LOTUS: Location-Aware Online Truthful Double Auction for Dynamic Spectrum Access

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***Abstract*—In dynamic spectrum auction, if a buyer locates in a "critical" place, interfering with many other buyers, its occupancy of the spectrum may deprive many others the transmission opportunity. In this paper, we for the first time propose a location-aware online double auction mechanism, LOTUS. The biggest challenge is: the spectrum requests are sporadic, an interference-harmful buyer may request spectrum earlier than others, making it difficult to decide whether to grant its spectrum request instantly. To address the challenge, we considers the opportunity cost of the spectrum allocation regarding location-based interference conditions. We introduce the "interference discount" to characterize the interference harmfulness. The simulation results show that LOTUS can improve buyers' utility by more than 110% over the static auction, and 25% over the online auction without considering interference discount. The spectrum utilization is enhanced by 120% over static auction, and 65.3% over the online auction without considering interference discount.**

I. Introduction

To increase the spectrum efficiency, under-utilized spectrums are being explored and exploited with the technical support of dynamic spectrum access [1]. The FCC has opened up a significant amount of TV whitespace for unlicensed usage in 2010. Through online auction, the whitespace databases operator can dynamically allocate spectrums based on buyers' temporal and spatial features [1].

To exploit the spectrum reusability can greatly enhance spectrum utilization. Static spectrum auction partly address the problem by grouping the interference-free users together. However, using existing static double auction mechanisms for dynamic spectrum allocation will cause potential utility loss. As shown in Fig.1, suppose that buyer 2 arrives at time slot 1, requesting for 3 time slots, and its true valuation is 1 per time slots. Later at time slot 2, buyer 1 and buyer 3 arrive, each requesting for 2 time slots, and their true valuations are 1 per time slots. If the auctioneer allocates the spectrum to buyer 2 at time slot 1, buyer 2 gets utility of 3. However, if the auctioneer rejects buyer 2's, leave the spectrum idle at time slot 1, then allocates the spectrum to buyer 1, buyer 3 concurrently at time slots 2, they get utility 4. This example shows the necessity of considering the spectrum request dynamics in spectrum allocation.

Most of the previous works only focused on static auction [2] [3], which will incur utility loss in case of dynamic spectrum allocation. Jonathan Bredin [4] *et al.* proposed online auction mechanism for dynamic resource and task allocation. However, spectrum allocation is different from the spectrum can be reused by multiple buyers. To the best of our knowledge, only one existing paper considered online spectrum auction [5]. However, the paper makes the simplified assumption that the interference graph is complete. While such an assumption makes the auction design easier, it does not capture the practical scenarios where some buyers are more interference harmful, and should be rejected or charged a higher price.

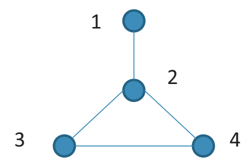

Fig. 1. A motivated example

This motivates us to design an online double auction framework that indeed takes into consideration the sporadic nature of spectrum request and buyers' geographic feature. The challenges facing us is two folds: 1) How to characterize the interference harmfulness of a certain buyer? This is difficult because, although a buyer may interfere with many other buyers, it is possible that one of its interfering neighbors interfere with even more buyers. 2) How to determine the influence of assigning spectrum to a buyer in the present time slot? This is difficult because spectrum request is random and it is hard to predict whether a buyer's interfering neighbors will request spectrum in later time slots. Apart from the above challenges, the proposed double auction mechanism should also be economic-robust, that is, truthful, individual rational and budget balanced for the auctioneer.

To address these challenges, in this paper, we propose LOTUS, a **L**ocation-aware **O**nline **T**ruthful do**U**ble auction **S**cheme, which aims at exploiting location heterogeneity to improve spectrum utilization while guaranteeing economic robustness. In Section IV, we answer the first question of how to characterize interference harmfulness, by introducing the concept of *interference discount*. If a buyer is considered to be severely interference harmful, its bid will be discounted heavily to compensate the potential utility loss it will cause. In Section V, we answer the second question of how to address

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spectrum request dynamics, by introducing the concept of *opportunity cost*. Opportunity cost is the utility difference of assigning spectrum to a buyer and to its interfering neighbors, considering potential spectrum request in the forthcoming time slots.

The paper makes the following contributions:

- To the best of our knowledge, we are the first to design a location-aware online spectrum auction mechanism which incorporates complicated interference relationship among buyers and can deal with dynamic spectrum requests.
- The proposed auction framework has nice economic properties, including truthfulness, individual rationality and budget balance for the auctioneer.
- Simulation results show that LOTUS can significantly improve buyers' and sellers' utility, as well as spectrum utilization, making it desirable for implementation in dynamic spectrum access scenarios.

The rest of the paper is organized as follows. We briefly review the related work in Section II. We give system model and basic assumptions in Section III, and explain the concept of interference discount in detail in Section IV. We introduce the proposed truthful double auction mechanism in Section V and analyze its economic properties in Section VI. We present the simulation results in Section VII, and finally summarize our work in Section VIII.

II. Literature Review

Spectrum auction has been widely studied for efficient spectrum allocation. Most of the previous works only focused on static auction, in which the spectrum request (either for the present time slot or for future time slots) is determined and submitted all at the same time. In simple forward auction [2], [6]–[8], spectrum reusability was exploited by assigning the same spectrum to non-interfering buyers. For the double auction, to guarantee truthfulness, a third-party auctioneer is needed [9]. The paper [10] [11] considered spectrum allocation in different locations but the allocation is static. The paper [3] considered buyers' requests for multiple time slots. However, the allocation is still static since all buyers submit their request at the first time slot, and the allocation is decided at the first time slot. As we discussed before, such static auction mechanisms will incur utility loss because it fails to consider the dynamic nature of spectrum request and the influence of early spectrum allocation on the later allocation decisions.

Online auction is considered by Jonathan Bredin [4] *et al.*. However, spectrum is different from other goods since it can be assigned to multiple buyers simultaneously. To the best of our knowledge, only one existing paper considered online spectrum auction. In [5], the authors proposed a truthful online double auction mechanism. However, the paper makes the simplified assumption that the interference graph is complete, thus avoid the spectrum reusability problem. While such an assumption makes the auction design easier, it does not capture the practical scenarios where some buyers are more interference harmful, and should be rejected or charged higher price.

III. Online Spectrum Auction Model

We consider a network of N potential buyers and M potential sellers. Each seller possesses one unit bandwidth of spectrum that is available during time slots $[1, T]$, and would like to lease the spectrum to buyers[1]. There is a third-party auctioneer who hosts the auction, who is in charge of collecting spectrum information from sellers, maintaining interference graph of buyers[2], and deciding the auction results at each time slot $t \in [1, T]$[3]. The $i-$th PU asks for price a_i per time slot, and its true valuation is $\widetilde{a}_i$. A PU may manipulate a_i in order to gain higher utility, so a_i is not necessarily equal to $\widetilde{a}_i$. We assume that sellers do not change their asking prices during $[1, T]$[4]. We assume that at each time slot t, a buyer wants to lease at most one unit bandwidth of spectrum from the sellers. The $i-$th buyer bids b_i per time slot for the spectrum usage, and its true valuation is $\widetilde{b}_i$. A buyer may also manipulate its bid to try to get higher utility, so b_i may not be equal to $\widetilde{b}_i$. In the following part of the paper, we refer to sellers' asking prices as "ask" and buyers' bidding prices as "bid". We assume that each buyer and each PU only knows their own bid or ask[5]. In this paper, we focus on the case where the spectrums are homogeneous to buyers, that is, a certain buyer has the same valuation and will bid the same price for every spectrum, regardless the differences such as central frequency, which mainly affects the radio propagation. The analysis can be easily extended to the heterogeneous spectrum case, where the buyers are willing to bid differently towards different spectrums. The auctioneer just has to calculate the discounted bid for each spectrum respectively. Apart from the bids, the $i-$th buyer also submits its desired time slots t_i to the auctioneer. If the buyer arrives at t, requests t_i time slots and wins, it will occupy the spectrum during $t \sim t + t_i$.

The auction results is represented by a 3-dimensional matrix $\mathcal{X}_{i,j}(t), i \in [1, M], j \in [1, N], t \in [1, T]$. At time slot t, if the $i-$th PU's spectrum is won by the $j-$th buyer for t_j time slots, then $x_{i,j}(t) = 1, x_{i,j}(t{+}1) = 1, ..., x_{i,j}(t{+}t_i) = 1$; otherwise, $x_{i,j}(t) = 0$. In the former case, the PU gets paid from the auctioneer $p_i^{PU}(t)$ per time slot, and in total $p_i^{PU}(t) * t_i$; the buyer pays the auctioneer $p_j^{SU}(t)$ per time slot, and in total $p_i^{SU}(t) * t_i$. The two unit prices are not necessarily the same: when the aggregated payment from the buyers exceeds the payment to the sellers, the auctioneer is able to maintain a

[1] Although we assume that all the seller's spectrums are available during $[1, T]$, it can be easily applied to the case where some spectrums are unavailable in certain time slots by setting its instant ask price to be infinity or simply removing the spectrum from any buyer's available spectrum set.

[2] When buyers register in the database, the auctioneer can make it compulsory for them to report their locations, and then construct the interference graph based on certain propagation models.

[3] In this paper, the spectrum allocation is centrally decided through the auctioneer. Distributed spectrum allocation can be an interesting future direction.

[4] It is possible that sellers may change their asking prices according to the demand, e.g., higher price when demand is high. In this paper, we focus on the temporal and spatial features of buyers' spectrum request, and consider the sellers' price dynamics as a future direction.

[5] In this paper, we do not consider the case of collusion since it is hard for buyers to collude when they may have spectrum request at different time slots.

balanced budget. All the spectrum allocation and payments are decided by the auctioneer.

At time t, the utility of the $j-$th buyer is its achieved valuation from the auction minus its payment:

$$U_j^{SU}(t) = \widetilde{b}_j * \sum_i x_{i,j}(t) - p_j^{SU}(t). \quad (1)$$

in which $\sum_i x_{i,j}(t) = 1$, if the buyer wins spectrum from any PU, and $\sum_i x_{i,j}(t) = 0$, if the buyer fails to win any spectrum.

At time t, the utility of the $i-$th PU is its payment from the auctioneer minus its true valuation:

$$U_i^{PU}(t) = p_i^{PU}(t) - \widetilde{a}_i * \sum_j x_{i,j}(t) \quad (2)$$

in which $\sum_j x_{i,j}(t) = 1$, if the PU successfully sells spectrum to any buyer, and $\sum_j x_{i,j}(t) = 0$, if the PU fails to sell the spectrum to any buyer.

The auctioneer's budget at time t is the payment obtained from all the buyers minus the payment to all the sellers:

$$U^{AU}(t) = \sum_{j=1}^{N} p_j^{SU}(t) - \sum_{i=1}^{M} p_i^{PU}(t) \quad (3)$$

In this paper, we assume that the objective of the auctioneer is to maintain a balanced budget[6] at each time slot, that is $U^{AU}(t) \geq 0, t = 1, 2, \cdots, T$.

One of the unique features of spectrum is its reusability, that is, two buyers can reuse the same spectrum if they are far away from each other. However, if two nearby buyers use the same spectrum, they end up interfering with each other. By leveraging the spectrum reusability, we can improve the spectrum utilization, which is essential to solve the seemingly spectrum scarcity problem. Based on the location information of the buyers, the auctioneer is able to construct an interference graph to denote the interference relationship among buyers. Let $G = (V, E)$ denote the interference graph, in which V is the set of all buyers. If two buyers are potential interferers[7], there is an edge between them, otherwise, there is no edge between them. We define the interfering neighbors and interference degree as follows.

Definition 1: Interfering neighbors. The interfering neighbors of the $i-$th buyer is the set of buyers who interfere with the buyer, i.e., sharing an edge in E with the buyer.

Definition 2: Interference degree. The interference degree of the $i-$th buyer is the number of its interfering neighbors, that is, the degree of its representing vertex in the interference graph G.

We use $N(i)$ to denote the set of interfering neighbors of the $i-$th buyer and θ_i to denote its interference degree. We assume that the interference relationship is static during $[1, T]$, so that the interference graph remains unchanged. Dynamic interference relationship with time-variant interference graph, which captures buyers' mobility, is a future direction.

[6]An alternative assumption is that the auctioneer aims at maximizing its revenue. Revenue maximization in forward auction is considered in [12]–[14], but is not the focus of this paper.

[7]The interference relationship can be derived based on radio propagation models [15].

We made the following assumptions throughout the paper.

Assumption 1: Buyers' spectrum request follows identical, independent distribution (i.i.d), more specifically, Poisson distribution with arrival rate λ.

Assumption 2: Buyers' bids are i.i.d, and the distribution is known by the auctioneer.

Assumption 3: The requested time of each buyer is i.i.d, and the distribution is known by the auctioneer.

Assumption 1 is a common assumption for traffic in wireless network ([5] [16]). Assumption 2 is reasonable as the auctioneer can estimate the trend of buyers' bids either from the bidding history or by evaluating the value of the spectrum in an open market. Assumption 3 is reasonable as the time for performing common tasks in wireless communication can be estimated based on historical data.

In this paper, we focus on the following three economic properties in designing our auction mechanism:

Truthfulness. Truthfulness is the most fundamental property of an auction mechanism. The buyers and sellers are selfish and rational players, who will manipulate their asks and bids to maximize their own utilities. The property of truthfulness ensures that neither the sellers nor the buyers can get higher payoff by misreporting their true valuation, thus avoiding market manipulation. For the online spectrum auction, we have to consider the truthfulness at each time slot. Let $\widetilde{U}_i^{PU}(t)$ and $U_i^{PU}(t)$ denote the utility of the $i-$th PU when asking truthfully and untruthfully respectively. In a truthful auction, $U_i^{PU}(t) \leq \widetilde{U}_i^{PU}(t)$ always holds. As for the buyers, the truthfulness includes two parts: 1) buyers report their true desired time slots. 2) buyers bid their true valuation. We assume that the buyers do not lie about their requested time slots. The reason is: if the $i-th$ buyer requests a shorter time slots $t_i' < t_i$, it cannot finish its task, and will get negative utility; if it requests a longer time slots $t_i' > t_i$, it has to pay for the extra time slots, and will get negative utility. Therefore, we only consider the truthfulness regarding buyers' bidding price. Assume that the $j-$th buyer gets $\widetilde{U}_j^{SU}(t)$ and $U_j^{SU}(t)$ when bidding truthfully and untruthfully respectively. In a truthful auction, $U_i^{SU}(t) \leq \widetilde{U}_i^{SU}(t)$ always holds.

Individual rationality. Individual rationality ensures that all buyers and sellers achieve non-negative utility through the auction so that they have incentive to participate in the first place. In an individual rational auction, the PU is paid more than its ask, i.e., $p_i^{PU}(t) \geq a_i$; the buyer pays less than its bid, i.e., $p_j^{SU}(t) \leq b_j$.

Budget balance. Budget balance means that the auctioneer maintains non-negative budget at each time slot, i.e., $U^{AU}(t) > 0, \forall t \in [1, T]$. If the auctioneer is a profit institution, it is important to keep a balanced budget to avoid bankruptcy[8].

[8]It is possible that the governmental bodies or non-profit groups may assume the responsibility of the spectrum allocation, and even subsidize the auction to promote efficient spectrum usage.

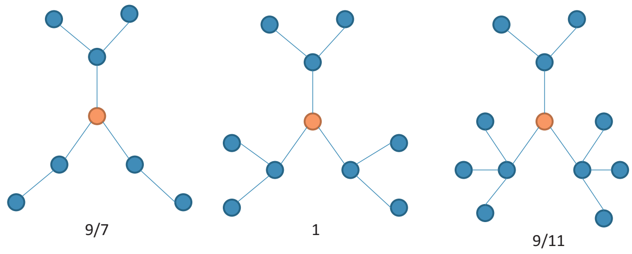


Fig. 2. Comparison of interference degree

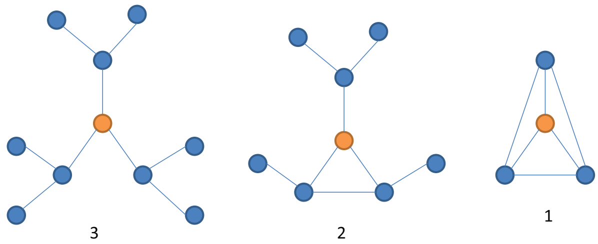


Fig. 3. Reuse degree of interference neighbors

In this paper, the objective of the proposed online spectrum auction mechanism is: ***Given the temporal and spatial features of the buyers' spectrum request, how to find a truthful, individually rational and budget balanced double auction mechanism to dynamically allocate the spectrum at each time slot?***

IV. Interference Discount

We have shown in Fig.1 that, if we simply run an existing single-round double auction mechanism for each time slot where the auctioneer considers only the spectrum request at that time slot, it will incur utility loss. The reason is that, if the $i-$th buyer wins a certain spectrum at time t, it will occupy the spectrum from $t \sim t+t_i$. Its interfering neighbors cannot use the same spectrum when they request at a later time $t+1 \sim t+t_i$ even if they bid higher prices, or can reuse the spectrum more efficiently. An important observation is: granting the spectrum request of an buyer who is located in a "critical" place which interferes with a lot of other buyers, may generate high potential utility loss. Therefore, we have to take into consideration buyers' spatial feature, especially their interfering relationship, in order to improve spectrum utilization.

We analyze the interference harmfulness of an buyer from two aspects. First, we compare the interference degree of an buyer with the average interference degree of its interference neighbors. Second, we consider the reusability degree of an buyer's interference neighbors. We combine the two factors to decide the influence of assigning the spectrum to a certain buyer on the spectrum utilization.

A. Comparison of Interference Degree

For the $i-$th buyer, we consider two choices: assign a certain spectrum to it or one of its interfering neighbors[9]. In the former case, none of its interfering neighbors can use the same spectrum. In the later case, none of the interfering neighbors of its interfering neighbors can use the same spectrum. Such exclusivity analysis has a ripple effect, which may extend to the entire interference graph. To make the problem tractable, we only consider 3 layers, from the buyer to the interference neighbors of its interference neighbors. Let r_i denote the ratio of the interference degree of the $i-$th buyer and the average interference degree of its interfering neighbors.

$$r_i = \frac{\theta_i}{(\sum_{j \in N(i)} \theta_j)/\theta_i} = \frac{\theta_i^2}{\sum_{j \in N(i)} \theta_j} \tag{4}$$

If r_i is large, to assign the spectrum to the $i-$th buyer will have negative effect on more buyers than to assign the spectrum to its interfering neighbors, vice versa. Hence, r_i indicates how "harmful" an buyer is in terms of its interference degree.

The r_i in different cases is shown in Fig.2, and the central node in orange color denotes the $i-$th buyer. In the first subgraph, the buyer's interfering neighbors have smaller average interference degree, so $r_i > 1$, which means that the buyer is quite interference harmful. In the second subgraph, the buyer's interfering neighbors have exactly the same average interference degree, so $r_i = 1$. In the third subgraph, the buyer's interfering neighbors have larger average interference degree, so $r_i < 1$, which means that the buyer may be a good candidate for spectrum allocation than its interfering neighbors.

B. Reusability Degree of Interfering Neighbors

It is not enough to only consider the interference degree. As shown in Fig.3, the central node in orange color denotes the $i-$th buyer. By definition, $r_i = 1$ is true for all three cases. However, in the first subgraph, since the interfering neighbors can reuse the same spectrum, to assign the spectrum to the central buyer causes more utility loss than the case in the last subgraph, where none of the interfering neighbors can reuse the same spectrum. Therefore, we have to take into consideration the spectrum reusability degree among an buyer's interfering neighbors.

To determine the reusability degree among a set of buyers is not easy. For instance, in Fig.3, consider the interfering neighbors of the central buyer, some can reuse the spectrum, but some cannot. The average reusability degree should consider all possible subset of the 3 buyers. When the number of buyers increases, the number of subsets increases exponentially. For simplification, we only consider the maximum reusability degree (denoted by m_i), that is, the maximum number of buyers that can reuse the spectrum[10].

[9]The third choice is not to assign the spectrum to either the $i-$th buyer or any of its interfering neighbors, but it is unreasonable since we can always improve spectrum utilization by assigning the spectrum to either party without violating the interference constraint.

[10]To derive a more precise calculation for average reusability degree is a future direction.

In Fig.3, the first subgraph shows the case in which all the interference neighbors are interference free, so $m_i = 3$. In the second subgraph, the maximum number of interference neighbors who can reuse the spectrum is 2, so $m_i = 2$. In the last subgraph, which is a complete graph, $m_i = 1$. As we discussed before, to assign the spectrum to the central buyer causes the most utility loss in the first subgraph as its interfering neighbors can use the spectrum most efficiently.

C. Interference Discount

We define the *Interference Discount* as follows.

Definition 3: Interference Discount. Interference discount is the discount factor that shrinks an buyer's bid by the comparison of interference degree and the reusability degree of its interference neighbors. The interference discount of the $i-$th buyer is

$$I_i = \frac{1}{r_i w_i} \tag{5}$$

In other words, if the $i-$th buyer bids b_i, the auctioneer discounts the bid as $I_i * b_i$ when comparing its bid with its opportunity cost.

V. The Proposed Auction Mechanism

At $t = 0$, all sellers submit their asks $a_i, i = 1, 2, \cdots, M$ to the auctioneer, which remains unchanged throughout T time slots. At the start of time t, the auctioneer gets spectrum requests from buyers, including their bids and desired time slots. Then the auction proceeds in two phases:

- *Candidate screening*. In a static auction, since all buyers' spectrum requests arrive at the same time, it is possible for the auctioneer to perform the spectrum allocation in an "optimal" way. However, in the online auction, the spectrum requests happen sporadically. The spectrum requests in the early time slots influence those in the later time slots. It is necessary for the auctioneer to screen the request according to 1) the discounted bids, and; 2) the opportunity cost. We further explain the opportunity cost in the following part.
- *Spectrum allocation determination*. After screening, the valid candidate buyers become the buyers in the auction. Then the auctioneer conducts the auction mechanism to determine which PU's spectrum to assign to which buyer, and how much each PU and buyer has to pay. The auction mechanism should be truthful, individual rational and budget balanced.

A. Candidate Screening

Now we explain how to calculate the opportunity cost of assigning the spectrum to the $i-$th buyer at time t. At time $t = 0$, the auctioneer derives the expected valuation matrix $\mathcal{V}$. Entry $v_i(t), i \in [1, N], t \in [1, T]$ denotes the expected value of assigning the spectrum to the interfering neighbors of the $i-$th buyer from time t on. Let T_τ denote the event that the desired time slots are τ, $B_i = \sum_{j \in N(i)} b_j I_j / |N(i)|$ denote the average discounted bids among the interfering neighbors. Then we can calculate $v_i(t)$ as follows:

$$v_i(t) = \sum_{\tau=1}^{T-t} \mathbf{Pr}(T_\tau)(B_i \tau + v_i(t+\tau)) \tag{6}$$

in which $\mathbf{Pr}(\cdot)$ denote the probability of an event. $\mathcal{V}$ is an $N \times T$ matrix and can be calculated using dynamic programming.

When the $i-$th buyer requests the spectrum at time t for t_i time slots, its opportunity cost is

$$C_i(t, t_i) = v_i(t) - v_i(t + t_i). \tag{7}$$

If the $i-$th buyer requests a spectrum at time t, the auctioneer compares the opportunity cost $C_i(t, t_i)$ with the discounted bid $b_i * I_i$.

- If $C_i(t, t_i) > I_i * b_i * t_i$, meaning that the $i-$th buyer does not bid enough to compensate the potential utility loss, the auctioneer rejects its request directly in the screening phase.
- If $C_i(t, t_i) \leq I_i * b_i * t_i$, meaning the $i-$th buyer's bid exceeds the potential utility loss, the auctioneer considers it as a valid candidate for auction winner, and derives its available spectrum set $\mathcal{A}_i(t)$ as follows.
  - Assign all the spectrums to $\mathcal{A}_i(t)$.
  - Eliminate from $\mathcal{A}_i(t)$ the spectrum of the $k-$th PU if there exists $j \in N(i)$ and $x_{k,j}(t) = 1$, which means that at least one of the interfering neighbors won the spectrum at time prior to t and occupies the spectrum at time t.

After the screening phase, each valid candidate buyer gets an available spectrum set. Then the auctioneer moves on to the spectrum allocation determination phase.

B. Spectrum Allocation Determination

The spectrum allocation includes two parts: grouping and matching.

1) Grouping: buyers who can reuse the same spectrum can be grouped together, and assigned the same spectrum in order to improve spectrum utilization. In the previous double auction design [9]–[11], it is assumed that all the spectrums are available to each buyer, so the grouping process can be easily done by partitioning the interference graph into multiple independent sets. However, this no longer works in our settings when different buyers have different available spectrum sets. Two buyers who do not have any common available spectrums cannot be grouped together even if they do not interfere with each other. To solve this problem, we propose the algorithm 1 for grouping.

- We add virtual edges in the interference graph to make sure that buyers with no common available spectrum will not be grouped together.
- When computing the group bid, we first calculate the minimum bid per time slot. This is to make sure that no buyer will pay more than its bid, thus guaranteeing individual rationality. We sacrifice the buyer who bids the

minimum bid (by reduce the group size by 1) in order to guarantee truthfulness.

Algorithm 1 Grouping

1: Get the interference subgraph of the valid candidate buyers.
2: Add edges between any two buyers who do not have a common spectrum in their available spectrum sets.
3: Partition the resulting graph into L independent sets: $G_1, G_2, \cdots, G_L$. GS_l is the set of common available spectrums for all buyers in group G_l.
4: The group bid of G_l is

$$\Phi_l = \frac{\min_{i \in G_l}(b_i t_i)}{\max_{j \in G_l} t_j} \times (|G_l| - 1)$$

in which $|G_l|$ is the size of G_l, $\max_{j \in G_l} t_j$ is the maximum requested time slots in G_l.

2) Matching: We propose the algorithm 2 for matching sellers with buyer groups. After sorting the sellers and buyer groups, we find the threshold k, where the $k-$th PU's ask is less than the $k-$th group bid, but the $(k+1)-$th PU's ask is greater than the $(k+1)-$th group bid. We assume that the $i-$th PU is matched to the group $G_i^{\mathcal{M}}$.

Algorithm 2 Matching

1: $\mathbb{A}$ = sorted sellers by asks in increasing order.
2: $\mathbb{B}$ = sorted groups by the group bids in decreasing order.
3: Construct bipartite graph $\mathbb{A} \cup \mathbb{B}$. Construct an edge between G_l and every $j \in GS_l$.
4: Find k such that

$$k = \arg\{a_k \leq \Phi_k, a_{k+1} > \Phi_{k+1}\} \tag{8}$$

5: Find the maximal matching $\mathcal{M}$ on the first $(k-1)$ vertices in $\mathbb{A}$ and the first $(k-1)$ vertices in $\mathbb{B}$.

3) Winner Determination: we propose the algorithm 3 for determining winning sellers and buyers as well as their payment.

Algorithm 3 Winner determination

1: Each PU in $\mathcal{M}$ is a winner, and the $i-$th PU is paid $a_k \times \max_{j \in G_i^{\mathcal{M}}} t_j$.
2: Each buyer group in $\mathcal{M}$ is a winning group. Each group member except the one with the minimum bid (i.e., $\min_{i \in G_l} b_i t_i$) is a winner. The winning group G_l pays a total amount of $\Phi_l \times \max_{j \in G_l} t_j$, which is equally shared by the winning buyers.

VI. Theoretical Analysis

In this section, we will prove that the proposed auction mechanism is economic robust in terms of 1) Truthfulness; 2) Individual rationality; and 3) Budget balanced.

A. Individual Rationality

Theorem 1: The proposed double auction mechanism is individual rational for both sellers and buyers.

Proof: Since sellers are sorted by asks in increasing order, for any $i-$th PU in $\mathcal{M}$, $a_i < a_k$ as $i < k$. Also, the spectrum of the $i-$th PU is occupied for at most $\max_{j \in G_i^{\mathcal{M}}} t_j$ time slots. So the $i-$th PU is paid more than its ask multiplying the occupied time slots. In winning group G_l, each winning member pays $\Phi_l \times \max_{j \in G_l} t_j / (|G_l| - 1) = \min_{i \in G_l}(b_i t_i)$, which is less than its bid multiplying the occupied time slots. ■

B. Budget Balanced

Theorem 2: The proposed double auction mechanism is budget balanced for the auctioneer.

Proof: The auctioneer's budget at time t is

$$\begin{aligned} U^{AU}(t) &= \sum_{l \in \mathcal{M}} \Phi_l \times \max_{j \in G_l} t_j - \sum_{i \in \mathcal{M}} a_k \times \max_{j \in G_i^{\mathcal{M}}} t_j \\ &= \sum_{G_l = G_i^{\mathcal{M}}} (\Phi_l - a_k) \times \max_{j \in G_l} t_j \\ &\geq \sum_{G_l = G_i^{\mathcal{M}}} (\Phi_k - a_k) \times \max_{j \in G_l} t_j \geq 0 \end{aligned}$$

Therefore, the auctioneer's budget is always non-negative. ■

C. Truthfulness

In this paper, we only consider the truthfulness after the screening phase[11]. We first analyze the truthfulness on the buyer side. We already explain that the buyers do not have incentive to lie about their requested time slots, so we focus on the truthfulness of buyers' bids.

Lemma 1: If the $i-$th buyer wins, it pays the same price regardless of its bid b_i.

Proof: If the $i-$th buyer wins in group G_l, it is not the one with the minimum bid and it will pay $\min_{j \in G_l}(b_j t_j)$, in which $j \neq i$. So b_i does not affect the payment. ■

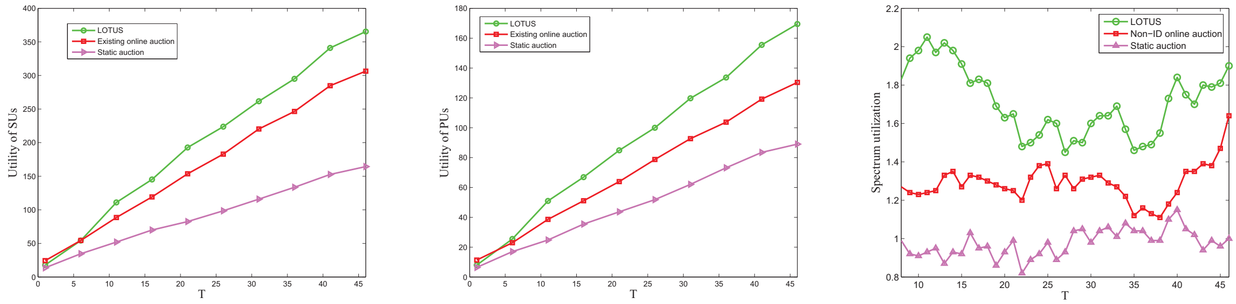
Lemma 2: If the $i-$th buyer wins by bidding b_i, it will also win by bidding $b_i' > b_i$.

Proof: Assume that the $i-$th buyer is in group G_l (the grouping result is independent of b_i). An increase in the bid of any group member will make the group bid Φ_l increase or remain unchanged. Therefore $\Phi_l' \geq \Phi_l$. If G_l is in the first $(k-1)$ vertices in $\mathbb{B}$ when the group bid is Φ_l, it will also be in the first $(k-1)$ vertices in $\mathbb{B}$ when the group bid is Φ_l'. So G_l will also win at Φ_l'. Since the $i-$th buyer is not the one with minimum bid in the group (otherwise, it loses when bidding b_i), it is still a winner. ■

Theorem 3: The proposed double auction mechanism is truthful for the buyers.

Theorem 3 can be proved based on Lemma 1 and 2. Due to page limitation, we ignore the proof here.

[11] In fact, an buyer's bid also affects whether it can enter the spectrum allocation determination phase since the bid affects the comparison between discounted bid and opportunity cost. To incorporate truthfulness in the screening phase is a future direction.



(a) Utility of buyers (b) Utility of sellers (c) Spectrum utilization

Fig. 4. Performance comparison of 1) LOTUS; 2) Online auction without interference discount (referred to as Non-ID online auction in the figure) and; 3) Static auction

Now we consider the truthfulness on the PU side.

Lemma 3: If the $i-$th PU wins, it gets paid the same price regardless of its ask a_i.

Lemma 4: If the $i-$th PU wins by asking a_i, it will also win by asking $a_i' < a_i$.

Theorem 4: The proposed double auction mechanism is truthful for the sellers.

Theorem 4 can be proved based on Lemma 3 and 4. Due to page limitation, we ignore the proofs here.

VII. Performance Evaluation

A. Simulation Setup

We consider the network where buyers scatter randomly within a square area layout of size $100m \times 100m$ area. The transmission range of secondary devices is $35m$. The number of buyers is 100 at default. The probability that an buyer requests the spectrum is 0.5, and the average requested time is 5 at default. The number of sellers (and correspondingly their contributed spectrums) is fixed as 10. Sellers' asks and buyers' bids both follow independent uniform distribution in the range (0,1]. We consider a lasting time period consisting of 50 time slots. All the results are averaged over 100 rounds.

Firstly, we compare the performance of LOTUS with the following cases:

- *Static auction without opportunity cost consideration*. In this case, the influence of the spectrum assignment at early time slot on the spectrum request at later time slots is totally ignored. The auctioneer does not calculate the opportunity cost, thus there is no screening phase. All the buyers making spectrum requests at time t will enter the spectrum allocation determination phase.
- *Online auction without interference discount consideration*. In this case, the influence of the spectrum assignment at early time slot on the spectrum request at later time slots is considerer. But when calculating the opportunity cost, no interference discount is applied. Actually, this can be viewed as a special case where the interference graph is a complete graph, thus each buyer's interference discount equals 1.

Secondly, we study the key factors that affect LOTUS' performance, including

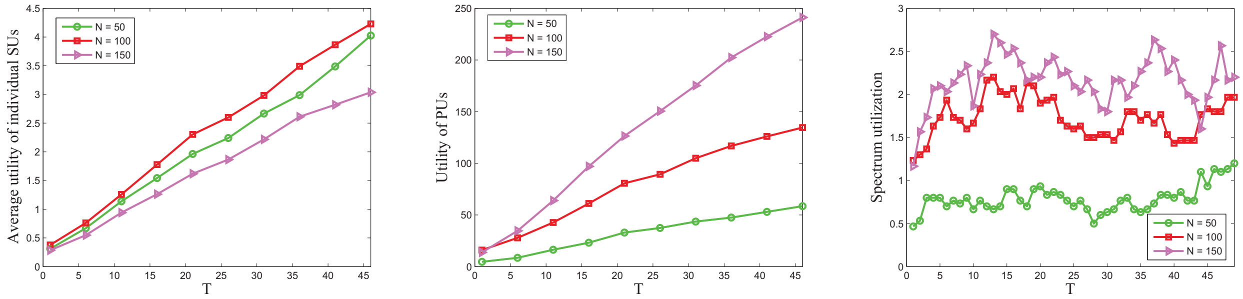
- *The density of buyers*, which has a great influence on the interference relationships among buyers;
- *The average frequency of buyers' spectrum request*, which is represented by the probability that a buyer makes spectrum request.
- *The average requested time slots*, which affects how long a winning buyer will occupy the spectrum, thus depriving its interfering neighbors the access opportunity.

B. Comparison of Different Auction Mechanisms

Fig. 4 shows the results of the comparison of different auction mechanisms. It is verified that LOTUS outperforms both existing online auction and static auction mechanisms. Subfigure (a) shows that the total utility of all buyers can be improved by as high as 25.5% over the existing online auction, 134% over the static auction; and subfigure (b) shows that the total utility of all sellers can be improved by as high as 33% over the existing online auction, 106% over the static auction. Subfigure (c) shows the spectrum utilization at each time slot. The spectrum utilization may exceeds 1 since the same spectrum can be reused by several buyers. The LOTUS improves spectrum utilization by as high as 65.3% over the existing online auction, 120% over the static auction. The static auction performs the worst, which again verifies the necessity of introducing online auction.

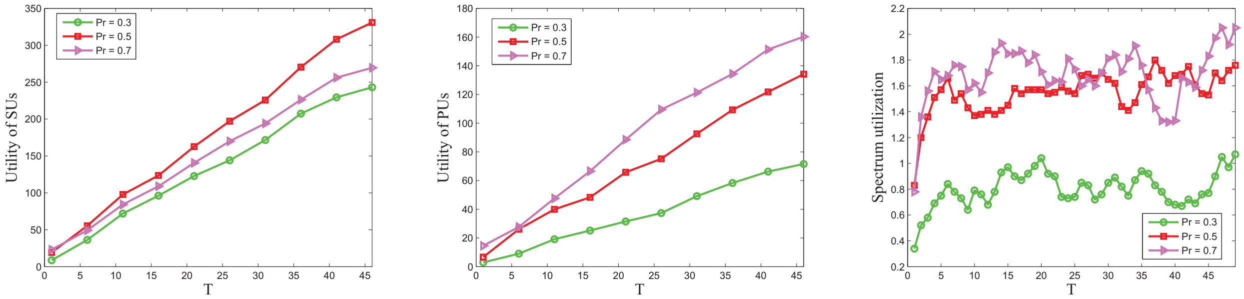
C. Factors Affecting LOTUS Performance

1) Impact of buyer Density: Fig.5 shows the impact of buyer density. Subfigure (a) is the total utility of buyer divided by the number of buyer. We can see that when the number of buyer increases from 50 to 100, the average utility increases. This is because more groups can be formed with larger size, which increases the group bid and the chances of winning the spectrum. However, when the number of buyer further increases from 100 to 150, the average utility decreases. The reason is that the network becomes crowded with many buyers interfering with each other, and the ratio of winning buyers decreases. But for sellers, the utility will always increase when the number of buyers increases, because the demand increases. The spectrum utilization when there are few number of buyers ($N = 50$) is low. The improvement of spectrum utilization



(a) Average utility of individual buyer (b) Utility of sellers (c) Spectrum utilization

Fig. 5. Impact of buyer density.



(a) Utility of SUs (b) Utility of sellers (c) Spectrum utilization

Fig. 6. Impact of buyer request frequency.

for $50 \rightarrow 100$ change is much more significant than that for $100 \rightarrow 150$ change, due to the worsened interference situation in the latter case.

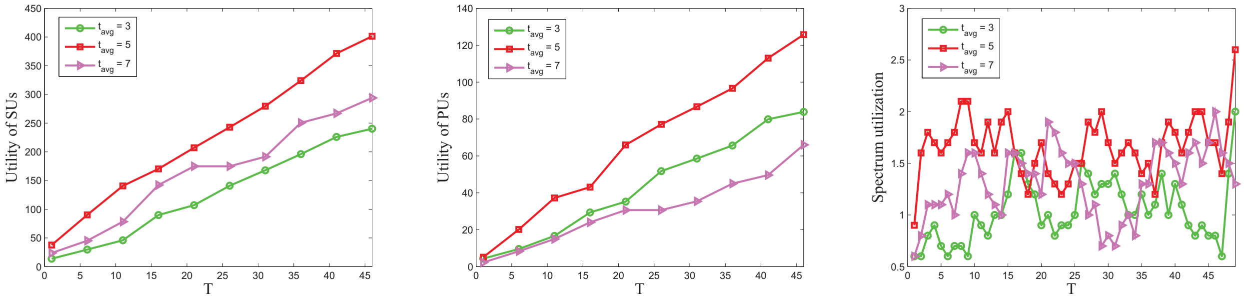
2) Impact of buyer Request Frequency: Fig.6 shows the impact of buyer request frequency. When the spectrum request happens quite infrequent ($Pr = 0.3$), which means that the traffic is very light, the total utility of buyer is low because few buyer participates in the auction. The increase in the request frequency ($Pr = 0.5$) increases buyers' utility but a further increase ($Pr = 0.7$) will degrade the utility since the network is too crowded and many buyers' request cannot be satisfied. Also, the opportunity cost for an buyer increases since the chance of its interfering neighbor requesting the spectrum in later time becomes higher. Therefore, more buyers will be rejected in the screening phase. The utility of sellers keep increasing due to higher demand. The spectrum utility change has similar trend to that in Fig.5.

3) Impact of Average Requested Time Slots: Fig.7 shows the impact of the average requested time slots. Spectrum utilization is almost not affected since the number of arriving buyers in each time slot is nearly the same. So the number of buyers participating in the auction and the final winners is nearly the same, too. When the average requested time slots increases from 3 to 5, the utilities of both buyers and sellers increase. The reason is that more available time slots are requested and assigned to buyers, making them better off and generating higher revenue for the sellers. On the contrary, when the average requested time slots increases from 5 to 7, the utilities of both buyers and sellers decrease. This is because the buyers arrive at early time slots occupy the spectrum for a long period, during which the unpredicability of later spectrum requests increases, making the calculation of opportunity cost less reliable. There is higher probability that an buyer with higher valuation cannot access the spectrum because its interfering neighbors still occupy the spectrum. This reduces not only buyers' utility but also buyers' utility.

VIII. Conclusion

In this paper, we proposed a location-aware online truthful double auction framework for spectrum allocation. We determine the interference harmfulness of a buyer by reducing its bid by a factor defined as *interference discount*. The interference discount depends on a buyer's geographic location and its interference relationship with others. If an buyer has a larger interfering degree than its interfering neighbors, and if many of its interfering neighbors can reuse the same spectrum, then the interference discount will greatly reduce this buyer's bid. For a buyer, if its discounted bid is greater than its opportunity cost, the buyer is considered as a participant in the spectrum auction. The auctioneer then executes the auction mechanism, which is proved to be truthful, individual rational and budget balanced. The simulation results show that LOTUS is effective in improving the utility of sellers and buyers, as well as the spectrum efficiency.

For the future work, there are many interesting directions since online auction is an important but rather untapped



(a) Utility of buyers (b) Utility of sellers (c) Spectrum utilization

Fig. 7. Impact of average requested time slots.

area. We can consider spectrum heterogeneity where different buyers have different preference towards different spectrums. Collusion in online spectrum auction is another interesting topic. buyers who arrive at the same time slot may collude, and buyers who arrive at different time slots may also collude. It has to be defined how buyers may collude in the later case and how to avoid it.

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