Optimization Models for Streaming in Multihop Wireless Networks

Zongpeng Li
Department of Computer Science
University of Calgary
zongpeng@cpsc.ucalgary.ca

Baochun Li, Mea Wang
Department of Electrical and Computer Engineering
University of Toronto
{bli, mea}@eecg.toronto.edu

***Abstract*— Wireless spectrum is a scare resource, while media streaming usually requires high end-to-end bandwidth. Media streaming in wireless ad hoc networks is therefore a particularly challenging problem, especially for the case of streaming to multiple receivers. In this paper, we design linear optimization models for computing a high-bandwidth routing strategy for media multicast in wireless networks, which targets near-optimal throughput, given constraints including network topology, radio capacity, and link contention. We study both the directional antenna and omni-directional antenna cases and point out their connections. We also combine the classic forward error correction techniques with the novel network coding techniques to provide error control in a timely fashion. Simulation results show that our solutions indeed achieve high streaming rates, and prompt error recovery under a wide range of link failure patterns.**

Keywords: Media streaming, Network coding, Multicast, Ad hoc networks, Directional antenna.

I. Introduction

Ad hoc networks are multi-hop wireless networks that may self-organize into the functioning mode without the support of a wireline infrastructure. Due to its flexibility and ease of deployment, vast opportunities in both military and civilian applications are expected [1], [2], [3]. Compared to wireline networks, wireless networks provide relatively low bandwidth. For example, Gupta and Kumar [4] show that the available end-to-end bandwidth for a certain pair of nodes may approach zero, as the network size grows. Peraki and Servetto [5] further show that the introduction of directional antennas does not change the pessimistic picture. On the other hand, the transmission of live media streams often requires high data rates. Therefore, media streaming in wireless ad hoc networks becomes a particularly challenging research problem. The situation is further exacerbated by the unreliable nature and high error rates of wireless communication links.

In this paper, we study the problem of media multicast in ad hoc networks with either directional or omnidirectional antennas. Compared with omni-directional antennas, which introduces a more complex interference model [6], [7], directional antenna essentially eliminates local interference by having the sender node forming a beam towards the intended receiver only [8], [5]. Even without interference, the maximum-rate multicast topology problem is still in general NP-hard, with traditional multi-path routing and data replication techniques considered only [9]. However, it has been recently realized that the novel network coding techniques [10], [11] may dramatically reduce the computational complexity of this problem, by transforming it into linear network optimization [12], [9].

We propose to utilize network coding in our media streaming scheme. The advantages of applying network coding in wireless media multicast are three-fold. First, each node in a wireless ad hoc network is usually a fully functional computer, as opposed to routers and switches in the IP core network. Therefore network coding support is naturally feasible. Second, as just mentioned, considering coded multicast leads to efficient algorithms for computing the maximum-rate transmission scheme. Third, as we will show later in the paper, combining network coding with forward error correction (FEC) provides high robustness and prompt error-recovery, which is of particular interests to multimedia applications. Both network coding and FEC codes (*e.g.*, Tornado codes [13] and Reed-Solomon codes [14]) employ symbol-wise linear coding operations over finite fields such as $GF(2^8)$ or $GF(2^{16})$, therefore they may co-exist in our solution in harmony.

We consider three models of ad hoc networks: (M1) the single directional antenna case, (M2) the multiple directional antenna (antenna array) case, and (M3) the omni-directional antenna case. (M1) and (M2) correspond to *simple directional transmissions* and *complex directional transmissions* in Peraki and Servetto [5], respectively. In (M1), each node is equipped with one directional antenna, shared among transmissions to and from all of its neighbors. In (M2), each node is equipped with multiple directional antennas, such that one is dedicated to communication with each neighbor. In (M3), each node is equipped with an omni-directional antenna, which can broadcast to all neighbors within a radium of r. In all cases, we show how the maximum-rate routing topology can be computed as a linear network optimization problem, with a node-centric linear program provided for (M1) and (M3), and a link-centric linear program provided for (M2). We apply subgradient optimization methods to derive distributed solutions for these linear programs. The solution we obtain is a multicast topology with a flow rate specified between each pair of nodes, which is ready to serve a coded multicast session.

Finally, we adopt the simple, light-overhead randomized code assignment algorithm [15] to determine the content of information flows being transmitted between each pair of nodes. We combine FEC coding at the source node and network coding at each relay node in the multicast routing topology. The result is a coded multicast strategy that achieves very high level of error-resilience, as verified by both theoretical analysis [11] and our simulation results. Furthermore, error recovery does not involve sender-receiver message exchanges, and does not introduce extra overhead at the receiver beyond regular decoding operations necessary in any network coding scheme. Therefore, the entire multicast scheme achieves high throughput, high robustness, as well as prompt error handling, and constitutes an ideal solution for media multicast in ad hoc networks, which are characterized by low link capacity and high error rates.

The remainder of the paper is organized as follows. We discuss related work in Sec. II, present optimization models for computing the maximum-rate multicast topology in Sec. III. We then provide details on randomized code assignment and robust data transmission in Sec. IV. Sec. V concludes the paper.

II. Related Work

Recent research in information theory discovers that, routing alone is not sufficient to achieve maximum information transmission rate across a data network [10], [11]. Rather, applying encoding and decoding operations at relay nodes in addition to the sender and receivers is in general necessary in an optimal transmission strategy. Coding at intermediate relay nodes is referred to as *network coding*. The coding process uses *linear codes* in the Galois field, and includes two basic operations: the $+$ and $\cdot$ operations in the Galois field GF(2^k). Since elements in a Galois field have a fixed-length representation, bytes in flows do not increase in length after being encoded.

The pioneering work of Ahlswede *et al.* [10] and Koetter *et al.* [11] proves that, in a directed network with network coding support, a multicast rate is feasible if and only if it is feasible for a unicast from the sender to each receiver independently. Li *et al.* [16] prove that linear codes usually suffice in achieving the maximum transmission rate. They also provide the first code assignment algorithm, which performs exponentially many linear independence tests. Sanders *et al.* [17] improve their result by providing a polynomial time algorithm for exact code assignment. Ho *et al.* [15] then point out that randomized code assignment, in which each node generates outgoing flows by linearly combining its incoming flows with random coefficients, actually constitutes another attractive solution. The probability of conflict due to randomly chosen code coefficients is negligibly small, with mild assumptions on code length and network configurations.

Recent work [18], [19] study the problem of computing maximum unicast transmission rates, in wireless ad hoc networks with known topologies. Both our work and [18], [19] allow multi-path routing for higher transmission rates. The focus in [18], [19] is on unicast transmission with omni-directional antennas, where the main challenge is scheduling of local wireless transmissions. We consider multicast transmission with directional antennas, where the main challenge is on computing the optimal multicast topology.

Traditionally, with only data replication and multi-path routing considered, the maximum multicast rate problem is known to be equivalent to the steiner tree packing problem, and is therefore NP-hard to solve [20], [21], [9]. We showed that [21], [12], taking the unique encodable property of information flows into consideration can dramatically reduce the complexity of the problem, which can now be solved as linear network optimization. We provide [21] linear programming formulations of the maximum transmission rate problem for various wireline communication scenarios, and provide [12] an efficient, distributed solution to the single multicast session case where each link is undirected and each node has network coding capability. In this paper, we show the results can be extended into wireless networks.

Guo *et al.* [22] propose to apply network coding in overlay media streaming. Their work is similar to ours in that both abandon the traditional multicast tree approach for multicast routing, and adopt network coding to improve transmission rates. A major difference between their work and ours is that, we take a fixed wireless network topology as input, and target near-optimal multicast rate, while in Guo *et al.* [22] the optimality of the achieved rate is not a design goal and is not studied.

Directional antenna has recently attracted a considerable amount of research interests in the wireless networking community [5], [8], [23], [24]. Compared with omni-directional antennas, directional antennas may direct energy radiation towards the intended receiver only. This leads to drastically reduced local interference, as well as larger communication ranges. In this paper, we design effective media multicast strategies for ad hoc networks, based on the nice interference model of directional antennas.

As a special case of multiple description codes that are used for robust, adaptive multimedia streaming, FEC is a sender based error recovery technique where encoded redundancy is transmitted together with original data. The study of FEC codes (*a.k.a.* source erasure codes) dates back to the seminal work of Reed and Solomon [14], and has enjoyed ubiquitous application in areas including bulk data storage, satellite communication, reliable multicast, and wireless transmission. The power of FEC codes can be summarized into the following fact: an original data item with n equal-sized uncoded blocks can be coded into $m > n$ coded blocks, such that any n of the m coded blocks are sufficient to recover the original data item.

III. Computing high-bandwidth streaming topology: LP Models

In this section, we present linear programming (LP) models that compute the maximum-rate multicast topology, which will be used in Sec. IV to construct the coded multicast streaming scheme. We also discuss how these LPs can be effectively solved.

We assume that each node is equipped with one or multiple *ideal* antennas, which may be used for communication between a pair of nodes within a certain distance from each other; in the directional antenna cases, local transmissions are subject to no interferences with each other. We model the logical topology of the ad hoc network as an undirected network $G = (V, E)$. In the single antenna cases, $C \in Q_+^V$

denotes the radio capacity available at each node, where Q_+ is the set of non-negative rational numbers. In the antenna array case, $C \in Q_+^E$ denotes the available capacity of each wireless link, which is the smaller radio capacity at the two radios forming the link. The set $A = \{\vec{uv}, \vec{vu}\ | uv \in E\}$ denotes the set of directed links that are possible to be utilized in a routing topology. The media source is denoted as S, and destination nodes are denoted as $T_1, \ldots, T_k$.

A. M1: The single directional antenna scenario

We first consider the case where each node has one directional antenna equipped only, which is shared among transmissions to and from each of its neighbors. In this scenario, capacity allocation is node-centric. The linear program that maximizes the multicast rate is given below, based on the fact that, a coded multicast rate is feasible in a directed network, if and only if it is feasible for a unicast from the sender to each receiver independently [10], [11].

Maximize χ
Subject to:

$$\begin{cases} \chi \le f_i(\vec{T_iS}) & \forall i & (1) \\ \sum_{v\in N(u)}(f(\vec{uv}) + f(\vec{vu})) \le C(u) & \forall u & (2) \\ \sum_{v\in N(u)}(f_i(\vec{uv}) - f_i(\vec{vu})) = 0 & \forall i, \forall u & (3) \\ f_i(\vec{uv}) \le f(\vec{uv}) & \forall i, \forall\ \vec{uv} \neq \vec{T_iS} & (4) \end{cases}$$

$$f(\vec{uv}), f_i(\vec{uv}), \chi \ge 0 \qquad \forall i, \forall\ \vec{uv}$$

In the above LP, variable χ is the achievable multicast rate, and is also the objective function being maximized. $N(u) = \{v | uv \in E\}$ denotes the set of neighbor nodes of u. Each $f_i \in Q_+^A$ is a network flow from sender S to receiver T_i. We have added virtual links $\vec{T_iS}$ with infinite capacity, in order to ease the concise presentation of the linear program. We now require flow balance at every node, including sender S and receivers T_i (3). Consequently, flow rate on each virtual link, $f(\vec{T_iS})$, represents the network flow rate from S to T_i. The overall throughput of the media streaming session, χ, is upper-bounded by each of these individual network flow rates (1). The total capacity required on a directed link $\vec{uv}$ is the maximum network flow through it (4), and total capacities of links incident to a node should not exceed the radio capacity at that node (2).

The dual of the above LP is given below, where we have variables representing node prices (x) and the objective is to minimize aggregated node capacity-price product. We discuss an efficient and distributed solution algorithm based on the dual LP in the Appendix.

Minimize $\sum_u C(u)x(u)$
Subject to:

$$\begin{cases} x(u) + x(v) \ge \sum_i y_i(\vec{uv}) & \forall\ \vec{uv} & (5) \\ y_i(\vec{uv}) + p_i(v) \ge p_i(u) & \forall i, \forall\ \vec{uv} \neq \vec{T_iS} & (6) \\ p_i(T_i) - p_i(S) \ge z_i & \forall i & (7) \\ \sum_i z_i \ge 1 & & (8) \end{cases}$$

$$x(uv), y_i(\vec{uv}), z_i \ge 0 \qquad \forall i, \forall\ \vec{uv}$$

B. M2: The antenna array scenario

Now assume each node is equipped with multiple directional antennas, such that one antenna is dedicated for transmissions between each neighbor and itself. Local wireless transmissions between directional antenna pairs do not interfere with each other, therefore all links in G may be active concurrently. We need only to guarantee that for each link uv, data rate of the $\vec{uv}$ transmission and data rate of the $\vec{vu}$ transmission together do not exceed the capacity of either radio connecting u and v. Therefore the wireless network may be modelled as an undirected, link-capacitied network. Consequently, we can formulate the maximum streaming rate problem as a linear optimization problem, given that network coding is supported at each node:

Maximize χ
Subject to:

$$\begin{cases} \chi \le f_i(\vec{T_iS}) & \forall i & (9) \\ f_i(\vec{uv}) \le c(\vec{uv}) & \forall i, \forall\ \vec{uv} \neq \vec{T_iS} & (10) \\ \sum_{v\in N(u)}(f_i(\vec{uv}) - f_i(\vec{vu})) = 0 & \forall i, \forall u & (11) \\ c(\vec{uv}) + c(\vec{vu}) \le C(uv) & \forall uv \neq T_iS & (12) \end{cases}$$

$$c(\vec{uv}), f_i(\vec{uv}), \chi \ge 0 \qquad \forall i, \forall\ \vec{uv}$$

It is interesting to note that the LP above is essentially the same as the cFlow LP for computing maximum multicast throughput in an undirected network [21]. $C(uv)$ denotes the total available bandwidth between nodes u and v, which is the smaller capacity of the two radios dedicated for transmissions between u and v. This capacity is further allocated for transmissions in both directions, and leads to directed link capacities $c(\vec{uv})$ and $c(\vec{vu})$. Similar to the LP for the single antenna case, f_i is a network flow from source S to receiver T_i, and each $\vec{T_iS}$ is a virtual link inserted for concise formulation of the linear program.

While the LP for M1 is node-centric, with variables representing node capacities and prices in the primal and dual respectively, the LP for M2 is link-centric with variables for link capacities and prices. The algorithm we present in the Appendix finds the optimal multicast flow f^* for M1 directly. A subgradient algorithm for M2 [12] will yield an optimal network routing instead, from which the optimal flow can be further computed by solving network flow problems.

C. M3: The omni-directional antenna scenario

Optimizing throughput in multihop wireless networks is in general hard due to the interference among nearby transmissions, which are broadcast in nature. An optimal schedule of node transmissions that avoids interference is equivalent to the graph coloring problem and is NP-hard. In order to obtain feasible solutions, we adopt here a simplified interference model, which is called the *primary interference model* or *node-exclusive spectrum sharing* in the literature [25], [26], [27]: at any given time point, a node can communicate with at most one neighbor only.

A unique feature of omni-directional antennas is their local broadcast nature. The same packet can be transmitted from a node to multiple neighbors in one broadcast. This feature is especially beneficial in multicast routing, and is known as *the wireless multicast advantage*. However, modelling the multicast advantage in optimal multicast routing is by no means straightforward [28]. Below we apply the *shadow node* technique from [28], which models local broadcasts using regular graph edges by inserting virtual broadcast nodes into the network, and show how the node-centric LP in the directed antenna case can be simply modified to optimize multicast throughput with omni-directional antennas.

TABLE I
NETWORK TRANSFORMATION UNDER PRIMARY INTERFERENCE

Input: Original network $G = (V, E)$ and $C \in Q_+^V$
Output: New network $G' = (V', E')$ and $C' \in Q_+^{E'}$
Initialize: $V' = \phi$, $E' = \phi$

$\forall u \in V$:
  $V' \leftarrow V' \cup \{u, u'\}$
  $C'(u) = C(u)$, $C'(u') = \infty$
  $E' \leftarrow E' \cup \{\overrightarrow{uu'}\}$
  $\forall\ \overrightarrow{uv} \in E$:
    $E' \leftarrow E' \cup \{\overrightarrow{u'v}\}$

As shown in Table I, for each node u in the original network, we create its *shadow* u' with infinite capacity, and redirect all edges emanating from u to from u'. An example is illustrated in Fig. 1.

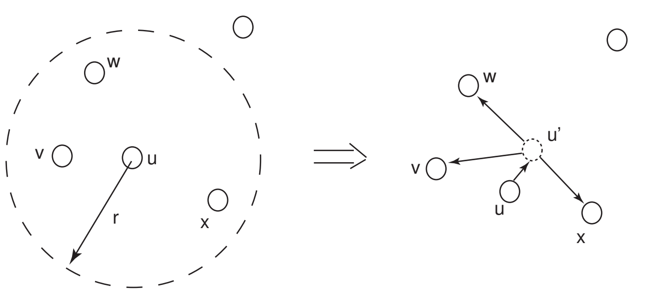

Fig. 1. Constructing the network topology for the cFlow LP under the primary interference model.

A local broadcast is achieved by a transmission from u to its shadow u' and then to all $u's$ neighbors. The entire procedure consumes only the amount of bandwidth required for one transmission, which is in line with the wireless multicast advantage. To further model the fact that a local broadcast consumes bandwidth at a neighbor even if it is not interested in receiving the packet, we require that the shadow u' relays exactly the same amount of flow to all neighbors as the rate it receives from u. The final linear program in the omnidirectional antenna case is then:

Maximize χ

Subject to:

$$
\begin{cases}
\chi \le f_i(\overrightarrow{T_i S}) & \forall i & (13)\\
\sum_{v\in N(u)} (f(\overrightarrow{uv}) + f(\overrightarrow{vu})) \le C'(u) & \forall u \in V' & (14)\\
\sum_{v\in N(u)} (f_i(\overrightarrow{uv}) - f_i(\overrightarrow{vu})) = 0 & \forall i, \forall u \in V' & (15)\\
f_i(\overrightarrow{uv}) \le f(\overrightarrow{uv}) & \forall i, \forall\ \overrightarrow{uv} \ne \overrightarrow{T_i S} & (16)\\
f(\overrightarrow{uu'}) = f(\overrightarrow{u'v}) & \forall u \in V, \forall v \in N(u) & (17)
\end{cases}
$$

$$f(\overrightarrow{uv}), f_i(\overrightarrow{uv}), \chi \ge 0 \qquad \forall i, \forall\ \overrightarrow{uv}$$

D. Discussions

The three LPs can be similarly solved using the subgradient method; details for the LP for M1 is given in the Appendix. Each step of the subgradient algorithm can be decomposed into computations performed at individual nodes, based on local information maintained. The main computation consists of solving a series of max-flow/min-cut problems, for which totally distributed algorithms exist, such as the push-relabel algorithm [29] or the ϵ-relaxation algorithm [30].

Fig 2 shows the convergence speed of the subgradient algorithm on random networks generated by BRITE [31], in with network sizes up to 1000 nodes, and with 2, 5, and 10 receivers, respectively. As we can see, the optimal solution is usually approached within 10 iterations, regardless of the network size or the multicast group size. However, the computation time for large networks or for large multicast groups are longer, due to the fact that the time taken by each single max-flow/min-cut computation is roughly proportional to $|V|^3$, and that the number of max-flow/min-cut computations in each iteration is proportional to the multicast group size.

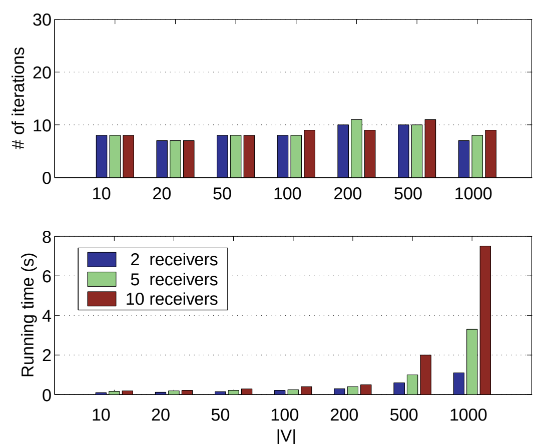

Fig. 2. Convergence speed in random networks.

In the antenna array scenario, the optimal solution of the linear program provides the exact maximum achievable multicast streaming rate. In the single directional antenna case, however, the optimal solution of the linear program may be slightly higher than the real achievable rate, if the multicast topology contains an odd cycle. This is introduced by difficulties in wireless link scheduling. Detailed discussions on a similar phenomenon about *odd holes* and *odd anti-holes* are provided in the work of Jain *et al.* [18] on unicast rate maximization in ad hoc networks. A close, rather than exact, upper-bound is attained in the single antenna case. The situation is similar for the case of omni-directional antennas since we considered primary interference only and ignored secondary interference. This does not introduce a serious problem to our solution, since in the coded multicast streaming, as will be discussed in Sec IV, we utilize only a portion of the achievable bandwidth for streaming original media flows, and target a near-optimal rate rather than an absolutely optimal one. Given the high network dynamics that are present in wireless ad hoc networks, it is impractical to target absolutely optimal performance anyway.

IV. Coded media streaming

The multicast topology computed by our LP models is in the form of combined network flows, rather than a collection of multicast trees. Correspondingly, it is not feasible to stream uncoded raw data along each network flow to the receivers. Network coding is required to handle bottleneck link sharing among different network flows, to make the multicast transmission feasible. Beside network coding, we also need to consider error correction codes, in order to achieve robust media streaming in the highly dynamic wireless settings with unreliable communication links. Below we compare FEC codes with other error-recovery techniques for media streaming, and conclude in favor of the former. We then describe in more details how to combine network coding and FEC coding into a unified coding framework.

There exist three categories of error recovery techniques for multimedia streaming: (a) sender&receiver-based techniques, (b) receiver-based techniques, and (c) sender-based techniques. In (a), each receiver sends explicit *ACK* and/or *NACK* messages back to the sender, which then re-transmits a missing packet to a requesting receiver. This approach has two major drawbacks. First, it does not scale well as the number of receivers grows, since the numerous *ACK* or *NACK* messages eventually flood links around the sender. Second, the round trip time introduced by the *ACK/NACK*-retransmission round is usually intolerable for multimedia applications. In (b), a receiver applies heuristics such as linear interpolation to substitute data not successfully received. Although it eliminates both problems in (a), the result of recovery is usually sub-optimal, and may lead to sensible distortion in the audio/visual output at the receiver. In (c), a small amount of redundant information is added by the sender, and then transmitted to the receivers along with the original data. A receiver may then recover its missed data from the redundant information received. Both layered coding and multiple description coding belong to this category.

We choose a special case of multiple description codes, FEC code, to provide error-resiliency in our media streaming solution. The reason is that FEC coding and network coding can be based on coding operations over the same finite field, and may consequently be unified into a common coding framework. We partition the source data into a number n of data streams, from which k FEC-coded redundancy streams are generated. Then these $n+k$ data streams are sent out from the sender S, and are further coded and relayed hop-by-hop until arriving at the receivers.

The study of Koetter *et al.* [11] shows that, such a coded transmission stream achieves the highest robustness possible. Their original theorem says that, if a multicast rate χ is feasible in a directed network G, let *F* be the set of link failure patterns such that χ is still feasible in G-*f*, for all $f \in F$, then there exists a common network coding scheme that achieves rate χ, and is resilient to any link failure pattern in *F*. In other words, there always exists a static coding scheme that may handle every link failure scenario possible to handle. Our inter-stream FEC coding, combined with randomized network coding, constitutes precisely such a coding scheme.

More specifically, after computing the maximum multicast rate χ^* in Sec. III, we use $(1-\alpha)\chi^*$ of the capacity to transmit raw data streams, while reserve $\alpha\chi^*$ of the capacity for coded redundancy. Here α is a small fractional number, and is adjustable depending on specific network conditions and application requirements. We normalize rates χ^*, $(1-\alpha)\chi^*$, and $\alpha\chi^*$ to $n+k$, n, and k number of information flows, respectively, with a unit flow rate of our choice. Then sender S sends out n original media flows, plus k coded flows, each of which is encoded from the n original flows using FEC codes, *i.e.*, is a linear combination of the n original flows. The effect is that any n flows out of the $n+k$ source flows are linearly independent, and can be used to recover the n original flows. Each relay node linearly combines its incoming streams to construct its outgoing streams in a random fashion. Each receiver may then recover the n original streams by decoding any n coded streams it receives.

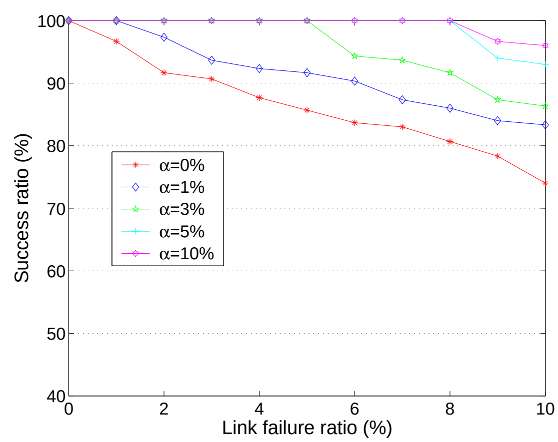

Fig. 3. Robustness of multicast streaming.

We tested the robustness of our coded transmission scheme in a network with 200 nodes, and 19 receivers in the multicast group. We randomly pick a certain percentage of links on the multicast streaming routes to fail, and compute how many flows can still arrive at each receiver. If the number is less than n, then that receiver will not be able to successfully recover the source data. In Fig. 3, x-axis corresponds to the percentage of active links that fail, and y-axis corresponds to the percentage of receivers that can still successfully recover all data streams. We can see that with 3% redundancy in source FEC coding, even if 5% of the currently utilized links fail, *all* receivers can still receive at least n independent coded data streams, and can successfully recover original data being multicast by the sender.

In our coded streaming scheme, each node on the multicast topology needs to perform encoding operations, and each receiver needs to perform decoding operations. Note that from the computation point of view, there is no essential difference between encoding and decoding operations. In the former, symbols from different incoming data streams form a vector, which is multiplied with the encoding matrix. In the latter, the original data streams are recovered by multiplying symbols from incoming flows at the receiver with a decoding matrix. Upon session setup, each node randomly generates its encoding matrix, which remains unchanged during the streaming session. The accumulated overall transformation matrix at the receiver is inversed to obtain the decoding matrix, which is also fixed throughout the streaming session.

TABLE II
CODING TIME OVER DIFFERENT PACKET SIZES.

packet size (KB)	1	5	10	15	20	25	30	35
computation time (μs)	18	90	177	263	353	440	532	619

In order to determine the latency introduced by the coding process on each node, we have completed an implementation of network coding over base field GF(2^8). To be more realistic, we place six parallel instances of the coding implementation in one AMD Opteron 2.4 GHz node, each coding two incoming streams. We then measure the average coding time for data packets of various sizes. The table above shows the results of our measurements. It takes 18 μs to code 1 KB packets, and around 0.5 ms with 30 KB packets. Such a light overhead should not be a serious issue in most scenarios. However, considering the fact that coding delay accumulates along each hop during the transmission, it may be desirable to adopt relatively smaller packet sizes for interactive media applications in networks with a large diameter.

V. CONCLUSIONS

In this paper, we propose solutions to overcome two difficulties for multicast media streaming in ad hoc networks, low bandwidth and high error rate, with either directional or omni-directional radio antennas. We design linear optimization models and subgradient algorithms for computing and achieving high bandwidth multicast routing in such settings. The algorithm is inspired by linear programming duality theory. However, it consists of mostly combinatorial computations, and is consequently very efficient, as verified by simulation studies.

We utilize the optimal multicast topology computed to stream coded data to the destinations, with a small number of data streams reserved for redundancy. By results in network coding research, such a coded transmission achieves the highest possible resiliency against link failures. Furthermore, data error at a receiver is handled by forward error correction, therefore it is possible to meet the stringent delay requirements of multimedia applications.

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Appendix

To design the subgradient algorithm for the LP in model M1, We relax constraints in (5), and introduce "prices" in vector f accordingly. The objective function with the penalty added, then becomes:

$$\begin{aligned} & \sum_u C(u)x(u) + \sum_{\vec{uv}} f(\vec{uv})(\sum_i y_i(\vec{uv}) - x(u) - x(v)) \\ = \quad & \sum_u (C(u) - \sum_{v\in N(u)}(f(\vec{uv}) + f(\vec{vu})))x(u) \\ & + \sum_{\vec{uv}} f(\vec{uv}) \sum_i y_i(\vec{uv}) \end{aligned}$$

Note that primal feasibility requires $\sum_{v\in N(u)}(f(\vec{uv})+f(\vec{vu})) \leq C(u)$, $\forall u$. Therefore we obtain the Lagrangian dual problem:

Maximize $\quad L(f)$
Subject to:

$$\begin{cases} \sum_{v\in N(u)}(f(\vec{uv}) + f(\vec{vu})) \leq C(u) & \forall u \\ f(\vec{uv}) \geq 0 & \forall\ \vec{uv} \end{cases}$$

where

$$L(f) = \text{Min}_P \sum_i \sum_{\vec{uv}} f(\vec{uv}) y_i(\vec{uv}) \qquad (A.1)$$

with P being the polytope:

$$P: \begin{cases} y_i(\vec{uv}) + p_i(v) \geq p_i(u) & \forall i, \forall\ \vec{uv} \neq \vec{T_i S} \\ p_i(T_i) - p_i(S) \geq z_i & \forall i \\ \sum_i z_i \geq 1 & \\ y_i(\vec{uv}), z_i \geq 0 & \forall i, \forall\ \vec{uv} \end{cases}$$

Lagrangian duality theory assures that, the objective function value in an optimal solution to the Lagrangian dual problem is equal to that in an optimal solution to the primal linear program, which models our maximum streaming rate problem. Therefore we may solve the primal LP by solving the above Lagrangian dual. We start by choosing a set of initial values for f. Any legal vectors that satisfy node capacity bounds are fine, although they may lead to different speeds of convergence. One feasible choice is to try to distribute radio capacity at each node u to all its incident links evenly, *i.e.*, set $f[0](\vec{uv}) = \min(\frac{C(u)}{2|N(u)|}, \frac{C(v)}{2|N(v)|})$, $\forall\ \vec{uv}$.

After initialization of f, the major part of the subgradient algorithm iteratively updates y and f, until f converges to its optimal value. To update y, we take current values in f fixed, and solve sub-problem (A.1). Note that, in an optimal solution of the original dual LP, we must have $\sum_i z_i = 1$, since complementary slackness conditions require $\chi(\sum_i z_i - 1) = 0$. Consequently, (A.1) can be decomposed into k min-cut computations. We can compute a min-cut from S to each T_i, taking $f[k]$ as the link capacity vector:

$$L(c) = \text{Min}_P \sum_i \sum_{\vec{uv}} f(\vec{uv}) y_i(\vec{uv}) = \text{Min}_i[\text{Min}_{P^i} \sum_{\vec{uv}} f(\vec{uv}) y(\vec{uv})]$$

where P^i is the standard min-cut polytope for the S-T_i cut:

$$P^i: \begin{cases} y(\vec{uv}) + p(v) \geq p(u) & \forall\ \vec{uv} \neq \vec{T_i S} \\ p(T_i) - p(S) \geq 1 & \\ y(\vec{uv}) \geq 0 & \forall\ \vec{uv} \end{cases}$$

Now let $y_i^* = \text{argmin}_{y\in P^i} \sum_{\vec{uv}} f[k](\vec{uv}) y(\vec{uv})$, and let $j = \text{argmin}_i \sum_{\vec{uv}} f[k](\vec{uv}) y_i^*(\vec{uv})$, we can update y by setting $y_j[k] = y_j^*$, and $y_i[k] = 0$, $\forall i \neq j$.

After new values in y are computed, vector f is then updated in two steps. We first compute a new vector f' according to a sequence of prescribed step sizes θ:

$$f' = f[k] + \theta[k] \sum_i y_i[k],$$

f' may not be feasible, and is then projected into the feasibility simplex, *e.g.*, the nearest feasible point:

$$f[k+1] = \text{argmin}_{f\geq 0, \sum_{v\in N(u)}(f(\vec{uv})+f(\vec{vu}))\leq C(u)} ||f - f'||.$$

After updating both y and f, the next iteration starts. Since the objective function and constraints in our original problem are all linear, it is guaranteed that choosing step sizes satisfying the following conditions will have the subgradient algorithm converge to the optimal solution:

$$\theta[k] \geq 0, \lim_{k\to\infty} \theta[k] = 0, \text{and} \sum_{k=1}^{\infty} \theta[k] = \infty. \qquad (A.2)$$

Finally, we observe that the vector f upon convergence, f^*, provides exactly the desired optimal multicast routing strategy.