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Spectrum Matching

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***Abstract*—Dynamic spectrum access (DSA) redistributes spectrum from service providers with spare channels to those in need for them. Existing works on such spectrum exchange mainly focus on double auctions, where an auctioneer centrally enforces a certain spectrum allocation policy. In this paper, we take a different and new perspective, proposing to use *matching* as an alternative tool to realize DSA in a distributed way for a free market, which consists of only buyers and sellers, but no trustworthy third-party authority. Compared with conventional many-to-one matching problems, the spectrum matching problem is distinctively challenging due to the interference bound between buyers: the same channel can be reused by an unlimited number of non-interfering buyers, but must be exclusively occupied by only one of interfering buyers. In this paper, we firstly formulate the spectrum matching problem as a many-to-one matching with peer effects, *i.e.*, a buyer's utility is affected by other buyers who are matched to the same seller. We then present a two-stage distributed algorithm that converges to an interference-free and Nash-stable matching result. Simulations show that the proposed distributed matching algorithm can achieve 90% of the social welfare from the optimal matching result.**

I. Introduction

To support the ever-increasing volume of wireless traffic with limited spectrum availability, dynamic spectrum access has been proposed to better leverage the underused channels [1], [2]. A wireless service provider can sell spare spectrum to others when her traffic demand is light, and buy additional spectrum from others when her demand becomes heavy. Conventionally, such spectrum exchange is assumed to be achieved via double auctions, where a third-party auctioneer determines the spectrum allocation in a centralized manner, based on auction participants' bids and certain optimization objectives. Unfortunately, the need for a trustworthy auctioneer makes it impossible to apply double auctions to free spectrum markets with only buyers and sellers but no third-party rule-enforcing authorities.

Taking a completely different viewpoint, we propose to leverage *matching* as an alternative framework for spectrum redistribution. The seminal work of Gale and Shapley [3] pioneers the research on matching items in two different sets with stability. The concept of stable matching, compared with optimal matching (*i.e.*, the social welfare of buyers and sellers is the highest), matters more in free spectrum markets for two reasons. Firstly, stable matching ensures that no buyer or seller is willing to deviate from the current matching result. Optimal matching, if unstable, will not be obeyed by buyers and sellers, unless it is enforced by a third-party authority. Secondly, stable matching can be realized through *deferred acceptance*, an algorithm that is both efficient and fully distributed. Stable matching has been widely applied to computer science, such as resource management in the cloud [4], user association in small cells [5], as well as resource sharing in device-to-device communication [6]. In this paper, the challenge of finding a stable matching in free spectrum markets is referred to as the *spectrum matching* problem.

Unfortunately, the *spectrum matching* problem is quite different from traditional matching problems, such as the well-known college admission problem [3]. In the college admission problem, each student can attend only one college, and each college can admit multiple students, but subject to a fixed quota. In spectrum matching, the fundamental constraint is no longer fixed quotas but *interference:* non-interfering buyers can freely reuse the same channel, while interfering buyers have to operate on separate channels. In other words, the "quota" of a channel is infinite for non-interfering buyers, but reduced to one for interfering buyers. The unique feature of spectrum reusability has been widely discussed in spectrum auctions, yet has never been considered within a stable matching framework.

In this paper, we make the first attempt to introduce the spectrum matching framework as a new economic model for distributed spectrum exchange in free spectrum markets (Section II). Sellers who own multiple channels and buyers who demand multiple channels are represented by corresponding numbers of dummies. In this way, spectrum exchange can be formulated as a many-to-one matching problem, where a buyer can be matched to no more than one seller, and a seller can be matched to multiple non-interfering buyers (spectrum reuse). To address spectrum heterogeneity, different interference graphs are constructed for different channels to determine spectrum reuse.

In this context, we propose a two-stage distributed algorithm to achieve the objective of stable spectrum matching (Section III). Stage I is inspired by the deferred acceptance (Gale-Shapley) algorithm, which we adapt to enable spectrum reuse and avoid interference among buyers. Stage I converges to an interference-free but unstable matching, due to the complicated interference relationship among buyers. Therefore, we

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introduce Stage II, which allows buyers to transfer to more-preferred channels and sellers to invite previously-rejected buyers, if the interference condition permits. The final matching result is proved to be individual rational and Nash-stable.

To address the synchronization problems in the real implementation of the proposed two-stage algorithm, *i.e.*, buyers and sellers are not coordinated to end Stage I and enter Stage II simultaneously, we design rules for individual buyers and sellers to independently decide the timing of their stage transition (Section IV).

Through extensive simulations (Section V), we demonstrate that our proposed distributed algorithm can achieve more than 90% of the maximum social welfare obtained by centralized optimal (but unstable) matching. We also show the influence of different parameters on the final matching results. One interesting finding is that, if buyers have diverse utilities in using different channels, the overall social welfare will be higher, because more buyers will be matched to their desired channels.

II. System Model

A. Spectrum Market

In a free spectrum market, service providers with spectrum supply or demand serve as sellers or buyers, respectively. Spectrum reuse must conform to interference constraints.

Market participants. Assume that there are I sellers and J buyers in the spectrum market. Seller i owns m_i channels, and buyer j requests n_j channels. Let $\sum_{i=1}^{I} m_i = M$ and $\sum_{j=1}^{J} n_j = N$ denote the total numbers of supplied and demanded channels, respectively. Inspired by the idea in [7], we create m_i and n_j dummies for seller i and buyer j, respectively. Hence, there are M virtual sellers and N virtual buyers, and each virtual buyer or seller can trade only one channel. In the remainder of this paper, we omit the term "virtual" without confusion[1]. We also use the index of a seller for her channel, e.g., seller i's channel is referred to as channel i.

Utility of buyers and sellers. A service provider obtains different utilities when operating on different channels. We assume that buyer j has a utility vector $B_j = (b_{1,j}, b_{2,j}, ..., b_{M,j})$, in which $b_{i,j}$ is the utility for her to use channel i. The higher $b_{i,j}$ is, the more valuable channel i is to buyer j, and buyer j is willing to pay more for channel i. We assume that $b_{i,j}$ is also the price that buyer j offers to seller i. A seller's utility equals the total offered price of all buyers matched to her.

Interference relationship. The key feature of the spectrum resource is interference-restricted reuse. To characterize interference heterogeneity of different channels [7], we construct a series of interference graphs $\{G^i = (V, E^i)\}_{i=1}^{M}$, in which each node $v \in V$ represents a buyer, and each edge $e^i \in E^i$ connects a pair of interfering buyers on channel i. If two virtual buyers originate from the same buyer, they are viewed as interfering buyers, since they should not be matched to the same channel. Let $e^i_{j,j'} \in \{0, 1\}$ denote the interference status between buyers j and j' regarding channel i.

[1]For simplicity, we assume that channels are independent from each other. Therefore, the value of a combination of channels is exactly the sum of values of each individual channel. We will consider that channels may be complementary or substitute goods (e.g., in a combinatorial auction) in the future.

B. Optimal Matching

We introduce the optimal matching as a benchmark to be compared later with our proposed stable matching. The optimal matching maximizes social welfare while complying with the interference constraint. Following the norm of spectrum auctions, we define social welfare as the sum of buyers' utility from acquiring the channels through spectrum matching. Let $\{x_{i,j}\}_{i=1,j=1}^{i=M,j=N}$ denote the matching result. $x_{i,j} = 1$ if and only if buyer j is matched to seller i. The optimal matching is the solution to the following centralized maximization problem:

$$\max_{x_{i,j}} \quad \sum_{i=1}^{M}\sum_{j=1}^{N} b_{i,j}x_{i,j}, \tag{1}$$

$$\text{subject to} \quad \sum_{i=1}^{M} x_{i,j} \leq 1, \forall j, \tag{2}$$

$$x_{i,j} \cdot x_{i,j'} = 0, \text{if } e^i_{j,j'} = 1, j \neq j', \forall i, j, j', \tag{3}$$

$$x_{i,j} \in \{0, 1\}, \forall i, j. \tag{4}$$

The first constraint indicates that each (virtual) buyer can only get one channel. The second constraint restricts that no interfering buyers can be matched to the same channel. The optimal matching, if unstable, can only be implemented by a third-party authority, who solves the above non-linear integer programming problem, which is NP-hard. In comparison, our proposed distributed matching algorithm can reach an interference-free stable matching in linear time, and our simulation results show that it can achieve 90% of maximum social welfare yielded by the optimal matching.

III. Spectrum Matching

A. Preliminaries

We formally define spectrum matching as follows.

Definition 1. *(Spectrum Matching). Given the set of sellers $\mathcal{M}$ and the set of buyers $\mathcal{N}$, a spectrum matching is a function μ from $\mathcal{M} \cup \mathcal{N}$ to subsets of $\mathcal{M} \cup \mathcal{N}$, such that*

- *For every buyer $j \in \mathcal{N}$, $\mu(j) = \{i\}$ if buyer j is matched to seller i, and $\mu(j) = \{j\}$ if buyer j is unmatched;*
- *For every seller $i \in \mathcal{M}$, $\mu(i) \subseteq \mathcal{N}$;*
- *For every seller i and buyer j, $\mu(j) = \{i\}$ if and only if $j \in \mu(i)$.*

In the traditional college admission problem, a preference profile is built for each student, indicating her willingness of attending different colleges; a preference profile is also built for each college, indicating its willingness of accepting different students. In spectrum matching, however, a buyer's willingness to be matched to a channel depends not only on her utility of using the channel, but also on whether her interfering neighbors are matched to the same channel. In comparison, in the college admission problem, a student's willingness of attending a college will not be affected by other students at the same college. We assume that if a buyer is matched to a channel without her interfering neighbors, she obtains full utility; otherwise, she gets zero utility because of interference. To deal with such peer effects in spectrum matching, we construct preference profiles on spectrum coalition, rather than individual buyers or sellers. A spectrum coalition consists of a seller and the buyers matched to this seller, or includes a single unmatched seller or buyer.

For buyer j, we can construct a complete, reflexive, and transitive preference relation $\triangleright$ over all spectrum coalitions, based on her utility in the coalition. Assume that buyer j is a member of two coalitions C^{i_1} and C^{i_2}, containing seller i_1 and seller i_2, respectively. Buyer j prefers C^{i_1} to C^{i_2} in two cases: 1) buyer j prefers channel i_1 to channel i_2, and buyer j does not have any interfering neighbor in C^{i_1}; 2) buyer j has some interfering neighbors in C^{i_2}. We implicitly assume that a buyer is indifferent towards two coalitions both involving her interfering neighbors, and she is indifferent towards a coalition of herself (unmatched) and a coalition with her interfering neighbors.

$$C^{i_1} \triangleright_j C^{i_2} \Longleftrightarrow \begin{cases} \forall j' \in C^{i_1}, e^{i_1}_{j,j'} = 0, b_{i_1,j} > b_{i_2,j}, \text{or} \\ \exists j' \in C^{i_2}, e^{i_2}_{j,j'} = 1. \end{cases} \tag{5}$$

For seller i, we can also build a preference relation over all coalitions, based on her utility in the coalition. Assume that seller i is a member of two coalitions C^i and C'^i, containing different groups of buyers. Seller i prefers C^i to C'^i in two cases: 1) the total offered price of buyers in C^i is higher than that of buyers in C'^i, and buyers in C^i are all non-interfering; 2) there are interfering buyers in C'^i. We also assume that a seller is indifferent towards two coalitions both involving interfering buyers, and she is indifferent towards a coalition of herself (unmatched) and a coalition with interfering buyers.

$$C^i \triangleright_i C'^i \Longleftrightarrow \begin{cases} \forall j, j' \in C^i, e^i_{j,j'} = 0, \sum_{j \in C^i} b_{i,j} > \sum_{j \in C'^i} b_{i,j}, \text{or} \\ \exists j, j' \in C'^i, e^i_{j,j'} = 1. \end{cases} \tag{6}$$

B. Distributed Matching Algorithm

1) Stage I: Adapted Deferred Acceptance: The traditional deferred acceptance algorithm, designed to solve the college admission problem, runs as follows [3]. There is a set of students to be admitted to a set of colleges, each with a fixed quota. In the first round, each student applies to her favorite college. Among all applicants, a college with a quota q temporarily puts the top q students in the waiting list, or all students if the number of applicants is smaller than q, and rejects others. In the following rounds, each rejected student applies to her most-preferred college which has never rejected her before. Each college updates its waiting list by selecting the top q students among the current applicants and those in the previous waiting list. This process is repeated until all students have exhausted their applications.

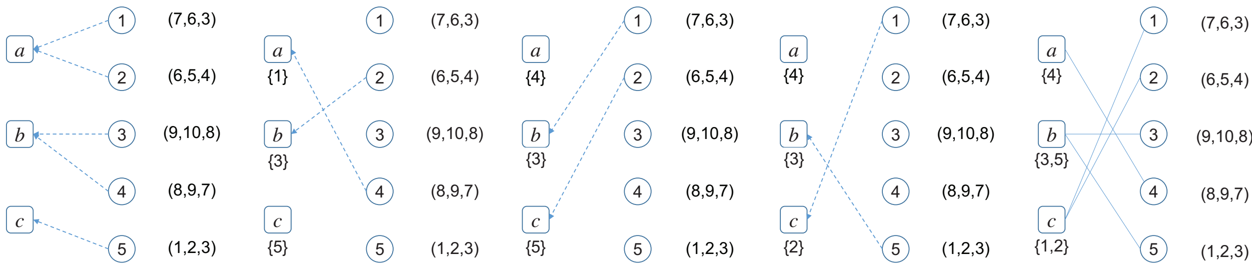
Algorithm 1 Stage I: Adapted Deferred Acceptance

Initialization
$\forall i \in \mathcal{M}$, the waiting list $\mathcal{L}_i = \Phi$, the current proposer list $\mathcal{P}_i = \Phi$.
$\forall j \in \mathcal{N}$, the unproposed seller list $\mathcal{A}_j = \mathcal{M}$.
while $\exists$ unmatched buyer with non-empty unproposed seller list **do**
  for all Unmatched buyer j with non-empty unproposed seller list **do**
    i = the most-preferred seller in $\mathcal{A}_j$.
    Buyer j proposes to seller i.
    Add buyer j to seller i's current proposer list, $\mathcal{P}_i = \mathcal{P}_i \cup \{j\}$.
    Remove seller i from the unproposed seller list, $\mathcal{A}_j = \mathcal{A}_j \setminus \{i\}$.
  end for
  for all Seller i with non-empty current proposer list **do**
    Form the most-preferred coalition $C^i \subseteq \mathcal{L}_i \cup \mathcal{P}_i, \forall C'^i \subseteq \mathcal{L}_i \cup \mathcal{P}_i, C^i \triangleright_i C'^i$.
    Set the waiting list as $\mathcal{L}_i = C^i$.
  end for
end while
for all $i \in \mathcal{M}$ **do**
  $\mu(i) = \mathcal{L}_i$.
end for
for all $j \in \mathcal{N}$ **do**
  if $\exists i, j \in \mathcal{L}_i$ **then**
    $\mu(j) = \{i\}$.
  else
    $\mu(j) = \{j\}$.
  end if
end for

We adapt the original deferred acceptance algorithm for spectrum matching in Algorithm 1. Unlike the college admission problem, we are not able to determine the "quota" of each seller, since the "quota" depends on the interference relationship among buyers. Instead, at each round, we let a seller to form her most-preferred spectrum coalition, *i.e.*, to select a group of buyers who do not interfere with each other according to the interference graph, and whose total offered price is the highest. To find a group of non-interfering buyers with maximum offered price is equivalent to finding a maximum weighted independent set on the interference graph, which is NP-hard. Greedy algorithms have been proposed to solve the maximum weighted independent set (MWIS) problem in linear time [8], which we will adopt in our algorithm.

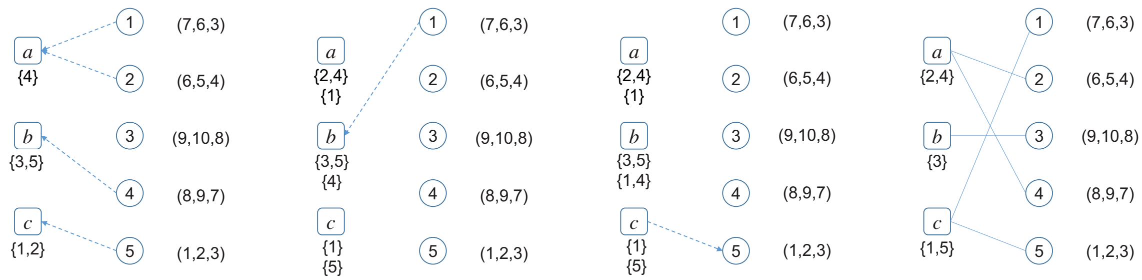
Proposition 1. *Algorithm 1 is guaranteed to converge, and the running time is $O(MN)$.*

Proof. Each time an unmatched buyer makes a proposal, she will remove one seller from her unproposed seller list.



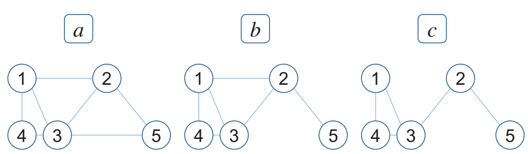
(a) 1st round proposal. (b) 2nd round proposal. (c) 3rd round proposal. (d) 4th round proposal. (e) Matching result.

Fig. 1. Stage I: Adapted deferred acceptance.

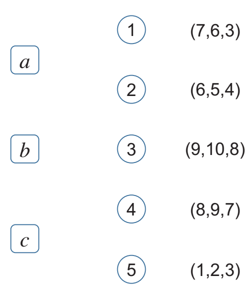


(a) 1st round transfer. (b) 2nd round transfer. (c) 1st round invitation. (d) Final matching result.

Fig. 2. Stage II: Transfer and invitation.



(a) Interference graph.



(b) Preference relations.

Fig. 3. Toy example.

Therefore, each buyer will eventually become matched or exhaust her unproposed seller list, thus Algorithm 1 comes to an end.

The worst case is that the interference graph on every channel is a complete graph. Then, the spectrum matching problem is reduced to a one-to-one matching problem, and Algorithm 1 is equivalent to the original deferred acceptance algorithm, whose running time is $O(MN)$. □

Toy Example. As shown in Fig. 3, there are five buyers $\{1, 2, 3, 4, 5\}$ and three sellers $\{a, b, c\}$. The interference graph on each channel is shown in Fig. 3(a); the buyers' utility vectors are shown in Fig. 3(b). We make the simplified assumption that $f(b_{i,j}) = b_{i,j}$. Fig. 1 shows the process of the adapted deferred acceptance algorithm. In the first round, buyer 1 and buyer 2 propose to seller a; buyer 3 and buyer 4 propose to seller b; buyer 5 proposes to seller c, as shown in Fig. 1(a). After the first round, all sellers' waiting lists are shown in Fig. 1(b). Then, the unmatched buyer 2 and buyer 4 propose to seller b and seller a, respectively. Seller a evicts buyer 1 to form a better coalition with buyer 4 in the waiting list. Fig. 1(c)(d) show the following rounds, and the final matching result is shown in Fig. 1(e), with a social welfare of 27.

2) Stage II: Transfer and Invitation: The deferred acceptance algorithm produces a stable matching result for college admission problem [9], *i.e.*, there is no pair of student and college who both prefer each other to their current choices. However, the matching result of the adapted deferred acceptance algorithm is not stable for spectrum matching. For instance, in Fig. 1(e), buyer 2 can be matched to seller a without interfering with buyer 4, and both buyer 2 and seller a are better off. This instability is a result of the peer effect of spectrum matching, caused by complicated interference relationship among buyers. Buyer 2 has a better chance of being chosen by seller a in the presence of her non-interfering buyer 4, but a worse chance in the presence of her interfering neighbor 1 (e.g., in the first round).

To improve the matching result and achieve a stable matching, we propose a transfer and invitation algorithm as a second stage, as shown in Algorithm 2. At Stage II Phase 1, buyers send transfer applications to sellers who are more preferred than their currently matched sellers. This means that buyers send transfer applications to sellers whom they have proposed to in Stage I. But unlike Stage I, sellers cannot evict any currently matched buyers in Stage II. Hence a buyer's transfer application to a seller can only be accepted if she does not interfere with any buyers matched to the seller. In Stage I, we do not allow buyers to re-propose to sellers who have rejected them, because this may lead to Ping-Pong effect, where buyers continuously make proposals and the algorithm never converges. In Stage II, such Ping-Pong effect won't happen because each buyer can only send transfer application *once* to each seller who is more preferred than her currently matched seller, and the number of such sellers is *limited*.

At Stage II Phase 2, as a seller's previously matched buyers may have transferred to other sellers, she can invite some of the buyers whom she has rejected in Phase 1. In simulations, we find that the invitation opportunities are scarce, but Stage II Phase 2 has to be included to guarantee the stability of the final matching result.

Toy example. Fig. 2 shows the process of our transfer and invitation algorithm. Given the matching result in Fig. 1(e), buyer 1 and buyer 2 send transfer applications to seller a; buyer 4 sends transfer application to seller b; buyer 5 sends transfer application to seller c. Buyer 2's application is granted while other buyers are rejected and added to the sellers' rejecting lists, shown under the matching lists. After Phase 1, seller c sends invitation to buyer 5, and the final matching result is shown in Fig. 2(d), with a social welfare of 30.

Proposition 2. *Algorithm 2 is guaranteed to converge, and the running time of Phase 1 is $O(M)$, of Phase 2 is $O(N)$.*

Proof. Each buyer sends transfer application once to each more-preferred seller, so Phase 1 will terminate in a limited number of rounds. In Phase 2, each seller has a finite invitation list and only sends invitation at most once to each buyer in the list, thus Phase 2 will also end in a limited number of rounds.

In Phase 1, each buyer has at most M more-preferred sellers, so the running time is $O(M)$. Unlike Stage I, no buyer will be evicted in Stage II, therefore the running time is $O(M)$ rather than $O(MN)$. In Phase 2, each seller has at most N buyers in the invitation list, so the running time is $O(N)$. □

C. Properties

In this section, we prove that the matching result of the proposed distributed algorithm is individual rational and Nash-stable.

Definition 2. *(Individual rational).*

Algorithm 2 Stage II: Transfer and Invitation

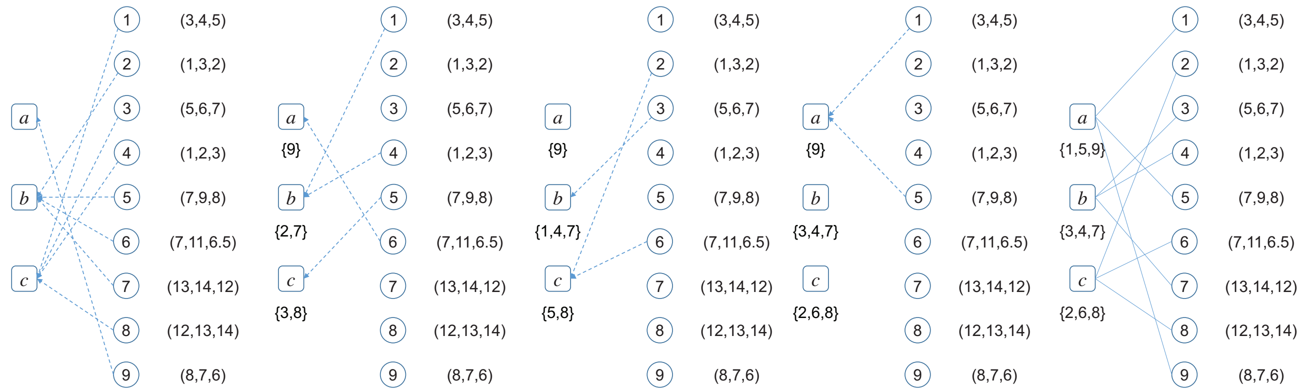
Initialization
$\forall i \in \mathcal{M}$, the current applicant list $\mathcal{D}_i = \Phi$, the invitation list $\mathcal{R}_i = \Phi$.
$\forall j \in \mathcal{N}$, the unapplied seller list $\mathcal{T}_j = \{i | b_{i,j} > b_{\mu(j),j}\}$.
Phase 1: Transfer
while $\exists$ buyers with non-empty unapplied seller list **do**
  for all Buyer j with non-empty unapplied seller list **do**
    i = the most-preferred seller in $\mathcal{T}_j$.
    Buyer j sends a transfer application to seller i.
    Add buyer j to seller i's current applicant list $\mathcal{D}_i = \mathcal{D}_i \cup \{j\}$.
    Remove seller i from the unapplied seller list $\mathcal{T}_j = \mathcal{T}_j \setminus \{i\}$.
  end for
  for all Seller i with non-empty current applicant list **do**
    Select the most-preferred coalition $C^i = \mu(i) \cup \mathcal{S}, \mathcal{S} \subseteq \mathcal{D}_i, \forall C'^i = \mu(i) \cup \mathcal{S}', \mathcal{S}' \subseteq \mathcal{D}_i, C^i \triangleright_i C'^i$.
    Update the matching, $\mu(i) = C^i$, $\forall j \in C^i, \mu(j) = i$.
    Set the invitation list as $\mathcal{R}_i = \mathcal{R}_i \cup \mathcal{D}_i \setminus \mathcal{S}$.
  end for
end while
Phase 2: Invitation
for all $i \in \mathcal{M}$ **do**
  Screen non-interfering buyers in the invitation list $\mathcal{R}_i = \{j | j \in \mathcal{R}_i, \forall j' \in \mu(i), e^i_{j,j'} = 0\}$.
end for
while $\exists$ seller with non-empty invitation list **do**
  for all Seller i with non-empty invitation list **do**
    j = buyer with the highest offered price.
    Seller i send invitation to buyer j.
    if Seller i is more preferred than $\mu(j)$ **then**
      Buyer j accepts invitation.
      Update the matching, $\mu(i) = \mu(i) \cup \{j\}, \mu(j) = i$.
      Remove buyer j's interfering neighbors in the invitation list $\mathcal{R}_i = \mathcal{R}_i \setminus \{j | j' \in \mathcal{R}_i, e^i_{j,j'} = 1\}$.
    end if
    Remove buyer j from the invitation list $\mathcal{R}_i = \mathcal{R}_i \setminus \{j\}$.
  end for
end while

A matching result is blocked by seller i if she prefers not to be matched to some of her currently matched buyers. In other words, $\exists \mathcal{S} \subseteq \mu(i), \mathcal{S} \neq \Phi, C^i = \{i\} \cup \mu(i), C'^i = \{i\} \cup \big(\mu(i) \setminus \mathcal{S}\big), C'^i \triangleright_i C^i$.

A matching result is blocked by buyer j if she prefers being unmatched to being matched to the current seller. In other words, $\{j\} \triangleright_j \big(\mu(j) \cup \mu(\mu(j))\big)$.

A matching result is individual rational if it is not blocked by any buyer or seller.

Proposition 3. *The matching result of the proposed distributed algorithm is individual rational.*



(a) 1st round proposal. (b) 2nd round proposal. (c) 3rd round proposal. (d) 4th round proposal. (e) Final matching result.

Fig. 4. Counter example, proposed spectrum matching algorithm.

Proof. For seller i, the matching result $\mu(i)$ of the proposed algorithm is interference-free. In this case, removing any buyers in $\mu(i)$ will reduce the total offered price, and result in a less-preferred matching result.

For a matched buyer j, she does not have interference neighbors in $\mu(\mu(j))$. Therefore, her utility is positive, higher than the utility of being unmatched, which is zero. □

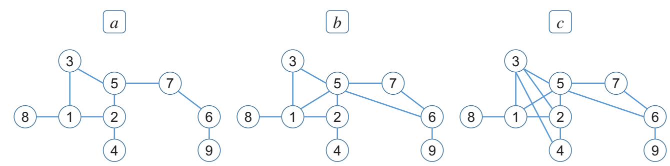


Fig. 5. Counter example, interference graph.

Deferred acceptance algorithm can achieve pairwise stability for the college admission problem, *i.e.*, no pair of detached student and college would both be better off if they are matched together. Due to peer effects in spectrum matching, pairwise stability cannot be preserved, whereas we can guarantee a weaker form of stability.

Definition 3. *(Nash-stable). A matching result is Nash-stable if no buyer prefers to be a member of another spectrum coalition rather than stay in the current spectrum coalition, i.e.,* $\forall j \in \mathcal{N}, i = \mu(j), i' \neq i, \big(i \cup \mu(i)\big) \rhd_j \big(i' \cup \mu(i') \cup j\big)$.

Proposition 4. *The matching result of the proposed distributed algorithm is Nash-stable.*

Proof. We prove this by contradiction. Assume that the final matching result is not Nash-stable. There exists at least one buyer j who prefers to join another spectrum coalition with seller $i \neq \mu(j)$. This implies that in Stage II Phase 1, buyer j must have sent a transfer application to seller i because buyer j prefers i to $\mu(j)$. Seller i must have rejected j and put j in her rejecting list. In Stage II Phase 2, seller i can not have sent an invitation to buyer j; otherwise, j will accept the invitation and be matched to seller i. This implies that buyer j must interfere with some buyers in $\mu(i)$. Therefore, buyer j would not like to join seller i's spectrum coalition because her utility will become zero, which contradicts the assumption. Hence, the matching result of the proposed algorithm is Nash-stable. □

D. Discussions

In this section, we discuss the limitations of the proposed matching algorithm. The deferred acceptance can achieve a matching result that is pairwise stable and student-optimal for the traditional college admission problem[2]. Unfortunately, our two-stage algorithm based on adapted deferred acceptance cannot achieve these two ideal properties, which may inspire future research in this direction.

Definition 4. *(Pairwise stability). A matching is blocked by a pair of seller i and buyer $j \notin \mu(i)$, if there exists $\mathcal{S} \subseteq \mu(i)$ such that:*

- *Non-interfering condition:* $\forall j' \in \mathcal{S}, e^i_{j,j'} = 0$.
- *Seller improvement:* $\big(\{i\} \cup \{j\} \cup \mathcal{S}\big) \rhd_i \big(\{i\} \cup \mu(i)\big)$.
- *Buyer improvement:* $\big(\{i\} \cup \{j\} \cup \mathcal{S}\big) \rhd_j \big(\mu(j) \cup \mu(\mu(j))\big)$.

A matching is pairwise stable if it is not blocked by any seller-buyer pair.

Unfortunately, the proposed spectrum matching algorithm cannot ensure pairwise stability. A counter example is given in Fig. 5 and Fig. 4. We ignore Stage II since the matching result will not change in this example. Given the final matching result in Fig. 4(e), seller b and buyer 2 form an unstable pair. According to Definition 4, we know that $\mathcal{S} = \{3, 7\}$; seller b prefers the spectrum coalition $\{b, 2, 3, 7\}$ to the current

[2]Note that the definition of the optimal matching here is different from that in Section II.

spectrum coalition $\{b, 3, 4, 7\}$; buyer 2 prefers seller b to the currently matched seller c. Seller b and buyer 2 have incentives to be matched together at the sacrifice of buyer 4, which is, however, not allowed by the proposed matching algorithm.

Another nice property of the deferred acceptance algorithm is that its matching result realizes an optimal assignment of students among all pairwise stable matching result, that is, no student can get admitted by a better college in another pairwise stable matching result[3]. As the spectrum matching can only achieve Nash-stability, we give the following definition for optimality regarding Nash-stable matching.

Definition 5. *(Optimality). A Nash-stable matching result μ is buyer-optimal, if there does not exist another Nash-stable matching result μ', in which no buyer is worse off, and some buyers are better off. In other words, $\forall j \in \mathcal{N}, \big(\mu(j) \cup \mu(\mu(j))\big) \triangleright_j \big(\mu'(j) \cup \mu'(\mu'(j))\big), \forall \mu'$ that is Nash-stable.*

Using the same counter example in Fig. 5 and Fig. 4, we show that the matching result of the proposed algorithm is not optimal. Swap buyer 2 and buyer 4 to seller b and seller c, respectively. It can be easily checked that the new matching result is Nash-stable. The new matching result is strictly better than the one produced by the proposed algorithm, in that not only buyer 2 and buyer 4 but also seller b and seller c are better off, and other buyers and sellers are unaffected. In Stage II Phase 1, such a swap cannot be accomplished because seller b is not aware that buyer 4 can transfer to seller c, as long as she accepts the transfer application from buyer 2, who is matched to seller c and interferes with buyer 4. How to enable such a swap, which requires a coordination among different sellers and buyers, is an interesting topic for future works.

IV. Implementation of Spectrum Matching

The proposed two-stage matching algorithm runs in a distributed fashion within each stage and each phase. Nevertheless, asynchronization problem arises during stage or phase transition. More specifically, Stage II commences when all buyers exhaust their proposals and start the transfer application. Unfortunately, it is impossible for a buyer to know whether all other buyers have stopped making proposals. Similar problem exists during phase transition in Stage II. Therefore, in this section, we specify practical rules for stage or phase transition, facilitating the implementation of the proposed two-stage matching algorithm in perfectly distributed manner.

Assume that each round in the proposed algorithm takes one time slot. Proposition 1 and Proposition 2 give the running time of Algorithm 1 and Algorithm 2, respectively, based on which we have the following default transition rule.

Default transition rule. From the very beginning, all buyers and sellers wait for MN time slots to transit to Stage II; then M time slots to transit to Stage II Phase 2; finally N time slots to end the matching process.

[3]When the students initiate the proposal, the matching result is optimal for students, but is not guaranteed to be optimal for colleges.

The default transition rule can be extremely inefficient. For instance, given the toy example in Fig. 3, according to the default transition rule, the whole matching process takes 23 time slots, but in fact, 7 time slots are enough to reach the final matching result as shown in Fig. 1 and Fig. 2. To tackle this problem, we design the following transition rules.

A. Stage Transition Rules on Buyers' Side

For a buyer, the risk of a premature entrance into Stage II is being evicted after she starts to send transfer applications but no more proposals. A transfer application has a lower chance of being accepted than a proposal, as the seller will not evict any currently matched buyers upon a transfer application. Therefore, a buyer should transit to Stage II only when her risk of being evicted by the currently matched seller is low. One observation is that a buyer only faces the threat from her interfering neighbors, so we have the following transition rule.

Stage transition rule I for buyers. A buyer can transit to Stage II if all her interfering neighbors have proposed to her currently matched seller.

Stage transition rule I for buyers guarantees that a buyer's matching result in Stage I will not change anymore, but the condition may be hard to meet. For example, in Fig. 1, buyer 4 will never detect buyer 3's proposal to seller a because buyer 3 never proposes to seller a.

To design a more operable rule, we estimate the probability of a buyer being evicted after she performs the stage transition. The smaller this probability is, the less risky for the buyer to enter Stage II. Assume that all buyers' prices follow identically independent distribution (i.i.d.) with a cumulative distributed function $F(\cdot)$.

Consider buyer j who is matched to seller i till the $(k-1)$th round, and n of her interfering neighbors have not proposed to seller i yet. Let p_x^k denote the probability that, at the kth round, x of buyer j's interfering neighbors propose to seller i, and at least one of their offered prices is higher than buyer j's.

$$p_x^k = \binom{n}{x}(\frac{1}{M})^x(1-\frac{1}{M})^{n-x}\big(1-F^x(b_{i,j})\big). \quad (7)$$

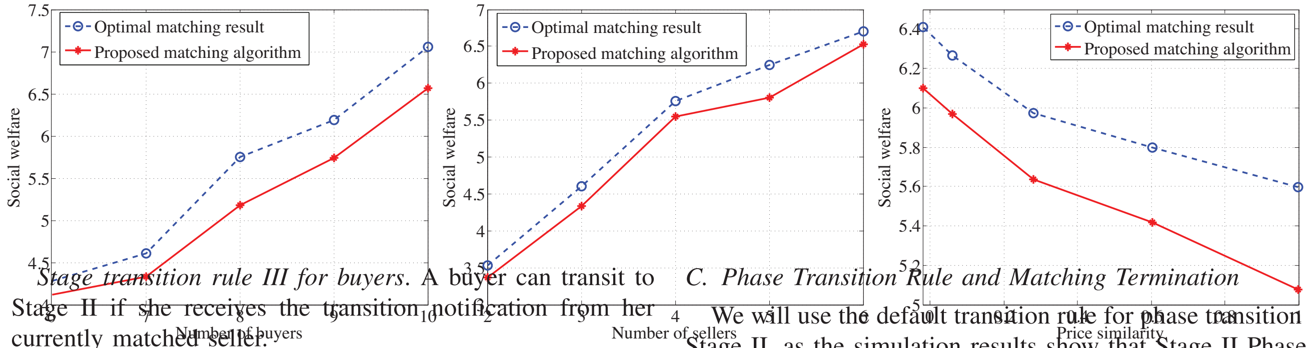
The probability of buyer j being evicted in the kth round can be estimated as $p^k = \sum_{x=1}^{n} p_x^k$. The probability of buyer j being evicted in the $(k+1)$th round but not in the kth round is $(1-p^k)p^k$. Following the same logic, the probability of buyer j being evicted through the kth round till the MNth round is:

$$\begin{aligned} P^k &= p^k + (1-p^k)p^k + \cdots + (1-p^k)^{MN-k}p^k \\ &= 1-(1-p^k)^{MN-k+1} \end{aligned} \quad (8)$$

P^k decreases with k, so it is more secure for a buyer to commence Stage II at a later round.

Stage transition rule II for buyers. A buyer can transit to Stage II at the kth round if P^k is less than a threshold.

We have one more transition rule for buyers that is incurred by sellers. When a seller determines to carry out stage transition, she will inform all her currently matched buyers, which ensures that the seller will no longer evict these buyers.



(a) (b) (c)

Fig. 6. Optimal matching result versus the proposed distributed spectrum matching algorithm. (a) $M = 4$; (b) $N = 8$; (c) $M = 5$, $N = 8$.

Stage transition rule III for buyers. A buyer can transit to Stage II if she receives the transition notification from her currently matched seller.

B. Stage Transition Rule on Sellers' Side

A seller has to make the stage transition decision if she receives no proposal but some transfer applications in the current time slot. After stage transition, a seller cannot grant proposals anymore. In other words, none of her currently matched buyers can be expelled to make room for new buyers. We estimate the probability of a seller getting better proposals after she makes the stage transition. If the probability is low, a seller may begin to process the transfer applications, thus completing the stage transition.

Consider seller i who is, at the $(k-1)$th round, matched to a group of buyers, among which buyer j has the lowest offered price of $b_{i,j}$. There are n buyers who haven't proposed to seller i yet. Let θ denote the probability that an un-proposed buyer does not interfere with anyone in $\mu(i)$ except buyer j. θ is an empirical value, which can be estimated by analyzing the interference relationship between buyers in and out of $\mu(i)$. Let q_y^k denote the probablility that, at the kth round, y buyers propose to seller i, and at least one of them offers a price higher than $b_{i,j}$, and this buyer do not interfere with anyone in $\mu(i)$ other than buyer j.

$$q_y^k = \binom{n}{y}(\frac{1}{M})^y(\frac{M-1}{M})^{n-y}\Big[1-\Big(F(b_{i,j}) + (1-\theta)(1-F(b_{i,j}))\Big)^y\Big]. \tag{9}$$

Similar to (8), the probability of seller i receiving better proposals through the kth round till the MNth round is $Q^k = 1-(1-q^k)^{MN-k+1}$, in which $q^k = \sum_{y=1}^{n} q_y^k$. Q^k also decreases with k, thus it is less likely for a seller to obtain more favorable proposals at a later round.

Stage transition rule for sellers. A seller can transit to Stage II at the kth round if Q^k is less than a threshold.

Upon stage transition, a seller will notify all her currently matched buyers, and these buyers will transit to Stage II as well, as specified by stage transition rule III for buyers.

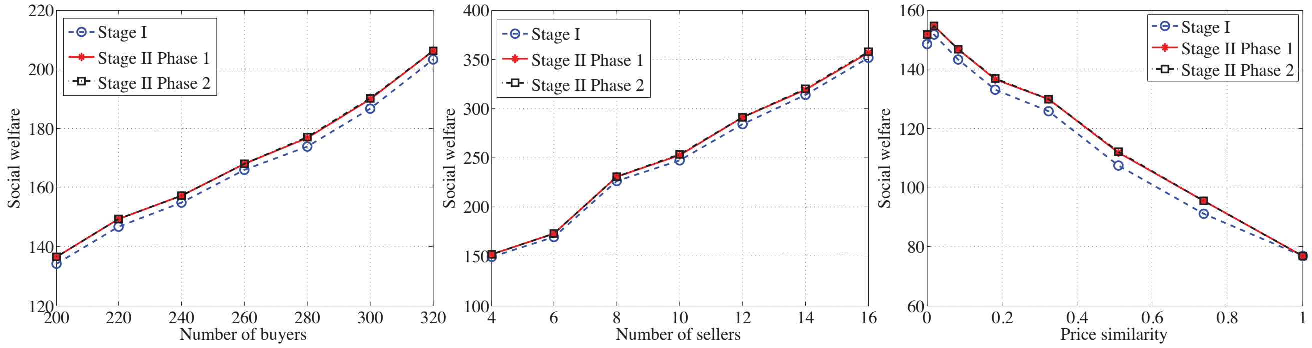
C. Phase Transition Rule and Matching Termination

We will use the default transition rule for phase transition in Stage II, as the simulation results show that Stage II Phase 1 lasts for approximately M rounds. Buyers and sellers have to decide when to terminate the matching process so that spectrum exchange can be finalized. The simulation results show that Stage II Phase 2 only runs a few rounds, as the opportunities for sellers to send invitations to buyers are rare. Nonetheless, the invitation phase is indispensable to guarantee the stability of the final matching result. We set the rule that each seller will put an end to the matching process when she has no invitation to make, and let the user to access her channel.

V. Simulation

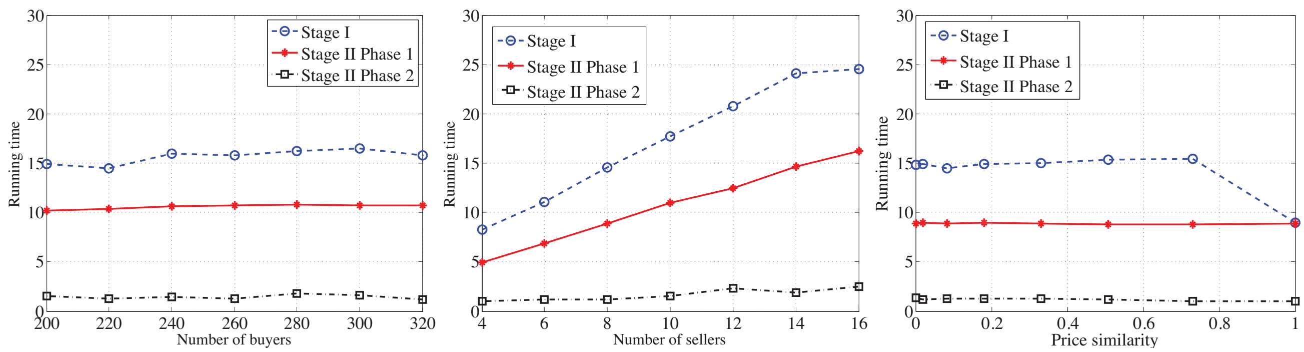
A. Simulation Settings

We assume that buyers are randomly located in a 10×10 area. The transmission range of each channel is randomly chosen in the range $(0, 5]$. The interference graph of each channel is established based on users' locations and the transmission range of the channel. Users' utility vectors are independently and identically distributed (i.i.d.), following a uniform distribution in $[0, 1]$. The numbers of buyers and sellers are specified in each simulation scenario. The similarity across buyers' utility vectors are quantified by the Spearman's rank correlation coefficient (SRCC) [10], which assesses whether the relationship between two variables can well be described as monotonic. We compute the SRCC for every pair of buyer's utility vectors, and obtain the average value. If the result is close to 1, buyers' utility vectors are perfectly similar to one another. If the result is close to 0, buyers' utility vectors are perfectly random and independent from one another. To study the utility similarity and its influence on the matching result, we maneuver buyers' utility vectors as follows. First, we sort all buyers' utilities in the ascending (or descending) order. In this way, the average SRCC is 1. Then, for each buyer, we randomly select m out of M items from her utility vector and perform an $m-$permutation. As m increases, the average SRCC will decrease, indicating that the buyers' utility vectors become more dissimilar to one another. When $m = M$, the SRCC is approximately 0.



(a) (b) (c)

Fig. 7. The social welfare of the two-stage distributed spectrum matching algorithm. (a) $M = 10$; (b) $N = 500$; (c) $M = 8, N = 300$.



(a) (b) (c)

Fig. 8. The running time of the two-stage distributed spectrum matching algorithm. (a) $M = 10$; (b) $N = 500$; (c) $M = 8, N = 300$.

B. Performance of the Proposed Matching Algorithm

We compare the social welfare of the matching result generated by the proposed algorithm and that of the optimal matching result derived by (1)[4], as shown in Fig. 6. Our proposed distributed matching algorithm can obtain more than 90% of the social welfare from the optimal matching result. Moreover, the running time of the proposed algorithm is only $O(MN)$ while the optimal matching problem in (1) is NP-hard. The social welfare grows with the number of buyers or sellers. If the buyers' utility vectors are similar to each other, multiple buyers will compete for the same channel, and it is hard to satisfy every buyer's requirement; on the contrary, if buyers' utility vectors are more diverse, they will pursue different channels, and the final matching result can satisfy more buyers.

[4]The optimal matching result is derived by the brute-force approach. As the running time exponentially increases with the numbers of buyers and sellers, we can only simulate small-scale spectrum markets.

C. Two-stage Distributed Algorithm

In Fig. 7 and Fig. 8, we demonstrate the social welfare and running time of different stages and phases in our proposed two-stage matching algorithm. Note that the social welfare is accumulated with each stage and phase, while the running time is separately counted for each stage and phase. Most of the social welfare improvement in Stage II comes from Phase 1, while Phase 2 makes a minor contribution. Nevertheless, Phase 2 is indispensable to guarantee the stability of the final matching result.

When the number of buyers is far greater than the number of sellers, the running time of Stage I is mostly influenced by the number of sellers. The running time of Stage II Phase 1 is theoretically $O(M)$: it linearly increases with the number of sellers, irrespective of the number of buyers and their offered prices, as shown in Fig. 8. Stage II Phase 2 only runs for a few rounds, as opportunities for sellers to send invitations to buyers are rare.

VI. Related Work

Matching-based resource allocation. Gale and Shapley first studied the problems of stable matching in [3], and proposed the deferred acceptance algorithm to achieve a stable matching in a distributed way. Afterwards, the research in economics explores all kinds of variants of matching problems [11], [12]. Matching has been widely used for resource allocation in computer science. In [4], online and offline algorithms were proposed to match virtual machines to heterogeneous sized jobs in the cloud. In [5], matching was used to associate users to small cells. In [6], Device-to-Device users were matched to cellular users for resource sharing. In [13], secondary users were matched to primary users for data relay. In [14], a friendly jammer was matched to a transmission pair to help protect them from eavesdropping. However, none of these frameworks can be applied to spectrum matching, where the complicated interference relationship among buyers makes it challenging to reach a stable matching result.

Spectrum auctions. Double auctions used to be the major spectrum allocation paradigm for dynamic spectrum access. A truthful spectrum double auction was first proposed in [15]. In [16], a multi-auctioneer progressive auction mechanism was designed, with all sellers assuming the role of auctioneers. In [17], local availability of the spectrum license and its influence on spectrum auctions were studied. In [18], [7], heterogeneous interference graphs for different channels were built for spectrum reuse. Apart from single-round auction mechanisms, dynamic spectrum auction mechanisms were proposed in [19], [20]. The major drawback of double auctions is the need for a third-party authority to enforce the spectrum allocation in a centralized way, which is not applicable for the free spectrum markets.

VII. Conclusion

In this paper, we have presented the first matching framework for distributed spectrum exchange in a free spectrum market. In stark contrast to prior double auction mechanisms, spectrum matching does not require the centralized management of a third-party authority. We have designed a two-stage distributed algorithm, with consideration of the interference constraint in spectrum matching. We have theoretically proved the convergence of our algorithm, as well as the stability of the matching result. Simulations have demonstrated the efficiency of the proposed distributed algorithm, as the final matching results can attain 90% of the maximum social welfare from optimal matching that needs centralized enforcement.

VIII. Acknowledgement

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References

[1] X. Feng, J. Zhang, and Q. Zhang, "Database-Assisted Multi-AP Network on TV White Spaces: Architecture, Spectrum Allocation and AP Discovery," in *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, 2011.

[2] X. Chen and J. Huang, "Database-Assisted Distributed Spectrum Sharing," *IEEE Journal on Selected Areas in Communications (JSAC)*, vol. 31, no. 11, pp. 2349–2361, 2013.

[3] D. Gale and L. S. Shapley, "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, pp. 9–15, 1962.

[4] H. Xu and B. Li, "Anchor: A Versatile and Efficient Framework for Resource Management in the Cloud," *IEEE Transactions on Parallel and Distributed Systems (TPDS)*, vol. 24, no. 6, pp. 1066–1076, 2013.

[5] W. Saad, Z. Han, R. Zheng, M. Debbah, and H. V. Poor, "A College Admissions Game for Uplink User Association in Wireless Small Cell Networks," in *IEEE International Conference on Computer Communications (INFOCOM)*, 2014.

[6] Y. Gu, Y. Zhang, M. Pan, and Z. Han, "Cheating in Matching of Device to Device Pairs in Cellular Networks," in *IEEE Global Communications Conference (GLOBECOM)*, 2014.

[7] Y. Chen, J. Zhang, K. Wu, and Q. Zhang, "TAMES: A Truthful Double Auction for Multi-Demand Heterogeneous Spectrums," *IEEE Transactions on Parallel and Distributed Systems (TPDS)*, vol. 25, no. 11, pp. 3012–3024, 2014.

[8] S. Sakai, M. Togasaki, and K. Yamazaki, "A Note on Greedy Algorithms for the Maximum Weighted Independent Set Problem," *Discrete Applied Mathematics*, vol. 126, no. 2, pp. 313–322, 2003.

[9] D. Gusfield and R. W. Irving, *The Stable Marriage Problem: Structure and Algorithms*. MIT press, 1989.

[10] J. H. McDonald, *Handbook of Biological Statistics*. Sparky House Publishing Baltimore, MD, 2009, vol. 2.

[11] M. Pycia, "Many-to-One Matching Without Substitutability," *MIT Industrial Performance Center Working Paper*, vol. 8, p. 2005, 2005.

[12] E. Bodine-Baron, C. Lee, A. Chong, B. Hassibi, and A. Wierman, "Peer Effects and Stability in Matching Markets," *Algorithmic Game Theory*, pp. 117–129, 2011.

[13] S. Bayat, R. H. Louie, Y. Li, and B. Vucetic, "Cognitive Radio Relay Networks With Multiple Primary and Secondary Users: Distributed Stable Matching Algorithms for Spectrum Access," in *IEEE International Conference on Communications (ICC)*, 2011.

[14] S. Bayat, R. H. Louie, Z. Han, B. Vucetic, and Y. Li, "Physical-Layer Security in Distributed Wireless Networks Using Matching Theory," *IEEE Transactions on Information Forensics and Security*, vol. 8, no. 5, pp. 717–732, 2013.

[15] X. Zhou and H. Zheng, "TRUST: A General Framework for Truthful Double Spectrum Auctions," in *IEEE International Conference on Computer Communications (INFOCOM)*, 2009.

[16] L. Gao, Y. Xu, and X. Wang, "MAP: Multi-Auctioneer Progressive Auction for Dynamic Spectrum Access," *IEEE Transactions on Mobile Computing (TMC)*, no. 99, pp. 1–1, 2010.

[17] W. Wang, B. Li, and B. Liang, "District: Embracing Local Markets in Truthful Spectrum Double Auctions," in *IEEE International Conference on Sensing, Communication, And Networking (SECON)*, 2011.

[18] X. Feng, Y. Chen, J. Zhang, Q. Zhang, and B. Li, "TAHES: A Truthful Double Auction Mechanism for Heterogeneous Spectrums," *IEEE Transactions on Wireless Communications (TWC)*, vol. 11, no. 11, pp. 4038–4047, 2012.

[19] S. Wang, P. Xu, X. Xu, S. Tang, X. Li, and X. Liu, "TODA: Truthful Online Double Auction for Spectrum Allocation in Wireless Networks," in *IEEE International Symposium on Dynamic Spectrum Access Networks (DySPAN)*, 2010.

[20] Y. Chen, P. Lin, and Q. Zhang, "LOTUS: Location-Aware Online Truthful Double Auction for Dynamic Spectrum Access," in *IEEE International Symposium on Dynamic Spectrum Access Networks (DySPAN)*, 2014.