

# Wi-Fi or Femtocell: User Choice and Pricing Strategy of Wireless Service Provider

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**Abstract**—Wi-Fi and femtocell are two competing and complementary technologies for delivering broadband wireless Internet access services. For the Wireless Service Provider (WSP) who owns both Wi-Fi and femtocell network, pricing for the coexisted heterogeneous network is a critical yet challenging problem due to different features of the two technologies: Wi-Fi is based on wireless local area network (WLAN), whose time delay is highly dependent on concurrent in-service users; femtocell is based on cellular technology, which provides guaranteed Quality of Service (QoS). In this paper, we consider a monopolist WSP who operates both Wi-Fi and femtocell network within a certain area, and offers corresponding services to a fixed pool of users. Users can freely choose either service that incurs less cost, which is a weighted sum of service price and time delay. Users with different sensitivity for price and time delay tend to choose different services. The major objective of WSP is to maximize its profit, which is the revenue from both services. We first analyze user distribution under different Wi-Fi and femtocell price combination, then study the optimal pricing strategy for WSP. Finally, we use simulation to evaluate the influence of Wi-Fi network quality on the WSP's utility and user distribution.

## I. INTRODUCTION

Ever increasing demand for faster and more convenient wireless communication has triggered a series of new technologies. Wi-Fi and femtocell are two major ways to provide users with high-speed mobile Internet access. Since Wi-Fi and femtocell are based on different standards, they have different features. Femtocell service provides guaranteed QoS (time delay), thanks to the exclusive licensing and centralized scheduling, while the time delay of Wi-Fi service is highly dependent on the number of concurrent in-service users (due to contentions and interference generated by sharing the same unlicensed bands) [2]. Users with different QoS requirements thus have different preferences over these two services.

How to manage the Wi-Fi and femtocell services so that maximum profits can be achieved remains a challenging problem for WSPs. In this paper, we consider a monopolist WSP who runs both Wi-Fi and femtocell network, and provides corresponding services to a fixed pool of users, who are free to choose either service (as shown in Fig.1). The revenue of WSP is the sum of service fee charged from users who choose either Wi-Fi or femtocell service. Since Wi-Fi and femtocell are two competing services, the number of users who choose Wi-Fi service will not only be affected by Wi-Fi's price and time delay, but also affected by femtocell's price and time delay,

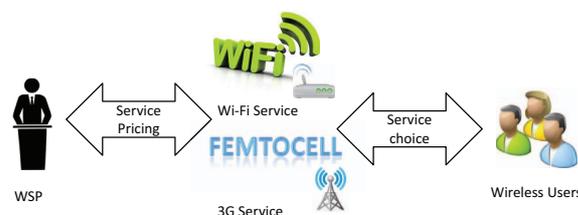


Fig. 1. Illustration of Mobile Service Model.

vice versa. More users will choose Wi-Fi (femtocell) service if Wi-Fi (femtocell) charges lower price and has shorter time delay, also if femtocell (Wi-Fi) charges relatively higher price and has longer time delay. In addition, the time delay of Wi-Fi relies on the number of users choosing Wi-Fi service: the more Wi-Fi users, the more congested the Wi-Fi network becomes, and the longer the time delay for Wi-Fi service. WSP has to be fully aware of the impact of Wi-Fi and femtocell service prices on users' choice in order to launch optimal pricing strategy which yields maximum profit.

The competitive and complementary relationship of Wi-Fi and 3G technology (for which femtocell is a representative) is surveyed in [2]. In [5], the authors built a pricing competition model between Wi-Fi and WiMAX, in which the two services are provided by different operators, which is different from our monopolistic WSP model. In addition, the paper [5] didn't analyze users' choice when facing different service prices and time delays. In [3] [4], the authors studied the interaction between a Wi-Fi service provider and a client and analyzed the pricing equilibrium under different utility functions. References [7] [6] jointly considered spectrum allocation and pricing strategies for service provider. Femtocell pricing schemes is studied by [8]. Users' different preference for QoS is taken into consideration. Reference [1] analyzed optimal pricing and spectrum allocation for monopolist WSP who owns both femtocell and macrocell services and tries to maximize its profit. However, these papers did not address the pricing issue for heterogeneous network where both Wi-Fi and femtocell are available.

In this paper, we first derive the user distribution under different Wi-Fi and femtocell price combinations. We consider a fixed number of rational wireless users who choose the service that incurs least cost. Some users are more price-sensitive whose decision will be more easily influenced by price difference. Some users care more about time delay and are willing to pay higher prices to avoid traffic congestion.

We jointly consider the prices and time delays of both Wi-Fi and femtocell services as they all have an influence on users' choice. We compute best pricing strategies and corresponding maximum profit for WSP. The main contributions of the paper are as follows:

- We propose a concrete economic framework for analyzing coexisted heterogeneous Wi-Fi and femtocell networks, which addresses fundamental Wi-Fi and femtocell service diversity.
- We analyze user behavior, jointly considering the influence of service price and time delay of both Wi-Fi and femtocell networks.
- We conduct simulations to study the impact of Wi-Fi network quality on the WSP's utility and user distribution. The results show that if the increase of Wi-Fi users does not cause much longer time delay, the WSP can obtain higher utility and more users will choose Wi-Fi network. While higher Wi-Fi service price leads to fewer Wi-Fi users, the number of femtocell users also decreases because Wi-Fi time delay is shortened due to fewer users, making the femtocell service less advantageous.

The rest of the paper is organized as follows. First, we introduce the detailed system model in section II. After that, we analyze user distribution in section III. In section IV, we compute WSP's optimal strategies and corresponding profit. In section V, we evaluate our model from various aspects through simulation. We finally summarize our work in section VI.

## II. SYSTEM MODEL

We consider a monopolist WSP who provides Wi-Fi and femtocell services to end users<sup>12</sup>. There are a total number of  $n$  users in the covered area, who are free to toggle between Wi-Fi and femtocell services<sup>3</sup>. We assume that WSP charges a flat rate of  $p_w$  per user who chooses Wi-Fi service and  $p_f$  per user who chooses femtocell service. Let  $n_f$  denote the number of users who will choose femtocell service and  $n_w$  denote the number of users who will choose Wi-Fi service.

Users' cost comprises of two parts: 1) *service cost*:  $p_w$  for Wi-Fi service and  $p_f$  for femtocell service; 2) *time delay cost*: defined as the time needed to finish a typical task<sup>4</sup> using either Wi-Fi or femtocell service. Let  $t_w$  and  $t_f$  denote the average time delay for Wi-Fi service and femtocell service respectively. Different users have different valuation for money and time. We use  $\gamma$  to represent users' sensitivity towards service price.  $\gamma \in [0, \gamma^{max}]$  can be viewed as the characterization of user

<sup>1</sup>We assume that the deployment costs of Wi-Fi and femtocell networks are fixed costs which are sunk and will not affect the pricing strategy of the WSP, so we exclude it from the analysis.

<sup>2</sup>In order to focus on the competitive and complementary relationship of Wi-Fi and femtocell, we only consider the geographic areas where users can access both services. Areas which are covered by either one service but not the other (thus users do not have multiple choices) are out of the scope of this paper. To further involve the macrocell network is our future direction.

<sup>3</sup>With the technology such as "Soft Sim" by Apple, users can switch between services without costs.

<sup>4</sup>It is flexible to define a typical task, for example, to upload/download 1MB file, to launch a search engine webpage.

type. The cumulative distribution function of  $\gamma$  is  $\Gamma$  and  $\Gamma(\gamma \leq 0) = 0, \Gamma(\gamma \geq \gamma^{max}) = 1$  are satisfied. A user of type  $\gamma$  is willing to pay price  $p$  for a reduction of  $\gamma p$  time delay (the user treats the price  $p$  and time delay  $\gamma p$  as equivalent). The higher  $\gamma$  is, the more sensitive the user towards money, and he is willing to endure longer time delay. Since femtocell belongs to cellular network and provides guaranteed QoS, we assume that  $t_f$  is fixed regardless of number of in-service users. However, Wi-Fi is based on the IEEE 802.11 standards, operating on unlicensed band and providing best-effort service. When the number of users increases, the contention and interference increases, resulting in longer time delay. We assume that  $t_w = \Phi(n_w)$  is a non-decreasing function of  $n_w$ . For a type  $\gamma$  user, if choosing femtocell service, the equivalent time delay cost is

$$c_f^\gamma = \gamma p_f + t_f; \quad (1)$$

if choosing Wi-Fi service, the equivalent time delay cost is

$$c_w^\gamma = \gamma p_w + \Phi(n_w). \quad (2)$$

Each user has a reserve cost  $c_0$ <sup>5</sup>. Users are rational and utility driven, choosing the service that gives them most benefits, namely, least cost. A type  $\gamma$  user's service choice is as follows:

$$\begin{cases} \text{Femtocell service,} & \text{if } c_f^\gamma < c_w^\gamma, c_f^\gamma < c_0 \\ \text{Wi-Fi service,} & \text{if } c_f^\gamma > c_w^\gamma, c_w^\gamma < c_0 \\ \text{No service,} & \text{otherwise} \end{cases} \quad (3)$$

The utility of WSP is the sum of revenue from femtocell service and Wi-Fi service :

$$U_{WSP} = p_f n_f + p_w n_w. \quad (4)$$

## III. USER DISTRIBUTION

In this section, we derive the user distribution, given that the prices of Wi-Fi and femtocell services  $p_w$  and  $p_f$  are announced by WSP and observed by end users.

For simplicity, we assume that the dependency of  $t_w$  on  $n_w$  is linear, i.e.,

$$t_w = \Phi(n_w) = \alpha n_w,$$

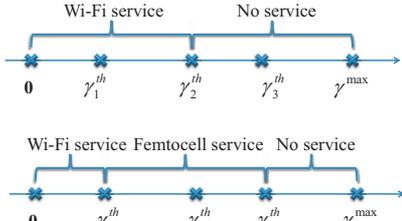
in which  $\alpha$  is a constant parameter representing by how much the time delay increases when an additional user joins the Wi-Fi network.

To better understand users' choice, we introduce three critical user type.

- *Indifferent to femtocell and no service user type*. There exists a critical user type  $\gamma_1^{th}$ , whose is indifferent to choosing femtocell service or no service at all. It satisfies

$$\gamma_1^{th} p_f + t_f = c_0 \Rightarrow \gamma_1^{th} = \frac{c_0 - t_f}{p_f}. \quad (5)$$

<sup>5</sup>We assume the same reserve cost for femtocell and Wi-Fi services, which indicates that users do not differentiate the Internet access service provided by either femtocell or Wi-Fi technology.


 Fig. 2. User Distribution when  $p_f < p_w$ .

Users of type  $\gamma < \gamma_1^{th}$  prefer femtocell service to no service.

- *Indifferent to Wi-Fi and no service user type.* There exists a critical user type  $\gamma_2^{th}$ , who is indifferent to choosing Wi-Fi service or no service at all. It satisfies

$$\gamma_2^{th} p_w + t_w = c_0 \Rightarrow \gamma_2^{th} = \frac{c_0 - t_w}{p_w} \quad (6)$$

Users of type  $\gamma < \gamma_2^{th}$  prefer Wi-Fi service to no service.

- *Indifferent to Wi-Fi and femtocell service user type.* There exists a critical user type  $\gamma_3^{th}$ , who is indifferent to choosing Wi-Fi service or femtocell service. It satisfies

$$\gamma_3^{th} p_w + t_w = \gamma_3^{th} p_f + t_f \Rightarrow \gamma_3^{th} = \frac{t_f - t_w}{p_w - p_f}. \quad (7)$$

Here, it is possible that  $\gamma_3 \leq 0$ . If  $p_w > p_f$ , users of type  $\gamma > \gamma_3^{th}$  prefer femtocell service to Wi-Fi service; otherwise, users of type  $\gamma < \gamma_3^{th}$  prefer femtocell service to Wi-Fi service.

*Proposition 1:* Given the Wi-Fi and femtocell service price as  $p_w$  and  $p_f$  respectively, the user distribution are as follows:

- 1) When

$$p_w \geq \frac{\alpha n(c_0 - t_f)}{t_f \gamma^{max}}, p_f \geq \frac{(\gamma^{max} p_w + \alpha n)(c_0 - t_f)}{c_0 \gamma^{max}} \quad (8)$$

the number of Wi-Fi users and femtocell users are:

$$n_w = \frac{n c_0}{\gamma^{max} p_w + \alpha n}, n_f = 0 \quad (9)$$

- 2) When

$$p_w \geq \frac{\alpha n(c_0 - t_f)}{t_f \gamma^{max}}, p_f \leq \frac{(\gamma^{max} p_w + \alpha n)(c_0 - t_f)}{c_0 \gamma^{max}}$$

$$\text{or } p_w \leq \frac{\alpha n(c_0 - t_f)}{t_f \gamma^{max}}, p_f \leq p_w, \quad (10)$$

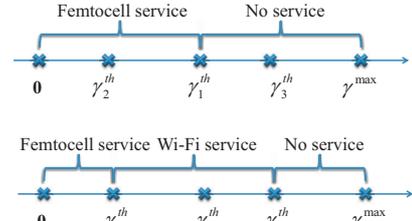
the number of Wi-Fi users and femtocell users are:

$$n_w = \frac{n t_f}{\gamma^{max} (p_w - p_f) + \alpha n} \quad (11)$$

$$n_f = \frac{n}{\gamma^{max}} \left[ \frac{c_0}{p_f} + \frac{p_f t_w - p_w t_f}{p_f (p_w - p_f)} \right]$$

- 3) When

$$p_w \leq \frac{\alpha n(c_0 - t_f)}{t_f \gamma^{max}}, p_f \geq \frac{(\gamma^{max} p_w + \alpha n)(c_0 - t_f)}{c_0 \gamma^{max}} \quad (12)$$


 Fig. 3. User Distribution when  $p_f > p_w$ .

the number of Wi-Fi users and femtocell users are:

$$n_w = \frac{n(p_f - p_w)c_0 + n p_w t_f}{\gamma^{max} (p_f - p_w) p_w + \alpha n p_f} \quad (13)$$

$$n_f = \frac{n(t_w - t_f)}{\gamma^{max} (p_f - p_w)}$$

*Proof:*

- 1) If  $p_f < p_w$ , it can be easily derived that

$$\gamma_2^{th} = \frac{p_f}{p_w} \gamma_1^{th} + \left(1 - \frac{p_f}{p_w}\right) \gamma_3^{th}$$

so there are only two possibilities:  $\gamma_1^{th} \leq \gamma_2^{th} \leq \gamma_3^{th}$  or  $\gamma_3^{th} \leq \gamma_2^{th} \leq \gamma_1^{th}$ .

- $\gamma_1^{th} \leq \gamma_2^{th} \leq \gamma_3^{th}$  as shown in the upper subfigure in Fig.2. In this case, no users are willing to choose the femtocell service, and  $n_f = 0$ .

$$n_w = \frac{n c_0}{\gamma^{max} p_w + \alpha n} \quad (14)$$

In order to satisfy that  $\gamma_1^{th} \leq \gamma_2^{th} \leq \gamma_3^{th}$ ,  $p_w$  and  $p_f$  have to satisfy that

$$p_w \geq \frac{\alpha n(c_0 - t_f)}{t_f \gamma^{max}}, p_w \geq p_f \geq \frac{(\gamma^{max} p_w + \alpha n)(c_0 - t_f)}{c_0 \gamma^{max}}$$

- $\gamma_3^{th} \leq \gamma_2^{th} \leq \gamma_1^{th}$  as shown in the lower subfigure in Fig.2. We first assume that  $\gamma_3 > 0$  and derive the number of users choosing Wi-Fi service as follows:

$$n_w = \frac{n t_f}{\gamma^{max} (p_w - p_f) + \alpha n} \quad (15)$$

We can check that  $t_w < t_f$ , so  $\gamma_3 > 0$  is true. The number of users choosing femtocell service is:

$$n_f = \frac{\gamma_1 - \gamma_3}{\gamma^{max}} n = \frac{n}{\gamma^{max}} \left[ \frac{c_0}{p_f} + \frac{p_f t_w - p_w t_f}{p_f (p_w - p_f)} \right] \quad (16)$$

In order to satisfy that  $\gamma_3^{th} \leq \gamma_2^{th} \leq \gamma_1^{th}$ ,  $p_f$  has to satisfy that

$$p_f \leq \min \left\{ p_w, \frac{(\gamma^{max} p_w + \alpha n)(c_0 - t_f)}{c_0 \gamma^{max}} \right\}$$

- 2) If  $p_f > p_w$ , it can be easily derived that

$$\gamma_1^{th} = \frac{p_w}{p_f} \gamma_2^{th} + \left(1 - \frac{p_w}{p_f}\right) \gamma_3^{th}$$

so there are only two possibilities:  $\gamma_2^{th} \leq \gamma_1^{th} \leq \gamma_3^{th}$  or  $\gamma_3^{th} \leq \gamma_1^{th} \leq \gamma_2^{th}$ .

- $\gamma_2^{th} \leq \gamma_1^{th} \leq \gamma_3^{th}$  as shown in the upper subfigure in Fig.3. In this case, no users are willing to choose

the Wi-Fi service, and  $n_w = 0$ . However, it can be checked that  $\gamma_3 = -t_f/(p_f - p_w) < 0$ . So  $\gamma_2^{th} \leq \gamma_1^{th} \leq \gamma_3^{th}$  actually will not happen.

- $\gamma_3^{th} \leq \gamma_1^{th} \leq \gamma_2^{th}$  as shown in the lower subfigure in Fig.3. We first assume that  $\gamma_3 > 0$  and derive the number of users choosing Wi-Fi service as follows:

$$n_w = \frac{\gamma_2 - \gamma_3}{\gamma^{max}} n = \frac{n}{\gamma^{max}} \left[ \frac{c_0 - t_w}{p_w} - \frac{t_w - t_f}{p_f - p_w} \right] \Rightarrow$$

$$n_w = \frac{n(p_f - p_w)c_0 + np_w t_f}{\gamma^{max}(p_f - p_w)p_w + \alpha n p_f} \quad (17)$$

To guarantee that  $\gamma_3 > 0$ , it must be satisfied that  $p_w \leq \alpha n(c_0 - t_f)/(t_f \gamma^{max})$ . We also check that when  $p_f > p_w$ ,  $\gamma_3^{th} \leq \gamma_1^{th} \leq \gamma_2^{th}$  is always true. In this case, the number of users choosing femtocell service is

$$n_f = \frac{\gamma_3}{\gamma^{max}} n = \frac{n(t_w - t_f)}{\gamma^{max}(p_f - p_w)} \quad (18)$$

If  $p_w \geq \alpha n(c_0 - t_f)/(t_f \gamma^{max})$ ,  $\gamma_3 < 0$ , then no users will choose femtocell service,  $n_f = 0$ . The number of users choosing Wi-Fi service is:

$$n_w = \frac{nc_0}{\gamma^{max}p_w + \alpha n} \quad (19)$$

Combine the results of the above analysis, we can get the user distribution in Proposition 1. ■

#### IV. REVENUE MAXIMIZATION FOR WSP

In this section, we consider the pricing strategy of WSP, which is constrained by  $\underline{p}_w \leq p_w \leq \bar{p}_w, \underline{p}_f \leq p_f \leq \bar{p}_f$ <sup>6</sup>.

*Proposition 2:* The optimal pricing strategy and optimal utility for WSP is as follows:

- If  $\bar{p}_w \geq \alpha n(c_0 - t_f)/(t_f \gamma^{max})$ , the optimal pricing strategy for WSP is

$$p_w = \bar{p}_w, p_f \geq \frac{(\gamma^{max}\bar{p}_w + \alpha n)(c_0 - t_f)}{c_0 \gamma^{max}}$$

And the optimal utility for WSP is

$$U_{WSP}^* = \frac{nc_0 \bar{p}_w}{\gamma^{max} \bar{p}_w + \alpha n}$$

- If  $\bar{p}_w \leq \alpha n(c_0 - t_f)/(t_f \gamma^{max})$ , the optimal pricing strategy for WSP is

$$\underline{p}_w \leq p_w \leq \bar{p}_w, p_f \geq \frac{(\gamma^{max}p_w + \alpha n)(c_0 - t_f)}{c_0 \gamma^{max}}$$

And the optimal utility for WSP is

$$U_{WSP}^* = \frac{n(c_0 - t_f)}{\gamma^{max}}$$

<sup>6</sup>The upperbound of the price can be the restriction from the government on the highest price that the WSP can charge. The lowerbound of the price can be the cost of providing the service, below which the WSP receives negative net profit.

*Proof:*

- 1) When condition (8) is satisfied. The WSP's utility is:

$$U_{WSP} = \frac{p_w n c_0}{\gamma^{max} p_w + \alpha n} = \frac{n c_0}{\gamma^{max} + \frac{\alpha n}{p_w}}$$

So the utility of WSP is an increasing function of  $p_w$ . The optimal Wi-Fi price is  $p_w^* = \bar{p}_w$  and the highest possible  $U_{WSP}$  is  $\bar{p}_w n c_0 / (\gamma^{max} \bar{p}_w + \alpha n)$ .

- 2) When condition (10) is satisfied. The WSP's utility is:

$$U_{WSP} = \frac{n}{\gamma^{max}} \left( c_0 - \frac{\alpha n t_f}{\gamma^{max}(p_w - p_f) + \alpha n} \right)$$

The first derivative of  $U_{WSP}$  with regard to  $p_f$  is

$$\frac{\partial U_{WSP}}{\partial p_f} = \frac{\alpha n^2 t_f}{[\gamma^{max}(p_w - p_f) + \alpha n]^2} \geq 0$$

So the utility of WSP is increasing with  $p_f$ .

- When  $p_w \geq \alpha n(c_0 - t_f)/(t_f \gamma^{max})$ ,  $p_f^* = (\gamma^{max} p_w + \alpha n)(c_0 - t_f)/(c_0 \gamma^{max})$ . The WSP's utility becomes:

$$U_{WSP} = \frac{nc_0 p_w}{\gamma^{max} p_w + \alpha n}$$

$U_{WSP}$  is increasing with  $p_w$ , so the optimal Wi-Fi price is  $p_w^* = \bar{p}_w$  and the highest possible  $U_{WSP}$  is  $nc_0 \bar{p}_w / (\gamma^{max} \bar{p}_w + \alpha n)$ .

- When  $p_w \leq \frac{\alpha n(c_0 - t_f)}{t_f \gamma^{max}}$ ,  $p_f^* = p_w$ . The WSP's utility becomes:

$$U_{WSP} = \frac{n}{\gamma^{max}} (c_0 - t_f)$$

So the utility of WSP is irrelevant of  $p_w$ , and the highest possible  $U_{WSP}$  is  $n(c_0 - t_f)/\gamma^{max}$ .

- 3) When condition (12) is satisfied. The WSP's utility is:

$$U_{WSP} = \frac{n}{\gamma^{max}} (c_0 - t_f)$$

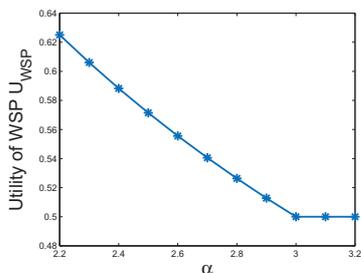
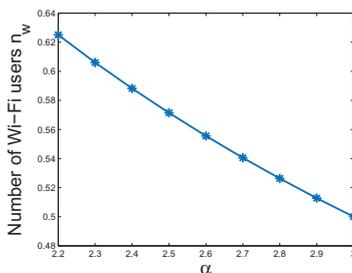
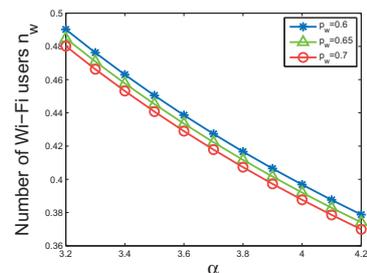
So the utility of WSP is irrelevant of  $p_f$  and  $p_w$  as long as condition (12) is satisfied, and the highest possible  $U_{WSP}$  is  $n(c_0 - t_f)/\gamma^{max}$ .

According to the analysis above, there are two highest possible value for  $U_{WSP}$ :  $n(c_0 - t_f)/\gamma^{max}$  and  $nc_0 \bar{p}_w / (\gamma^{max} \bar{p}_w + \alpha n)$ . By comparing these two values and their constraints, we can get the optimal pricing strategy for WSP as in Proposition 2. ■

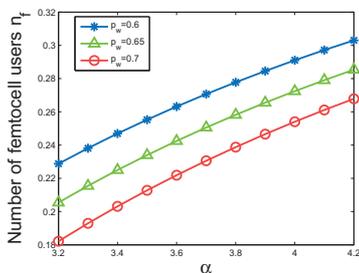
#### V. SIMULATION RESULTS

The parameter values used in the simulations are listed in Table I.

When  $\alpha$  is small, the time delay of Wi-Fi network is short, given the same number of Wi-Fi users. Therefore, when  $\alpha$  increases, fewer users will choose Wi-Fi service due to increased time delay as shown in Fig.5 and Fig.6. On the contrary, more users will choose femtocell service instead as shown in Fig.7. Increasing  $\alpha$  indicates worsened Wi-Fi

Fig. 4. Utility of WSP  $U_{WSP}$  versus  $\alpha$ .Fig. 5. Number of Wi-Fi users  $n_w$  versus  $\alpha$  when  $\alpha \leq \bar{p}_w t_f \gamma^{max} / (n(c_0 - t_f))$ .Fig. 6. Number of Wi-Fi users  $n_w$  versus  $\alpha$  when  $\alpha \geq \bar{p}_w t_f \gamma^{max} / (n(c_0 - t_f))$ .TABLE I  
SIMULATION PARAMETERS

Parameter	Description	Value
$n$	Total number of users	1
$t_f$	Femtocell time delay	1.5
$\gamma^{max}$	Upperbound of user type	1
$c_0$	Reserve cose	2
$\bar{p}_w$	Upperbound of $p_w$	1
$\underline{p}_w$	lowerbound of $p_w$	0.5

Fig. 7. Number of femtocell users  $n_f$  versus  $\alpha$  when  $\alpha \geq \bar{p}_w t_f \gamma^{max} / (n(c_0 - t_f))$ .

service quality, so the utility of WSP decreases due to user dissatisfaction as shown in Fig.4.

The reason why Fig.5 resembles the left part of Fig.4 is: when  $\alpha \leq \bar{p}_w t_f \gamma^{max} / (n(c_0 - t_f))$ , if WSP adopts the optimal pricing strategy in Proposition 2, the number of femtocell users will be zero as indicated by Proposition 1, so the revenue of WSP is the Wi-Fi service price ( $\bar{p}_w$ ) multiplies the number of users choosing Wi-Fi service.

When  $\alpha \geq \bar{p}_w t_f \gamma^{max} / (n(c_0 - t_f))$ , the prices of Wi-Fi and femtocell can be any value chosen from the ranges  $[\underline{p}_w, \bar{p}_w]$  and  $[(\gamma^{max} p_w + \alpha n)(c_0 - t_f) / (c_0 \gamma^{max}), \bar{p}_f]$ , without affecting the revenue of WSP. However, the price combination does affects the user distribution. In Fig.6 and Fig.7, we fixed the femtocell service price as 0.9 and vary Wi-Fi service price. When Wi-Fi service price increases, the number of users choosing Wi-Fi service decreases as expected since users prefer low price to high price (as shown in Fig.6). Surprisingly, the number of femtocell service also decreases (as shown in Fig.7). This is because fewer Wi-Fi users leads to shorter Wi-Fi time delay, making femtocell's guaranteed time delay less attractive to users.

## VI. CONCLUSION

In this paper, we propose an economic framework for coexisted heterogeneous Wi-Fi and Femtocell networks. We study the influence of users' different sensitivity for price and time delay on the demand of Wi-Fi and femtocell services. We derive user distribution, based on which deriving WSP's optimal strategy and maximum profit. The simulation results show that improvement of Wi-Fi network (reduce the parameter  $\alpha$  so that Wi-Fi time delay decreases) contributes to higher revenue for WSP. The number of Wi-Fi users is reversely related to Wi-Fi service price as expected, while surprisingly, the number of femtocell users is also reversely related to Wi-Fi service price. The reason is that the time delay of Wi-Fi service is reduced due to fewer users, which makes femtocell service less attractive.

## VII. ACKNOWLEDGEMENT

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