Core-Selecting Secondary Spectrum Auctions

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***Abstract*—In a secondary spectrum market, the utility of a secondary user often depends on not only whether it wins, but also which channels it wins. Combinatorial auctions are a natural fit here to allow secondary users to bid for combinations of channels. In this context, the VCG mechanism constitutes a generic auction that uniquely guarantees both truthfulness and efficiency. There also exists related auction design that relaxes efficiency due to perceived complexity issues, and focuses on truthfulness. Starting with new empirical evidences on the complexity issue, we propose to design core-selecting auctions instead, which resolve VCG's vulnerability to collusion and shill bidding, and improve seller revenue. While the VCG type of auctions are unique in guaranteeing both efficiency and truthfulness, we prove that our core-selecting auctions are unique in guaranteeing both efficiency and shill-proofness, and always outperform VCG auctions in terms of seller revenue generated. Employing linear programming and quadratic programming techniques, we design two payment rules for minimizing the incentives of bidders to deviate from truth telling.**

***Index Terms*—Truthful Auctions, Secondary Spectrum Allocation, Secondary Networks, Linear Programming**

I. Introduction

With the continuing growth of new wireless technologies and applications, the demand for radio spectrum escalates at a fast pace. A spectrum scarcity problem ensues, due to the *status quo* static allocation in both temporal and spatial domains: large chunks of spectrum remain idle while non-licensed new users are unable to access them. Secondary leasing is envisioned as a panacea to mitigate the problem: a licensed user, or a *primary user* (PU), pools its idling spectrum chunks for sale to the new users, or *secondary users* (SUs), with monetary remuneration in return.

In such a secondary spectrum market, auctions are a natural mechanism for a PU to efficiently relinquish its unused channels to SUs [1], [2]. As illustrated in Fig. 1, the spectrum auctioneer (PU) elicits bids from SUs for channel access, computes a channel allocation in an interference-free manner and a corresponding payment vector for the SUs. An important goal in spectrum auction design is to achieve *efficiency*, *i.e.*, to maximize *social welfare*, the aggregated 'happiness' of everyone in the system. An efficient auction tends to allocate channels to SUs who value them the most. Another useful property is *truthfulness*. In a truthful auction, bidding its true valuation of a channel is a dominant strategy, and an SU has no incentive to lie.

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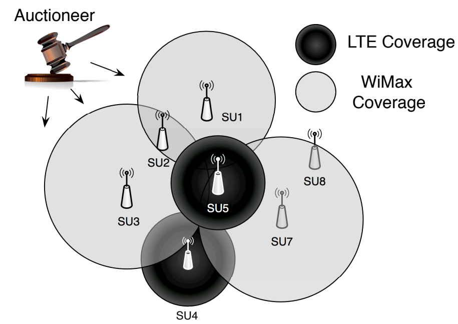


Fig. 1. An illustration of a secondary spectrum auction, with 7 SUs bid for paired (LTE) or unpaired (WiMax) channels. The PU acts as an auctioneer, computes (i) channel allocation results according to the bids and interference constraints of each channel, and (2) a corresponding payment vector for charging winning SUs.

Existing literature on secondary spectrum auctions often treats wireless channels as identical goods [1], [3]. Given the heterogeneity of channels and different technology requirements from real-world settings, secondary users are likely to desire combinations of channels in practice. For instance, the channels may experience different levels of fading at different locations, and two users may value the same channel quite differently. As another example, LTE and WiMAX require paired and unpaired channels respectively [4]. An SU aiming to provide an LTE-based service will be willing to bid for two paired channels, while a WiMax-based SU will not. If the PU holds multiple auctions to sell these heterogenous channels and paired channels, that usually undermines the efficiency in spectrum assignment [4]. Combinatorial auctions enable expressive bids for requesting bundles of channels, and are especially useful when the PU has no *a priori* information on how SUs plan to utilize the channels.

A classic auction that guarantees truthfulness is the celebrated VCG mechanism [5]–[7]. It is the only auction mechanism that is both truthful and efficient [8], [9]. Despite a myriad of interests in theoretical research, VCG mechanisms witness less enthusiasm in actual implementations [10]–[12]. Part of the hurdle was attributed to the requirement for solving the allocation problem to optimality, which is often NP-hard, as in interference-free channel allocation. This motivated the design of truthful spectrum auctions with a compromise in efficiency (social welfare) [1], [13]–[15]. However, our studies reveal that, given a representative secondary spectrum market, winner determination and channel allocation can be formulated into a linear integer program of modest size (on the order of 1000 variables and constraints), which can be solved to optimality in seconds over today's average computing platform. Sacrifices in efficiency are therefore less justified.

The VCG mechanism suffers from two other problems that are economic instead of computational. The first is that it turns to generate a low revenue for the auctioneer, under-exploiting the payment potential of bidders. The second is that a VCG mechanism is susceptible to a form of strategic bidding known as *shill bidding*, or *false-name bidding*, in which a single bidder desires a set of items impersonate multiple bidders, each bidding for a subset of those items [16]. For example, consider three SUs (SU1, SU2, SU3) bidding for two different channels $ch1$ and $ch2$, and they desire $\{ch1\}$, $\{ch2\}$ and $\{ch1, ch2\}$, respectively. Each SU is willing to pay \$10 for acquiring what it desires, and \$0 otherwise. The VCG mechanism allocates $ch1$ to SU1, and $ch2$ to SU2, with zero charges (because neither of them would cause any loss to social welfare by not participating in the auction, see Sec. V for details). The zero income is by no means satisfactory, given that each SU has expressed a willingness to pay up to \$10, manifesting the *low revenue problem.* Furthermore, assume that SU1 and SU2 are indeed controlled by a single SU0 who has a valuation of \$10 for $ch1$, \$10 for $ch2$, and \$20 for $\{ch1, ch2\}$. Knowing the rule of the auction, SU0 can reduce its payment for winning $\{ch1, ch2\}$ from \$10 (because SU3 would win the two channels if SU0 was ruled out, creating social welfare \$10, the payment for SU0 is \$20 − \$10) to \$0 via impersonation, manifesting the *shill bidding problem.*

The vulnerabilities of the VCG mechanism are so severe that it rarely made to a direct application in practice. Since VCG is the only truthful *and* efficient mechanism [8], [9], any other efficient auction aimed at addressing the two economic problems inherent in VCG will have to relax the requirement of absolute truthfulness. A promising direction of research is *core-selecting auctions* [17]–[19]. An auction outcome is *in-core* if no group of participants (including the auctioneer) are motivated to secede to settle for their own solution. Taking the group as the entire set of participants, this implies efficiency (social welfare is maximized). Given guaranteed efficiency, the auctioneer can further judiciously select from the core an auction (actually a payment rule, see Sec. V) that maximizes the likelihood of truthful bidding. Representing the state-of-the-art of a pragmatic combinatorial auction, core-selecting auctions recently enjoyed real-world applications, including spectrum auctions at the primary spectrum market level [4]. The primary and secondary spectrum markets differ fundamentally due to concerns on wireless interference — absent in the former and present in the latter.

In this paper, we first formulate a winner determination problem for the combinatorial spectrum auction to be applied in a secondary spectrum market. While previous research [1], [13], [15] concentrated on designing truthful auctions by relaxing efficiency, our design instead aims to guarantee efficiency for ensuring effective utilization of the scarce spectrum resources. We base our design on the emerging framework of core-selecting auctions, which is efficient by definition. Unlike previous core-selecting auctions in primary spectrum markets that view channels as commodities, our auctions take interference into consideration by assuming conflict graphs and enable frequency reuse. We prove that the core-selecting auctions we design are able to achieve a revenue that is at least on par with VCG mechanisms, and are robust against shills. The in-core property is proven to essentially forbid collusion, because bidders have no incentive to formulate coalitions. We further design tailored payment rules, using a pair of correlated linear program and quadratic program. These payment rules, beyond ensuring a core outcome, are proven to minimize the incentives of bidders to deviate from truthful bidding.

Extensive simulation studies were performed to examine the performance of our core-selecting auctions. Our results reveal that our auctions optimally guarantee social welfare efficiently on a platform with limited computing resources. Spectrum utilization can be largely improved with channel reuse. The core-selecting auctions achieve significantly higher revenues than the VCG mechanism. This is due to the fact that channels are reused geographically — every winner's payment is at least as large as its corresponding VCG price, which accumulates to a high revenue since channel reuse can accommodate more requests from bidders. The auctioneer, while maintaining optimal social welfare, can customize the ranking metric to achieve desired market outcomes, such as revenue versus bidder satisfaction.

In the rest of the paper, we discuss related work in Sec. II, and present preliminaries in Sec. III. In Sec. IV, we introduce and analyze the core-selecting auctions for a secondary spectrum market. In Sec. V, the payment rules of the VCG mechanism and of our core-selecting auctions are discussed. Simulation studies are presented in Sec. VI. Sec. VII concludes the paper.

II. Related Work

The unique auction that ensures both efficiency and truthfulness is the well-known VCG type of mechanisms due to Vickrey [5], Clarke [6] and Groves [7]. However, the VCG mechanism is rarely applied in practice directly, due to its low revenue and vulnerability to shills. Core-selecting auctions, originally proposed by Day and Milgrom [17], has attracted substantial attention in economics as a more robust and profitable alternative to the VCG mechanism. To minimize bidders' gain from deviating from truthful bidding, Day and Milgrom [19] propose a bidder-optimal pricing rule. A quadratic-core-pricing rule is presented by Day and Cramton [18] to make the core-selecting auction more robust. Recently, the necessary and sufficient conditions for the perfect Bayesian equilibrium of core-selecting auctions are characterized by Guler *et al.* [20]. The core-selecting auction mechanism further witnessed applications in the cloud computing market, for selling heterogeneous virtual machines [21].

Most existing research on spectrum auction design, starting from almost a decade ago, choose to relax the optimality in efficiency, and focus on a hard guarantee in truthfulness, *e.g.*, through ensuring bid independence in charges [14]. This is due to the challenge from wireless interference constraints, as highlighted by Huang *et al.* [22] and others. One of the first auctions in such a realm is VERITAS [1], which is based on a monotonic channel allocation rule. Zhou and Zheng propose TRUST [3], a truthful double auction with multiple sellers. Recently the first truthful privacy preserving spectrum auction mechanism, SPRING, is presented by Huang *et al.* [23]. Zhu *et al.* design auctions specifically for networked secondary users [13]. Chen *et al.* [24] improve uers' utilities while inducing only limited revenue loss by proposing a three-stage dynamic spectrum auction.

The aforementioned auctions suffer from a common drawback — channels are assumed to be identical and bidders are not allowed to bid for combinations of different channels, which may not be feasible for modern spectrum auction design [4]. Combinatorial auctions, instead, are expressive enough for SUs to bid for bundled items of interest. In this scope, Hoefer *et al.* propose a randomized combinatorial auction that is truthful in expectation, with a guaranteed performance bound on social welfare [15]. A recent solution due to Dong *et al.* employs a combinatorial auction as well [25], allowing bidders to have more flexible bids to require not only the channels, but also the time periods to use them. However, these auctions lose the optimality of efficiency as well, and their revenues are not guaranteed to outperform the VCG mechanism.

III. Network Model and Preliminaries

We start by introducing the settings of our problem. We consider a secondary spectrum market, where a primary user periodically pools the unused channels and conducts combinatorial auctions to lease them to secondary users in a round-by-round fashion. We focus on the design of a combinatorial auction for a specific round, to be repeated in different rounds. The system includes a set $\mathcal{K}$ of heterogeneous channels, and a set $\mathcal{N}$ of secondary users who are bidders in the auction. Bidders are geographically distributed in a region, each in possession of its base station. A *conflict graph* $\mathcal{G}_k(\mathcal{V}_k, \mathcal{E}_k)$ is defined for each channel $k \in \mathcal{K}$ and is known to the auctioneer. Broadcast stations and wireless access points are SUs, which are represented as nodes in a conflict graph. The wireless signal of the channel used by each SU covers a certain area, and two SUs i and j interfere with each other if they use the same channel k and $(i,j) \in \mathcal{E}_k$. Conflict graphs can be built using measurement-calibrated propagation models [26].

Each bidder $i \in \mathcal{N}$ has the flexibility to bid for bundles of channels. For each combination of channels $\mathcal{S} \subseteq \mathcal{K}$, $v_i(\mathcal{S})$ represents bidder i's valuation of $\mathcal{S}$. We adopt an *XOR bidding language* [19], *i.e.*, a bidder can submit as many bids as it wishes, but it can win a single bid only (a bidder's bids are *mutually exclusive*). Indicator variable $x_i(\mathcal{S})$ is 1 if bidder i wins bundle $\mathcal{S}$ in an auction and 0 otherwise. We assume each bidder i has a quasi-linear utility, defined as:

$$u_i = \begin{cases} v_i(\mathcal{S}) - p_i & \text{if agent } i \text{ wins the bundle } \mathcal{S}, \\ 0 & \text{otherwise.} \end{cases}$$

where p_i is the payment of bidder i if it wins bundle $\mathcal{S}$. Clearly, $v_i(\mathcal{S})$ is the maximum amount that i is willing to pay for $\mathcal{S}$. In a combinatorial spectrum auction, each bidder submits as many bids as it wishes, for channels or bundles of channels that it is interested in. Let $b_i(\mathcal{S})$ denote the bid submitted by bidder i for bundle $\mathcal{S}$. We denote by o the auctioneer and $u_o = \sum_{i \in \mathcal{N}} p_i$ represents the revenue of the auctioneer.

Bidders are assumed to be *individually rational* in that a bidder always prefers a higher utility. Consequently, a bidder is willing to participate in the auction only if it is guaranteed a nonnegative utility — the auctioneer cannot charge a bidder beyond its bid price.

$b_i(\mathcal{S})$	Bid submitted by SU i for bundle $\mathcal{S}$
$\mathcal{C}$	Coalition of SUs
Core($\mathcal{N}$)	Core with bidder set $\mathcal{N}$
$\mathcal{E}_k$	Edge set in conflict graph for channel k
$\mathcal{K}$	Set of heterogeneous channels
$\mathcal{N}$	Set of SUs
p_i	Payment of SU i
$\mathcal{S}$	Bundle of channels
$v_i(\mathcal{S})$	SU i's valuation of bundle $\mathcal{S}$
u_i	Quasi-linear utility of SU i
u_o	Revenue of the auctioneer
$\mathcal{W}$	Set of winning SUs
WD($\mathcal{N}$)	Winner determination problem with bidder set $\mathcal{N}$
$x_i(\mathcal{S})$	Indicator: SU i wins bundle $\mathcal{S}$ or not

After collecting all bids submitted, the auctioneer needs to compute both a channel allocation result and a corresponding payment vector. Unlike regular combinatorial auctions, in our settings, a channel can be awarded more than once in the region, but such channel reuse must be interference-free. The Winner Determination Problem (WDP) can be formulated accordingly:

$$\text{WD}(\mathcal{N}) = \max \sum_{i \in \mathcal{N}} \sum_{\mathcal{S} \subseteq \mathcal{K}} b_i(\mathcal{S}) x_i(\mathcal{S}) \tag{1}$$

subject to

$$\begin{aligned} \sum_{\mathcal{S} \subseteq \mathcal{K}} x_i(\mathcal{S}) \leq \quad & 1 \qquad \forall i \in \mathcal{N}; \\ x_i(k) + x_j(k) \leq \quad & 1 \qquad \forall (i,j) \in \mathcal{E}_k, \ \forall k \in \mathcal{K}; \\ x_i(\mathcal{S}) - x_i(k) \leq \quad & 0 \qquad \forall k \in \mathcal{S}, \forall \mathcal{S} \subseteq \mathcal{K}; \\ x_i(\mathcal{S}), \ x_i(k) \in \{0,1\} & \qquad \forall i \in \mathcal{N}, \ \forall \mathcal{S} \subseteq \mathcal{K}. \end{aligned}$$

In this WDP, the first set of constraints represent the employment of XOR bids, to make an individual bidder's bids mutually exclusive. The second set models interference of different channels, based on their conflict graphs. The third states that if a bidder wins a bundle $\mathcal{S}$ containing channel k, k is indeed allocated to it. The WDP is an integer programming problem, which is NP-hard. Despite its NP-hardness, the WDP can be solved in seconds on the order of a thousand variables and constraints on today's average computer.

As previously mentioned, an auction mechanism whose outcome optimally maximizes the total values in (1) is an *efficient* mechanism. Important notations are summarized in the table below for easy reference.

IV. Core Selection and Its Necessity

In this section, we define the concept of *the core*, and formally motivate the employment of core-selecting auctions in a secondary spectrum market.

A. The Core of An Auction

For a bidder i, we specify by $\mathcal{S}_i$ as the bundle allocated to i ($\mathcal{S}_i = \emptyset$ if i loses). An outcome is said to be *blocked* by coalition $\mathcal{C} \subseteq \mathcal{N}$ if there is some alternative outcome with awarded bundles $\{\mathcal{S}'_i\}_{i \in \mathcal{N}}$ and payment vector $\mathbf{p}'$, such that the corresponding $u'_i \geq u_i$ for all $i \in \mathcal{C}$, and for which $u'_o = \sum_{i \in \mathcal{N}} p'_i > u_o$. An outcome not *blocked* by any coalition is

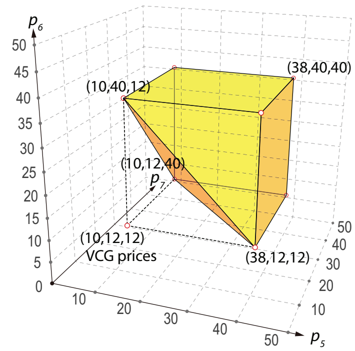


Fig. 2. A geometric illustration of the core.

in the core with respect to the submitted bids $\mathbf{b}$. It is worth noting that in our setting, the first price payment (pay what you bid) scheme is always in the core, and thus the core is always non-empty (in some other economical settings the core may not exist). Formally,

$$Core(\mathcal{N}) = \left\{\mathbf{u} \geq \mathbf{0} | \sum_{i\in\mathcal{N}\cup\{o\}} u_i = \mathrm{WD}(\mathcal{N}), \sum_{\substack{i\in\mathcal{C}\cup\{o\}\\ \forall\mathcal{C}\subseteq\mathcal{N}}} u_i \geq \mathrm{WD}(\mathcal{C})\right\} \quad (2)$$

For a simple example to illustrate the core of an auction, assume there are seven different SUs/bidders, $1, 2, 3, 4, 5, 6, 7$, bidding for three channels, A, B, C. All three channels share the same, complete conflict graph, so no channel reuse is allowed. The following bids are submitted:

$$b_1(A) = 10 \qquad b_2(B) = 12$$
$$b_3(C) = 12 \qquad b_4(A, B, C) = 62$$
$$b_5(A) = 38 \qquad b_6(B) = 40$$
$$b_7(C) = 40$$

Bidder 1 and bidder 5 are interested in channel A, bidder 2 and bidder 6 are interested in channel B, bidder 3 and bidder 7 are interested in channel C, and bidder 4 is interested in a combination of all the three channels, A, B and C.

It can be determined that the unique set of winners in an efficient allocation includes bidders 5, 6 and 7, generating a social welfare of 118 The core can be drawn in the payment space, shown in Fig. 2.

We note that, due to the simplicity of the example, the constraints defining the core are simply the bids of the losing bidders (together with the property of individual rationality of winning bidders). Therefore, each bid defines a half space of the payment space, the intersection of which formulates the core. In particular, since bidder 1 will always block if bidder 5 pays less than 10, we have the constraint $p_5 \geq 10$. Similarly, bidder 2 and bidder 3 dictate $p_6 \geq 12$ and $p_7 \geq 12$, respectively. Bidder 4 will block if all the winners, bidders 5, 6 and 7 together do not beat its bid on the channels they win, so we have $p_5 + p_6 + p_7 \geq 62$. Upper-bounds are given by winners' bids themselves, consistent with individual rationality.

B. Revenue Lower Bound of A Core-Selecting Auction

We have seen that the revenue from the VCG outcome can be too low to be acceptable to the auctioneer. How do we justify the use of a core-selecting auction in a secondary spectrum market, in terms of seller revenue, with consideration of shill bidding? In the following, we will treat the VCG revenue as a benchmark, and prove that a core-selecting auction always generates a higher revenue for the PU, through showing that it always leads to a total SU utility no higher than that in a VCG auction (recall that the total utility from both the PU and all SUs is constant, corresponding to an efficient channel allocation).

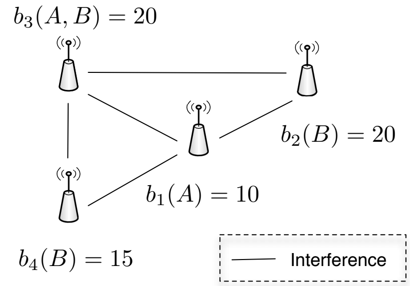


Fig. 3. Four SUs bidding for 2 channels.

We start by proving the following theorem.

Theorem 1. *In an efficient secondary spectrum auction with a WDP shown by (1), if there are bidders acting as shills, we have $WD(\mathcal{N}) \geq WD'(\mathcal{N})$, where $WD'(\mathcal{N})$ is the solution to the WDP when shills are treated as a coalition submitting a merged reported value.*

Proof: When $\mathcal{C} \subseteq \mathcal{N}$ is a coalition using shills, we treat these shills as a single bidder and take its merged report. Then the conflict graphs in our auction can be changed. That is, these bidders can be viewed as a single node in a conflict graph and the conflict relationship would change accordingly. Fig. 3 shows a small example. There are four bidders, 1, 2, 3 and 4 bidding for two channels A and B, with reported values shown in the figure. Assume that channels A and B share the same conflict graph. By solving the WDP of this example, we have $WD(\mathcal{N}) = 45$, with the result of allocating channel A to bidder 1, and channel B to bidders 2 and 3.

However, if bidders 1 and 2 are bidding as shills, they are a coalition hoping to obtain both A and B. In this case, we can view the conflict graph as shown in Fig. 4, in which bidder 2 and bidder 4 will also conflict because bidder 1 interferes with bidder 4. Note that $b_1(A)$ and $b_2(B)$ are merged to $b_c(A, B)$, with a value 30. The solution to the WDP of it will change to $WD'(\mathcal{N}) = 30$, by allocating both channels A and B to the coalition formed by bidders 1 and 2.

Therefore, if we treat a coalition as a single node, we will have more edges in the conflict graphs since extra edges are added to preserve the property of interference (removing the edges among shills does not change the result because their reported values are merged). As a result, the number of constraints in the WDP will increase. Then we have,

$$\mathrm{WD}'(\mathcal{N}) \leq \mathrm{WD}(\mathcal{N}) \quad (3)$$

where $\mathrm{WD}'(\mathcal{N})$ is the solution of the WDP if we treat the coalition $\mathcal{C}$ as a single bidder and take a merged report. Due to the potential additional constraints introduced into the WDP, it cannot be more than the original solution. ∎

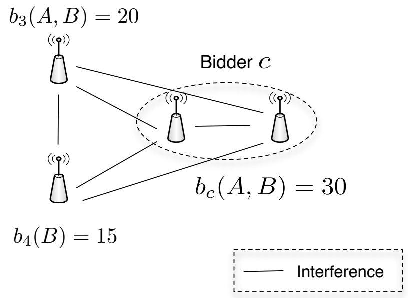


Fig. 4. Three SUs bidding for 2 channels, where bidder c is using shills. An edge between bidders 2 and 4 is introduced.

We are now ready to prove that in our secondary spectrum market, core-selecting auctions essentially form *the only type* of auctions that are robust against shill bidding.

Theorem 2. *An efficient secondary spectrum auction with a WDP shown by (1) has the following property: no bidder can gain more than its VCG utility by bidding with shills if and only if the auction is a core-selecting auction.*

Proof: Fix a set of players, including the set of bidders $\mathcal{N}$ and the auctioneer o. The core-selecting condition requires that these bidders cannot earn more utility than what they achieved in a VCG mechanism if they were to submit a merged report.

Then in a VCG mechanism, when $\mathcal{C} \subseteq \mathcal{N}$ is a coalition using shills, the marginal utility acquired by coalition $\mathcal{C}$ is,

$$\mathrm{WD}'(\mathcal{N}) - \mathrm{WD}'(\mathcal{N}\backslash\mathcal{C}) = \mathrm{WD}'(\mathcal{N}) - \mathrm{WD}(\mathcal{N}\backslash\mathcal{C}) \quad (4)$$

Our restriction is therefore

$$\sum_{i\in\mathcal{C}} u_i \leq \mathrm{WD}'(\mathcal{N}) - \mathrm{WD}(\mathcal{N}\backslash\mathcal{C}) \quad (5)$$

By Theorem 1, (5) can be further written as,

$$\sum_{i\in\mathcal{C}} u_i \leq \mathrm{WD}(\mathcal{N}) - \mathrm{WD}(\mathcal{N}\backslash\mathcal{C}) \quad (6)$$

This condition holds if and only if

$$\sum_{i\in(\mathcal{N}\backslash\mathcal{C})\cup\{o\}} u_i = \mathrm{WD}(\mathcal{N}) - \sum_{i\in\mathcal{C}} u_i \geq \mathrm{WD}(\mathcal{N}\backslash\mathcal{C}) \quad (7)$$

Since $\mathcal{C}$ is an arbitrary coalition of bidders, we have that for any coalition $\mathcal{T} = \mathcal{N}\backslash\mathcal{C}$, $\sum_{i\in\mathcal{T}\cup\{o\}} u_i \geq \mathrm{WD}(\mathcal{T})$, which means there is no blocking coalition in the auction. Combining this with efficiency, we derive $\mathbf{u} \in Core(\mathcal{N})$. ■

Since we assume a quasilinear utility function as in conventional auction theory, we have the following corollary.

Corollary 1. *In our secondary spectrum market, an efficient auction that selects a core outcome generates a revenue no less than that of the VCG mechanism.*

So far we have assumed that bids are truthful. While core-selecting auctions relax absolute truthfulness, we study how to maximize SUs' intention of truthful bidding in Sec. V.

V. Payment Rules

In this section, we discuss the payment rules that can be employed in efficient auctions, including the VCG auction (V-A) and two core-selecting auctions that we design — revenue-minimizing (V-B) and closest-to-VCG (V-C).

A. Payment Rule of the VCG Mechanism

The VCG mechanism [5]–[7] represents a general type of auctions that uniquely ensures both efficiency and truthfulness. Informally, the VCG mechanism first solves the WDP to obtain an optimal allocation with respect to (1), and asks each winning bidder to pay an amount equal to the externality it exerts on the other bidders. As a result, the utility of a winning bidder is actually the marginal contribution to the total values when it joins.

More specifically, the price charged by the auctioneer to the winning bidder i is:

$$\begin{aligned} p_i &= b_i - (\mathrm{WD}(\mathcal{N}) - \mathrm{WD}(\mathcal{N}\backslash\{i\})) \\ &= \mathrm{WD}(\mathcal{N}\backslash\{i\}) - (\mathrm{WD}(\mathcal{N}) - b_i), \end{aligned} \quad (8)$$

where $\mathrm{WD}(\mathcal{N}\backslash\{i\})$ is the result of solving the WDP again using bids from all bidders except i, and $(\mathrm{WD}(\mathcal{N}) - b_i)$ is the sum of all the winning bids by all bidders except i.

Under the above VCG mechanism, misreporting one's value for the item(s) is always dominated by truth telling. If all the bidders follow the truthful bidding strategy, the allocation outcome will be efficient.

Take the example in Section IV to compute the bidders' VCG payments. If we remove bidder 5, then the best assignment is allocating channel A to bidder 1, channel B to bidder 6 and channel C to bidder 7, generating 90 in total value. Thus the payment of 5 is $p_5 = 90 - (118 - 38) = 10$. The VCG payments of bidder 6 and bidder 7 can be computed similarly, and are $p_6 = 12$ and $p_7 = 12$ respectively. The point for these VCG prices is illustrated in Fig. 2, which is actually the intersection of the planes generated by constraints from bidders 1, 2 and 3. The total revenue generated by such a VCG mechanism is $10 + 12 + 12 = 34$ in this case.

B. Revenue-Minimization Rule

Despite the relaxation of absolute truthfulness in core-selecting auctions, there exist different measures that can be taken to promote bidder incentives for reporting valuations truthfully. It is important to find payment rules that minimize incentives for deviating from truthful-telling.

We start with the definition of *bidder-Pareto optimality*:

Definition 1. *A core outcome is bidder-Pareto optimal if there is no other core outcome that can improve at least one bidder's utility without reducing any other one's in a subset* $\mathcal{C} \subseteq \mathcal{N}$.

To evaluate bidders' incentive to deviate from truthful reporting, we introduce the definition of the *incentive profile* for a core-selecting auction [17].

Definition 2. *The incentive profile for a core-selecting auction* M *at* $\mathbf{v}$ *is* $\{\theta_i^M(\mathbf{v})\}_{i\in\mathcal{N}}$ *where* $\theta_i^M(\mathbf{v})$ *is* i*'s maximum utility gain by deviating from truthful reporting.*

The idea is to minimize these incentives to deviate from truthful bidding, subject to the core-selection rule. Note that the incentives are represented by a vector, and we use a Pareto-like criterion. That is, a core-selecting auction M provides *optimal incentives*, if there is no core-selecting auction M' such that for every bidder i, $\theta_i^{M'}(\mathbf{v}) \leq \theta_i^M(\mathbf{v})$ with strict inequality for some bidder.

Day and Milgrom [17] proved that a core-selecting auction provides optimal incentives if and only if for every $\mathbf{v}$, it chooses a bidder-Pareto optimal outcome. Suppose that a bidder i reports a bid b_i and there is an alternative efficient auction that allocates a bundle $\mathcal{S}_i$ and charges i p_i, instead of the VCG payment p_i^{VCG}. p_i is guaranteed to be greater than or equal to p_i^{VCG}. We then have the following lemma:

Lemma 1. *For any efficient auction that produces payments greater than or equal to the VCG payments, the amount that bidder i can benefit by unilaterally deviating from the truthful bidding strategy is no more than $p_i - p_i^{VCG}$.*

Proof: All efficient auctions, by definition, essentially share the same allocation outcome. Assume that bidder i submits a bid v_i and receives a bundle $\mathcal{S}_i$. By way of contradiction, assume Lemma 1 is not true. That is, there is an efficient auction that produces payments greater than or equal to the VCG payments, with the existence of some bid vector $(\tilde{b}_i, \mathbf{b}_{-i})$, such that

$$v_i(\tilde{\mathcal{S}}_i) - \tilde{p}_i - (v_i(\mathcal{S}_i) - p_i) > p_i - p_i^{VCG} \tag{9}$$

where $\tilde{\mathcal{S}}_i$ is the bundle awarded to bidder i given $(\tilde{b}_i, \mathbf{b}_{-i})$, and charged i $\tilde{p}_i$ under such an auction. After rearranging and cancelling, (9) is equivalent to

$$v_i(\tilde{\mathcal{S}}_i) - \tilde{p}_i > v_i(\mathcal{S}_i) - p_i^{VCG} \tag{10}$$

Since $\tilde{p}_i \geq \tilde{p}_i^{VCG}$, we have the following,

$$v_i(\tilde{\mathcal{S}}_i) - \tilde{p}_i^{VCG} \geq v_i(\tilde{\mathcal{S}}_i) - \tilde{p}_i \tag{11}$$

Combining (10) and (11), we have

$$v_i(\tilde{\mathcal{S}}_i) - \tilde{p}_i^{VCG} > v_i(\mathcal{S}_i) - p_i^{VCG} \tag{12}$$

which contradicts the truthfulness property of the VCG mechanism, and that concludes the proof. ■

We can now prove the following theorem:

Theorem 3. *A core-selecting auction provides optimal incentives for truthful bidding if and only if for every vector of reported values, it chooses a bidder-Pareto optimal outcome.*

Proof: From Lemma 1, the maximum benefit for bidder i to deviate is $p_i - p_i^{VCG}$ for a bidder-Pareto optimal, core-selecting auction that charges i p_i. Hence the auction is suboptimal exactly when there is another core-selecting auction with higher utilities for all bidders, contradicting the assumption that this core-selecting auction is bidder-Pareto optimal. ■

However, there may be a broad set of possible bidder-Pareto optimal outcomes in the core. We will employ the technique introduced by Day and Raghavan [19]: by minimizing the total payments over the core, one can guarantee bidder-optimality, which can narrow the field of possible outcomes further.

Corollary 2. *A core-selecting auction employing a revenue-minimization payment rule minimizes the incentive of bidders to deviate from truthful bidding.*

Recall the coalitional core constraint, we have

$$\sum_{i \in \mathcal{C} \cup 0} u_i \geq \mathrm{WD}(\mathcal{C}) \quad \forall \mathcal{C} \subseteq \mathcal{N} \tag{13}$$

After the WDP is solved, we substitute the set of winning bundles $\{\mathcal{S}_i\}_{i \in \mathcal{N}}$, cancel the payments that are duplicated in u_0, and obtain an alternative formulation of (13):

$$\sum_{i \in \mathcal{W}} p_i \geq \mathrm{WD}(\mathcal{C}) - \sum_{i \in \mathcal{C}} (b_i(\mathcal{S}_i) - p_i) \quad \forall \mathcal{C} \subseteq \mathcal{N} \tag{14}$$

which is equivalent to

$$\sum_{i \in \mathcal{W} \setminus \mathcal{C}} p_i \geq \mathrm{WD}(\mathcal{C}) - \sum_{i \in \mathcal{C}} b_i(\mathcal{S}_i) \quad \forall \mathcal{C} \subseteq \mathcal{N} \tag{15}$$

where $\mathcal{W}$ is the set of winning users.

Setting $\beta_{\mathcal{C}} = \mathrm{WD}(\mathcal{C}) - \sum_{i \in \mathcal{C}} b_i(\mathcal{S}_i)$, and denoting the vector of all $\beta_{\mathcal{C}}$ values as $\boldsymbol{\beta}$, we can reformulate Eq. (15) as

$$A\mathbf{p} \geq \boldsymbol{\beta} \tag{16}$$

where A is a $2^{|\mathcal{N}|-1} \times |\mathcal{W}|$ matrix. In each row $\mathbf{a}_{\mathcal{C}}^T$ of A, the i-th entry equals 0 if bidder i is in set $\mathcal{C}$ and equals 1 otherwise. Then we can find the minimal core payments by solving the following linear program:

$$\alpha = \min \mathbf{1}^T \mathbf{p} \tag{17}$$

subject to:

$$A\mathbf{p} \geq \boldsymbol{\beta}$$
$$\mathbf{p} \leq \mathbf{b}$$

C. VCG-Nearest Rule

There is still a lack of precision even if we minimize the total payments over the core to ensure bidder-optimality, because these points are not unique. A simple method to specify a point is to minimize the group's incentive to deviate from truthful-telling. That is, among all total-payment minimizing core points, select the one that minimizes the Euclidean distance from VCG [18].

Since the goal is to minimize the Euclidean distance from the VCG point, and minimize the total payment at the same time, we can formulate a quadratic program to determine the payment vector $\mathbf{p}$:

$$\min\ (\mathbf{p} - \mathbf{p}^{VCG})^T (\mathbf{p} - \mathbf{p}^{VCG}) \tag{18}$$

subject to:

$$A\mathbf{p} \geq \boldsymbol{\beta}$$
$$\mathbf{p} \leq \mathbf{b}$$
$$\mathbf{1}^T \mathbf{p} = \alpha$$

Let $\mathbf{p}^*$ be the optimal solution to (18). The Karush-Kuhn-Tucker (KKT) conditions [27] indicate that

$$\mathbf{p}^* - \mathbf{p}^{VCG} - A^T \boldsymbol{\lambda} + I\boldsymbol{\omega} + \mathbf{1}\nu = \mathbf{0} \tag{19}$$

where $\boldsymbol{\lambda} \geq \mathbf{0}$, $\boldsymbol{\omega} \geq \mathbf{0}$, and $\nu \geq 0$ are the *Lagrangian multipliers* associated with the constraints in (18), and I is the unit matrix of size $|\mathbf{p}|$. It is proved that $\boldsymbol{\omega} = \mathbf{0}$ [18]. The KKT conditions are necessary and sufficient as QP (18) is a convex optimization problem with differentiable constraints [27]. Linear system (19) can be decomposed as:

$$p_i^* = p_i^{VCG} + \sum_{\mathcal{C} \in \mathcal{W} \backslash \{i\}} \lambda_{\mathcal{C}} - \omega_i - \nu$$

We are now ready to prove the following:

Theorem 4. *The set of constraints* $\mathbf{p} \leq \mathbf{b}$ *is redundant for QP (18).*

Proof. Suppose that constraint $p_i \leq b_i(\mathcal{S}_i)$ is necessary for some $i \in \mathcal{W}$. Then $\exists\ \epsilon > 0$ such that the constraint relaxed by ϵ, *i.e.*, $p_i \leq b_i(\mathcal{S}_i) + \epsilon$, is still tight and the solution must change. Notice that after relaxation, bidder i bids $b_i(\mathcal{S}_i) + \epsilon$ for bundle $\mathcal{S}_i$, but the solution to the WDP remains intact. We argue that the KKT conditions still form the same linear system. The only affected condition is that of bidder i, *i.e.*, $p_i^* = p_i^{VCG} + \sum_{\mathcal{C} \in \mathcal{W} \backslash \{i\}} \lambda_{\mathcal{C}} - \omega_i - \nu$, which remains unchanged after relaxation. This is because $p_i^{VCG} = b_i(\mathcal{S}_i) - \mathrm{WD}(\mathcal{N}) + \mathrm{WD}(\mathcal{N} \backslash \{i\})$, and the increase of ϵ in $b_i(\mathcal{S}_i)$ is cancelled by the same amount of increase in $\mathrm{WD}(\mathcal{N})$. The solution to QP (18) remains the same as KKT conditions are sufficient and necessary. The contradiction completes our proof. □

Based on Theorem 4, QP (18) is equivalent to the following simplified form:

$$\min\ (\mathbf{p} - \mathbf{p}^{VCG})^T (\mathbf{p} - \mathbf{p}^{VCG}) \tag{20}$$

subject to:

$$A\mathbf{p} \geq \boldsymbol{\beta}$$
$$\mathbf{1}^T \mathbf{p} = \alpha$$

The VCG-nearest rule has a variation which finds an in-core payment point that is nearest to some constant point $\mathbf{p}'$, where $\mathbf{p}'$ is the reference point set by the auctioneer before auction begins. In consequence, the final payment vector highly depends on the auctioneer's assumption of $\mathbf{p}'$. When no information about the bids is known beforehand, it is a general practice to set $\mathbf{p}' = \mathbf{0}$. We call it a *zero-nearest rule* [18] when $\mathbf{p}' = \mathbf{0}$ and will compare it with the VCG-nearest rule in Section VI.

D. Algorithm for In-Core VCG-Nearest Payment

Note that in the payment rules mentioned above, evaluating each $\beta_{\mathcal{C}}$ requires the solution of a WDP, so there will be $2^{|\mathcal{N}|-1}$ non-empty coalitions to consider, which is formidable in practice. However, an in-core VCG payment generation procedure adapted from the core constraint generation algorithm [19] can be employed to reduce the complexity, as shown in Algorithm 1. Instead of enumerating all the possibilities of non-empty coalitions, it finds blocking coalitions effectively by raising payments from the VCG price point, thereby reducing the complexity.

This iterative algorithm actually continues to increase the payments made by the winning bidders, until no blocking coalition exists in the auction, reaching a core outcome. Note that for the case of no blocking coalition, the vector of payments generated by the VCG mechanism equals that of the revenue minimization rule and hence the VCG-nearest rule.

Algorithm 1: In-Core VCG-Nearest Payment Generation

1 Set $t := 0$, payment vector $\mathbf{p}^t := \mathbf{p}^{VCG}$, coefficient matrix $A^t := \emptyset$, and vector $\boldsymbol{\beta}^t := \emptyset$;
2 **while** *True* **do**
3     $t := t + 1$;
4   **for** *bidder* $i \in \mathcal{N}$ **do**
5   **for** *bundle* $\mathcal{S}$ *bid by* i **do**
6             $b_i^t(\mathcal{S}) := b_i(\mathcal{S}) - (b_i(\mathcal{S}_i) - p_i^{t-1})$;
7   **end**
8   **end**
9   Calculate $\mathrm{WD}^t(\mathcal{N})$ with $\mathbf{b}^t$, with the set of winning CUs $\mathcal{C}^t$ being the first violated coalition in the WDP;
10   **if** $\mathit{WD}^t(\mathcal{N}) \leq \mathbf{1}^T \mathbf{p}^{t-1}$ **then** break;
11     $\beta_{\mathcal{C}^t} := \mathrm{WD}^t(\mathcal{C}^t) - \sum_{i \in \mathcal{C}^t} b_i^t(\mathcal{S}_i)$;
12   Append the corresponding row $\mathbf{a}_{\mathcal{C}^t}^T$ and new entry $\beta_{\mathcal{C}^t}$ to A^{t-1} and $\boldsymbol{\beta}^{t-1}$ to form A^t and $\boldsymbol{\beta}^t$, respectively;
13   Solve LP (17) with A^t and $\boldsymbol{\beta}^t$, obtaining α^t;
14   Solve QP (20) with $A^t, \boldsymbol{\beta}^t$ and α^t, obtaining $\mathbf{p}^t$;
15 **end**
16 $\mathbf{p}^* := \mathbf{p}^{t-1}$ is the solution to QP (20).

The correctness of Algorithm 1 is established in the following theorem.

Theorem 5. *Payment vector generated by Algorithm 1 is the solution to QP (20).*

Proof. Algorithm 1 always terminates because: (i) the number of blocking constraints generated is bounded by $2^{|\mathcal{N}|-1}$, and (ii) the core, which contains the trivial first-price payment vector, is not empty.

We are going to prove that when the algorithm terminates, $\mathbf{p}^{t-1}$ is in the core. By way of contradiction, assume that $\mathbf{p}^{t-1}$ is not in the core, then Eq. (15) does not hold:

$$\sum_{i \in \mathcal{W} \backslash \mathcal{C}} p_i^{t-1} < \mathrm{WD}(\mathcal{C}) - \sum_{i \in \mathcal{C}} b_i(\mathcal{S}_i),\ \exists \mathcal{C} \subseteq \mathcal{N} \tag{21}$$

Line 6 of Algorithm 1 indicates that

$$\begin{cases} b_i^t(\mathcal{S}_i) = p_i^{t-1}, & i \in \mathcal{W} \\ b_i^t(\mathcal{S}) = b_i(\mathcal{S}), & i \notin \mathcal{W} \end{cases} \tag{22}$$

Combining Eq. (22) and the stopping criterion (Line 10), we get:

$$\mathrm{WD}^t(C) \leq \mathrm{WD}^t(\mathcal{N}) \leq \sum_{i \in \mathcal{W} \backslash \mathcal{C}} p_i^{t-1} + \sum_{i \in \mathcal{W} \cap \mathcal{C}} b_i^t(\mathcal{S}_i) \tag{23}$$

Letting $\tilde{\mathcal{W}}$ be the set of winners to $\mathrm{WD}(\mathcal{C})$ with bundle $\tilde{\mathcal{S}}_i$

allocated to bidder i, we have:

$$
\begin{aligned}
&\mathrm{WD}(C) - \mathrm{WD}^t(C) \\
&>_1 \Big(\sum_{i\in\mathcal{W}\setminus\mathcal{C}} p_i^{t-1} + \sum_{i\in\mathcal{C}} b_i(\mathcal{S}_i)\Big) - \Big(\sum_{i\in\mathcal{W}\setminus\mathcal{C}} p_i^{t-1} + \sum_{i\in\mathcal{W}\cap\mathcal{C}} b_i^t(\mathcal{S}_i)\Big) \\
&\geq_2 \sum_{i\in\mathcal{C}} (b_i(\mathcal{S}_i) - b_i^t(\mathcal{S}_i)) =_3 \sum_{i\in\mathcal{W}\cap\mathcal{C}} (b_i(\mathcal{S}_i) - p_i^{t-1}) \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{WD}(C) - \mathrm{WD}^t(C) \\
&\leq_4 \sum_{i\in\tilde{\mathcal{W}}} (b_i(\tilde{\mathcal{S}}_i) - b_i^t(\tilde{\mathcal{S}}_i)) \\
&=_5 \sum_{i\in\tilde{\mathcal{W}}\cap(\mathcal{W}\cap\mathcal{C})} (b_i(\tilde{\mathcal{S}}_i) - b_i^t(\tilde{\mathcal{S}}_i)) \\
&=_6 \sum_{i\in\tilde{\mathcal{W}}\cap(\mathcal{W}\cap\mathcal{C})} (b_i(\mathcal{S}_i) - p_i^{t-1}) \qquad (25)
\end{aligned}
$$

In the derivation above, $>_1$ holds due to inequalities (21) and (23). $\geq_2$ is due to the fact that $\sum_{i\in\mathcal{C}} b_i^t(\mathcal{S}_i) \geq \sum_{i\in\mathcal{W}\cap\mathcal{C}} b_i^t(\mathcal{S}_i)$, and $=_3$ is due to (22). The set of winners $\tilde{\mathcal{W}}$ with bundle $\tilde{\mathcal{S}}_i$ allocated to bidder i is a feasible solution to $\mathrm{WD}^t(\mathcal{C})$, and hence $\mathrm{WD}^t(\mathcal{C}) \geq \sum_{i\in\tilde{\mathcal{W}}} b_i^t(\tilde{\mathcal{S}}_i)$, which explains $\leq_4$. $=_5$ is due to Eq. (21) and the fact that $\tilde{\mathcal{W}} \subseteq \mathcal{C}$. Finally, $=_6$ is due to $b_i(\mathcal{S}) - b_i^t(\mathcal{S}) = b_i(\mathcal{S}_i) - p_i^{t-1}$ (derived from Line 6).

Inequalities (24) and (25) lead to:

$$
\sum_{i\in\mathcal{W}\cap\mathcal{C}} (b_i(\mathcal{S}_i) - p_i^{t-1}) < \sum_{i\in\tilde{\mathcal{W}}\cap(\mathcal{W}\cap\mathcal{C})} (b_i(\mathcal{S}_i) - p_i^{t-1}) \qquad (26)
$$

The inequality is obviously a contradiction as $b_i(\mathcal{S}_i) \geq p_i^{t-1}$ and our theorem is thus proved. □

VI. Simulation Results

In this section, we present results from simulation studies, for evaluating the performance of our core-selecting auctions. First we investigate the influence of interference, to evaluate the auction with or without channel reuse. Then we compare the payment rules for efficient auctions that can be employed, with respect to their achieved revenues. We examine the impact of different metrics and settings in the auction including the total bids submitted, extent of interference in the region and the distribution of bids.

A. Simulation Environment

We assume a single auctioneer that handles the auction, in which SUs act as bidders. These bidders are uniformly and randomly distributed in a 1×1 square region. For simplicity, we assume a distance-based interference model for generating the corresponding conflict graphs. Any two bidders located within distance $0.1 \times \Delta$ ($\Delta > 1$) conflict with each other, and hence cannot be allocated the same channel, leading to an edge between them in the conflict graph.

We use the combinatorial auction test suite (CATS) developed by Leyton-Brown et al. [28] to generate auction instances, including the number of channels to be auctioned, the number of bids, the bidders, etc. The CATS software suite simulates bidding behaviour in a number of realistic economic environments. For instance, bidders are interested in bundles of channels that are adjacent in frequency, which can be employed in our case. We allow the number of channels on sale to vary from 8 to 64, and the CATS number-of-bids parameter to vary from 20 to 160. We generate 100 instances for each set of auction parameters, *i.e.*, all the results are averaged over 100 times.

All the auction instances are executed using CPLEX 12.1 [29], on a 1.86 GHz Intel Core 2 Duo processor. The longest execution time of the instances takes no more than a few seconds.

We adopt the following three performance criteria:

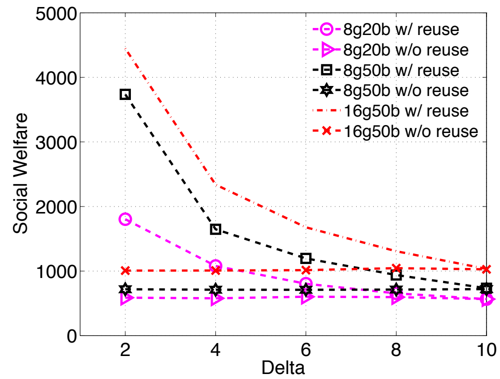
- *Social Welfare*: Efficiency of the auction, i.e., the sum of reported values from all the winning bidders.
- *Spectrum Utilization*: The sum of allocated channels of all the winning bidders.
- *Bidder Satisfaction*: The percentage of bidders that win.
- *Revenue*: The sum of payments from all the winners.

B. Allocation Results

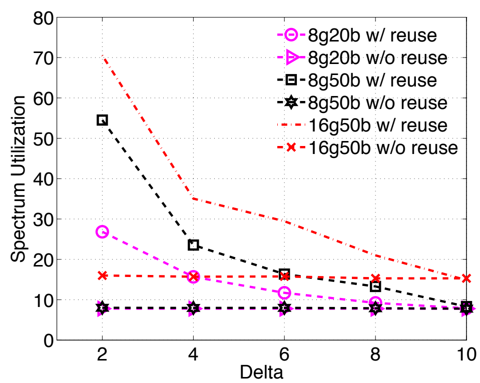
We first evaluate the auction in terms of allocation results. In this section, we use "XgYb" to denote the auction settings, where X represents the number of goods (channels) auctioned and Y represents the number of bids submitted. (Note that CATS adopts the number of bids as input instead of the number of bidders. We take this setting in our evaluation as well). A bidder can submit multiple bids, and the number of bidders is no more than the number of bids, Y; but due to our previously mentioned XOR-bid convention, a bidder can only win a single bid.

In Fig. 5, we plot the social welfare, spectrum utilization and bidder satisfaction under core-selecting auction environments, by changing the interference ratio Δ from 2 to 10. When traditional combinatorial auction settings are employed, where channels are treated as regular commodities and channel reuse is not considered, all the performance metrics do not apparently change. When channels can be reused, the performance is always higher, but degrades when Δ increases. When interference is severe in the region ($\Delta = 10$), most of the SUs interfere with one another (the interference range is 1, a little less than the maximum possible distance $\sqrt{2}$). We can observe that the two lines of the same colour converge at $\Delta = 10$, at which point almost no channel reuse is possible.

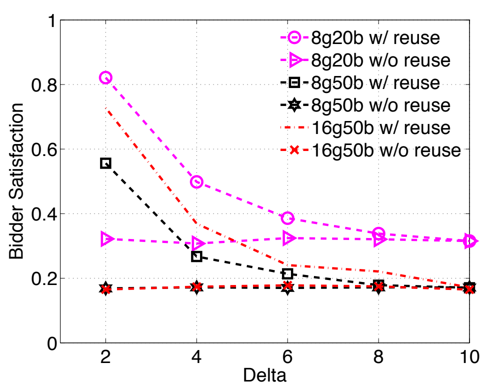
In Fig. 5(a), when interference is not high ($2 \leq \Delta \leq 4$), as is the case in many practical settings, the auction can achieve at least 2 times the social welfare as compared to when channel reuse is disabled. The ratio of spectrum utilization in Fig. 5(b) is even higher. From Fig. 5(b), we can see that all the channels are always sold out, in both cases of 8 channels and 16 channels. In our simulation, CATS generates sufficiently many bids and bidders to buy these channels, even if the auction only adopts XOR bids. This makes the spectrum utilization number to be no less than the number of channels, even when reuse is disabled. One interesting observation from Fig. 5(c) is that, without channel reuse, the bidder satisfaction ratio stays the same in the "8g50b" and "16g50b" cases. This is due to the fact that when there are enough participants in the auction, there exist a few bidders requesting many channels and submitting high bids to exclude other bidders. Because channels are not reused, only these users are able to win their bids. This is similar to monopolization. Therefore, from this



(a) Social welfare



(b) Spectrum utilization



(c) Bidder satisfaction

Fig. 5. Performance of the allocation result.

point of view, channel reuse is highly desired when designing secondary spectrum auctions.

C. Revenues of Core-Selecting Auctions

We next study the performance of core-selecting auctions by comparing its revenue with that of the VCG mechanism. We first investigate the impact of interference on revenues, as shown in Fig. 6. In this experiment, 16 channels are auctioned and 50 bids are submitted in every simulation instance. We can see that interference can largely deteriorate revenues when we use core-selecting auctions with the revenue minimization rule or the VCG-nearest rule. For example, when Δ rises from 2 to 4, the revenue drops by 41%. However, they are always better than the VCG revenue, especially when interference is moderate. When $\Delta = 2$, the core-selecting auctions can even achieve a 177% higher revenue. This high revenue is attributed to channel reuse. Because there will be more winners in the auction, and they are all charged at least their VCG payments, accumulating to a much higher aggregate revenue.

We then study how core-selecting auctions outperform the VCG mechanism in terms of revenues by changing the number

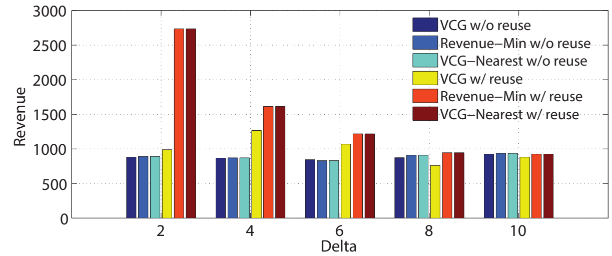
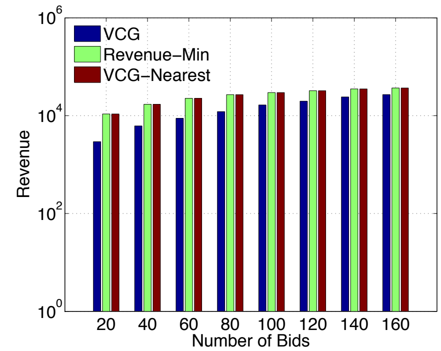
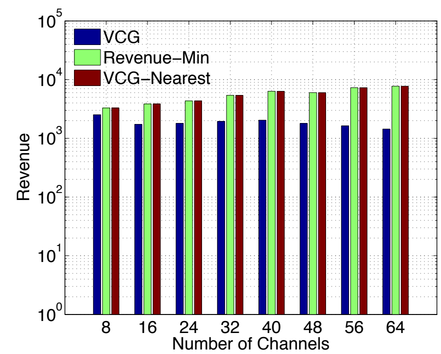


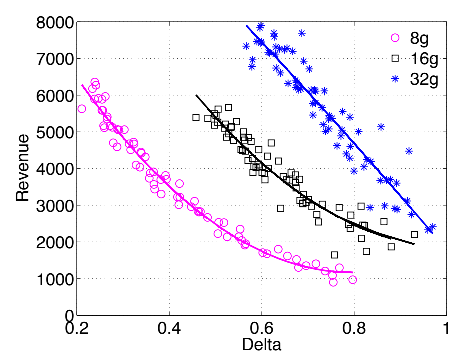
Fig. 6. Impact of interference on revenues.



(a) Revenue vs Number of bids



(b) Revenue vs Number of channels



(c) Revenue vs Bidder satisfaction

Fig. 7. Comparisons of revenues.

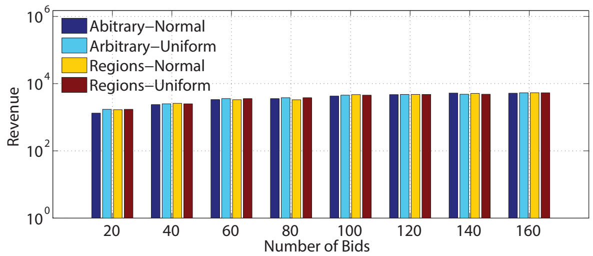


Fig. 8. Influence of bid distribution: Revenue vs Number of bids

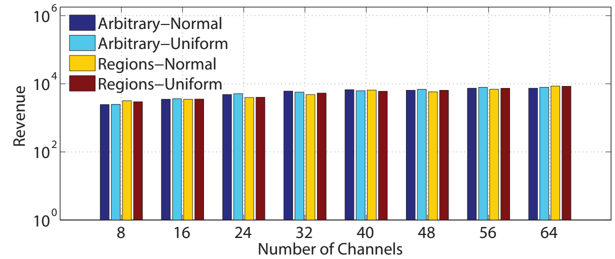


Fig. 9. Influence of bid distribution: Revenue vs Number of channels

of bids submitted and the number of channels auctioned. We first fix the number of auctioned channels to 16, and vary the number of bids. The results are shown in Fig. 7(a). One observation is that the revenue minimization rule and the VCG-nearest rule generate the same amount of revenues, verifying the relationship between (17) and (18). We can see that the core-selecting auctions always achieve higher revenues than the VCG mechanism. However, the difference between them dwindles when the number of bids becomes larger. This is due to the fact that in the VCG mechanism, in the presence of more bids, is more likely to charge a higher payment to a winner due to the latter's higher potential externality on other bidders. However, our auctions still manage to deliver a 36% higher revenue when there are 160 bids submitted (note that the vertical axis is on logarithmic scale). Then we fix the number of bids to 80 and vary the number of channels. Similarly, Fig. 7(b) shows that the core-selecting auctions improves revenues significantly, as compared to the VCG mechanism.

For a better illustration of revenues against bidder satisfaction of our core-selecting auctions, we plot Fig. 7(c). We fit the points with second order polynomial curves for 8, 16 and 32 channels, respectively. These curves describe trade-offs between revenues and bidder satisfaction, which can help the auctioneer leverage both in making long-term decisions.

In previous experiments, we assume bidders select channels rationally and reasonably as in reality (*e.g.*, choose channels that are spectrally adjacent). The amount of bids are drawn from a normal distribution. To investigate the influence on revenues by the bid distribution, we further assume bids are arbitrarily set and their amount are drawn from a uniform distribution for comparison. The four settings that we adopt are "Arbitrary-Normal", "Arbitrary-Uniform", "Regions-Normal" and "Regions-Uniform" (details available in [28]), respectively. From Fig. 8 and Fig. 9, we found that the performance of our auctions is quite stable. Even when the bidders arbitrarily bid for channels, the revenues do not deteriorate (we only show the results of the VCG-nearest rule, since the revenue-minimization rule has the same results).

D. *VCG-Nearest Rule* vs. *Zero-Nearest Rule*

It is observed that under the zero-nearest rule, the winner with high valuation shares less of the burden to conquer a coalitional blocking. For example, consider three SUs, $1, 2, 3$, bidding for two different channels A and B with the following bids:

$$b_1(A) = 40,\ b_2(B) = 20,\ b_3(A, B) = 50$$

Winners 1 and 2 have VCG payments $(30, 10)$ not in the core, and have to raise their combined payment to 50 to keep SU3 from blocking. Under the zero-nearest rule, SU2 will be responsible for this total payment increase with final payments $(30, 20)$. On the contrary, the VCG-nearest rule results in a sharing of this burden, with payments $(35, 15)$.

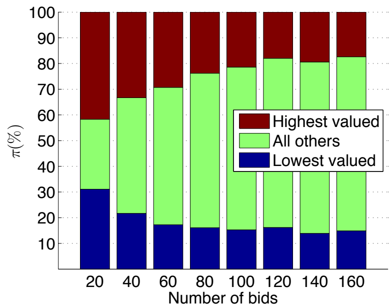
The above phenomenon is verified by Fig. 10. When VCG payment is not in the core, the *monetary burden* [18] of bidder j is defined as:

$$\pi_j = \frac{p_j - p_j^{VCG}}{\sum_{i \in \mathcal{N}} (p_i - p_i^{VCG})}$$

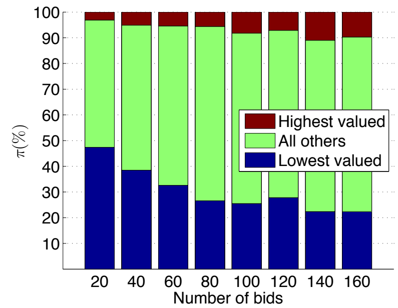
We fix the number of channels to 8 and vary the number of bids from 20 to 160. The highest valued winners shoulder more of the burden of conquering blocking coalitions under the VCG-nearest rule (25.5% on average) than under the zero-nearest rule (6.9% on average). On the contrary, the lowest-valued winners on average pay about 18.0% under a VCG-nearest rule, while they shoulder heavier a burden, paying about 30.4%, under the origin-nearest implementation. We also observe from Fig. 10 that the disparity under these two payment rules is most pronounced when the number of bids is small.

VII. Conclusions

Secondary spectrum auctions can serve as a promising approach to efficiently mitigate the scarcity of wireless spectrum. We find that, if the problem scale is limited, combinatorial auctions can be employed to enable expressive bids, while efficiency of the auction is still guaranteed. For the first time in the literature, we propose the use of core-selecting auctions in the secondary spectrum market, with the consideration of interference-free channel allocation. The advantages of core-selecting auctions over the VCG mechanism, including higher revenues and robustness against shill bidding, are rigorously proven. Due to the fact that channels can be reused in the region, accommodating more bidders compared to regular core-selecting auctions, our design can achieve significantly higher revenues. While absolute truthfulness is compromised, the payment rules of our tailored core-selecting auctions still tend to minimize any deviations from absolute truthfulness.



(a) VCG-nearest



(b) Zero-nearest

Fig. 10. When total payments are more than the total VCG payments, the figures show the monetary burden shouldered by the winner(s) under (a) the VCG-nearest rule, and (b) the zero-nearest rule.

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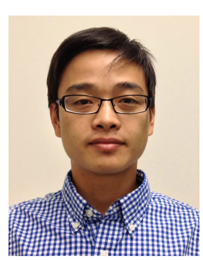

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