

Designing Truthful Spectrum Auctions for Multi-hop Secondary Networks

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Abstract—Opportunistic wireless channel access granted to non-licensed users through auctions represents a promising approach for effectively distributing and utilizing the scarce wireless spectrum. A limitation of existing spectrum auction designs lies in the over-simplifying assumption that every non-licensed *secondary user* is a single node or single-hop network. For the first time in the literature, we propose to model non-licensed users as *secondary networks* (SNs), each of which comprises of a multihop network with end-to-end routing demands. We use simple examples to show that such auctions among SNs differ drastically from simple auctions among single-hop users, and previous solutions suffer from local, per-hop decision making. We first design a simple, heuristic auction that takes inter-SN interference into consideration and is truthful. We then design a randomized auction framework based on primal-dual linear optimization, which is automatically truthful and achieves a social welfare approximation ratio that matches one achieved by cooperative optimization assuming truthful bids for free. The framework relieves a spectrum auction designer from worrying about truthfulness of the auction, so that he or she can focus on social welfare maximization while assuming truthful bids for free.

Index Terms—Truthful Auctions, Secondary Spectrum Allocation, Secondary Networks, Linear Programming, Primal-Dual Algorithms

I. INTRODUCTION

RECENT years have witnessed substantial growth in wireless technology and applications, which rely crucially on the availability of bandwidth spectrum. Traditional spectrum allocation is static, and is prone to inefficient spectrum utilization in both temporal and spatial domains: large spectrum chunks remain idling while new users are unable to access them. Such an observation has prompted research interest in designing a secondary spectrum market, where new users can access a licensed channel when not in use by its owner, with appropriate remuneration transferred to the latter.

In a secondary spectrum market, a spectrum owner or *primary user* (PU) leases its idle spectrum chunks (channels) to *secondary users* (SUs) through auctions [1], [2]. SUs submit bids for channels, and pay the PU a price to access a channel if their bids are successful. A natural goal of spectrum auction design is *truthfulness*, under which an SU's best strategy is to bid its true valuation of a channel, with no incentive to lie. A truthful auction simplifies decision making at SUs, and lays a foundation for good decision making at the PU. Another important goal in spectrum auction design is *social welfare*

maximization, i.e., maximizing the aggregated ‘happiness’ of everyone in the system. Such an auction tends to allocate channels to SUs who value them the most. The focus of this work is to advance the studies of spectrum auction design from serving single-hop secondary users to multi-hop secondary networks.

A unique feature of spectrum auction design is the need of appropriate consideration for wireless interference and spatial reuse of channels. A channel can be allocated to multiple SUs provided that they are far apart, with no mutual interference. Optimal channel assignment for social welfare maximization is equivalent to the graph colouring problem, and is NP-hard [3], even assuming truthful bids are given for free. Existing works on spectrum auctions often focus on resolving such a challenge (e.g., [4], [5]) while assuming the simplest model of a SU: a single node, or a single link, similar to a single hop transmission in cellular networks [2], [4], [5].

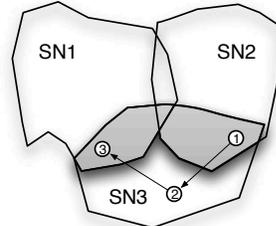


Fig. 1: A secondary spectrum market with 3 SNs and 2 channels.

After extensive research during the past five years, auction design for single-hop users, each requesting a single channel, has been relatively well understood. However, a practical SU may very well comprise of multiple nodes forming a multi-hop network, which we refer to as a *secondary network* (SN). These include scenarios such as users with multihop access to base stations, or users with their own mobile ad hoc networks. SNs require coordinated end-to-end channel assignment, and in general benefit from multi-channel diversity along its path. The SN model subsumes the SU model as the simplest special case.

Fig. 1 depicts three co-located SNs, SN1, SN2 and SN3, which have interference with one another, because their network regions overlap. The primary network (PN) has two channels, Ch1 and Ch2, which have been allocated to SN1 and SN2, respectively. Now SN3 wishes to route along a two-hop path $1 \rightarrow 2 \rightarrow 3$. Under existing single-channel auctions for SUs, SN3 cannot obtain a channel, because each channel interferes with either SN1 or SN2. However, a solution

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exists by relaxing the one channel per user assumption, and assigning Ch1 to link $1 \rightarrow 2$ and Ch2 to the link $2 \rightarrow 3$. In general, taking multichannel, multihop transmissions by SNs into consideration can apparently improve channel utilization and social welfare. Note here that the model in which an SN bids for multiple channels is inapplicable, because due to the unawareness of other SNs' information, an SN cannot know the number of channels to bid for, to form a feasible path.

Designing truthful auctions for SNs is an interesting problem, but by no means an easy one. We note that it is hard for an SN to decide by itself an optimal or good path to bid for. Such decision making requires global information on other SNs as well, and is naturally best made by the auctioneer, *i.e.*, the PN. Consequently, a bid from an SN includes just a price it wishes to pay, with two nodes it wishes to connect using a path. Furthermore, SNs now interfere with each other in a more complex manner. Not only that they transmit along multihop paths, but each path can be assigned with distinct channels at different links. The PN, after receiving bids, needs to make judicious joint routing and channel assignment decisions.

In this work, we first design a simple heuristic auction for spectrum allocation to SNs, which guarantees both truthfulness and interference-free channel allocation, providing winning SNs with end-to-end multihop paths, with a channel assigned to each hop. The heuristic auction enables multi-channel assignment along a path, thereby reducing the possibility that a path is blocked due to interference. To achieve truthfulness, we employ a greedy, monotonic allocation rule and design an accompanying payment scheme, by referring to Myerson's characterization of truthful auctions [6].

The heuristic auction provides no hard guarantee on social welfare. Inspired from recent linear programming (LP)-based techniques due to Lavi and Swamy [7] and Carr and Vempala [8] that decompose LP solutions into integer solutions, our main contribution is a randomized auction framework rooted in primal-dual optimization, which is proved to be truthful (in expectation), and can guarantee a social approximation ratio that is achievable by a cooperative social welfare optimization algorithm that assumes truthful bids for free. At a high level, a spectrum auction designer faces two challenges at the same time: that of carefully tailoring the allocation rules for eliciting truthful bids, and that of handling the high computational complexity of the social welfare maximization problem introduced by wireless interference and spectrum reuse. Given the randomized auction framework, the auction designer is essentially relieved from the former challenge and can focus on the latter.

The remainder of the paper is organized as follows. We discuss related work in Sec. II, and present preliminaries in Sec. III. A heuristic truthful auction is designed in Sec. IV. In Sec. V, we present and analyze a randomized auction framework. Simulation studies are presented in Sec. VI. Sec. VII concludes the paper.

II. RELATED WORK

Auctions serve as an efficient mechanism for distributing scarce resources to competing participants in a market. To

simplify the strategic behaviour of agents and hence encourage participation, truthfulness is desired. A celebrated work is the VCG mechanism due to Vickrey [9], Clarke [10], and Groves [11]. However, the VCG mechanism is only suitable when optimal solutions are computationally feasible, and is not directly applicable for secondary spectrum auctions, because interference-free channel allocation is NP-Hard. Our randomized auction design in this work employs a fractional VCG solution where the computational complexity of the VCG mechanism is circumvented by relaxing the integral requirement in the joint routing and channel assignment solution.

The efficiency of auctions to distribute spectrum has received considerable research attention recently. A main challenge here is that appropriate handling of wireless interference and optimal spatial reuse of channel spectrum often require solving computationally expensive problems. Early solutions include auctions that allocate power [12] and allocate a channel to each winning user [13]. These auctions are unfortunately not truthful. Truthfulness is first considered in VERITAS [1] based on the monotonic allocation rule. *Topaz* [14] is an online spectrum auction that is truthful in both agents' bids and their channel access time reports. Jia *et al.* [15] design a spectrum auction mechanism that not only encourages truthful behaviour but also computes approximately maximum revenue, which is an alternative goal to maximum social welfare.

For spectrum auctions that take interference among secondary users into consideration, Wu *et al.* [2] develop a semi-definite programming based mechanism, which is truthful and resistant to bidder collusion. Gopinathan *et al.* [5] propose auctions that incorporate fairness considerations into channel allocation. Their goal is to maximize social welfare, while ensuring a notion of fairness among bidders when the auction is repeatedly held. Double auctions, where buyers and sellers simultaneously submit bids and ask prices respectively to the auctioneer, are adopted in TRUST [16] and *District* [17]. A truthful and scalable spectrum auction enabling both sharing and exclusive access is proposed by Kash *et al.* [4]. This auction handles heterogeneous agent types with different transmission powers and spectrum needs. Al-Ayyoub and Gupta [18] design a truthful auction with approximate revenue guarantee in the wireless cellular network setting. All these works focus on single-hop users bidding, often for a single channel, with the exception of the last work. Our work essentially generalizes the problem to multi-hop users, which are characterized by multi-channel paths. We note that existing application of auctions in the multi-hop network setting are for the purpose of routing instead of spectrum allocation [19], [20]. We also note that while the majority of spectrum auctions in the literature are deterministic, the main contribution of this work is a randomized auction framework where the joint routing and channel assignment solution computation involves randomized decision making.

III. PRELIMINARIES

In this section, we first introduce some background in truthful auction design in Sec. III-A, then describe our system model in Sec. III-B.

A. Truthful Auction Design

Auction theory is a branch of economics that studies how people act in an auction and analyzes the properties of auction markets. We first introduce some basic and most related concepts, definitions and theorems from auction design.

An auction allocates items or goods (channels in our case) to competitive agents with bids and private valuations. We adopt w_i as nonnegative valuations of each agent i , which is often private information known only to the agent itself. Besides determining an allocation, an auction also computes payments/charges for winning bidders. We denote by $p(i)$ and b_i the payment and bid of agent i , respectively. Then the utility of i is a function of all the bids:

$$u_i(b_i, \mathbf{b}_{-i}) = \begin{cases} w_i - p(i) & \text{if } i \text{ wins with bid } b_i \\ 0 & \text{otherwise} \end{cases}$$

where \mathbf{b}_{-i} is the vector of all the bids except b_i . We first adopt some conventional assumptions in economics here. We assume that each agent i is selfish and rational. A *selfish* agent is one that acts strategically to maximize its utility. An agent is said to be *rational* in that it always prefers the outcome that brings itself a larger utility. Hence, an agent i may lie about its valuation, and bid $b_i \neq w_i$ if doing so yields a higher utility.

Truthfulness is a desirable property of an auction, where reporting true valuation in the bid is optimal for each agent i , regardless of other agents' bids. If agents have incentives to lie, other agents are forced to strategically respond to these lies, making the auction and its analysis complex. A key advantage of a truthful auction is that it simplifies agent strategies. Formally, an auction is *truthful* if for any agent i with any $b_i \neq w_i$, any \mathbf{b}_{-i} , we have

$$u_i(w_i, \mathbf{b}_{-i}) \geq u_i(b_i, \mathbf{b}_{-i}) \quad (1)$$

An auction is *randomized* if its allocation decision making involves flipping a (biased) coin. The payment and utility of an agent are then random variables. A randomized auction is *truthful in expectation* if (1) holds in expectation. Besides, we also prefer an auction to be *individually rational*, in which agents pay no more than their gain (valuations).

As discussed, the classic VCG mechanism for truthful auction design requires the optimal allocation to be efficiently computable, and is not practical for spectrum auctions, since optimal channel allocation is NP-hard. If we aim to design a tailored, heuristic truthful auction, then we may rely on the characterization of truthful auctions by Myerson [6].

Theorem 1. *Let $P_i(b_i)$ be the probability of agent i with bid b_i winning an auction. An auction is truthful if and only if the followings hold for a fixed \mathbf{b}_{-i} :*

- $P_i(b_i)$ is monotonically non-decreasing in b_i ;
- Agent i bidding b_i is charged $b_i P_i(b_i) - \int_0^{b_i} P_i(b) db$.

Given Theorem 1, we see that once the allocation rule $\mathbf{P}(\cdot) = \{P_i(b_i)\}_{i \in \mathcal{N}}$ is fixed (\mathcal{N} is the set of bidders), the payment rule is also fixed. For the case where the auction is deterministic, there are two equivalent ways to interpret Theorem 1: (i) there exists a minimum bid b_i^* , such that i will win only if agent i bids at least b_i^* , i.e., the monotonicity of $P_i(b_i)$ implies that, there is some *critical bid* b_i^* , such that

$P_i(b_i)$ is 1 for all $b_i > b_i^*$ and 0 for all $b_i < b_i^*$; (ii) the payment charged to agent i for a fixed \mathbf{b}_{-i} should be independent of b_i (formally, $p_i(b_i) = b_i - \int_{b_i^*}^{b_i} db = b_i^*$).

B. System Model

We assume there is a set of SNs, \mathcal{N} . Each SN has deployed a set of nodes in a geographical region, and has a demand for multihop transmission from a source to a destination. A PN has a set of channels, \mathcal{C} , available for auctioning in the region. We refer to SNs as *agents* and the PN as the *auctioneer*. Each node within an SN is equipped with a radio that is capable of switching between different channels. SNs do not collaborate with each other, and nodes from different SNs are not required to forward traffic for each other.

We assume nodes from each SN i form a connected graph $G^i(\mathcal{E}^i, \mathcal{V}^i)$, which also contains node locations. We use “node” and “link” for the connectivity graphs and “vertex” and “edge” for the conflict graph introduced later. To better formulate the joint routing-channel assignment problem, we incorporate the concept of network flows. Let u^i be a node in SN i and s^i, d^i be the source and the destination in SN i . We use l_{uv}^i to denote the link from node u^i to node v^i belonging to SN i , and f_{uv}^i to denote the amount of flow on link l_{uv}^i . Later we connect d^i back to s^i with a virtual feedback link l_{ds}^i , for a compact formulation of the joint optimization integer program (IP).

We define a *conflict graph* $H(\mathcal{E}_H, \mathcal{V}_H)$, whose vertices correspond to links from all the connectivity graphs. We use (l_{uv}^i, l_{pq}^j) to denote an edge in \mathcal{E}_H , indicating that link l_{uv}^i and link l_{pq}^j interfere if allocated a common channel. Before the auction starts, each SN i submits to the auctioneer a compound bid, defined as $\mathfrak{B}_i = (G^i(\mathcal{E}^i, \mathcal{V}^i), s^i, d^i, b_i)$. The auctioneer computes the conflict graph once all compound bids are collected. We denote by w_i the private valuation of SN i for a feasible path between s^i and d^i , and $p(i)$ its payment. b_i, w_i and $p(i)$ all represent monetary amounts. Note that we assume agents only have incentives to lie about their valuations, and assume topology information in a bid is truthful.

Let R_T and R_I be the transmission range and interference range of every node u^i , respectively, let $\Delta = \frac{R_I}{R_T}$ be the interference-to-communication-ratio, where $\Delta \geq 1$. Since no inter-SN collaboration is assumed, links from different SNs do not participate in joint MAC scheduling, and cannot be assigned the same channel if they interfere. As a result, two links l_{uv}^i and l_{pq}^j interfere if a node in $\{u, v\}$ is within the interference range of a node in $\{p, q\}$, and cannot be assigned the same channel if $i \neq j$. Formally, let a binary variable $x(c, l_{uv}^i) \in \{0, 1\}$ denote whether channel $c \in \mathcal{C}$ is assigned to link l_{uv}^i for user i . For the joint routing-channel assignment problem we have the *Channel Interference Constraints*:

$$x(c, l_{uv}^i) + x(c, l_{pq}^j) \leq 1, (l_{uv}^i, l_{pq}^j) \in \mathcal{E}_H, \forall c \in \mathcal{C} \quad (2)$$

An agent needs an end-to-end path, which corresponds to an end-to-end network flow of rate 1. Note that the flow rate on the virtual feedback link f_{pds}^i equals the end-to-end flow rate for SN_i . We further have *Flow Conservation Constraints*: at

any node in \mathcal{V}^i , the total incoming and outgoing flows equal:

$$\sum_{u \in \mathcal{V}^i} f_{uv}^i = \sum_{v \in \mathcal{V}^i} f_{vu}^i, \quad \forall v \in \mathcal{V}^i \quad (3)$$

Assuming each channel has the same unit capacity 1, we next have the *Capacity Constraints*:

$$\sum_{u \in \mathcal{V}^i \setminus \{d^i\}} f_{uv}^i \leq \sum_{c \in \mathcal{C}} x(c, l_{uv}^i) \leq 1 \quad (4)$$

which also ensures that a link can be assigned a single channel only.

Finally, let a sub-linear function $\gamma_i(\mathbf{f}, \mathbf{x})$ denote that valuation of SN_i on a solution (\mathbf{f}, \mathbf{x}) , which models the utility of SN_i on its end-to-end path specified in \mathbf{f} with channel assignment specified in \mathbf{x} (a vector that contains all $x(c, l_{uv}^i)$ values). The utility Ψ_i of SN_i is:

$$\Psi_i \leq \gamma_i(\mathbf{f}, \mathbf{x}) \quad (5)$$

We formulate the joint routing-channel assignment problem for SNs into an IP:

$$\text{maximize } O(\mathbf{w}) = \sum_{i \in \mathcal{N}} \Psi_i \quad (6)$$

subject to

$$\begin{aligned} x(c, l_{uv}^i) + x(c, l_{pq}^j) &\leq 1, & (l_{uv}^i, l_{pq}^j) \in \mathcal{E}_H, \forall c \in \mathcal{C} \\ \sum_{u \in \mathcal{V}^i} f_{uv}^i &= \sum_{v \in \mathcal{V}^i} f_{vu}^i, & \forall v \in \mathcal{V}^i \\ \sum_{u \in \mathcal{V}^i \setminus \{d^i\}} f_{uv}^i &\leq \sum_{c \in \mathcal{C}} x(c, l_{uv}^i) \leq 1, & \forall v \in \mathcal{V}^i \\ \Psi_i &\leq \gamma_i(f^i, x) & \forall i \\ f_{uv}^i, x(c, l_{uv}^i) &\in \{0, 1\}. \end{aligned}$$

where $O(\mathbf{w})$ denotes the objective function of the IP. Optimally solving IPs to optimal is NP-hard. In particular, the optimization in (6) involves interference-free scheduling that can be reduced into the NP-hard graph coloring problem. We first introduce a heuristic auction in Sec. IV, which is based on the technique of monotonic allocation and critical bids, and is simple and truthful but does not provide any performance bound. A more sophisticated, randomized auction with a proven bound is studied next, where the LP relaxation of IP (6) is solved as a first step.

IV. A HEURISTIC TRUTHFUL AUCTION

In this section, we design an auction with a greedy style allocation and a payment scheme to ensure truthfulness. The auction consists of two phases: Algorithm 1 determines the channel assignment and winning bidders, and Algorithm 2 computes the payments for winning agents. The auction design in this section is based on a well-known technique of truthful auction design: combining a greedy allocation rule with charging critical bids to the winners.

A. Channel Allocation

As discussed in Sec. III, the key to designing a truthful auction is to have a non-decreasing allocation rule. Prices can then be calculated by the critical bids to make the auction truthful. A natural method is to sort all agent bids in a non-decreasing order, and greedily assign channels to agents in this order, subject to interference constraints [21]. However, ranking agents only according to their bids is inefficient. An agent with high bid may be subject to severe interference, and assigning channels to it with higher priority is potentially detrimental to social welfare.

Our solution improves upon such a naive greedy algorithm by normalizing an SN's bid by its degree of interference with other SNs, as shown in Algorithm 1. Such scaled *virtual bids* were adopted in recent literature [1], [5], which shows virtual bids can help achieve a good approximation ratio for the weighted independent set problem. Assume channels are indexed by $1, 2, \dots, |\mathcal{C}|$. We first compute the shortest path for each agent as its end-to-end path. Let $I_s(i)$ be the set of SNs that interfere with i along the path, including i itself. We define the virtual bid of SN i as

$$\phi(i) = \frac{b_i}{|I_s(i)|} \quad (7)$$

Then we greedily assign available channels along the paths to each link that maximizes the path valuation, according to a non-increasing order of virtual bids $\phi(i)$.

Algorithm 1 A greedy truthful auction — channel allocation.

1. **Input:** Set of channels \mathcal{C} , all the compound bids $\mathfrak{B}_i = (G^i(\mathcal{E}^i, \mathcal{V}^i), s^i, d^i, b_i)$, conflict graph $H(\mathcal{E}_H, \mathcal{V}_H)$
 2. **for all** $i \in \mathcal{N}$ **do**
 3. $I_s(i) \leftarrow \{i\}$;
 4. Compute the shortest path P^i from s^i to d^i ;
 5. **for all** $i \in \mathcal{N}$ **do**
 6. **for all** l_{uv}^i **along path** P^i **do**
 7. $x(c, l_{uv}^i) \leftarrow 0 \quad \forall c \in \mathcal{C}$;
 8. **if** $(l_{uv}^i, l_{pq}^j) \in \mathcal{E}_H$ **then**
 9. $I_s(i) \leftarrow I_s(i) \cup \{j\}$;
 10. $\phi(i) \leftarrow \frac{b_i}{|I_s(i)|}$;
 11. $\text{Win}(i) \leftarrow \text{TRUE}$;
 12. **for** $i \in \mathcal{N}$ **in non-increasing order of** $\phi(i)$ **do**
 13. **for all** l_{uv}^i **along path** P^i **do**
 14. Let $\mathcal{T}_{uv}^i \leftarrow \mathcal{C}$;
 15. **for all** $c \in \mathcal{T}_{uv}^i$ **do**
 16. **if** $x(c, l_{pq}^j) = 1$ **with** $(l_{uv}^i, l_{pq}^j) \in \mathcal{E}_H, \forall p, q$ **then**
 17. $\mathcal{T}_{uv}^i \leftarrow \mathcal{T}_{uv}^i \setminus \{c\}$;
 18. **if** $\mathcal{T}_{uv}^i = \emptyset$ **then**
 19. $\text{Win}(i) \leftarrow \text{FALSE}$;
 20. **if** $\text{Win}(i) = \text{TRUE}$ **then**
 21. **for all** l_{uv}^i **along path** P^i **do**
 22. Assign channel c_m in \mathcal{T}_{uv}^i that maximizes valuation of P^i so far under $\gamma_i(\cdot)$;
 23. $x(c_m, l_{uv}^i) \leftarrow 1$;
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Fig. 2 shows an example to illustrate the channel assignment procedure. There are four SNs, a, b, c and d , where

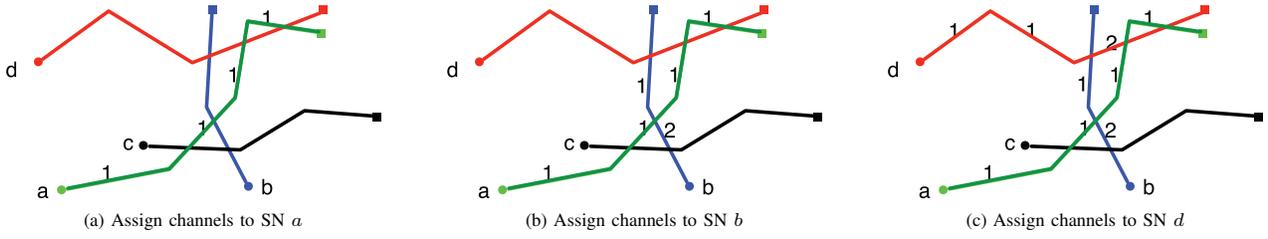


Fig. 2: Procedure of channel assignment. Dots and squares represent source and destination nodes respectively.

$\phi(a) > \phi(b) > \phi(c) > \phi(d)$. Two channels are available for allocation. In the figure, two intersecting links also interfere with each other, and cannot be allocated with the same channel if they belong to different SNs. The algorithm first assigns Channel 1 to SN a . As a result, it cannot assign Channel 1 to the first link of SN b , which receives Channel 2 instead, as shown in Fig. 2b, leaving SN c without a channel — it is impossible to assign either channel to c 's first link. However, SN d wins, and receives a channel assignment along its path without introducing interference to a or b .

We now prove that Algorithm 1 is monotonic:

Lemma 1. *In Algorithm 1, the probability of bidder i with bid b_i winning the auction is non-decreasing in b_i , and critical bids for winning agents exist.*

Proof: Bidding higher can only increase an agent's virtual bid, and therefore increase its rank in Algorithm 1. Hence, the probability of assigning a channel to the agent is non-decreasing. Besides, Algorithm 1 is deterministic, so a critical bid b_i^* exists for a winning bidder i , such that i always wins if it bids $b_i \geq b_i^*$. ■

B. Payment Calculation

Algorithm 2 computes payments for the winning agents. The payment scheme design is where we ensure the truthfulness of an auction. Algorithm 2 aims to find a critical bidder with critical bid b_i^* for a winning agent, such that i is guaranteed to win as long as i 's virtual bid $\phi(i) \geq \phi^*(i)$. Here $\phi^*(i) = \frac{b_i^*}{|I_s(i)|}$ is the critical virtual bid for i . If b_i^* is independent from b_i , then charging agent i b_i^* will ensure that the auction is truthful, which we will argue formally later.

Algorithm 2 first clears a winning agent i 's bid, and hence its virtual bid, to 0. Then Algorithm 1 is run, based on $(0, \mathbf{b}_{-i})$. In Algorithm 1, an agent i loses only if a link along its shortest path is unable to receive any channel. In that case there must exist at least one link along its shortest path whose neighbouring links (neighbouring vertices in the conflict graph) have used all the channels. From all the agents that block links of agent i , we identify an agent j with the minimum virtual bid, set it as i 's critical bidder, and compute i 's payment. We claim that $\phi(i) \geq \phi(j)$, because otherwise agent i would not be a winning agent among agents in $I_s(i) \cup \{i\}$. Agent i 's payment can be computed as:

$$p(i) = \phi^*(i)|I_s(i)| = \phi(j)|I_s(i)| \quad (8)$$

For the example in Fig. 2, we first set SN a 's bid to 0, and run Algorithm 1 based on the new bid vector. After assigning channels to agent c , we find that there are no available channels for the second link of agent a . Hence, agent c becomes the

critical bidder of agent a , which leads to a 's payment $p(a) = \phi(c)|I_s(a)|$. The rule applies to the other two winning agents b and d as well, where $p(b) = \phi(c)|I_s(b)|$ and $p(d) = 0$.

Algorithm 2 A greedy truthful auction — payment calculation.

1. **Input:** Set of channels \mathcal{C} , all the compound bids $\mathfrak{B}_i = (G^i(\mathcal{E}^i, \mathcal{V}^i), s^i, d^i, b_i)$, conflict graph $H(\mathcal{E}_H, \mathcal{V}_H)$, all the routing paths P^i and channel assignment from Algorithm 1.
 2. **for** $i \in \mathcal{N}$ in non-increasing order of $\phi(i)$ **do**
 3. $p(i) \leftarrow 0$;
 4. **if** $\text{Win}(i) = 1$ **then**
 5. Set $b'_i \leftarrow 0$;
 6. Run Algorithm 1 on (b'_i, \mathbf{b}_{-i}) ;
 7. **if** $\text{Win}(i) = \text{FALSE}$ **then**
 8. Let $\phi^*(i) \leftarrow +\infty$;
 9. **for all** l_{uv}^i along path P^i **do**
 10. Let $\mathcal{T}_{uv}^i \leftarrow \mathcal{C}$;
 11. **for all** $c \in \mathcal{T}_{uv}^i$ **do**
 12. **if** $x(c, l_{pq}^j) = 1$ with $(l_{uv}^i, l_{pq}^j) \in \mathcal{E}_H$ **then**
 13. $\mathcal{T}_{uv}^i \leftarrow \mathcal{T}_{uv}^i \setminus \{c\}$;
 14. **if** $\mathcal{T}_{uv}^i = \emptyset$ **then**
 15. $A \leftarrow \{j | (l_{uv}^i, l_{pq}^j) \in \mathcal{E}_H, \forall p, q; \text{Win}(j) = \text{TRUE}\}$;
 16. $\phi^*(i) \leftarrow \min(\phi^*(i), \min_{j \in A} \phi(j))$;
 17. $p(i) \leftarrow \phi^*(i) \times |I_s(i)|$;
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Theorem 2. *The auction in Algorithms 1 and 2 is individually rational and truthful.*

Proof: Assume agent i wins by bidding b_i , and let j be the critical bidder of i . Then we have $\phi(i) \geq \phi(j)$, so $p(i) = \phi(j)|I_s(i)| \leq \phi(i)|I_s(i)| = b_i$.

Furthermore, Algorithms 1 is monotone, and the allocation is binary (0 or 1). In this case, the critical-value based payments computed by Algorithm 2 matches the payments described in Theorem 1. Therefore, following Theorem 1, we can claim that the greedy auction is truthful. ■

V. A RANDOMIZED AUCTION FRAMEWORK

The greedy auction in Sec. IV, while simple and truthful, attempts to maximize social welfare in a heuristic manner, without providing any guarantee. We next design a randomized auction framework that translates any (cooperative) solution to the social welfare maximization problem, where truthful bids from SNs are given for free, to a randomized auction that is truthful in expectation. A risk-neutral bidder who is rational will bid truthfully under a randomized auction that

is truthful in expectation. The most attractive property of the framework is that the resulting auction can guarantee the same approximation ratio on social welfare as the cooperative solution does. We first present the framework and analyze its properties in Sec. V-A, then present in Sec. V-B its main enabling technique that is an LP-duality based solution decomposition, and discuss necessary plug-in algorithm modules utilized in Sec. V-C.

The randomized auction framework achieves truthfulness by employing a fractional VCG auction mechanism. The approximate social welfare guarantee is enabled by the LP-based decomposition technique due to Lavi and Swamy [7] and Carr and Vempala [8], which can help us decompose a fractional routing-channel assignment solution into a convex combination of integral solutions with guaranteed approximate expected social welfare.

A. The Randomized Auction Framework

As shown in Algorithm 3, the randomized auction framework contains the following key steps. First, we run the fractional VCG mechanism, and obtain fractional VCG routing and channel allocation solutions for each SN, as well as their corresponding VCG payments. Second, we apply the LP duality based decomposition technique to be detailed in Sec. V-B for decomposing the fractional VCG solution into a weighted combination of integral solutions, each with its associated probability. Finally, we randomly choose an integral solution from the combination, with weights taken as probabilities, as the result of the auction, and scale down VCG payments by a factor of Λ to be payments requested from the SNs. Here Λ is the gap between an integral algorithm A and the optimal fractional solution to the social welfare maximization problem in (6).

Algorithm 3 A Randomized Auction Framework

1. **Input:** Set of channels \mathcal{C} , all the compound bids $\mathfrak{B}_i = (G^i(\mathcal{E}^i, \mathcal{V}^i), s^i, d^i, b_i)$, conflict graph $H(\mathcal{E}_H, \mathcal{V}_H)$.
 - 2.
 3. Run fractional VCG auction on input, obtain solution (f^*, x^*) and payments p^F ;
 4. — apply plug-in algorithm A for solving LPR
 - 5.
 6. Decompose (f^*, x^*) into weighted integral solutions $\{(f(l), x(l), \rho(l))\}$;
 7. — LP duality based decomposition
 8. — employs plug-in algo B with integrality gap Λ
 9. — guarantees: $\sum_l (f(l), x(l))\rho(l) = \frac{1}{\Lambda}(f^*, x^*)$
 - 10.
 11. **Output:** Routing and channel assignment, and payments/prices:
 12. — select each $(f(l), x(l))$ with probability $\rho(l)$;
 13. — set prices $p = \frac{1}{\Lambda}p^F$;
-

The Fractional VCG Auction. The VCG mechanism is well known for providing truthfulness when the underlying social welfare maximization problem can be solved to optimal. We

first assume the existence of a plug-in algorithm A, which can solve the fractional version of (6) (the LPR) to optimal. Let (f^*, x^*) denote the optimal solution computed by algorithm A, which is the outcome of the fractional VCG auction and contains a fractional routing and channel assignment solution for the SNs. Let (f', x') be the optimal fractional solution to (6) when SN_i bids zero. The the VCG payment for each SN_i is computed as the *eternality* exposed by SN_i on other SNs' aggregated utilities :

$$p^F(i) = \sum_{i' \neq i} \gamma_{i'}(f', x') - \sum_{i' \neq i} \gamma_{i'}(f^*, x^*) \quad (9)$$

Properties of the Auction Framework.

Theorem 3. *The randomized auction framework defined in Algorithm 3 is truthful in expectation, and achieves at least $\frac{1}{\Lambda}$ of optimal social welfare.*

Proof: The property that $\sum_l (f(l), x(l))\rho(l) = \frac{1}{\Lambda}(f^*, x^*)$ is indeed a strong one, which coupled with the framework defined in Algorithm 3 guarantees the correctness of the theorem. Essentially, Algorithm 3 successfully outputs an integral solution, while scaling down both social welfare and SN payments by a factor of Λ from those of the fractional VCG auction. In particular, the expected utility of SN_i in Algorithm 3 is:

$$\begin{aligned} \sum_l \gamma_i(f_i(l), x(l))\rho(l) &\geq \gamma_i\left(\sum_l (f_i(l), x(l))\rho(l)\right) \\ &\geq \gamma_i(f_i^*, x^*) \end{aligned} \quad (10)$$

In the above derivations, the first inequality is due to the linear or sub-linear property of the utility function γ_i , and the second inequality follows from $\sum_l (f(l), x(l))\rho(l) = \frac{1}{\Lambda}(f^*, x^*)$ and the fact that γ_i is non-decreasing.

Now that we know Algorithm 3 scales down the fractional VCG payment p^F by exactly a factor of Λ , and at the same time scales down each SN 's utility by at most a factor of Λ , we can derive the individual rationality, truthfulness in expectation from the corresponding properties of the fractional VCG mechanism. Given truthfulness, one may safely assume that SNs will place truthful bids, and the $\frac{1}{\Lambda}$ -approximate social welfare guarantee follows. ■

Limitations of the Framework. Applying the randomized auction in Algorithm 3, one can successfully transfers a cooperative algorithm that provides an approximate guarantee on the social welfare into a mechanism that deals with selfish SNs while still providing the same approximate guarantee. This strong result does not come without assumptions and limitations, which we outline here for the reader's reference. Three assumptions are critical for the framework in Algorithm 3 to work properly. First, there is an efficient algorithm A that solves the LPR of (6) to optimal. Second, the utility function γ_i is linear or sub-linear, and is non-decreasing. Third, there is an efficient algorithm that computes an integral solution for (6), providing a guaranteed upper-bound on the integrality gap. The first and the third requirements are further

discussed in Sec. V-C. The approximation on social welfare guaranteed by Algorithm 3 is limited by that of the cooperative approximation algorithm B. The former is loose whenever the latter is, *e.g.*, in scenarios where a complicated wireless interference model is adopted. However, this is inevitable since an auction mechanism dealing with selfish agents can never outperform a cooperative algorithm that assumes truthful bids for free.

B. LP Duality Based Solution Decomposition

The LPR of (6) allows the integer variables f_{uv}^i , $x(c, l_{uv}^i)$ to take fractional values in $[0, 1]$. Let $S(\gamma)$ to denote the objective function of the LPR under input valuation function vector γ , and let (f^*, x^*) be the optimal solution of the LPR, which also contains agents' winning/losing information. Assume the existence of an algorithm B that verifies an integrality gap of Λ for (6) and the LPR, we now show how the LP duality technique due to Carr and Vempala [8] and Lavi and Swamy [7] from theoretical computer science for decomposing the fractional solution (f^*, x^*) into a weighted combination of integral solutions of polynomial size. That is, we will have $\rho(l)$ values such that $\frac{1}{\Lambda}(f^*, x^*) = \sum_{l \in \mathcal{I}} \rho(l)(f(l), x(l))$, where $\mathbb{Z}(\mathcal{F}\mathcal{X}) = \{(f(l), x(l))\}_{l \in \mathcal{I}}$ is the set of all integer solutions, \mathcal{I} is its index set, $\rho(l) \geq 0$, $\sum_{l \in \mathcal{I}} \rho(l) = 1$, and $\mathcal{F}\mathcal{X}$ denotes the feasibility region of the LPR. The integrality gap is then:

$$IG_{\mathcal{F}\mathcal{X}} = \sup_{\gamma} \frac{\max_{(f,x) \in \mathcal{F}\mathcal{X}} \sum_i \gamma_i(f, x)}{\max_{l \in \mathcal{I}} \sum_i \gamma_i(f(l), x(l))} \quad (11)$$

The crux of the decomposition technique is to compute $\rho(l)$ values that satisfies $\frac{1}{\Lambda}(f^*, x^*) = \sum_{l \in \mathcal{I}} \rho(l)(f(l), x(l))$, where $\rho(l) \geq 0$, $\sum_{l \in \mathcal{I}} \rho(l) = 1$. Then one can view this convex combination as specifying a probability distribution over the integer solutions, where a solution $(f(l), x(l))$ is selected with probability $\rho(l)$. Such a vector ρ can be computed through solving the following pair of primal-dual LPs. For each primal/dual constraint, we list its corresponding dual/primal variable for ease of reference.

<Decomposition LP - primal:>

$$\text{minimize } \sum_{l \in \mathcal{I}} \rho(l) \quad (12)$$

subject to:

$$\begin{aligned} \sum_{l \in \mathcal{I}} \rho(l) \Psi_i(l) &= \frac{1}{\Lambda} \Psi_i^* & \forall i \in \mathcal{N} & \longleftrightarrow \eta^i \\ \sum_{l \in \mathcal{I}} \rho(l) &\geq 1 & & \longleftrightarrow z \\ \rho(l) &\geq 0 & \forall l \in \mathcal{I} & \end{aligned}$$

The primal decomposition LP (12) has an exponential number of variables and takes exponential time to solve with a simplex or interior-point algorithm. The way we get around with this is to consider the dual LP (13), and apply the Ellipsoid algorithm together with a separation oracle (the plug-in algorithm A that verifies an integrality gap of Λ for any input valuation function vector γ of (6)) for identifying a polynomial sized set of dual constraints that is equivalent to

the original set. This indicates a corresponding polynomial sized set of primal variables (candidate integral solutions $(f(l), x(l))$) for consideration in the primal LP, which can then be solved using standard LP solution methods such as the simplex method or the interior point method.

<Decomposition LP - dual:>

$$\text{maximize } \frac{1}{\Lambda} \sum_{i \in \mathcal{N}} \eta^i \Psi_i^* + \lambda \quad (13)$$

subject to:

$$\begin{aligned} \sum_{i \in \mathcal{N}} \eta^i \Psi_i(l) + \lambda &\leq 1 & \forall l \in \mathcal{I} & \longleftrightarrow \rho(l) \\ \lambda &\geq 0 \\ \eta^i &\text{ unconstrained} & \forall i \in \mathcal{N} & \end{aligned}$$

The dual variable η^i can be viewed as a linear scaling factor that scales a valuation function $\gamma_i(\cdot)$ into $\gamma'_i(\cdot) = \eta^i \gamma_i(\cdot)$. In particular, if $\gamma_i(\cdot)$ is linear or sub-linear as we assumed, so is the linearly scaled function $\gamma'_i(\cdot)$. A potential problem is that the η^i values could be negative, leading to negative valuation functions $\gamma'_i(\cdot)$, whereas the plug-in algorithm A is only for non-negative valuations. However, one can instead use A with the non-negative valuations $\gamma(\cdot)^{(+)}$ given by $\gamma(\cdot)^{i(+)} = \max(\gamma(\cdot)^i, 0)$, and this yields a separation oracle [7].

Claim 1. Let $\gamma = \{\gamma^i(\cdot)\}_{i \in \mathcal{N}}$ be any vector of valuation functions. $\gamma(\cdot)^{i(+)} = \max(\gamma(\cdot)^i, 0)$. Given any integer solution $(\tilde{\mathbf{f}}, \tilde{\mathbf{x}}) \in \mathbb{Z}(\mathcal{F}\mathcal{X})$, one can obtain $(\mathbf{f}(l), \mathbf{x}(l)) \in \mathbb{Z}(\mathcal{F}\mathcal{X})$ such that $\sum_{i \in \mathcal{N}} \gamma^i(f^i(l), x^i(l)) = \sum_{i \in \mathcal{N}} \gamma^{i(+)}(\tilde{f}^i, \tilde{x}^i)$.

Proof: We first exploit the packing property. That is, if $\mathbf{a}_1 \in \mathbb{Z}(\mathcal{F}\mathcal{X})$ and $\mathbf{a}_2 \leq \mathbf{a}_1$ is integral then $\mathbf{a}_2 \in \mathbb{Z}(\mathcal{F}\mathcal{X})$. Now we set $(f^i(l), x^i(l)) = (\tilde{f}^i, \tilde{x}^i)$ if $\gamma^i(\cdot) \geq 0$ and 0 otherwise. Clearly, $\sum_{i \in \mathcal{N}} \gamma^i(f^i(l), x^i(l)) = \sum_{i \in \mathcal{N}} \gamma^{i(+)}(\tilde{f}^i, \tilde{x}^i)$. Since $(f^i(l), x^i(l)) \leq (\tilde{f}^i, \tilde{x}^i)$ is integral, by the packing property $(\mathbf{f}(l), \mathbf{x}(l)) \in \mathbb{Z}(\mathcal{F}\mathcal{X})$. ■

Now we are ready to show the following lemma:

Lemma 2. An optimal solution ρ^* to LP (12) satisfies $\sum_{l \in \mathcal{I}} \rho^*(l) = 1$.

Proof: We show that the optimal value of (13) is 1, and hence the lemma follows by strong LP duality. If we simply set $\lambda = 1$, $\eta^i = 0$ for all $i \in \mathcal{N}$, it provides a feasible solution with value 1. We then prove that the optimal value is at most 1 by way of contradiction. Let $(\eta^{(*)}, \lambda^{(*)})$ denote the optimal solution to (13). Assume, by way of contradiction, that $\frac{1}{\Lambda} \sum_{i \in \mathcal{N}} \eta^{i(*)} \Psi_i^{(*)} + \lambda^{(*)} > 1$. Using Algorithm A and Claim 1, we can compute a social welfare maximizing feasible solution $(\mathbf{f}(l), \mathbf{x}(l))$, such that

$$\sum_{i \in \mathcal{N}} \gamma^i f_{ds}^i(l) \geq \frac{1}{\Lambda} \sum_{i \in \mathcal{N}} \gamma'_i(f^{i(*)}, x^{i(*)}) \quad (14)$$

which leads to:

$$\sum_{i \in \mathcal{N}} \eta^{i(*)} \Psi_i(l) \geq \frac{1}{\Lambda} \sum_{i \in \mathcal{N}} \eta^{i(*)} \Psi_i^{(*)} \quad (15)$$

The inequality above further implies:

$$\sum_{i \in \mathcal{N}} \eta^{i(*)} \Psi^{i(*)} + \lambda > 1 \quad (16)$$

which violates the first constraint in the dual decomposition LP (13), and hence contradicts the feasibility of $(\boldsymbol{\eta}^{(*)}, \lambda^{(*)})$ ■

The above lemma shows that without being more restrictive, the inequality $\frac{1}{\lambda} \sum_{i \in \mathcal{N}} \eta^i \Psi^{i(*)} + \lambda \geq 1$ can be added to the dual (13). We will run the ellipsoid method to solve this dual LP. The first set of inequalities of (13) will be the violated inequalities returned by the separation oracle during the execution of the ellipsoid method. The separation oracle is, at a point $(\boldsymbol{\eta}, \lambda)$, if $\frac{1}{\lambda} \sum_{i \in \mathcal{N}} \eta^i \Psi^{i(*)} + \lambda > 1$, then we can use Algorithm A and Claim 1 to find an $(\mathbf{f}(l), \mathbf{x}(l))$ for which the constraints of (13) is violated; otherwise, we use the half space $\frac{1}{\lambda} \sum_{i \in \mathcal{N}} \eta^i \Psi^{i(*)} + \lambda \geq 1$ to cut the current ellipsoid. Since the ellipsoid method is guaranteed to take at most a polynomial number of steps, it will return a set of solutions $\{\mathbf{f}_{ds}(l)\}_{l \in \mathcal{I}}$ that is polynomial in size. Then we can plug back these solutions to (12), leading to a linear program with a polynomial number of variables and constraints, which is solved to recover $\rho(l)$'s that sum to 1.

C. Plug-in Algorithms for The Auction Framework

The randomized auction framework we presented in Algorithm 3 is intended to be a general framework where different versions for algorithms A and B can be plugged in, for solving the LPR and approximately solving (6), respectively. The resulting randomized auction is always truthful in expectation, regardless of detailed design choices within algorithms A and B. The eventual social welfare guarantee matches the approximation ratio of algorithm B. We next discuss the possibilities of algorithms A and B under different network settings, as well as their limitations in terms of handling wireless interference from practice.

Plug-in Algorithm A: Solving the LPR

At the high level, Algorithm 3 essentially attempts to scale down a fractional VCG mechanism. In order to obtain the fractional VCG mechanism first, we need to solve the LPR of (6) to optimal, using some plug-in algorithm A.

The valuation function $\gamma_i(\cdot)$ is assumed to be linear or sub-linear. In the linear case, the LPR is a normal linear program, which can be solved using any general LP solution technique, including the simplex method and the interior-point method. When $\gamma_i(\cdot)$ is sub-linear but not linear, then the problem becomes maximizing a convex function over a convex set, which unfortunately does not have general solution algorithms that run in polynomial time. The best solution will be problem specific, and may or may not run in polynomial time. When the network connection desired by an SN_i is not data-intensive, the corresponding valuation function $\gamma_i(\cdot)$ does not depend on intra-SN interference and can be modeled as the end-to-end throughput of the path scaled linearly by a constant weight w_i , making $\gamma_i(\cdot)$ linear. When the connection is data-intensive, SN_i wishes to take intra-SN interference into consideration when evaluation its end-to-end path. We point out that all

natural models for such interference should satisfy the sub-linear property of $\gamma_i(\cdot)$, as defined below:

$$\begin{cases} \text{positive homogeneity: } \gamma_i(c \cdot (f, x)) = c \cdot \gamma_i(f, x), \forall c \geq 0 \\ \text{subadditivity: } \gamma_i((f, x) + (f', x')) \leq \gamma_i(f, x) + \gamma_i(f', x') \end{cases}$$

In particular, for the first requirement, the constant c can be viewed as a scaling factor for the time fraction of (f, x) . The valuation should scale linearly with the time fraction a path is active and hence the first requirement is satisfied. For the second requirement, note that given two different solutions to the LPR, (f, x) and (f', x') , $\gamma_i((f, x) + (f', x')) = \gamma_i(f, x) + \gamma_i(f', x')$ when (f, x) and (f', x') are entirely interference free under the interference model of choice, and $\gamma_i((f, x) + (f', x')) < \gamma_i(f, x) + \gamma_i(f', x')$ otherwise. Hence the second requirement is also satisfied.

Plug-in Algorithm B: Approximately Solving (6)

The randomized auction framework in Algorithm 3 requires an efficient algorithm that approximately solves the optimization problem in (6), which is the classic multi-hop multi-channel wireless routing problem that has witnessed a plethora of studies during the past decade (e.g., [22]). The best design of such a joint routing-channel assignment algorithm is a research problem of its own right and is beyond the scope of this paper. The framework in Algorithm 3 can work with any such approximation algorithm plugged in. Essentially, our framework allows an auction mechanism designer to be worry-free on truthfulness and focus on the approximation algorithm design (in the cooperative paradigm). For the sake of completeness of the framework, below we assume the greedy LP-rounding Algorithm B that successively picks interference-free integral solutions from the LPR solution, and show its performance bound .

In the LPR, fractional channel allocation is directly related to link flows, which can be viewed as the fraction of time a specific link is active. Similarly, we can turn constraint (2) into the following *Link Scheduling Constraint*, for any given channel

$$f_{uv}^i + \sum_{l_{pq}^j: (l_{pq}^j, l_{uv}^i) \in \mathcal{E}_H} f_{pq}^j \leq 1. \quad (17)$$

The following lemma exploits the intuitive result that for a path, the number of neighboring paths that are pairwise interference-free is upper-bounded by a linear function of L , the number of hops of that path.

Lemma 3. *For any channel $c \in \mathcal{C}$ and an SN i with a path that has L hops, there are at most $g(L)$ interference-free SNs among $I_s(i)$, where $g(L)$ is a linear function of L .*

Proof: Consider the circumference area formed by the L -hop path of SN i , by taking the union of the interference disk of each node along the path. For an SN j that interfere with i , the closest node u_j in its path lies in this area. Furthermore, if we scale the circumference by a factor of 2, then the disk centered at u_j with radius equal to the interference range lies entirely within the scaled circumference. Any pair of interference-free SN neighbors must have their disks disjoint from each other. The number of such disks we can “pack”

in the scaled circumference is upper-bounded by the ratio of the scaled circumference area and the interference disk area, which in turn is upper-bounded by a linear function of L . ■

Theorem 4. Assume that a SN's path is at most L_{\max} -hops. Then the integrality gap between the IP (6) and the LPR is at most $\Lambda = g(L_{\max}) + 1$.

Proof: For an SN i and a single channel c , we know from Lemma 3 that there are at most $g(L)$ interference free SNs among $I_s(i)$. In the worst case, the integral solution picks only one SN from at most $g(L) + 1$ SNs. Since this is true for any SN and $g(L)$ is an increasing function of L , the lemma follows for a single channel case.

If there are $|\mathcal{C}|$ channels, for an SN i , we can imagine that the maximum independent set of a link is duplicated into $|\mathcal{C}| - 1$ copies, so that the integral solution will pick SNs from less than $(|\mathcal{C}| - 1)g(L)L + (|\mathcal{C}| - 1)L + 1 = (|\mathcal{C}| - 1)(g(L) + 1)L$ SNs. Since the integral solution picks at least $(|\mathcal{C}| - 1)L + 1$ SNs (picks i , and $|\mathcal{C}| - 1$ SNs per link along i 's path), the integrality gap is at most

$$\begin{aligned} & \frac{(|\mathcal{C}| - 1)(g(L_{\max}) + 1)L_{\max} + 1}{(|\mathcal{C}| - 1)L_{\max} + 1} \\ & \leq \frac{(|\mathcal{C}| - 1)(g(L_{\max}) + 1)L_{\max}}{(|\mathcal{C}| - 1)L_{\max}} \\ & \leq g(L_{\max}, \Delta) + 1 \end{aligned} \quad (18)$$

The greedy Algorithm B modifies a fractional flow to 1 among $i \cup I_s(i)$, and “verifies” an integrality gap of Λ of (6), leading to a social welfare approximation factor of Λ for the resulting randomized auction. If a more sophisticated Algorithm B is designed, guaranteeing a better integrality gap, the social welfare approximation ratio of the resulting randomized auction improves accordingly.

VI. SIMULATION RESULTS

In this section, we present simulation results for evaluating our auctions. Since our randomized auction is performance-guaranteed, we will mainly focus on the heuristic auction. For each SN, we randomly distribute a number of nodes in a 1×1 region. Two nodes are connected if their Euclidean distance is at most 0.05. The largest connected component is used as the connected graph for the corresponding SN. All bids are taken from a uniform distribution over the range $[40, 100]$. All data are averaged over 100 simulation.

A. Auction Efficiency

We assume the non-data intensive scenario and let the valuation function $\gamma_i(f, x) = w_i f_{ds}^i$, i.e., end-to-end throughput of SN_i 's path scaled by a constant weight w_i . Since the auction has already been proven to be truthful and the optimal social welfare is hard to obtain, we evaluate its performance in terms of auction efficiency, which reflects the portion of SN demands that are satisfied, weighted by w :

$$\vartheta = \frac{\sum_{i \in \mathcal{N}} w_i f_{ds}^i}{\sum_{i \in \mathcal{N}} w_i} \quad (19)$$

We then vary the number of channels in the simulations to study the performance of the auction.

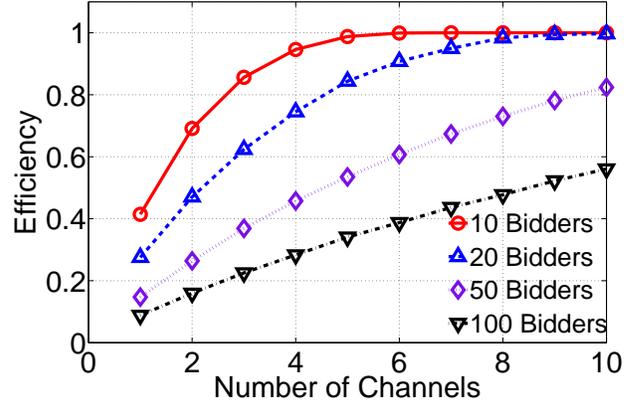


Fig. 3: Auction efficiency with different numbers of bidders enrolled.

We observe that, in general, as the number of channels increases, the auction efficiency increases as well, which verifies the intuition that the more channels, the higher probability for a bidder to win. First we change the number of bidders (SNs), while fixing $\Delta = 4$ and the number of nodes for each SN at 300. From Fig. 3, we can see that our auction in general effectively exploits the increasing number of channels available. Even in the extremely interfering case where there are 100 SNs in the region, the efficiency increases approximately linearly with the number of channels.

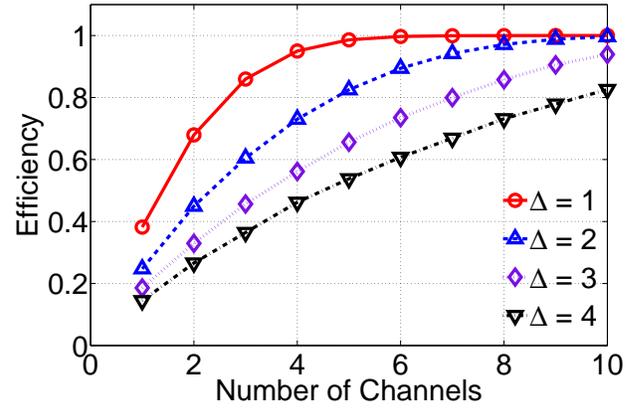


Fig. 4: Auction efficiency under different interference situations.

Fig. 4 (with 50 bidders and 300 nodes for each SN) and Fig. 5 (with 50 bidders and $\Delta = 4$) also show the performance of our auction in terms of the severity of interference, by changing Δ and the size of SNs. We can see that the change of Δ 's does not hurt the performance too much. However, large sizes of SNs may increase interference significantly, thereby decreasing the auction efficiency, where the connected graph for an SN with 500 nodes distributed can contain more than 150 nodes.

We then compare the performance of our auction with two other approaches. One is a greedy auction that only assigns one channel to an SN, in which the same channel cannot be

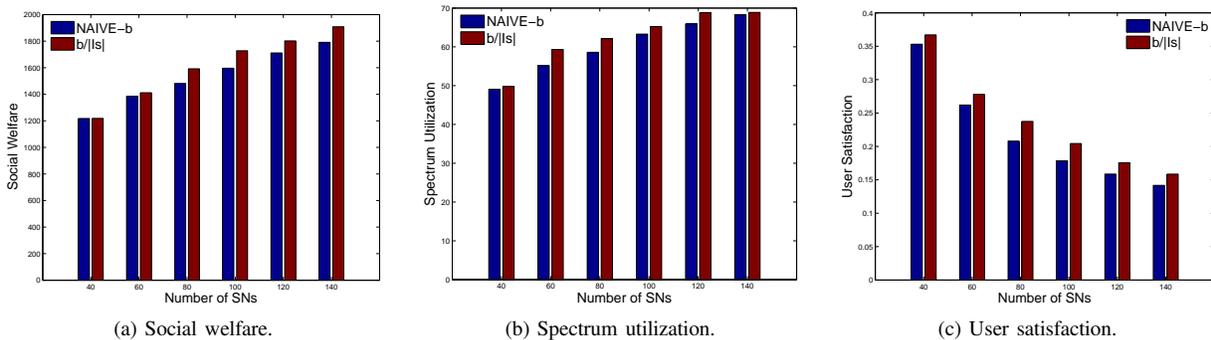


Fig. 7: Comparing our heuristic auction with the NAIVE-b auction, under different evaluation metrics.

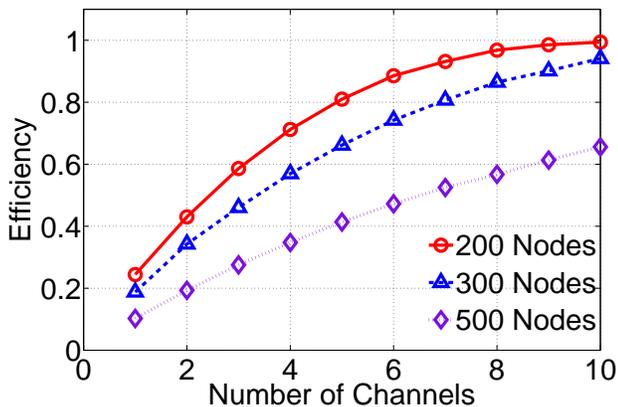


Fig. 5: Auction efficiency with different sizes of networks.

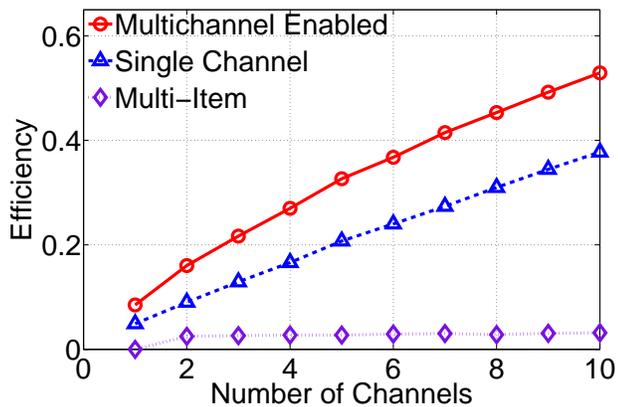


Fig. 6: Comparison of three different auction settings.

assigned to SNs who interfere with one another. The other one is a multi-item auction that greedily assigns channels to links in each SN, without global vision of forming an end-to-end path. We fix $\Delta = 4$, the number of potential nodes for each SN is 450, and the number of bidders is 50. We can see from Fig. 6 that our auction and the single-channel auction perform much better than the multi-item auction. Another observation is that the efficiency of our auction increases faster than the single-channel one as the number of channels increases. This justifies the use of multichannel assignment for each SN.

B. Comparison with the Naive Greedy Auction

In Sec. VI-A, we have shown the performance of our heuristic auction in terms of auction efficiency, under different settings. Now we will compare our heuristic auction to the naive greedy auction discussed in Sec. IV-A, where bidders are allocated channels according to a simple ranking of their bids. We denote this auction as NAIVE- b , and ours as $b/|I_s|$. We consider the following three performance metrics.

- *Spectrum Utilization*: The total number of channels successfully allocated to winning agents.
- *User Satisfaction*: The ratio of winning agents.
- *Social Welfare*: The sum of utilities of the auction participants.

We compare the two auctions under different deterministic, random and clustered topologies.

Fig. 7a - Fig. 7c compare NAIVE- b and our heuristic auction in terms of social welfare, spectrum utilization and user satisfaction, in random topologies where one channel is auctioned. The nodes of each SN are uniformly distributed in an 1×1 square. We can observe from the above results that, by taking interference into consideration, our auction outperforms the naive greedy one in all three metrics. However, the observed performance differences here are not large, since given the random nature in which we distributed the nodes and selected the SN communication terminals, the degree of interference are rather centered for different SNs. The normalized virtual bid technique only makes a difference at a small selected set of SNs.

C. The Effect of Intra-SN Interference

We now consider intra-SN interference, and evaluate two versions of the heuristic auction, one selects end-to-end path without considering intra-SN at all, one tailored for the end-to-end path utility function that evaluates a path without neighboring links sharing a common channel to 1.0, and to 0.5 otherwise. An SN without a path has utility 0. Efficiency here is taken as the ratio of aggregated SN utility over the number of SNs in the system.

As shown in Table I, the efficiency of auctions are generally lower, as compared to those seen in Sec. VI-A, since some paths that are evaluated to 1 in the previous simulations are now evaluated to 0.5 due to intra-SN interference. The degradation is more severe for the non-adapted version of the

TABLE I: Efficiency of adapted, non-adapted heuristic auction, and of NAIVE-b

$ \mathcal{C} $	2	4	6	8	10
adapted	0.138	0.225	0.306	0.412	0.497
non-adapted	0.124	0.195	0.254	0.309	0.348
NAIVE-b	0.131	0.191	0.242	0.298	0.335

heuristic auction, often more than half of whose selected paths are subject to intra-SN interference.

D. Illustration for the Randomized Auction Performance

Fig. 8 shows the distribution of the fractional solution to the LPR for one simulation instance (with 50 bidders, 300 nodes for each SN, one channel and $\Delta = 4$), where the sum of all flows equals 5.67. The agents with relatively large non-zero flows in the solution are shown in Table II. We can see that there is only one agent guaranteed to win and most of them almost always lose (the amount of flows is approximately 0). In the worst case, our randomized auction will select agents

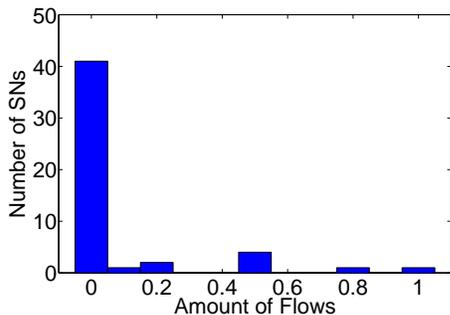


Fig. 8: A histogram of fractional solutions of the LPR.

with flow amount 1, 0.85, 0.54, and one of agents with flow amount 0.5, and another agent with fractional flow. Note that two agents with flow amount larger than 0.5 must not interfere with each other. Hence, in this experiment, algorithm A can actually achieve $\frac{1}{1.13}$ of the solution to the LPR, hence raising the social welfare with our randomized auction.

TABLE II: Agents with non-zero fractional flows

Agent	2	5	9	12	21	31	35	47
Flow	0.54	0.15	0.46	0.85	0.15	1	0.5	0.5

VII. CONCLUSION

Secondary spectrum auctions are emerging as a promising approach to efficiently distributing and sharing scarce wireless spectrum. For the first time in the literature, we propose the concept of a *secondary network*, relaxing the over-simplifying assumption on secondary users in existing research. We designed two auctions for spectrum allocation among SNs. The first is a simple, greedy style deterministic auction that heuristically maximizes social welfare. The heuristic auction is truthful due to its monotone allocation rule. The second is a randomized, linear optimization based auction that is

not only truthful (in expectation), but also provides proven guarantees on social welfare. For future work, one may further improve the performance guarantee of the randomized auction, by proving a tighter bound on social welfare approximation, and extend it to handle intra-SN interference as well as inter-SN interference.

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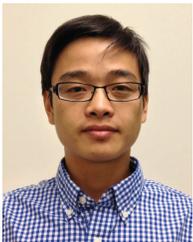
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