Incentive Mechanism for Hybrid Access in Femtocell Network with Traffic Uncertainty

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***Abstract*—Femtocell refers to a new class of low-power, low-cost base stations (BSs) which can provide improved indoor coverage and higher voice/data Quality of Service (QoS). Hybrid access in two-tier macro-femto networks is regarded as the most ideal access control mechanism to help offload macrocell traffic to femtocell, thus enhancing overall network performance. However, without suitable incentive mechanism, the Femtocell Service Providers (FSPs) are not willing to share their femtocell resource with the Macrocell Service Provider (MSP). To address this problem, in this paper, we propose an ACcess Permission (ACP) transaction framework, in which a single MSP purchases ACP from multiple FSPs in various locations throughout T timeslots, and FSPs who have overlapped coverage compete with each other for selling their ACP. However, we are facing the challenge that the demand of MSP in each location dynamically changes at each timeslot. At the start of each timeslot, FSPs are unaware of the demand of MSP, which impedes them to choose an ideal strategy that yields high payoff. To address the problem of information incompleteness, we propose an adaptive strategy updating algorithm, which is based on online learning process and enables FSPs to obtain guaranteed payoff. We conduct simulations to evaluate the payoff and the payoff gap of the FSPs when the MSP's demand is constant, quasi-constant or probabilistic. We also show that the payoff of the FSPs is affected by the learning speed of the proposed algorithm.**

I. Introduction

Femtocell is a fast-growing technology addressing the problem of poor indoor coverage and wireless capacity limitation [1]. The deployment and operating cost of femtocell is much lower than that of macrocell, which gives new business opportunity to small operators (Femtocell Service Providers (FSP)) who can invest in and manage femtocell-only network, referred to as stand-alone femtocell network [2]. The stand-alone femtocell network can not only serve registered femtocell users, but also assist macrocell in traffic offloading. Whether macrocell users are allowed to access femtocell BSs is decided by the access control mechanism adopted by FSP.

There are three mainstream access control mechanisms [3]: 1) *Closed access*. Only femtocell users can access femtocell BSs. 2) *Open access*. Any users, femtocell or macrocell users, can access femtocell BSs. 3) *Hybrid access*. Femtocell BSs accept macrocell users with certain conditions. In closed access, when macrocell users are very close to femtocell BSs, both macrocell users and femtocell users receive severe interference[1]. In open access, the QoS of femtocell users can not be guaranteed. Therefore, the most ideal way is hybrid access.

Despite its desired features, the implementation of hybrid access in femtocell networks is challenging. Interference mitigation schemes have been proposed in [4]–[6] from a technical perspective. However, performance concerns alone do not provide sufficient incentive to drive the hybrid access forward. In [7], [8], economic frameworks are proposed to optimize the utility of service providers who is in charge of both macrocell and femtocell networks. Nevertheless, when the MSP and the FSP are different operators (e.g. stand-alone FSP) who care about their own utility, considering the associated cost, FSP may not allow hybrid access of the macrocell user. In [9], a utility-aware refunding framework is proposed for the MSP to compensate the FSPs for taking over macrocell traffic, but the work does not consider the dynamics of femtocell and macrocell traffic. This motivates us to design an incentive mechanism to motivate FSPs to give hybrid access to macrocell users, which is able to deal with traffic uncertainty of both macrocell and femtocell networks. In doing so, we are facing the following challenges:

1) Overlapped femtocell coverage. Femtocell BSs can be densely deployed by different FSPs, and their coverage may overlap. At one spot, MSP can choose from multiple FSPs' femtocell BSs, which leads to a fierce sales competition among FSPs.
2) Unpredictable macrocell traffic. The dynamic change of macrocell traffic makes it difficult for FSPs to adjust their strategy: if the demand happens to be low in a certain location, the competition will be intense; if the demand happens to be high in a certain location, FSPs with a low price may lose potential revenue since MSP is willing to accept high price due to traffic pressure.
3) Variant FSP's supply. In hybrid access, FSPs should first guarantee the traffic demand of femtocell users, then use the extra resource to serve macrocell users. As femtocell users' traffic is dynamic, the remaining capacity[2] also changes accordingly.

To address the above problems, in this paper, we propose a novel femtocell ACcess Permission (ACP) trading framework,

[1]We consider the co-channel macro-femto network, which is proved to be of highest spectral efficiency.

[2]The capacity here is the total capacity of femtocell BS minus the capacity consumed by femtocell users. By monitoring the traffic of registered femtocell users, FSP can easily determine the capacity left for possible hybrid access of macrocell users.

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where a single MSP wants to procure hybrid access to femtocell network for its macrocell users by purchasing ACP from FSPs at different locations throughout T timeslots. We propose an adaptive strategy updating algorithm, which is based on online learning process, for FSPs to solve the problem of information incompleteness. FSPs "learn" from the historical results to get a better understanding of the market. At the start of each timeslot, the FSP selects a strategy according to the probability distribution. After each timeslot, the FSP adjusts its strategy-choosing probability based on the payoff in this timeslot. The payoff gap between the proposed algorithm and the optimal one is proved to be bounded.

The paper makes the following key contributions.

- We propose an incentive mechanism for the FSPs to implement hybrid access by trading ACP with the MSP.
- To deal with the problem that the FSPs are unaware of the MSP's demand and other FSP's strategy, we propose an adaptive strategy updating algorithm for individual FSP to dynamically adjust its strategy selection probability. We theoretically prove the bound of the payoff between the proposed algorithm and the optimal one.
- The simulation results verifies the negative influence of information incompleteness on the FSPs' payoff. It is also shown that either high or low learning speed in the adaptive strategy algorithm will reduce FSPs' payoff.

The rest of the paper is organized as follows. In section II, we describe the ACP transaction model in detail. Then we formulate the price competition among the FSPs as a non-cooperative game in section III. We present the adaptive strategy updating algorithm and prove the bound of payoff gap in section IV. We present simulation results in section V and finally summarize our work in section VI.

II. SYSTEM MODEL

We consider a single macrocell service provider (MSP) and a bunch of femtocell service providers (FSP). We use $\{F_1, F_2, ..., F_N\}$ to denote the N FSPs. There are K locations in consideration. Let $\{L_1, L_2, ..., L_K\}$ represent these locations. Femtocell BSs owned by a specific FSP cover various locations. In each timeslot, F_i informs MSP the ACP supply in each location. We use $q_{i,k}(t)$ to denote the capacity of femtocell BS owned by F_i located at L_k in timeslot t. The supply profile of F_i is $Q_i(t) = (q_{i,1}(t), q_{i,2}(t), ..., q_{i,K}(t))$. If F_i's femtocell BSs cover location L_k, $q_{i,k} > 0$; otherwise, $q_{i,k} = 0$.

MSP's demand for ACP is highly time-dependent. We consider a total time duration of T timeslots[3]. Let $d_k(t)$ denote the traffic demand in location k at timeslot t. We specifically study the following traffic demand pattern:

- **Constant Demand**: the demand of macrocelll users remains unchanged during the entire T timeslots. $d_k(t) = d_k$, which is time-independent, but still unknown to the FSPs. The situation of constant demand happens when the mobility of macrocell users in each location is relatively limited, e.g., in the office during work hours.
- **Quasi-constant Demand**: in timeslots $[1, t]$, the traffic demand remains constant; in timeslots $[t + 1, T]$, the traffic changes to a new state. The quasi-constant demand happens when people gather in one location for meeting or other social events, then back to their work place[4].
- **Probabilistic Demand** : at timeslot t, the value of $d_k(t)$ is determined by a probability distribution.

In each timeslot t, given the ACP supply and price by all FSPs, the MSP always chooses to buy from the FSP with the lowest price, then the second lowest price, third lowest price... until the demand is fully satisfied.

III. PROBLEM FORMULATION

A. Strategy Space for FSPs

We use $\mathcal{S}_i$ to denote the strategy space of F_i. In timeslot t, F_i's strategy is $s_i(t) = (p_{i,1}(t), p_{i,2}(t), ..., p_{i,K}(t))$, in which $p_{i,k}(t)$ is the price in location L_k. We assume that the price can be chosen from set $\{p_i^0, p_i^1, ..., p_i^m\}$, in which p_i^0 is the reserve price of F_i and p_i^m is the reserve price of MSP for F_i's ACP[5]. We assume that MSP does not differentiate the ACP from any FSP; so $p_1^m = p_2^m = ... = p_N^m = p^m$. We also assume that the available price choice and reserve price for all FSPs is the same, so we can ignore the subscript and re-write the price set as $\{p^0, p^1, ..., p^m\}$. The strategies chosen by all FSPs at timeslot t form a strategy profile $\{\mathcal{S}_1(t), \mathcal{S}_2(t), ..., \mathcal{S}_N(t)\}$. We use $\mathcal{S}_{-i}(t) = \{\mathcal{S}_1(t), \mathcal{S}_2(t), ..., \mathcal{S}_{i-1}(t), \mathcal{S}_{i+1}(t), ..., \mathcal{S}_N(t)\}$ to denote the strategy profile of FSPs excluding F_i.

B. Payoff for FSP

F_i's payoff does not only depend on the strategy chosen by F_i itself, but also relies on the strategy profile of all other FSPs. If a lot of FSPs provide large amount of ACP in one location, the competition will be intense, and FSPs with high price cannot sell their ACP. On the contrary, if rarely any FSPs offer ACP in one location (for instance, in remote places where the femtocell coverage is sparse), there may be a supply shortage, and as long as the proposed price does not exceed MSP's reserve price, the ACP will get sold. F_i's payoff at timeslot t is its revenue.

$$r_i(t) = \sum_{k=1}^{K} [p_{i,k}(t) * x_{i,k}(t)]. \quad (1)$$

in which $x_{i,k}(t) \leq q_{i,k}(t)$ is the actual amount of ACP sold in location L_k.

FSPs are rational and selfish, whose purpose is to maximize its own aggregated payoff throughout all timeslots. Let $\mathcal{S}_I = (s_i(1), s_i(2), ..., s_i(T))$ denote the strategy sequence

[3]The granularity of timeslots can be tailored to characterize all kinds of demand change. We assume that MSP is well aware of the ACP demand in each location by constantly monitoring the macrocell traffic.

[4]This one-change demand model can be easily extended to the multiple-change demand situation.

[5]The reserve price of F_i is the lowest price F_i is willing to accept, possibly to cover its cost; The reserve price of the MSP is the highest price the MSP is willing to pay, usually lower than serving the user via macrocell.

adopted by F_i throughout the entire T timeslots. The total payoff procured by F_i is the sum of the payoff in each timeslot.

$$R_{\mathcal{S}_I}(T) = \sum_{t=1}^{T} r_i(t). \tag{2}$$

Since the ACP demand dynamically changes throughout time, the Nash Equilibrium (NE) analysis in static non-cooperative game is not applicable here, because it is very likely that the NE already changes (due to ACP demand change) before the game converges to the previous NE. Therefore, instead of seeking FSPs' best response[6] that leads to NE, we focus on finding the adaptive strategy that can generate guaranteed payoff for FSPs.

We introduce the concept of the "best per timeslot strategy", based on which the "optimal static strategy" is defined.

Definition 1: **Best per timeslot strategy**. The best per timeslot strategy for femtocell owner F_i refers to the strategy $s_i = (p_{i,1}, p_{i,2}, ..., p_{i,K})$ which fixes the price in each location and yields the maximum aggregated payoff throughout the entire T timeslots.

Definition 2: **Optimal static strategy payoff**. The optimal static strategy payoff is the payoff received by F_i, if F_i plays the best per timeslot strategy throughout the entire T timeslots without changing.

Let $R_i^*(T)$ represent the optimal static strategy payoff of F_i. The payoff gap $G_i(T)$ is defined as the difference between maximum payoff $R_i^*(T)$ and the actual payoff $R_{\mathcal{S}_I}(T)$ over T period.

$$G_i(T) = R_i^*(T) - R_{\mathcal{S}_I}(T). \tag{3}$$

Our major objective is to find the strategy sequence $\mathcal{S}_I$ that has bounded payoff gap $G_i(T)$.

C. Multi-armed Bandit Problem Formulation (MBP)

The strategy choice of each FSP can be formulated as a non-stochastic multi-armed bandit problem [10], in which each possible strategy is an "arm", and the payoff of each FSP is not only influenced by the ACP demand of MSP but also depends on the whole strategy profile of rivalry FSPs. The size of strategy space is $N_s = N_p^K$, in which N_p is the number of price choices.

To express the strategy of FSPs more clearly, we use $(p,k), p \in [p^0, p^m]$ to denote the pairwise price and location, which we define as the "action" of F_i. At timeslot t, we say that $(p,k) \in s_i(t)$, if $p_{i,k}(t) = (p,k)$. Therefore, $s_i(t)$ is a combination of a sequence of actions (p,k), in which the element k traverses all locations $k = 1, 2, ..., K$ once and only once. The total number of action choices is $N_a = KN_p$. The number of actions is far less than the number of strategies, $N_a \ll N_s$. So instead of directly searching for optimal strategy, we will look for actions that yield maximum payoff.

[6] At timeslot t, given the ACP demand and the strategy profile of all FSPs other than F_i, the best response of F_i refers to the optimal strategy that yields maximum payoff for F_i.

IV. Adaptive Strategy Updating Algorithm

We consider two situations in terms of the degree of information incompleteness: 1) *Weakly incomplete information*: F_i does not know the MSP's demand or other FSPs' strategies at the start of each timeslot; but at the end of the timeslot, F_i can observe other FSPs' strategies; 2) *Strongly incomplete information*: F_i does not know the MSP's demand or other FSPs' strategy either at the start or the end of each timeslot.

A. Strongly Incomplete Information

The adaptive strategy updating algorithm for F_i in case of strongly incomplete information is shown in Algorithm 1.

Algorithm 1 Adaptive strategy updating algorithm for F_i (Strongly incomplete information)

Initiation phase $t = 0$;
Set the weight of each $w_{(p,k)}$ as equal.

$$w_{(p,k)}(0) = 1, \forall p \in [p^0, p^m], k = 1, 2, ..., K \tag{4}$$

Set the weight of each strategy as equal.

$$w_s(0) = 1, \forall s \in \mathcal{S}_i \tag{5}$$

Calculate the sum of strategy weight as

$$W(t) = \sum_{i=1}^{N_s} w_s(t) \tag{6}$$

Adaptive strategy updating
for all $t = 1, 2, .., T$ **do**
  F_i updates its strategy probability distribution as

$$d_s(t) = (1-\alpha)\frac{w_s(t-1)}{W(t-1)} + \frac{\alpha}{N_s} \tag{7}$$

  F_i randomly chooses a strategy s to be $s_i(t)$ according to the calculated probability distribution.
  F_i submits the supply $Q_i(t)$ and the price profile $s_i(t)$ to the MSP, receiving payoff $r_{(p,k)}(t)$ at location L_k.
  Compute the expected payoff of (p,k) as

$$\overline{r}_{(p,k)}(t) = \begin{cases} \frac{r_{(p,k)}(t)}{e_{(p,k)}(t)} & \text{if } (p,k) \in s_i(t) \\ 0 & \text{otherwise} \end{cases}$$

  in which

$$e_{(p,k)}(t) = \sum_{s:(p,k)\in s} d_s(t); \tag{8}$$

  is the probability of choosing action (p,k).
  F_i updates all the weights as

$$w_{(p,k)}(t) = w_{(p,k)}(t-1)e^{\beta \overline{r}_{(p,k)}(t)} \tag{9}$$

$$w_s(t) = \prod_{(p,k)\in s} w_{(p,k)}(t) \tag{10}$$

end for

In Algorithm 1, parameters α and β are used to control the speed of the learning process. If $\alpha = 1$, there is no strategy updating and the probability of choosing any strategy remains unchanged. If $\alpha = 0$, the strategy probability distribution at timeslot t merely depends on the strategy weight of the previous timeslot. In equation (9), the weight of action (p, k) is updated by multiplying $e^{\beta \overline{r}_{(p,k)(t)}}$. The higher the expected payoff of action (p, k) is, the greater it will weigh.

We give the upper bound of the payoff gap between the payoff generated by the proposed algorithm and the optimal static strategy payoff as follows:

Theorem 1: In case of strongly incomplete information, given the total time duration T, the number of locations K, and the strategy space of all femtocell owners, the gap between the payoff of adaptive strategy updating algorithm and the payoff of the optimal static strategy is no more than

$$G(T) \leq 2\sqrt{ATN_s \ln(N_s)} \quad (11)$$

in which $A = [1+M(e-2)]M$; $M = \max_s r_s$ is the maximum possible payoff in one timeslot.

Proof: We calculate the lower bound of $W(T)/W(0)$ as follows. For any strategy s,

$$\ln \frac{W(T)}{W(0)} \geq \ln \frac{\max_s w_s(T)}{N_s} = \beta \max_s \sum_{t=1}^{T} \overline{r_s}(t) - \ln(N_s)$$

Use the fact that $E[\overline{r_s}(t)] = r_s(t)$, we have

$$\ln \frac{W(T)}{W(0)} \geq \beta R_i^*(T) - \ln(N_s)$$

Then we calculate the upper bound of $W(T)/W(0)$.

$$\frac{W(t+1)}{W(t)} = \frac{\sum_{s=1}^{N_s} w_s(t+1)}{W(t)} = \frac{\sum_{s=1}^{N_s} w_s(t) e^{\beta \overline{r_s}(t)}}{W(t)}$$

Since $e^x \leq 1 + x + (e-2)x^2$, we have

$$\frac{W(t+1)}{W(t)} \leq 1 + \frac{\sum_{s=1}^{N_s} w_s(t)\beta \overline{r_s}(t)}{W(t)} + \frac{\sum_{s=1}^{N_s} w_s(t)(e-2)\beta^2 \overline{r_s}^2(t)}{W(t)}$$

Note that $\frac{w_s(t)}{W(t)} \leq \frac{d_s(t)}{1-\alpha}$, so we can get

$$\frac{W(t+1)}{W(t)} \leq 1 + \frac{\beta r_{s_i}(t)}{1-\alpha} + \frac{\sum_{s=1}^{N_s} M(e-2)\beta^2 \overline{r_s}(t)}{1-\alpha}$$

in which $M = \max r_s$. Knowing that $\ln(x) \leq 1 + x$, we have

$$\ln \frac{W(T)}{W(0)} \leq \frac{\beta R_{s_I}(T)}{1-\alpha} + \frac{M(e-2)\beta^2 N_s R_i^*(T)}{1-\alpha}$$

Combine the upper bound and lower bound, we can get

$$R_{s_I}(T) \geq [1 - \alpha - M\beta(e-2)N_s]R_i^*(T) - \frac{(1-\alpha)\ln(N_s)}{\beta}$$

Therefore,

TABLE I
SIMULATION PARAMETERS

Parameter	Description	Value
K	Number of locations	3
p^m	The MSP's reserve price	5
p^0	The FSPs' reserve price	1
β	Updating speed parameter	0.005
α	Learning speed parameter	0.3
T	Total number of timeslots	1000

$$G_i(T) \leq [\alpha + M\beta(e-2)N_s]MT + \frac{\ln(N_s)}{\beta}$$

Let $\beta = \frac{\alpha}{N_s}, \alpha = \sqrt{\frac{N_s \ln(N_s)}{AT}}$, in which $A = [1 + M(e - 2)]M$, we get

$$G_i(T) \leq 2\sqrt{ATN_s \ln(N_s)}$$

■

B. Weakly Incomplete Information with Feedback

In case of weakly incomplete information with feedback, after the ACP transaction ends at each timeslot, the demand of MSP and strategies of other FSPs will be disclosed. Therefore, the potential payoff of action (p, k) can be derived even if it is not contained in $s_i(t)$. With $r_{(p,k)}(t)$ for all possible (p, k), we modify the calculation of expected payoff of (p, k) in Algorithm 1 as

$$\overline{r}_{(p,k)}(t) = \frac{r_{(p,k)}(t)}{e_{(p,k)}(t)}, \forall (p, k) \quad (12)$$

V. PERFORMANCE EVALUATION

In this section, we evaluate the payoff and the payoff gap of FSPs from various aspects. Without loss of generality, we assume that there are two FSPs in the market, and we focus on the payoff and payoff gap of one of the FSPs. The key parameters are presented in Table I. The supply of each FSP is randomly derived from range $[1, 10]$ in each location.

In constant demand, the demand in each location is randomly drawn from $[1, 10]$. In quasi-constant demand, for the first $1 \sim T/2$ timeslots, the demand remains invariant, then the demand changes to a new set of values during the $T/2 + 1 \sim T$ timeslots. In probabilistic demand, at each timeslot, the demand in each location is randomly drawn.

Fig.1 shows that the MSP's payoff is the highest when its demand remains constant, and is the lowest when its demand changes frequently. This is because when the MSP's demand remains unchanges, an FSP's strategy updating process will converge to the optimal strategy. The FSPs are able to gain high utility by exploiting the optimal strategy in every coming timeslot. However, in case of probabilistic demand, such optimal strategy for each FSP does not exist at all. The FSPs have to adjust their strategy all the time, and the payoff is therefore less than that of constant demand. Interestingly, the payoff gap of the quasi-constant demand is the highest as shown in Fig.2. The possible reason is that during the first half of the time, the FSP's strategy approaches the optimal

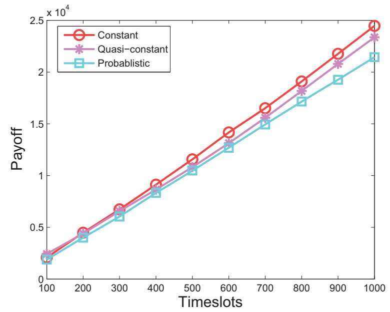

Fig. 1. The FSP's payoff in case of 1) Constant; 2) Quasi-constant; 3) Probabilistic demand.

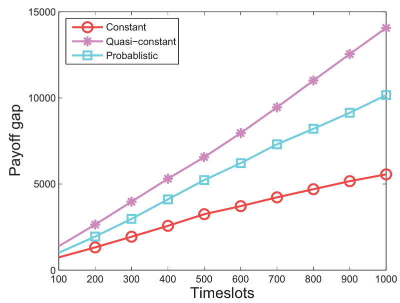

Fig. 2. The FSP's payoff gap in case of 1) Constant; 2) Quasi-constant; 3) Probabilistic demand.

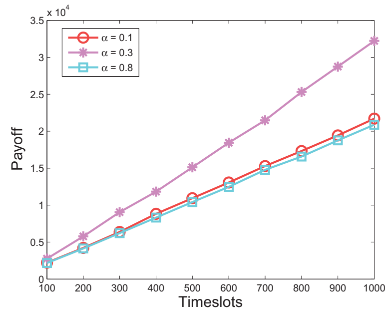

Fig. 3. The FSP's payoff in case of different learning speed α.

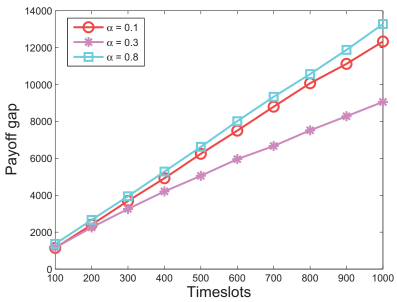

Fig. 4. The FSP's payoff gap in case of different learning speed α.

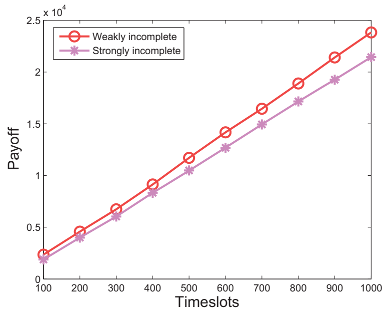

Fig. 5. The FSP's payoff in case of strongly and weakly incomplete information.

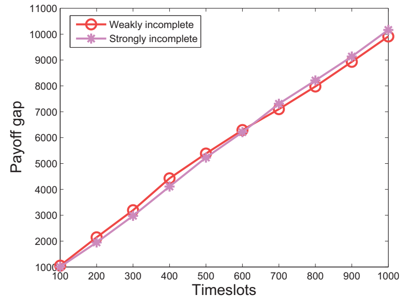

Fig. 6. The FSP's payoff gap in case of strongly and weakly incomplete information.

one. When the MSP's demand changes, it is hard for the FSPs to quickly adjust its strategies since $N_s - 1$ of the values in its strategy distribution probability are nearly zero.

In Fig.3 and Fig.4 we assume the demand is probabilistic. It is shown that both high and low learning speed α will harm the FSP's payoff. If the learning speed is too low, the FSP cannot swiftly adjust its strategy according to the payoff feedback from the previous timeslot. If the learning speed is too high, since the MSP's demand may not be the same as in the previous timeslot, aggressively adjusting the strategy will also reduce the FSP's payoff.

Fig.5 and Fig.6 show that the FSP can gain higher payoff if it can get "more complete" information. This verifies the negative influence of information incompleteness on FSPs' decision-making and payoff.

VI. Conclusion

In this paper, we propose an ACP transaction mechanism to motivate FSPs to adopt hybrid access and assist MSPs in traffic offloading. In order to deal with the information incompleteness, where FSPs do not know MSP's demand, we propose an adaptive strategy updating algorithm for individual FSP to adjust its strategy based on the history of information. We prove in theory that the payoff gap between the proposed algorithm and the optimal static strategy is bounded. Simulation results show that with more information, FSPs can better adjust their strategy and gain higher payoff. A suitable learning speed is helpful in helping FSPs to gain higher payoff. To seek for an optimal learning speed is our future direction.

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