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Optimal Pricing and Spectrum Allocation for Wireless Service Provider on Femtocell Deployment

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***Abstract*—Femtocell technology is regarded as a promising way to deal with poor indoor coverage and increase spectrum spatial reuse. In this paper, we focus on the scenario that macro and femto base stations are deployed by the same Wireless Service Provider (WSP), which treats the revenue maximization as its ultimate target. In such a system, there are several design factors will affect the overall revenue, which include price decision and resource allocation between macrocell and femtocell. In this paper, we propose an economic framework, where users choose either macrocell or femtocell service to optimize their own utility and the monopolistic WSP tries to maximize its revenue via pricing and spectrum allocation strategy. Theoretical results of optimal prices for macrocell and femtocell are given. Extensive theoretical analysis is carried out to determine the spectrum allocation strategy and evaluate the revenue of the WSP. The system capacity and the ratio of macrocell and femtocell users are also discussed. The results have indicated that the revenue of the WSP is significantly improved by combining the pricing strategy and the spectrum allocation strategy.**

I. Introduction

The unrelenting desire for higher wireless capacity has triggered a new wave of research focusing on increasing in-home link quality. Femotocell technology emerges to encourage the installation of short-range low-power indoor base stations , called femto base station (femto-BS), which provides better coverage and capacity [1]. The femto-BS communicates with the cellular network over a broadband connection such as DSL, cable modem, or a separate RF backhaul channel [2]. Wireless service providers (WSP) will benefit from this new technology owing to increased spatial reuse, lightened traffic load on the macrocell network and reduced user turnover.

One of the key challenges femtocell technology development faces is how to merge it with existing macrocell service so that win-win situation can be created for both WSP and wireless users. While price and spectrum should be assigned to the new femtocell service, there are two fundamental aspects that should be taken into consideration when we are trying to answer the following essential questions: 1) How can the WSP exploit highest profit from deploying femtocell; 2) What's in femtocell for the users to benefit from. First, the prices for the macro and femto service have impacts on the decisions of the users and the revenue of the WSP. If the price of the femtocell service is too high compared to that of macrocell service, fewer users will be attracted to adopt femtocell. On the contrary, if the price is too low, although the throughput increases, the outcome may still be unprofitable for WSP. In both case, WSP has no incentive to promote the new femtocell business. Second, there is compelling need for the WSP to make full utilize of its limited resources, especially spectrums. The smaller size of femtocells creates abundant opportunities for spatial reuse [3]. However, if the femtocell grabs too much spectrum from macrocell, the performance of macrocell will degrade, resulting in higher users' churn rate. Therefore, it is important to make the tradeoff of balancing macrocell and femtocell service quality by managing spectrum allocation efficiently and effectively.

Femtocell deployment compatible with existing macrocell infrastructure has been studied by [3] [4], treating femtocell users as secondary users (SU) and macrocell users as primary users (PU) and trying to optimize SU's performance as well as guarantee the quality of service (Qos) of PU. Actually, there's no reason to discriminate against femtocell users, who should be granted the same priority to access the network as long as they pay the corresponding fee. A majority of the conventional works focus on the technical performance of the network but seldom address economic issues. As far as we know, only one work explores the economic framework for femtocell analysis. In [5], the authors assume that users have a linear distributed valuation for the data throughput and the femtocell throughput is constant, which does not accord with reality. Moreover, the model proposed by [5] only considered fixed spectrum allocation between macrocell and femtocells but the spectrum distribution ratio is also a fundamental factor that determines the ultimate revenue. In this paper, we aim at revenue maximization for WSP, not only via leveraging price, but also adjusting spectrum allocation. In addition, we think of users' strategy making and interaction with WSP based on service quality and pricing, which are rarely touched by previous studies.

In this paper, we articulate a simple quantitative revenue-maximization model for WSP, combining pricing strategy and spectrum allocation strategy. The contributions are three fold:

- We advocate an economic framework for the heterogeneous networks, where macro and femto belongs to the same WSP and the WSP aims at maximizing its revenue by deciding the price and resource allocation. The model also takes users's response on the price into consideration.
- We theoretically give the best price and resource allocation strategy for WSP.
- Extensive simulations are carried out to evaluate the joint price and spectrum allocation strategy. The results have verified that our proposal yields the highest revenue for

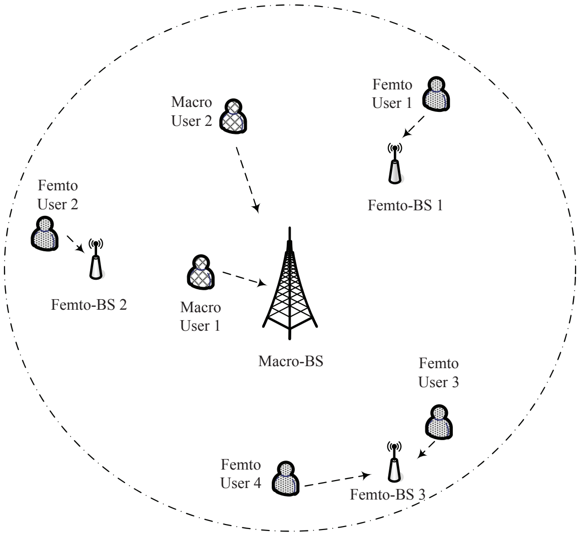

Fig. 1. System model for two-tier macro-femto network.

WSP. In addition, the ratio of macrocell and femtocell users and the total capacity of macrocell and femtocell service are explored.

The rest of the paper is organized as follows. Section II presents the design of macro-femto system and the corresponding model. In section III, we conduct theoretical analysis to determine the optimal pricing strategy. Simulation results are given in section IV and the whole work is summarized in Section V.

II. System Model

We consider a two-tier macro-femto network, consisting of one Macro base station (BS) and n femto BSs as is shown in Fig.1. A monopolist WSP is in charge of the entire network, offering two kinds of services, namely macrocell or femtocell.The WSP charges p_m (or p_f) from users for each unit of capacity served by macrocell service(or femtocell service). WSP use a fixed spectrum band to serve all the users. As a monopolist, the WSP set its ultimate goal as maximizing its profit M via pricing and spectrum allocation.

Users are free to access either service as long as they pay the corresponding fee. Once the prices and spectrum distribution are determined, users make their decision in order to optimize their utility. The users always prefer high capacity and low price. Let γ represents the user's valuation for capacity. Therefore, the utility function of each user taking service $j = m, f$ is defined to be achievable benefit from capacity subtracted by the payment for the service:

$$U_j = \gamma c_j - p_j. \tag{1}$$

Users are utility driven, that is, they will always choose the service that engender the most utility. There is a reservation utility U_0, below which users will not take any type of service whatever the capacity or the price. Given the capacity and the price, users compared the utility they can obtain from femtocell and macrocell service, choosing the one that generate the higher utility if the utility exceeds the reservation utility. Users response to the price and QoS like voting for their favorable service by adopting and paying for that service.

Apart from price, spectrum allocation also exerts an influence on users' determination through the capacity. Two principal spectrum schemes have been proposed in previous works: 1) Split spectrum, in which the spectrum is divided for dedicated use for femtocells and macrocell; 2) Common spectrum, in which the femtocells share a certain proportion of spectrum with macrocell [6]. In this paper, we presume that WSP acquires split spectrum scheme, where macro-BS and femto-BSs operate on different frequencies and do not interfere. All the femto-BSs share the same frequency, utilizing integrated channel allocation strategy to avoid femto-femto interference. We assume that femtocell acquires Fixed Channel Allocation Strategy (FCA), partitioning the total spectrum into k equivalent sub-spectrum for exclusive usage in each cell so that any femto-BSs that reuse the same sub-spectrum are far enough to be prevented from femto-femto interference. k can only take integer values $1, 3, 4, 7, 9, 12, \ldots$ according to certain expression [7]. The WSP owns a fixed amount of spectrum resource S to distribute between macrocell and femtocell. Let S_m and S_f represent the spectrum that is assigned to macrocell and femtocell respectively. Define

$$S_m = \alpha S, S_f = (1-\alpha)S. \tag{2}$$

Here we define capacity as the information rate that can be achieved. Users adopt time-division multiple access (TDMA) to transmit information. In the rest of the paper, we will restrict our attention to the downlink transmission analysis only. A similar analysis for the uplink may be performed.

III. Theoretical Analysis

In this section, we define the utility function for users and analyze how users' choice diversifies based on the utility function. Then, the optimal pricing and spectrum allocation strategy for WSP can be derived to achieve maximum revenue.

A. Revenue Maximization

After the price and spectrum allocation is determined, the revenue of WSP can be derived as the sum of money it collects from selling macrocell and femtocell service.

$$M = p_m E(C_m) + p_f E(C_f). \tag{3}$$

In (3), C_m and C_f are the total capacity for macrocell and femtocell service respectively. Let c_m and c_f denote the capacity that each user can achieve from macrocell and femtocell service. Further, we assume that c_m and c_f independently follow exponential distribution with parameter λ_m and λ_f.The distribution is on the interval $[0, +\infty)$ and the probability density function (pdf) is as follows:

$$f(c_j;\lambda_j) = \begin{cases} \lambda_j e^{-\lambda_j c_j} & c_j \geq 0 \\ 0 & c_j < 0 \end{cases} \tag{4}$$

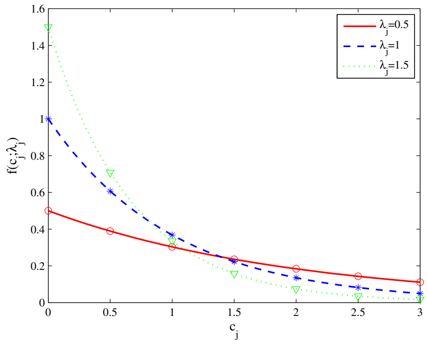

Fig. 2. Probability density function of exponential distribution with different parameters.

Fig.2 shows the pdf of exponential distribution with different λ.

The capacity varies with the user's location, the quality of channel, and the transmission power. We can obtain the user's expected capacity $E(c_j)$ from adoption of the service j by taking the expectation over all possible capacity trajectories:

$$E(c_j) = \frac{1}{\lambda_j}. \tag{5}$$

For the macrocell, we make the simplifying assumption that $E(c_m)$ is a function of spectrum S_m only. For the femtocell, we assume that $E(c_f)$ is proportional to the employed spectrum and inversely proportional to the user density. For the femtocell, we assume the capacity is also proportional to the number of femto-BSs but inversely proportional to the spectrum reuse factor. As a result, the parameter λ_m, λ_f is determined by:

$$\begin{aligned} E(c_m) &= \frac{1}{\lambda_m} = \frac{\beta\alpha S}{\rho}, \\ E(c_f) &= \frac{1}{\lambda_f} = \frac{\beta n(1-\alpha)S}{k\rho}. \end{aligned} \tag{6}$$

The parameter β represents the spectral efficiency. The spectral efficiency can be viewed as bits per second per hertz supported by the system [8]. It can be estimated by summing over all users in the system, divided by the channel bandwidth. Spectral efficiency is affected not only by the single user transmission technique, but also by multiple access schemes and radio resource management techniques utilized. We make the simplifying assumption that β is a pre-determined constant.

Once the prices are fixed, in order to attract users to take wireless service of any type, the utility gained must beyond the reservation utility. For this reason, the minimum capacity of macrocell and femtocell service, denoted by c_{m0} and c_{f0} respectively, is determined by forcing (1) to be the reservation utility:

$$c_{j0} = \frac{1}{\gamma}(U_0 + p_j), j = m, f. \tag{7}$$

With the capacity satisfying (7), if the utility provided by macrocell service surpasses that of femtocell service, users are willing to pay for macrocell service, vice versa. (1) and (7) also implies that if the price of femtocell service drops, users will swap macrocell service for femtocell service. At the same time, the minimum capacity required for femtocell service becomes smaller.

The expected gross capacity for each service is obtained by adding together the capacity of every user that acquires that service:

$$\begin{aligned} E(C_m) &= \int_{c_{f0}}^{+\infty} f(c_f;\lambda_f)[\int_{U_m>U_f}^{+\infty} c_m f(c_m;\lambda_m)c_m]dc_f, \\ E(C_m) &= \int_{c_{m0}}^{+\infty} f(c_m;\lambda_m)[\int_{U_f>U_m}^{+\infty} c_f f(c_f;\lambda_f)dc_f]dc_m. \end{aligned} \tag{8}$$

B. Optimal Price

In this section, we will derive the optimal price for WSP to achieve maximum revenue.

Substitute the total capacity in (3) with the expectation in (8). In case of a certain spectrum allocation, it is the adjustment of prices that leads to different choices of service, the expectation of total capacity and the revenue M.

Theorem 1: When the following conditions

$$\begin{aligned} &C_1 > 0, \\ &C_2 > 0, \\ &\frac{-B_1 + \sqrt{B_1^2 + 4AC_1}}{A} > \frac{\lambda_f}{\lambda_m * \sum_{j=m,f} \lambda_j}, \\ &\frac{-B_2 + \sqrt{B_2^2 + 4AC_2}}{A} > \frac{\lambda_m}{\lambda_f * \sum_{j=m,f} \lambda_j} \end{aligned} \tag{9}$$

are satisfied, for a given spectrum allocation ratio α, the corresponding parameter being (λ_m, λ_f), the WSP maximizes its revenue when the prices p_m and p_f are set to be the following optimal values:

$$\begin{aligned} p_m &= p_m^* = \gamma * \frac{-B_1 + \sqrt{B_1^2 + 4AC_1}}{2A}, \\ p_f &= p_f^* = \gamma * \frac{-B_2 + \sqrt{B_2^2 + 4AC_2}}{2A}, \end{aligned} \tag{10}$$

TABLE I
SIMULATION PARAMETER

Parameter	Description	Value
γ	user's valuation for capacity	1
U_0	Reservation utility	0
β	Spectral efficiency	1
S	Total spectrum resource	1
k	Spectrum reuse factor	3
ρ	User density	1

where

$$A = \prod_{j=m,f} \lambda_j^2 * \sum_{j=m,f} \lambda_j^3,$$
$$B_1 = \lambda_f^3 \lambda_m (\lambda_f^2 + \lambda_f \lambda_m - \lambda_m^2),$$
$$B_2 = \lambda_m^3 \lambda_f (\lambda_m^2 + \lambda_f \lambda_m - \lambda_f^2),$$
$$C_1 = \frac{(\lambda_m - \lambda_f)[(\sum_{i=0}^{6} +3\sum_{i=1}^{5} + \sum_{i=2}^{4} +5\sum_{i=3}^{3})\lambda_f^i \lambda_m^{6-i}] + 2\lambda_m^4 \lambda_f^3}{4\lambda_f * \sum_{j=m,f} \lambda_j},$$
$$C_2 = \frac{(\lambda_f - \lambda_m)[(\sum_{i=0}^{6} +3\sum_{i=1}^{5} + \sum_{i=2}^{4} +5\sum_{i=3}^{3})\lambda_m^i \lambda_f^{6-i}] + 2\lambda_f^4 \lambda_m^3}{4\lambda_m * \sum_{j=m,f} \lambda_j}. \tag{11}$$

Proof: Calculate the first order partial derivative of M in respect with p_m and p_f respectively. When (9) is satisfied, the second order derivative of M in respect with p_m and p_f is always negative. Therefore, both p_m and p_f can be calculated by assigning the first order partial derivative to be 0. The maximum M is achieved by setting the price p_m and p_f according to (10).

Proof is finished.

C. Optimal spectrum allocation

In this section, we analyze the optimal spectrum allocation strategy for WSP. Substitute (10) into (3), we can get the maximum revenue with reference to (λ_m, λ_f). Given a particular spectrum splitting ratio α, (λ_m, λ_f) is determined by (6). Then, we run through all the possible value from 0.01 to 0.99 with the interval of 0.01 to seek for the α that delivers the maximum revenue.

IV. SIMULATION RESULTS

Now, we conduct simulation to determine the optimal spectrum distribution ratio α and evaluate both pricing and spectrum allocation strategies proposed in the previous section.

We carry out numerical analysis on MATLAB with the related parameter values listed in Table I. We varied the number of femto-BSs to assess the maximum revenue, the optimal price, optimal spectrum distribution ratio α, the total capacity for each service and the ratio of macrocell users and femtocell users.

We compare our joint price and spectrum allocation scheme (JPSA) with the fixed spectrum allocation scheme (FSA). Fig.3 shows that when the number of femto-BSs is around 33, JPSA outperforms FSA by approximately $1.6 \sim 14$ times according to different fixed spectrum distribution ratio. The revenue of FSA approaches that of JPSA at the beginning because the fixed spectrum allocation ratio advances to the optimal one. However, the revenue degraded rapidly as the fixed spectrum allocation ratio drifts away from the optimal one. If the spectrum allocation is fixed, the maximum revenue for WSP only increases when there are a few femto-BSs but drops dramatically when there are large numbers of femto-BSs.Nevertheless, in circumstance of JPSA, the maximum revenue keeps augmenting with the increase of femto-BSs. One of the reasons is that the potential of spectrum spatial reuse is well exploited by femtocell. In consequence, it provides strong economic incentive for the WSP to promote the femtocell service.

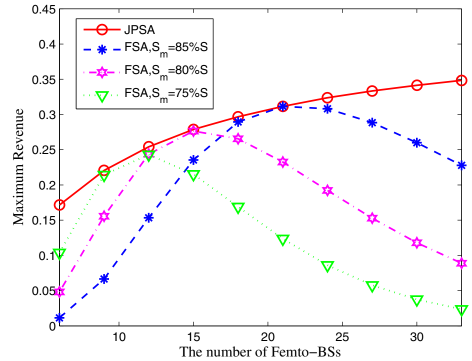

Fig. 3. The comparison of maximum revenue between (a) Optimal pricing and spectrum allocation; (b) Optimal pricing only, fixed spectrum allocation.

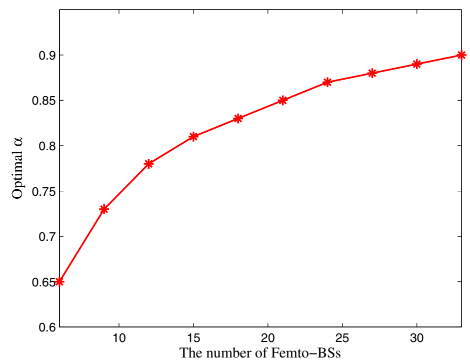

Fig. 4. The optimal spectrum ratio of macrocell and femtocell services versus the number of femto-BSs.

Fig.4 shows the optimal spectrum allocation ratio between macrocell and femtocell services. The optimal ratio α increases along with the number of femto-BSs, that is, less spectrum is allocated to the femtocell because that it can be reused more efficiently. As more spectrum is spared from femtocell to macrocell, macrocell can further enhance its capacity and generate more revenue.

Fig.5 displays the optimal prices for macrocell and femtocell

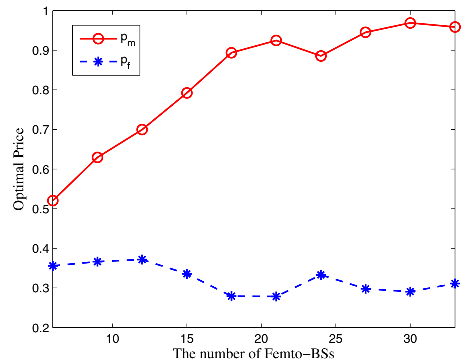

Fig. 5. The optimal price of macrocell and femtocell services versus the number of femto-BSs.

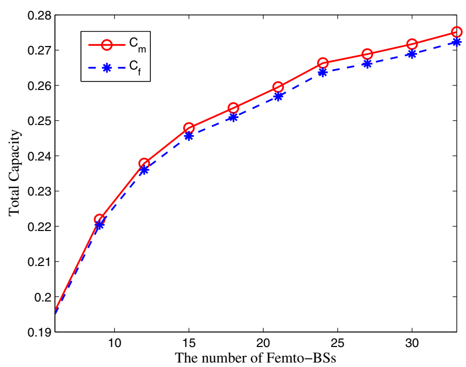

Fig. 6. The total capacity of macrocell and femtocell services versus the number of femto-BSs.

service respectively. On the one hand, the price of macrocell service keeps going up because it can render higher capacity thanks to more spectrum that is assigned to macrocell service according to Fig.4. On the other hand, although there are more and more femto-BSs, the spectrum distributed to femtocell keeps diminishing. Due to these two factors, the price of femtocell service is rather stable. Fig.5 also implies that femtocell tends to charge lower price than macrocell in all situations, which correspond to the current business situations. As discussed before, users will find it's a good bargain to adopt femtocell service. Hence, users are strongly motivated to acquire the femtocell service, which contributes to the booming of femtocell technology.

Fig.6 and Fig.7 appraise the total capacity for each service and the ratio of users taking each service. Fig.6 indicates that with the accretion of femto-BSs, the capacity for each service increases considerably. The capacity for macrocell increases owing to more spectrum assigned to macrocell service by WSP while the capacity for femtocell rises because of increased number of femto-BSs that can reuse the spectrum more efficiently. Although Fig.7 implies that the ratio of user taking wireless service drops slightly, yet more revenue can be exploited thanks to the notable accretion of capacity depicted in Fig.6.

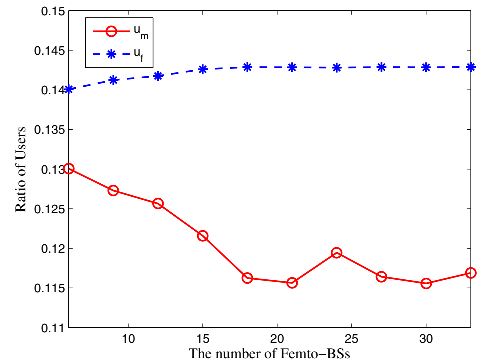

Fig. 7. The ratio of users adopting macrocell and femtocell services versus the number of femto-BSs.

V. Conclusion

In this paper, we propose an economic framework for analyzing two-tier macro-femto network, targeting at exploiting maximum revenue for WSP. As far as we are concerned, it is the first time that economic framework jointly considering pricing and spectrum allocation strategy is established for studying femtocell. The model we built also reflects on the users' strategy making based on price and QoS. Theoretical results for optimal price and spectrum allocation are given. Numerical analysis is carried out on MATLAB, verifying that JPSA outperforms the FSA in bringing WSP higher revenue and also motivate users to adopt femtocell service. Thereupon, we believe that femtocell has a bright market prospect.

VI. Acknowledgement

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