetration factor. In our model, the spectrums offered by spectrum owners may consist of a wide range of frequencies. For example, in the German spectrum auction held in 2010, the highest frequency (2.6GHz) are more than

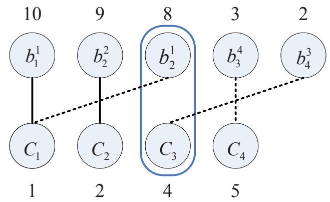

Fig. 1: Buyer b_2 can manipulate its non-winning bid to increase the utility

three times higher than the lowest frequency (800MHz)[12]. This huge gap leads to non-identical interference relationships among spectrum buyers in different channels. However, based on our knowledge, no existing auction schemes address non-uniform interference relationships among buyers caused by frequency heterogeneity.

C. Market Manipulation

In double auction for homogeneous items, the two well-known auction algorithms VCG[2]-[4] and McAfee[11] can both ensure truthfulness. But truthfulness cannot be guaranteed if we directly apply these algorithms for heterogeneous items.

In the McAfee double auction, the auctioneer sorts the buyers' bids in non-increasing order and the sellers' bids in non-decreasing order: $B_{i_1} \geq B_{i_2} \geq \cdots \geq B_{i_N}$ and $C_{i_1} \geq C_{i_2} \geq \cdots \geq C_{i_M}$. Then the auctioneer finds the largest k such that $B_{i_k} \geq C_{j_k}$. The McAfee scheme discard (B_{i_k}, C_{j_k}) which is the least profitable transaction and set the uniform price for buyers to be B_{i_k} and the uniform payment for sellers to be C_{j_k}. In double auction for heterogeneous items, although one buyer can win at most one item, it can manipulate its bids to achieve higher utility. In Fig. 1, there are four buyers among which buyer w_2 has two non-zero bids b_2^2 and b_2^1. Both the buyers and sellers' bids have been sorted according to the McAfee mechanism. We can see that the index of the least profitable transaction marked by the blue frame is $k = 3$. So the price charged for the winning group is $b_2^1 = 8$. If the buyer w_2 lowers its bid for seller s_1 to make $b_2^1 < 8$, say $\widetilde{b}_2^1 = 7$, this misconduct will not change the results of the auction, but can contort the price from 8 to 7. As a result, the utility of w_2 can be increased.

Since the VCG double auction uses a similar way to determine winners and prices, it suffers the same defect.

IV. Auction Design: TAHES

In this section, we propose TAHES, a **T**ruthful double **A**uction for **HE**terogeneous **S**pectrum.

A. Overview

To handle both spectrum heterogeneity and spectrum reusability, we design three key steps in TAHES:

(1) Buyer Grouping:

Spectrum can be reused by non-conflict buyers in different locations. By non-conflict, we mean when two buyers w_i and w_j are using the same channel h, they are out of $h's$ interference range of each other. However, the conflict relationships between one pair of buyers are non-identical in different frequencies. In this step, the auctioneer uses a grouping algorithm considering non-identical interference graphs to form non-conflict buyer groups so that they can purchase the same channel. Our buyer grouping algorithm is bid-independent. The input of the grouping algorithm is the channel availability information of each buyer. This kind of information can be calculated according to the path loss model given the locations of buyers and sellers or can be obtained from a geo-location database[13]. We assume the auctioneer can get such location information of buyers and sellers. The bid-independent property of the grouping algorithm is critical to ensure truthfulness in the auction[9].

(2) Matching:

After the first step, each buyer group may still purchase channels from multiple sellers if the buyers in a group have more than one common channel. While indeed, each group can at most win one channel. We have shown that only one bid in a bid vector should be kept for further winner determination. Otherwise, the auction can be vulnerable to market manipulation. Some buyers can strategically change some of their bids to lower the group bids so that they can change the price they need to pay and increase their utility. In this matching step, the auctioneer chooses one conventional matching algorithm to match each buyer group to only one buyer based on only the channel availability for each group. Therefore, the matching step is also bid-independent.

(3) Winner Determination and Pricing:

After buyer grouping and matching, the remaining problem of winner determination and pricing are similar to that in the McAfee double auction design. We can directly apply the McAfee's scheme to use the k'th pair of buyer group and seller to determine the winners.

B. Auction Procedure

TAHES comprises the following steps:

1) Buyer Grouping: Suppose the set of channels from sellers is $H = \{h_1, h_2, \cdots, h_M\}$. h_i's communication range is $R(h_i)$. Without loss of generality, we assume $R(h_1) \leq R(h_2) \leq \cdots \leq R(h_M)$. Let $A = \{a_{i,j} | a_{i,j} \in \{0,1\}_{N\times M}\}$, an N by M matrix, represent the buyers' channel availability. $a_{i,j} = 1$ means that channel h_j is available for buyer w_i. Let $E = \{e_{i,j,k} | e_{i,j,k} \in \{0,1\}_{M\times N\times N}\}$, an M by N by N matrix, represent the conflict relationships between buyers in each channel. $e_{i,j,k} = 1$ means that buyers w_j and w_k are conflict in h_i.

In this step, the inputs are A and E, which are both bid-independent. After grouping, we get a set of l ($l \leq n$) buyer groups denoted as $G = \{g_1, g_2, \cdots, g_l\}$ and the candidate channel set for each group denoted as $F = \{f_1, f_2, \cdots, f_l\}$. f_i contains the channels that g_i can purchase, which is assigned by the auctioneer. G and F are the outputs of the grouping algorithm. The grouping algorithm should satisfy the following constraints:

Common Channel Existence Constraint: There exists at least one channel that is available for all buyers in the same

Algorithm 1 Buyer-Grouping(A, E, H)

1: // L represents the set of grouped buyers
2: $L = \emptyset$, $G = \emptyset$, $F = \emptyset$
3: **while** $L \neq W$ **do**
4:   **for all** $h_i \in H$ **do**
5:     Candidate buyers to be grouped: $Q = \emptyset$
6:     **for all** $w_j \in \{w_k | A_{k,i} = 1 \wedge w_k \notin L\}$ **do**
7:       $Q = Q \bigcup w_j$
8:     **end for**
9:     Find independent set IS_i on buyer set Q based on E_i.
10:   **end for**
11:   Find IS_i, such that $|IS_i|$ is maximized
12:   $g = \emptyset$
13:   **for all** $w_j \in IS_i$ **do**
14:     $g = g \bigcup w_j$, $L = L \bigcup w_j$
15:   **end for**
16:   $f = \emptyset$
17:   **for all** $h_j \in H$ **do**
18:     **if** $R(h_j) <= R(h_i)$ **then**
19:       $f = f \bigcup h_j$
20:     **end if**
21:   **end for**
22:   $G = G \bigcup g$, $F = F \bigcup f$
23: **end while**
24: return (G, F)

group.

$$\forall g_i, \exists h_j, s.t. \forall w_k \in g_i \Rightarrow h_j \in f_i \wedge A_{k,j} = 1 \quad (4)$$

Conflict Free Constraint: Any two buyers in the same group do not mutually interfering with each other in any channel in the candidate channel set.

$$\forall w_j, w_k \in g_i, \forall h_l \in f_i \wedge A_{j,l} = A_{k,l} = 1 \Rightarrow E_{l,j,k} = 0 \quad (5)$$

The grouping algorithm first finds an independent set of buyers in each channel. Then it selects one such set with maximum group size and continues to find the next group until all buyers are classified into one group.

The grouping procedure is shown in Algorithm 1.

In Algorithm 1, we can use any existing algorithms to find independent sets, for example, the algorithms described in [14]. From the procedure of Algorithm 1, it's obvious that all buyers in IS_i have a common available channel h_i. And from line 16-20, we only consider candidate channel with smaller or equal communication range of h_i. Therefore, the grouping results G and F also satisfy the Conflict Free Constraint.

Theorem 1. *The buyer groups and candidate channel sets returned by Algorithm 1 satisfy the Common Channel Existence Constraint and the Conflict Free Constraint.* □

2) Matching: After step 1, we have formed a group set G. Suppose the number of buyers in group g_i is n_i and the group bid vector is $\delta_i = (\delta_i^1, \delta_i^2, \cdots, \delta_i^M)$. We follow the idea in [9] and assigned the group bid to be the minimum bid times the group size as:

$$\delta_i^j = \min\{b_k^j | w_k \in g_i\} \cdot n_i \quad (6)$$

Algorithm 2 Buyer-Group-Matching(G, F, S)

// Let Δ be an $|G|$ by $|S|$ matrix representing the weighted adjacent matrix between G and S
$\Delta = \{0\}_{M \times N}$
for all $g_x \in G, s_y \in S$ **do**
  if $\delta_x^y > 0$ and $h_y \in f_x$ **then**
    $\Delta_{x,y} = \Delta_{y,x} = |g_x|$
  end if
end for
$(G_C, S_C, \sigma) = MATCH(X, Y, \Delta)$
return (G_C, S_C, σ)

Here, we can imagine each buyer group as one *super buyer*. In δ_i, there may be more than one non-zero entries, say $\delta_i^{j_1}$ and $\delta_i^{j_2}$. If not well designed, the auction may be untruthful, as shown in Section III-C. Such type of market manipulation is caused by the multiple non-zero group bids. To tackle this challenge, in TAHES, we apply a channel matching scheme to match one buyer group to an identical seller. We call the results after matching the candidate winning group set G_C and candidate winning sellers set S_C. The matching procedure is shown in Algorithm 2.

In this algorithm, σ records the matching result. For example, $\sigma(g_x) = s_y$ indicates that buyer group g_x is assigned with seller s_y. $MATCH(X, Y, \Delta)$ matches nodes set X to Y with weighted edges in Δ. It can be any matching algorithm for bipartite graphs specified by the auctioneer, for example, maximum matching or maximum weighted matching. This matching step here is also independent of bids of both buyers and sellers.

3) Winner Determination and Pricing: In the winner-determination and pricing stage, we can apply the mechanism used in McAfee and directly use the buyer group-seller pair (g_{i_k}, s_{j_k}) to determine winners and price. The detailed algorithm is shown in Algorithm 3.

C. Economic properties

Theorem 2. *TAHES is individually rational.* □

Proof: For each buyer w_i in winning group g_k:

$$p_i^w = \frac{p^w}{n_k} \leq \frac{\delta_k^{\sigma(k)}}{n_k} \leq \frac{n_k \cdot b_i^{\sigma(k)}}{n_k} = b_i^{\sigma(k)}$$

For each winning seller s_j: $P_j^s \geq C_k \geq P^s$ ■

Theorem 3. *TAHES is budget-balanced.* □

Proof: According to the sorting in the winner determination algorithm, we have $p^w \geq P^s$ and $|G_W| = |S_W|$, therefore the budget for the auctioneer is: $|G_W| \times p^w - |S_W| \times P^s \geq 0$ ■

Theorem 4. *TAHES is truthful.* □

Algorithm 3 Winner-Determination-and-Pricing(G_C, S_C, σ)

```
G_W = ∅, S_W = ∅ // The set of winning groups and sellers
// Sort all buyer groups in G_C and sellers in S_C
Construct X = {g_{i_1}, g_{i_2}, ..., g_{i_l}},
    such that δ_{i_1}^{σ(i_1)} ≥ δ_{i_2}^{σ(i_2)} ≥ ... ≥ δ_{i_l}^{σ(i_l)}
Construct Y = {s_{j_1}, s_{j_2}, ..., s_{j_M}},
    such that C_{j_1} ≤ C_{j_2} ≤ ... ≤ C_{j_M}
Find the largest k, s.t. δ_{i_k}^{σ(i_k)} ≥ C_{j_k}
if k < 2 then
    return (G_W, S_W, 0, 0)
end if
G_W = groups δ_{i_1} to δ_{i_{k-1}}
S_W = sellers C_{j_1} to C_{j_{k-1}}
// Determine the price and payment
p^w = δ_{i_k}^{σ(i_k)}, P^s = C_{j_k}
for Any buyer w_i in g_{τ(i)} ∈ G_W do
    p_i^w = p^w / n_{τ(i)}
end for
for all Any seller s_j ∈ S_W do
    P_j^s = P^s
end for
return (G_W, S_W, P^S, p^w)
```

Due to limitation of space, we do not present the proof in this paper.

V. Related Works

Auction has been extensively studied in the scope of spectrum allocation[6]-[9]. However, most existing works failed to consider spectrums as non-identical items. In [9], Zhou and Zheng first address spectrum reusability in their auction design: TRUST. In [8], the authors also consider spectrum reusability for buyers, and they assume buyers can have multiple radios. The proposed TAHES scheme also consider spectrum reusability, moreover, TAHES can tackle the case when spectrums are heterogeneous.

In [16], an auction design for heterogeneous TV white space spectrums is proposed. In that paper, the spectrum allocation problem has been defined as an optimization problem where maximum payoff of the central trading entity (called spectrum broker) is the optimization goal. However, [16] is not a double auction scheme and its design goal is different from TAHES. Recently, in [19], Yang et. proposed a double auction design for cooperative communications with heterogeneous relay selections. However, there is no reusability in their scenario.

Different from our single-round auction model, recently, there are also works considering spectrum auction in an online fashion[15][17]. In an online spectrum auction, buyers may arrive in different time and they can request the spectrum for a particular duration. However, existing online double auction schemes consider only homogeneous spectrum.

VI. Conclusions

In this paper, we have designed TAHES, a truthful double auction scheme for heterogeneous spectrums. TAHES allows multiple spectrum owners with available spectrums in different locations and different frequency bands to participate in the spectrum leasing to secondary service providers. TAHES can not only solve unique challenges caused by spectrum heterogeneity but also perverse nice economic properties: Truthfulness, Budget Balance and Individual Rationality.

Acknowledgement

This research was supported in part by Hong Kong RGC grants 623209, 622410, Huawei-HKUST joint lab, National Natural Science Foundation of China with grants no. as 60933012, 61173156, by a grant from NSFC/RGC under the contract N_HKUST610/11, by grants from HKUST under the contract RPC11EG29, SRFI11EG17-C and SBI09/10.EG01-C, by a grant from Huawei Technologies Co. Ltd. under the contract HUAW18-15L0181011/PN, by a grant from Guangdong Bureau of Science and Technology under the contract GDST11EG06.

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