A Bayesian Game Model for Joint Pricing and Spectrum Allocation Strategy of Femtocell Service Providers

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***Abstract*—For wireless service providers (WSP), the emerging femtocell market brings new opportunities as well as challenges. It is difficult for a WSP to have complete information of the technical strength of other WSP in the market, and it is hard to estimate the user demand, which is influenced by the price and service quality of all competing WSP in the market. To address these problems, in this paper, we propose an economic framework for the WSP to maximize their utility, via a joint pricing and spectrum allocation strategy, under the condition of incomplete information of other rival WSP in the market. We study two scenarios: 1) all WSP enter the market at the same time; 2) the WSP enter the market at different times. In both scenario, we formulate the problem as a Bayesian game, and derive the Bayesian Nash equilibrium. The simulation results verify that the proposed joint pricing and spectrum allocation strategy outperforms sole pricing or sole spectrum allocation strategy. Interestingly, when the WSP enter the market at different times, the later entered WSP chooses more aggressive pricing and spectrum allocation strategy than the early entered WSP.**

I. Introduction

It is predicted that mobile data traffic is expected to grow exponentially in the coming years [1]–[3]. At the same time, the demand for improved coverage and capacity is unprecedentedly high. Huge amount of wireless traffic in hot spot puts great pressure on the existing cellular networks.

Femtocell can provide high quality coverage in homes or offices, creating an exciting and promising market for WSPs, who make profit from the new femtocell service offerings and increased macrocell user satisfaction due to traffic offloading. Though WSPs have strong motivation to exploit the femtocell market, there are several challenges they are facing.

1) *Complicated user choice due to competition among multiple WSPs.* When multiple WSPs launch femtocell services at the same time, users will drift away from a WSP if his rival WSP sets a lower price or provides a higher QoS.
2) *Incomplete information from competitors.* The Femtocell market is immature and WSPs lack adequate information to analyze the competence of their main opponents. Thus traditional game model based on complete information is not applicable.
3) *Price and spectrum allocation decision-making.* If the price is low, though the user demand increases, per-user profit decreases. If more spectrum is allocated to femtocell service, a higher QoS can be achieved, and more users will be attracted, but the WSP has to pay a higher price for the spectrum.
4) *Impact of market entry sequence.* Pioneering WSPs may take the lead in exploiting the femtocell market and expanding its market share. Nevertheless, their technical information can be studied by the later new-comers to make better decisions. Therefore, WSPs who enter the market at different phases should adopt different strategies.

To deal with the above challenges, WSPs have to cope with information incompleteness and jointly consider pricing and spectrum allocation strategies in order to maximize their profits. However, most of the existing work only considers either spectrum allocation [4]–[6] or the pricing [7]–[13] strategy. And to the best our knowledge, no work considers the market entry sequence and its impact on the WSPs' decision-making.

In this paper, we build a concrete framework for duopoly femtocell market based on Bayesian game model[1], in which the WSP leverages the probabilistic estimation of the other WSP's private information to maximize expected payoff. We investigate two market entry scenarios: 1) *Concurrent market entry.* Both WSPs enter the femtocell market at the same time. 2) *Sequential market entry.* One WSP enters the market first (which we refer to as the incumbent) and the other WSP follows afterwards (which we refer to as the new-comer). We formulate the first scenario as a static Bayesian game and the second one as a two-stage dynamic Bayesian game.

The main contributions of the paper are:

- To the best of our knowledge, we are the first to jointly consider pricing and spectrum allocation strategies for the competitive femtocell market with incomplete infor-

[1]To gain the insight of the interactions among WSPs, we focus on the duopoly market and will study oligopoly market in our future work.

mation.

- We formulate the competition of two WSPs as static or dynamic Bayesian games based on their market entry sequence, and study how the intrinsic and extrinsic factors influence WSPs' best response and the equilibrium of the femtocell market.
- The simulation results show that the proposed joint pricing and spectrum allocation strategy outperforms either sole pricing or sole spectrum allocation strategy in bring WSPs higher utility. The interesting observation in market with sequential entry is that the new-comer adopts more aggressive strategy than the incumbent, which verifies that information incompleteness truly places barriers for WSPs' decision making.

The rest of the paper is organized as follows. In section II we give a detailed analysis of the femtocell market. We study the concurrent market entry of two WSPs in section III, and the sequential market entry in section IV. We evaluate the proposed framework through simulation in section V. We finally summarize our work in section VI. Due to page limitation, we ignore all the proofs and give them in the online technical report.

II. Characterization of Femtocell Market

We consider a duopoly market where there are two WSPs $I = \{1, 2\}$ competing with each other as shown in Fig.1.

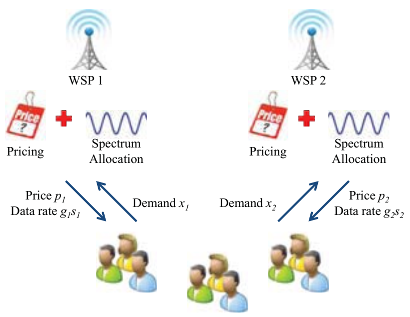


Fig. 1. Network model for the duopoly femtocell WSPs.

A. Strategy Space for WSPs

WSPs have two primary strategies: pricing and spectrum allocation. Strategy space of a WSP comprises its all possible price and spectrum allocation combination.

Consider a user j, when choosing the service from WSP i, his transmission rate is:

$$R(j, i) = A(s_i)g(SINR(j, i)), \tag{1}$$

in which $A(s_i)$ represents the influence of spectrum allocation and $g(SINR(j, i))$ represents the influence of Signal to Interference plus Noise Ratio (SINR). We assume that $A(s_i)$ is a linear function of s_i: $A(s_i) = \alpha s_i$. Let $P_{rx}(j, i)$ denote the signal power received at user j, $P_{noise}(j)$ denote the received noise power, $I(j)$ denote the received interference power from other transmitters at user j. Thus $SINR(j, i)$ can be modeled as

$$SINR(j, i) = \frac{P_{rx}(j, i)}{P_{noise}(j) + I(j)}.$$

The expected received power $P_{rx}(j, i)$ is mainly determined by the transmission power $P_{tx}(j, i)$ and the distance $r(j, i)$ between the femtocell BS and the user [14].

$$E[P_{rx}(j, i)] = E[B(j, i)]P_{tx}(i)(1 + r(u, i))^{-a}$$

in which $B(j, i)$ is a random variable that captures the channel fading effect, a is the path loss exponent whose typical value is $2 \sim 4$. Therefore, the expected transmission rate $R(j, i)$ is

$$E[R(j, i)] = \alpha s_i g(E[SINR(u, i)]) = s_i g_i$$

Here we assume $g_i = \alpha g(E[SINR(u, i)])$ is the spectral efficiency of WSP i. Spectral efficiency characterizes the transmission rate that can be achieved over a unit bandwidth. It is dependent on the WSPs' transmission rate, network topology and scheduling policy, which is private information and concealed from their rivals. Spectral efficiency can be viewed as the *type* of each WSP in the game.

B. User Behavior Analysis

Let x_i denote the demand for WSP i's femtocell service. We adopt the following demand function for each WSP [9], [15], [16]:

$$x_i = a_i(g_i s_i)^{\frac{1}{2}} - b_i(g_j s_j)^{\frac{1}{2}} - c_i p_i + d_i p_j, i \in I, j \in I\backslash\{i\} \tag{2}$$

in which $a_i, b_i, c_i, d_i, i \in I$ are all positive constants.

C. Influence of Femtocell Market on Macrocell

When the price of a femtocell service is low, more users are willing to access femtocell BSs. As a results, more traffic can be transferred from macrocell to femtocell, and the macrocell can support more users. Therefore, we assume that the increase in WSP i's macrocell demand Δx_i^m is a decreasing function of p_i:

$$\Delta x_i^m = x_i^0 - \frac{\theta_i}{p_i^m} p_i \tag{3}$$

where x_i^0 is the highest possible increase in demand, θ_i is the sensitivity of macrocell user demand towards p_i, p_i^m is the price for macrocell service. For simplification, we assume that $x_1^0 = x_2^0 = x_0, \theta_1 = \theta_2 = \theta$. We further assume that the macrocell market is perfect competitive so that p_i^m is the same for both WSPs, $p_1^m = p_2^m = p_m$.

D. Utility of WSP

WSP i's payoff is the revenue from femtocell plus the increment in macrocell revenue , minus the payment for spectrum acquisition.

$$M_i = p_i x_i + p_m \Delta x_i^m - p_s s_i \tag{4}$$

in which p_s is the unit spectrum price.

III. Static Competition with Concurrent Entry

In this section, we formulate the competition between two WSP, who concurrently enter the femtocell market as newcomers, as a static Bayesian game [17], [18].

A. Best Response

In a Bayesian game, since a player has incomplete information about the other player, it can not get a precise payoff of its own. Therefore, a rational risk-neutral player would seek a strategy that can maximize its own expected payoff.

Definition 1: Best Response. We define the best response in the Bayesian game as the strategy that generates the highest expected payoff for a WSP of all possible types, given its belief about the type and corresponding strategy of the other WSP.

$$\phi_i^*(\phi_{-i}|g_i) = \{\phi_i \,|\, E[M_i(\phi_i, \phi_{-i}|g_i)] \geq E[M_i(\phi_i', \phi_{-i}|g_i)], \forall \phi_i' \in \Phi_i\} \quad (5)$$

We make the simplified assumption that F_i is the same uniform distribution, i.e., $g_i \sim U(\underline{g}, \overline{g}), i \in I$.

Proposition 1: The Best response of WSP i. When the following conditions

$$\begin{aligned} &d_i p_j - b_i s_j^{\frac{1}{2}} g_e - \theta > 0, \\ &4 p_s c_i - a_i^2 g_i > 0 \end{aligned} \quad (6)$$

are satisfied, the best response of WSP i with spectral efficiency g_i is

$$\begin{aligned} p_i^*(p_j, s_j\,|g_i) &= \frac{2p_s(d_i p_j - b_i s_j^{\frac{1}{2}} g_e - \theta)}{4p_s c_i - a_i^2 g_i} \\ s_i^*(p_j, s_j\,|g_i) &= \frac{a_i^2 g_i}{4p_s^2}(p_i^*)^2 \end{aligned} \quad (7)$$

in which $g_e = E[g_j^{\frac{1}{2}}] = \frac{2(\overline{g}^{\frac{3}{2}} - \underline{g}^{\frac{3}{2}})}{3(\overline{g} - \underline{g})}$.

we show the payoff of the WSP with a particular spectral efficiency in Fig.2, given the price and spectrum allocation strategy of its rival WSP. It is shown that both price and spectrum allocation have considerable influence on the payoff. When condition (6) is satisfied, there exists an optimal joint price and spectrum point which generates the highest payoff for the WSP - the best response. Fig.3 better illustrates the best response. For each price there is a local optimal spectrum allocation value. The global optimal point is also the local optimal point when the price equals the optimal price as shown in Fig.3. It is also indicated that if only the price is set according to the best response, there can be no guarantee that the payoff will be higher than those of other price settings. In Fig.3 we can see, if the price is 2.69 (the best response) but the spectrum is 1, the payoff is less than the case where the price is 2.5 and the spectrum is 1. This reinforces the necessity for jointly considering both price and spectrum allocation.

The best response of a WSP is a function of the competitor's strategy, naturally affected by a change in the price and spectrum allocation of the rivalry WSP. In Fig.4(a), WSP

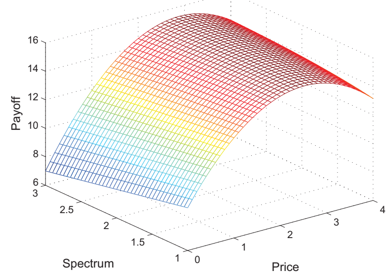


Fig. 2. Payoff of WSP vs the spectrum allocation and price.

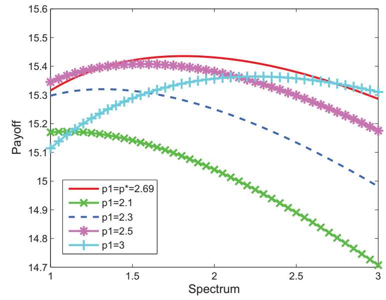


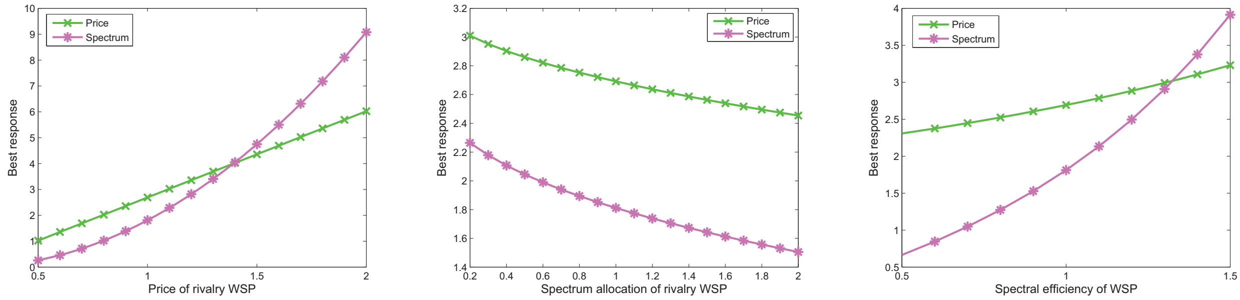
Fig. 3. Payoff of WSP vs the spectrum allocation under different prices.

increases its price when its opponent increases price because user demand will not decrease due to a stable price gap between two WSPs and the high unit price brings more revenue. As for spectrum, there are two possibilities: 1) The spectrum decreases because the competitor's price rise makes it possible for WSP to maintain user demand with lower QoS; 2) The spectrum increases because more users can be attracted and charged a higher unit price. Fig.4(a) shows that the second conjecture is valid. The spectrum allocation of a rival WSP also has a considerable influence on the best response of a WSP as shown in Fig.4(b). When the competitor increases itsspectrum allocation, the WSP has to reduce price in order to maintain user demand. However, instead of increasing spectrum allocation to further stabilize user demand, the WSP cuts down its spectrum allocation. From Fig.4(a) and Fig.4(b) we can see that, a WSP adjusts its price and spectrum allocation in response to a competitor's strategy, but the adjustment tends to have the opposite influence on the user demand. In general, users get higher QoS for a higher price and lower QoS for a lower price.

Best response is conditional on a WSP's spectral efficiency. When the spectral efficiency is high, a WSP is more willing to invest in spectrum (as shown in Fig.4(c)) since a small increase in spectrum allocation results in a larger increase in user demand. WSP also charges a higher price (as shown in Fig.4(c)) due to its technological competitive edge.

B. Bayesian Nash Equilibrium for Static Competition with Concurrent Entry

Definition 2: Bayesian Nash equilibrium. The strategy profile (ϕ_1, ϕ_2) is a (pure strategy) Bayesian Nash equilibrium if



(a) Price of rivalry WSP. (b) Spectrum allocation of rivalry WSP. (c) Spectral efficiency of WSP

Fig. 4. Influential factor for best response

for both WSPs $i \in I$ and for all possible spectral efficiencies (types) g_i of player i, $\phi_i(\phi_{-i}|g_i)$ is the best response of WSP i.

According to definition 2, we can derive the Bayesian Nash equilibrium of the static competition model.

Proposition 2: When the following conditions

$$1 - g_E^i > 0,$$
$$4p_s(c_i - d_i g_E^i) > (a_i g_i^{\frac{1}{2}} - g_E^i g_e b_j)^2, i \in I \tag{8}$$

are satisfied. There exists a (pure strategy) Bayesian Nash equilibrium where WSP i maximizes its revenue via joint pricing and spectrum allocation strategies:

$$p_i^*(g_i) = \frac{2p_s(1 - g_E^i)\theta}{4p_s(c_i - d_i g_E^i) - (a_i g_i^{\frac{1}{2}} - g_E^i g_e b_j)^2}$$
$$s_i^*(g_i) = \frac{(a_i g_i^{\frac{1}{2}} - g_E^i g_e b_j)^2}{4p_s^2} p_i^{*2} \tag{9}$$

where $g_E^i = \frac{2p_s(2c_j b_i - a_j d_i)}{a_j^3} \ln(\frac{a_j^2 \overline{g} - 4p_s c_j}{a_j^2 \underline{g} - 4p_s c_j}) + \frac{b_i}{a_j}$.

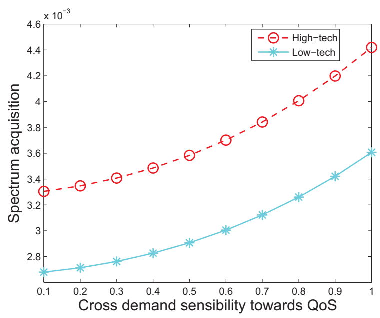


Fig. 8. Bayesian Nash equilibrium spectrum allocation vs cross demand sensibility in static competition.

In Fig.5 and Fig.6, the real spectral efficiency of WSP 1 is higher than that of WSP 2, which means that WSP 1 has a technological advantage over WSP 2. We refer to WSP 1 as a High-tech WSP and WSP 2 as a Low-tech WSP. The rise in the cross price elasticity of demand drives the price down for both WSPs as shown in Fig.5. The price for WSP 2 is lower compared with WSP 1 because WSP 2 has to cut its price to compensate for its technological weakness. Correspondingly, WSP 2 purchases less spectrum for its femtocell services as shown in Fig.6. In summary, a High-tech WSP adopts a "high price, high QoS" strategy while a Low-tech WSP adopts a "low price, low QoS" strategy.

When cross demand sensibility towards QoS increases, a WSP has to invest more in spectrum acquisition and enhance its own QoS in order to keep users from choosing its competitor's femtocell service. Users' high cross demand sensibility towards QoS forces WSPs to strengthen their technical competence, which requires more spectrum acquisition as shown in Fig.8 and accordingly a higher price as shown in Fig.7. WSP 1 tends to buy more spectrum and also sets a higher price compared with WSP 2, which indicates that High-tech WSPs stick to the "high price, high QoS" strategy while Low-tech WSPs choose the "low price, low QoS" strategy.

IV. Dynamic Competition with Sequential Entry

In this section, we study a two-stage sequential Bayesian game between two WSPs who enter the femtocell market at different times. One WSPs (WSP 1), which we refer to as the incumbent, determines its strategy first. The other WSP (WSP 2), which we refer to as the new-comer, observes the strategy of the incumbent and chooses its own strategy. We assume that WSP 2 knows the spectral efficiency of WSP 1, but WSP 1 does not know the spectral efficiency of WSP 2. This is reasonable since WSP 1 has been in the femtocell market for some time so that WSP 2 is able to better estimate WSP 1's spectral efficiency. This assumption can also be loosened to the case where the new-comer has an incomplete but more accurate spectral efficiency belief, i.e., $F_2(g_1) \sim U(\underline{g}', \overline{g}')$, where $\underline{g}' \geq \underline{g}, \overline{g}' \leq \overline{g}$. We use the backward induction method to derive the optimal strategy for both WSPs.

A. Best response of WSP 2

We assume that the spectral efficiency of WSP 1 is g, which is known to WSP 2.

Proposition 3: When the conditions

$$4p_s c_2 - a_2^2 g_2 > 0,$$
$$d_1 p_1 - b_2 s_1^{\frac{1}{2}} g^{\frac{1}{2}} - \theta > 0$$

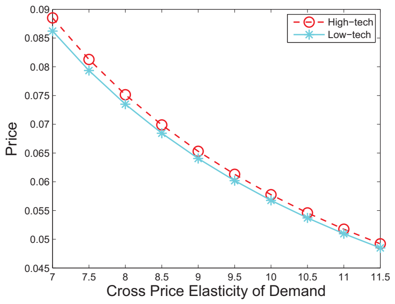


Fig. 5. Bayesian Nash equilibrium price vs cross price elasticity in static competition.

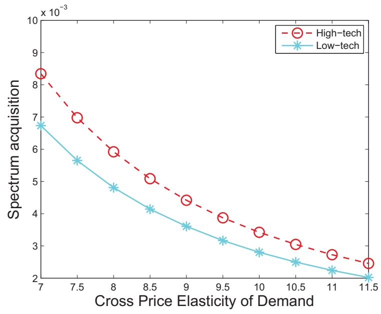


Fig. 6. Bayesian Nash equilibrium spectrum allocation vs cross price elasticity in static competition.

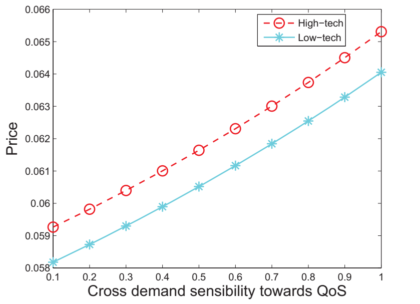


Fig. 7. Bayesian Nash equilibrium price vs cross demand sensibility in static competition.

are satisfied, WSP 2's best response is

$$\begin{aligned} p_2^*(p_1, s_1 \,|g_2) &= \frac{2p_s(d_1p_1 - b_2s_1^{\frac{1}{2}}g^{\frac{1}{2}} - \theta)}{4p_sc_2 - a_2^2g_2} \\ s_2^*(p_1, s_1 \,|g_2) &= \frac{a_2^2g_2}{4p_s^2}(p_2^*)^2 \end{aligned} \tag{10}$$

B. Optimal strategy of WSP 1

Proposition 4: When the following conditions

$$X > 0, Y > 0 \tag{11}$$

are satisfied, W_1 maximizes its expected revenue if and only if the price and spectrum allocation are determined as

$$\begin{aligned} p_1^* &= \frac{2p_sX}{Y} \\ s_1^* &= \frac{a_1^2g}{4p_s^2}(p_1^*)^2 \end{aligned} \tag{12}$$

in which

$$\begin{aligned} X &= \theta(b_1a_2\varepsilon_b - 2d_1p_s\varepsilon_a - 1), \\ Y &= 4p_sc_1 - a_1^2g + (2p_sd_1\varepsilon_a - a_2b_1\varepsilon_b)(a_1b_2g - 2p_sd_1) \\ \varepsilon_a &= E(\frac{1}{4p_sc_2 - a_2^2g_2}) = \frac{1}{a_2^2}\ln(\frac{a_2^2\underline{g} - 4p_sc_2}{a_2^2\overline{g} - 4p_sc_2}) \\ \varepsilon_b &= E(\frac{g_2}{4p_sc_2 - a_2^2g_2}) = -(\frac{1}{a_2^2} + \frac{4p_sc_2}{a_2^2}\varepsilon_a) \end{aligned} \tag{13}$$

V. Simulation Results

In this section, we compare three strategies: 1) The proposed joint pricing and spectrum allocation strategy; 2) The pricing without spectrum allocation strategy (referred to as sole pricing strategy); and 3) The spectrum allocation without pricing strategy (referred to as sole spectrum allocation strategy).

Fig.9 and Fig.10 show that the new-comer always sets a higher price and assigns more spectrum to its femtocell service. The new-comer acts more aggressively because it has a better understanding of its competitor. The incumbent only relies on its belief to make decisions so it acts more conservatively even though its spectral efficiency is exactly the same as its competitor's. Both concurrently entered WSPs know that their competitor is also restricted by a lack of information so their price and spectrum allocation are higher than those of the incumbent who is suffering from asymmetric information.

Fig.11 and Fig.12 show that if WSPs have complete information, they will set a higher price and purchase more spectrum. This indicates that information incompleteness hampers WSPs to some extent for adopting the most favorable strategy. The change in price and spectrum allocation in accordance with the change in cross demand sensibility is more dramatic with complete information than that with incomplete information. This is because WSPs are more responsive when they have handy information to help them make decisions.

It is obvious that the payoff for WSPs with joint strategy is always higher than that of either the fixed-price-optimal-spectrum allocation strategy or the optimal-price-fixed-spectrum allocation strategy as shown in Fig.13 and Fig.14, verifying that joint strategy outperforms sole strategy. As we stated before, if WSPs sets the price to be optimal, but randomly allocate spectrum, it is possible that the payoff is lower than the case when neither the price nor the spectrum allocation is optimal. Therefore, it is necessary for WSPs to auction for the right amount of spectrum in the spectrum market and then set the right price at the same time in order to gain maximum benefit from the femtocell market.

VI. Conclusion

In this paper, we study the newly emerged femtocell market, where multiple WSPs lack certain information about their competitors, yet trying to maximize their payoff through joint pricing and spectrum allocation strategies. We consider two scenarios. The first one is the static case where the two WSPs simultaneously enter the market. Based on Bayesian game theory, we derive the best response of each WSP and the Bayesian Nash equilibrium of the game. The second scenario is sequential market entry with one WSP as the incumbent and the other as the new-comer. The simulation results have shown that the new-comer adopts more aggressive strategies than the incumbent. This indicates that information actually helps WSPs make well-informed decisions. It is also verified that the joint strategy is better than either the sole pricing or sole spectrum allocation strategy.

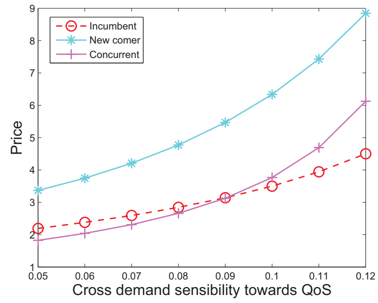


Fig. 9. Price of 1) Concurrent entry; 2) Incumbent; 3) New-comer.

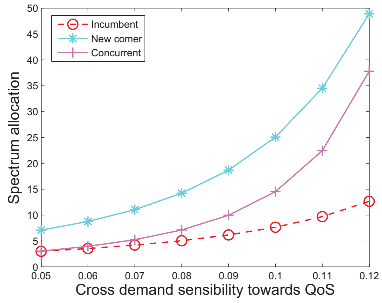


Fig. 10. Spectrum allocation of 1) Concurrent entry; 2) Incumbent; 3) New-comer.

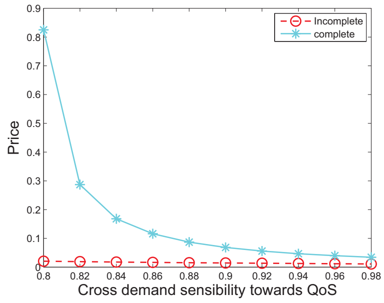


Fig. 11. Price in case of 1) Complete information; 2) Incomplete information.

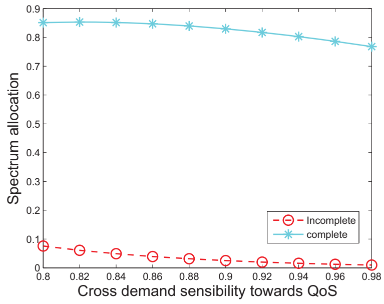


Fig. 12. Spectrum allocation in case of 1) Complete information; 2) Incomplete information.

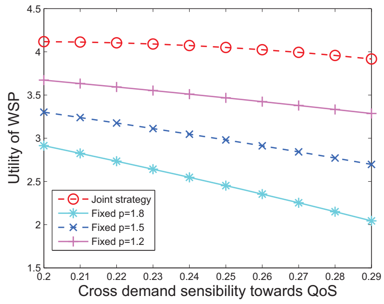


Fig. 13. Joint strategy vs sole spectrum allocation strategy.

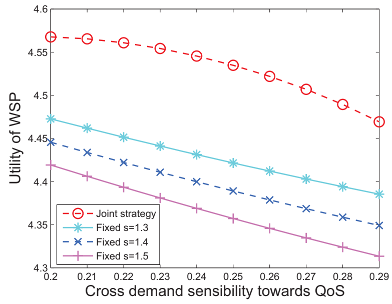


Fig. 14. Joint strategy vs sole pricing strategy.

VII. Acknowledgement

The research was support in part by grants from 973 project 2013CB329006, China NSFC under Grant 61173156, RGC under the contracts CERG 622410, 622613 and HKUST6/CRF/12R, the grant from Huawei-HKUST joint lab, Program for New Century Excellent Talents in University (NCET-13-0908), Guangdong Natural Science Funds for Distinguished Young Scholar (No.S20120011468), New Star of Pearl River on Science and Technology of Guangzhou (No.2012J2200081), Guangdong NSF Grant (No.S2012010010427), and China NSFC Grant 61202454.

References

[1] V. Chandrasekhar, J. Andrews, and A. Gatherer, "Femtocell networks: a survey," *Communications Magazine, IEEE*, vol. 46, no. 9, pp. 59–67, 2008.

[2] H. Claussen, L. Ho, and L. Samuel, "An overview of the femtocell concept," *Bell Labs Technical Journal*, vol. 13, no. 1, pp. 221–245, 2008.

[3] "Connecting When it Counts: the Role of Femtocells in Emergency Calls," *Intrado Technical Report*, 2008.

[4] S. Sengupta, M. Chatterjee, and S. Ganguly, "An economic framework for spectrum allocation and service pricing with competitive wireless service providers," in *DySPAN 2007*, pp. 89–98.

[5] A. Subramanian, M. Al-Ayyoub, H. Gupta, S. Das, and M. Buddhikot, "Near-optimal dynamic spectrum allocation in cellular networks," in *DySPAN 2008*, pp. 1–11.

[6] Y. Xu, J. Lui, and D. Chiu, "On oligopoly spectrum allocation game in cognitive radio networks with capacity constraints," *Computer Networks*, vol. 54, no. 6, pp. 925–943, 2010.

[7] H. Inaltekin, T. Wexler, and S. Wicker, "A duopoly pricing game for wireless ip services," in *SECON'07*, pp. 600–609.

[8] Y. Xing, R. Chandramouli, and C. Cordeiro, "Price dynamics in competitive agile spectrum access markets," *Selected Areas in Communications, IEEE Journal on*, vol. 25, no. 3, pp. 613–621, 2007.

[9] J. Jia and Q. Zhang, "Competitions and dynamics of duopoly wireless service providers in dynamic spectrum market," in *MobiHoc 2008*, pp. 313–322.

[10] V. Gajic, J. Huang, and B. Rimoldi, "Competition of wireless providers for atomic users: Equilibrium and social optimality," in *Communication, Control, and Computing, 47th Annual Allerton Conference on*. IEEE, 2009, pp. 1203–1210.

[11] J. Acharya and R. Yates, "Service provider competition and pricing for dynamic spectrum allocation," in *GameNets' 2009*, pp. 190–198.

[12] G. Kasbekar and S. Sarkar, "Spectrum pricing games with bandwidth uncertainty and spatial reuse in cognitive radio networks," in *Proceedings of the eleventh ACM international symposium on Mobile ad hoc networking and computing*. ACM, 2010, pp. 251–260.

[13] L. Duan, J. Huang, and B. Shou, "Duopoly competition in dynamic spectrum leasing and pricing," *Mobile Computing, IEEE Transactions on*, no. 99, pp. 1–1, 2011.

[14] J. Proakis, *Digital communications*. McGraw-hill, 1987, vol. 1221.

[15] N. Singh and X. Vives, "Price and quantity competition in a differentiated duopoly," *The RAND Journal of Economics*, vol. 15, no. 4, pp. 546–554, 1984.

[16] D. Niyato and E. Hossain, "Competitive pricing for spectrum sharing in cognitive radio networks: Dynamic game, inefficiency of nash equilibrium, and collusion," *Selected Areas in Communications, IEEE Journal on*, vol. 26, no. 1, pp. 192–202, 2008.

[17] J. Harsanyi, "Games with incomplete information played by" bayesian" players, i-iii. part i. the basic model," *Management science*, pp. 159–182, 1967.

[18] W. Armbruster and W. Böge, "Bayesian game theory," *Game theory and related topics, North-Holland, Amsterdam*, vol. 17, p. 28, 1979.