An Incentive Mechanism for Crowdsourcing Systems with Network Effects

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In a crowdsourcing system, it is important for the crowdsourcer to engineer extrinsic rewards to incentivize the participants. With mobile social networking, a user enjoys an intrinsic benefit when she aligns her behavior with the behavior of others. Referred to as *network effects*, such an intrinsic benefit becomes more significant as more users join and contribute to the crowdsourcing system. But should a crowdsourcer design her extrinsic rewards differently when such network effects are taken into consideration? In this article, we incorporate network effects as a contributing factor to intrinsic rewards, and study its influence on the design of extrinsic rewards. We show that the number of participating users and their contributions to the crowdsourcing system evolve to a steady equilibrium, thanks to subtle interactions between intrinsic rewards due to network effects and extrinsic rewards offered by the crowdsourcer. Taken network effects into consideration, we design progressively more sophisticated extrinsic reward mechanisms, and propose new and optimal strategies for a crowdsourcer to obtain a higher utility. Through simulations and examples, we demonstrate that with our new strategies, a crowdsourcer is able to attract more participants with higher contributed efforts; and the participants gain higher utilities from both intrinsic and extrinsic rewards.

CCS Concepts: • **Networks → Network economics**; *Network dynamics*;

Additional Key Words and Phrases: Crowdsourcing, incentive mechanism, intrinsic rewards, network effects

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1 INTRODUCTION

Crowdsourcing combines the collective efforts of the crowd to accomplish a specific task or collect large volumes of data, which is otherwise extremely costly or even unattainable. Crowdsourcing systems, such as Uber [10], Waze [19], and Amazon Mechanical Turk (MTurk) [1], have gained wide popularity. Another excellent example is ResearchKit, recently launched by Apple as a mobile application framework that supports a crowdsourcing platform for medical research [9].

The success of a new crowdsourcing platform relies on the scale of user participation, as well as the contribution from each individual participant. To recruit and maintain a large number of users, the crowdsourcer usually provides users with monetary compensations, referred to as *extrinsic rewards*. In contrast to such extrinsic rewards offered by the crowdsourcer, a participant often enjoys a reward that is derived from a sense of satisfaction, social status, or honor, and such rewards are inherently *intrinsic*. In ResearchKit, for example, it has been reported that only a few hours after its release, over 7,000 people has voluntarily enrolled in a study on Parkinson's disease without any extrinsic rewards, and the largest study ever was only 1,700 [14].

Such intrinsic rewards, however, do not typically remain unchanged as the size of participant population grows. When a user's behavior aligns with other users, she will obtain higher intrinsic rewards, usually due to social factors. This is known as the *network effects* [7]. In the example of Waze, a mobile crowdsourcing platform for sharing traffic information, a driver can get a better route if more users join and contribute their local traffic data. It is intuitively conceivable that a growing population of the crowdsourcing platform—with more intrinsic rewards due to network effects—would help reduce the amount of extrinsic rewards that a crowdsourcer will need to provide. Considering the network effects, the user participation level will evolve through a dynamic process. At each time stage, users observe the (current) community popularity, estimate the network effects, and decide whether or not to join the crowdsourcing platform. It is quite popular for crowdsourcing systems to publish their statistics on their website or APP. For instance, the Waze project provides a live map for interested visitors that demonstrates the real-time subscriber number and their individual location around a specific place. After users take actions, the resulting participation level will trigger user responses in the next time stage. Finally, the network size will reach equilibrium with a stable user participation level.

Unfortunately, existing works in the literature have not yet considered the influence of intrinsic rewards, as well as the exquisite and subtle interactions between intrinsic and extrinsic rewards. Reverse auctions (e.g., [25]) and Stackelberg games (e.g., [17]) are typically used to model extrinsic rewards in existing works. With reverse auctions, users submit bids with their desired extrinsic rewards, and the crowdsourcer chooses the users based on their bids, with a selection process that is typically NP-hard and impractical. With Stackelberg games, the crowdsourcer first announces her policy on extrinsic rewards, and the users would then make decisions on their contribution levels. In both models, the user population is assumed to be fixed, without considering intrinsic rewards and network effects.

In this article, we bring intrinsic rewards into the spotlight, with a focus on how network effects affect the mechanism design when a crowdsourcer provides extrinsic rewards to incentivize crowdsourcing systems. In particular, we study the dynamics of users' participation level and contributed efforts as a result of the interaction between intrinsic rewards incurred by network effects and extrinsic rewards from the crowdsourcer. Based on our analyses, we propose extrinsic reward mechanisms that take advantage of the intrinsic rewards to boost user participation and contributed efforts, as well as increase the crowdsourcer's utility.

To begin with, we assume that every user experiences homogeneous network effects (Section 2). Under this assumption, we first present a simple mechanism with fixed extrinsic rewards

(Section 2.2), which is easy to implement. Thanks to intrinsic rewards, the participation level is above-zero even without extrinsic rewards. Given a certain extrinsic reward, the participation level will evolve to a stable equilibrium. Targeting the most profitable participation level, we are able to compute the corresponding optimal fixed extrinsic reward.

Then, we proceed to design a flexible extrinsic reward mechanism, where the extrinsic reward of a user is a function of her effort level (Section 2.3). To specify the extrinsic reward function is challenging, since we do not have any foreknowledge of its form (e.g., linear or logarithmic). Moreover, different extrinsic reward functions, via a complex interaction with intrinsic rewards, lead to different equilibrium participation levels. To tackle this problem, we first focus on a certain participation level, and obtain the extrinsic reward function that achieves the highest utility for the crowdsourcer under this participation level. We then choose the best participation level that yields the maximum utility, and derive the corresponding optimal extrinsic reward function.

Taking it a step further, we analyze the scenario where network effects have heterogeneous influence on different users, depending on the intensity of their social relationships with other users (Section 3). In this case, we start with the flexible extrinsic reward mechanism (Section 3.2), and prove that it is the best for the crowdsourcer to involve every user, i.e., reach a full participation level. Under heterogeneous network effects, the crowdsourcer is willing to subsidize users who strongly affect other users and save money on extrinsic rewards to users who are receptive to the influence from other users. Interestingly, if the social relationship is symmetric, i.e., any two users exert the same amount of influence on each other, the optimal flexible extrinsic rewards are irrelevant of the network effects.

Finally, we turn our attention to the optimal fixed extrinsic reward mechanism, which may lead to partial user participation (Section 3.3). As the fixed extrinsic reward gradually decreases, users will drop out of the crowdsourcing system one after another. Therefore, we propose an algorithm, which iteratively seeks for the optimal extrinsic reward for a targeted set of participating users.

Via simulations and examples, we demonstrate the efficiency of our designed mechanisms. With the help of intrinsic rewards, the crowdsourcer is able to reach a higher participation level and obtain a higher utility. Stronger network effects will contribute to higher intrinsic rewards, thus more beneficial to the crowdsourcer. Compared with the fixed extrinsic reward mechanism, the flexible extrinsic reward mechanism is more effective in soliciting more user contributions. Better still, the flexible extrinsic reward mechanism improves both the crowdsourcer's and the users' utilities, resulting in a win-win situation. This is because users' higher contributions not only earn themselves higher extrinsic rewards but also become a valuable asset to the crowdsourcer.

A preliminary version of this article is published as Reference [4]. In this journal version, we extend the previous theoretical analysis by considering heterogeneous network effects on users, and its impact on the optimal extrinsic rewards.

2 HOMOGENEOUS NETWORK EFFECTS

In this section, we assume that the intrinsic rewards from network effects are homogeneous for all users. Due to page limitation, we ignore all proofs in this section, and interested readers can refer to Reference [4].

2.1 System Model

By participating in a crowdsourcing system, a user receives both extrinsic rewards from the crowdsourcer, and intrinsic rewards due to the benefits or social status she obtains. More formally, user i exerts an effort of x_i. $x_i \in [\underline{x}, \overline{x}]$, in which $\underline{x}$ is the minimum effort, for example, a user has to register and fill in the basic information; $\overline{x}$ is the maximum effort due to limitations such as time, battery life and manpower. For simplicity, we assume that all users have the same minimum and maximum efforts. In fact, users usually have different upper and lower bounds of their efforts due to limitations of their devices and environments. In the future, we will study the influence of users' heterogeneous effort levels on the extrinsic reward design of the crowdsourcing platform. To contribute an effort of x_i, the user incurs a cost of $c_i x_i$, where c_i is user i's unit cost.

Intrinsic rewards. On one side, a user benefits from her own effort, for example, a healthcare crowdsourcing platform enables a user to get a better understanding of her health condition by keeping track of her diet, exercise and heart rate. On the other side, a user enjoys the social advantage of a large crowd owing to network effects. Therefore, a user's intrinsic reward is $v_i x_i + E(n)$, in which v_i is the unit value a user gets from her own effort[1], and $E(n) = \theta n^\gamma$ is the network effects. $n \in [0, 1]$ represents the normalized participation level; $\theta \in [0, \inf)$ and $\gamma \in [0, 1]$ are constant parameters to characterize the intensity of the network effects. Exponential functions are commonly used to model network effects in the existing literature [3, 5, 7]. It can be checked that the network effects are a concave function of the participation level, monotonically increasing with the participation level, but the marginal return decreases. In this section, we assume that network effects $E(n)$ are the same for all users. In Section 3, we will study the case where users obtain heterogeneous rewards from network effects.

Extrinsic rewards. The crowdsourcer provides users with an extrinsic reward of $P(x_i)$, satisfying $P(0) = 0$. In the fixed extrinsic reward mechanism, $P(x_i) = p$, irrespective of the users' effort levels; in the flexible extrinsic reward mechanism, however, $P(\cdot)$ is a function of x_i.

User i's utility u_i is the sum of intrinsic and extrinsic rewards minus the cost:

$$u_i = v_i x_i + E(n) + P(x_i) - c_i x_i. \tag{1}$$

We combine $v_i x_i$ and $c_i x_i$ as they have the common term x_i,

$$u_i = E(n) + P(x_i) - \alpha_i x_i, \tag{2}$$

in which $\alpha_i = c_i - v_i$ is defined as the *net cost* of user i. For some users, $v_i > c_i$, so the net cost α_i is negative. Even without extrinsic rewards, these self-motivated users have incentives to participate in the crowdsourcing system. These pioneers help attract others via network effects. The net cost is $\alpha_i \in [\underline{\alpha}, \overline{\alpha}]$ is a random variable, with a cumulative distribution function $F(\alpha)$, and a probability density function $f(\alpha) = F'(\alpha)$. We have $\underline{\alpha} < 0$ and $\overline{\alpha} > 0$, as users may have negative or positive net costs.

Aiming at maximizing her utility in Equation (2), a user's optimal effort level x_i^* is a function of her net cost α_i, i.e., $x_i^* = g(\alpha_i)$. She will drop out if her utility is always negative whatever the effort level is.

The crowdsourcer makes profit from users' contributions, while having to pay extrinsic rewards. Her utility U is the total aggregated contribution from all participants minus the total extrinsic rewards,

$$U = \mu \int_\alpha \ln(1 + g(\alpha))\mathrm{d}F(\alpha) - \int_\alpha P(g(\alpha))\mathrm{d}F(\alpha), \tag{3}$$

in which μ is the equivalent monetary worth of users' contributions. Note that $g(\alpha)$ is a user's effort. We use logarithmic function $\ln(\cdot)$ to transform a user's effort to the perceived utility by the crowdsourcer, which features the law of diminishing return: A user's contribution increases with her effort level but the marginal return decreases. If a user inputs zero effort, then the utility received by the crowdsourcer is $\ln(1 + 0) = 0$.

[1]In this article, we assume that the unit cost c_i and unit value v_i are constants for a specific user i. In future works, we will explore the scenario where c_i and v_i are variables and derive user types based on their distribution.

The crowdsourcer's objective is to maximize her utility in Equation (3) by determining the optimal extrinsic rewards. In the fixed extrinsic reward mechanism, the crowdsourcer has to decide the optimal uniform extrinsic reward p^*; in the flexible extrinsic reward mechanism, the crowdsourcer has to design the optimal extrinsic reward function $P^*(\cdot)$.

The extrinsic reward mechanism can be formulated as a two-stage Stackelberg game. In the first stage, the crowdsourcer chooses the optimal extrinsic rewards that maximize her utility in Equation (3). In the second stage, given the extrinsic rewards, each user will choose the optimal effort level that maximizes her utility.

We can use backward induction to deal with the above game model. To begin with, we compute the optimal effort level of each user, assuming that the extrinsic reward $\boldsymbol{p}$ is known. Then, being aware of the influence of extrinsic rewards on users' choices, we can obtain the optimal extrinsic reward mechanism for the crowdsourcer.

2.2 Fixed Extrinsic Reward Mechanism

In this section, we first study how the interplay of a fixed extrinsic reward and network effects lead to participation levels at equilibrium, based on which we derive the optimal value of the fixed extrinsic reward.

2.2.1 Equilibrium Participation Level. Given a fixed extrinsic reward, a user's utility becomes

$$u_i = E(n) + p - \alpha_i x_i. \tag{4}$$

If $\alpha_i < 0$, then a user will definitely participate with maximum effort $\overline{x}$; otherwise, she will participate with minimum effort $\underline{x}$ if positive utility can be obtained. Given an expected participation level n^e and corresponding network effects $E(n^e)$, the marginal user, who is indifferent to the choices of participating or not, has a utility of zero. Let α_n denote the net cost of the marginal user. We have

$$\alpha_n(p) = \frac{1}{\underline{x}}\left(E(n^e) + p\right), \tag{5}$$

where $\alpha_n(p)$ is upward sloping in p, and the sloping straight line will shift up if n^e increases. This implies that the users with higher net costs will participate if either extrinsic or intrinsic rewards go up. Since $F(\alpha_n) = n$, we have

$$n = F\left(\frac{E(n^e) + p}{\underline{x}}\right). \tag{6}$$

At equilibrium, the expected participation level equals the real participation level, that is, $n = n^e$. With $p > \underline{x}\overline{\alpha} - E(1)$, $n = 1$ will be an equilibrium,[2] but the crowdsourcer will never set a p that is more than enough to achieve full participation. Thus, we stipulate that $p \leq \underline{x}\overline{\alpha} - E(1)$.

Proposition 1. ***Existence of an Equilibrium Participation Level.*** *For any extrinsic reward p, Equation (6) has at least one root.*

Define $\Phi(n) = F(\frac{E(n)+p}{\underline{x}}) - n$, $\Phi(n)$ is continuous in $[0, 1]$. Figure 1 shows the value of $\Phi(n)$ under different participation levels, and the condition $\Phi(n) = 0$ pinpoints the equilibria. Given a certain extrinsic reward, there are multiple equilibria, but they have different stability attributes.

(1) *Stable equilibria*, such as n_B and n_D in Figure 1. Suppose there is a small perturbation Δn upwards at n_B, $\Phi(n_B + \Delta n) < 0$, i.e., $\beta_{n_B+\Delta n}\underline{x} > E(n_B + \Delta n) + p$. The participation level will be pushed downwards back to n, because the net costs of the new participants are greater than their rewards, so they will leave. Similarly, suppose there is a small perturbation Δn downwards at n_B,

[2] If $p \leq \underline{x\alpha}$, then $n = 0$ will be an equilibrium. Nevertheless, $\underline{\alpha} < 0$, so this will not happen.

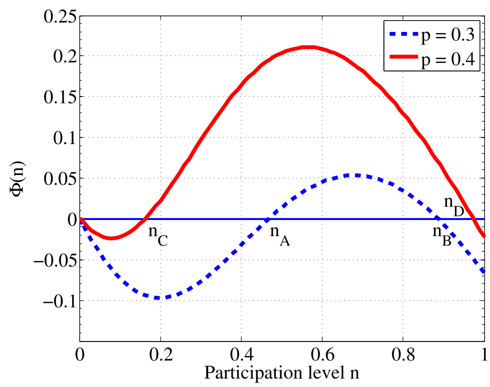


Fig. 1. Multiple equilibria under a certain extrinsic reward. $E(n) = \sqrt{n}, \underline{x} = -1, F(\cdot) \sim N(1, 0.2)$.

$\Phi(n_B - \Delta n) > 0$, i.e., $\beta_{n_B - \Delta n}\underline{x} > E(n_B - \Delta n) + p$. The participation level will be pushed upwards back to n, because users whose net costs are smaller than their rewards will rejoin.

(2) *Unstable equilibria*, such as n_A and n_C in Figure 1. Suppose there is a small perturbation Δn upwards at n_A, $\Phi(n_A + \Delta n) > 0$, which implies that more users will rush in, and the participation level will be pushed further up to n_B. Similarly, suppose there is a small perturbation Δn downwards at n_A, $\Phi(n_A - \Delta n) < 0$, therefore, more users will leave, and the participation level will be pushed further down to 0.

In summary, we have the following lemma to characterize the stability of an equilibrium.

LEMMA 1. ***Stable Equilibrium.*** *An equilibrium participation level is stable if* $\Phi'(n) < 0$.

PROPOSITION 2. ***Existence of a Stable Equilibrium Participation Level.*** *For any extrinsic reward p, there exists at least one stable equilibrium participation level. In particular, the highest equilibrium participation level is stable.*

2.2.2 Optimal Fixed Extrinsic Reward. The fixed extrinsic reward p leads to the equilibrium participation level, which in turn, affects the crowdsourcer's utility.[3] Since users with $\alpha_i \in [\underline{\alpha}, 0]$ make an effort of $\overline{x}$ and users with $\alpha \in (0, \alpha_n]$ make an effort of $\underline{x}$, the crowdsourcer's utility becomes

$$\begin{aligned} U &= \mu \int_{\underline{\alpha}}^{0} \ln(1+\overline{x}) \mathrm{d}F(\alpha) + \mu \int_{0}^{\alpha_n} \ln(1+\underline{x}) \mathrm{d}F(\alpha) - np \\ &= \mu F(0) \ln(1+\overline{x}) + \mu(n - F(0)) \ln(1+\underline{x}) - np \\ &= \mu F(0) \ln \frac{1+\overline{x}}{1+\underline{x}} + [\mu \ln(1+\underline{x}) - p]n. \end{aligned} \tag{7}$$

The extrinsic reward p determines the equilibrium participation level n according to Equation (6). Therefore, finding the optimal extrinsic reward p is equivalent to finding the optimal participation level n induced by p:

$$\max_p U \Rightarrow \max_n \left[\mu \ln(1+\underline{x}) + E(n) - \underline{x}F^{-1}(n)\right] n. \tag{8}$$

[3]For tractability, we only consider the highest stable equilibrium participation level. In the future, we will study the possibilities of other equilibria, and their influence on the design of extrinsic rewards.

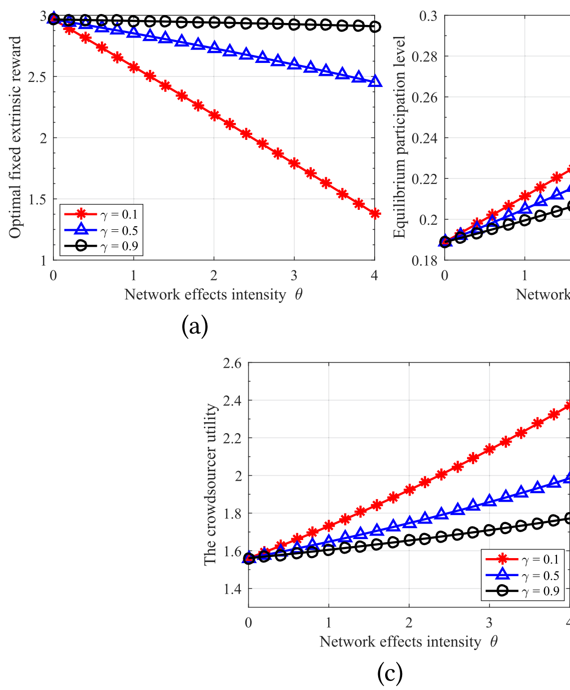


Fig. 2. The fixed extrinsic reward mechanism: $x \in [1, 10]$, $F(\cdot) \sim \mathrm{UNIF}(-1, 15)$.

Proposition 3. ***Optimal Fixed Extrinsic Reward.*** *Given the optimal equilibrium participation level as the solution of Equation (8), the optimal fixed extrinsic reward is*

$$p^* = \underline{x}F^{-1}(n^*) - E(n^*). \tag{9}$$

Figure 2 shows the optimal extrinsic reward, the equilibrium participation level, and the crowdsourcer's utility under the fixed extrinsic reward mechanism. Stronger network effects (a larger θ and a smaller γ) yield higher intrinsic rewards. The crowdsourcer therefore can take advantage of this to obtain a higher equilibrium participation level (Figure 2(b)) with a lower fixed extrinsic reward (Figure 2(a)), and her utility rises as well (Figure 2(c)). Neglecting network effects ($\theta = 0$) will potentially cause a significant amount of loss to the crowdsourcer.

An anecdotal real-world example that substantiates the above observations is that, according to its co-founder Elon Musk, when PayPal Inc. was started in 1998, a fixed monetary reward was paid for each user to sign up for the service. As the number of users grew, the amount of the reward was gradually reduced to zero, without affecting the growth of the user population due to network effects [11].

2.3 Flexible Extrinsic Reward Mechanism

In this section, we first analyze the users' strategic behavior in response to an extrinsic reward function, which leads to an equilibrium participation level. Given a targeted participation level, we can then derive the extrinsic reward function that helps the crowdsourcer achieve the highest possible utility (referred to as the conditional optimal extrinsic reward function). Finally, we compute the optimal participation level that maximizes the crowdsourcer's utility, and the corresponding global optimal extrinsic reward function.

2.3.1 Users' Optimal Effort Level. Given an extrinsic reward function instead of a fixed extrinsic reward, users will strategically choose their effort levels to maximize their utilities, rather than toggling between $\overline{x}$ and $\underline{x}$.

Proposition 4. ***Optimal Effort Level.*** *The optimal effort level of user i, based on her net cost α_i, i.e., $x_i = g(\alpha_i)$, is implicitly given by the following equation:*

$$E(n) + P(x_i) - \alpha_i g(\alpha_i) = \int_{\alpha_i}^{F^{-1}(n)} g(x)\mathrm{d}x. \tag{10}$$

Given a complicated extrinsic reward function $P(\cdot)$, it is difficult to solve equation (10) to obtain the optimal effort level function $g(\alpha)$. Fortunately, we show in the following section that, as the crowdsourcer intentionally designs $P(\cdot)$ for utility maximization, $g(\alpha)$ has a closed-form expression.

2.3.2 Optimal Extrinsic Reward Function. If a user cannot gain positive utility even with the optimal effort level, then she will not participate at all. Therefore, different extrinsic reward functions will result in different equilibrium participation levels. Based on this knowledge, the crowdsourcer can design the conditional optimal extrinsic reward function for a targeted participation level.

Proposition 5. ***Conditional Optimal Extrinsic Reward Function.*** *Given a targeted participation level n and the corresponding marginal user's net cost $\alpha_n = F^{-1}(n)$, under conditions that $\widetilde{\alpha} > 0$ and $E(n) \leq \underline{x}\alpha_n$, the crowdsourcer's conditional optimal extrinsic reward function, and the users' optimal effort level are given as follows, in which $\widehat{\alpha}$ satisfies $\widehat{\alpha} + \frac{F(\widehat{\alpha})}{f(\widehat{\alpha})} = \frac{\mu}{1+\underline{x}}$, and $\widetilde{\alpha}$ satisfies $\widetilde{\alpha} + \frac{F(\widetilde{\alpha})}{f(\widetilde{\alpha})} = \frac{\mu}{1+\overline{x}}$.*

- *If $\alpha_n \in [\underline{\alpha}, \widetilde{\alpha})$, then the conditional optimal extrinsic reward function is*

$$P(x) = \alpha_n x - E(n). \tag{11}$$

The users' optimal effort level is

$$g(\alpha) = \overline{x}, \alpha \in [\underline{\alpha}, \alpha_n]. \tag{12}$$

- *If $\alpha_n \in [\widetilde{\alpha}, \widehat{\alpha})$, then the conditional optimal extrinsic reward function is*

$$P(x) = g^{-1}(x)x - E(n) + \int_{x}^{g(\alpha_n)} \chi\,\tilde{}\,\mathrm{d}g^{-1}(\chi). \tag{13}$$

The users' optimal effort level is

$$g(\alpha) = \begin{cases} \overline{x}, & \alpha \in [\underline{\alpha}, \widetilde{\alpha}], \\ \frac{\mu f(\alpha)}{\alpha f(\alpha)+F(\alpha)} - 1, & \alpha \in [\widetilde{\alpha}, \alpha_n]. \end{cases} \tag{14}$$

In particular, $g^{-1}(\overline{x}) = \widetilde{\alpha}$.

- *If $\alpha_n \in [\widehat{\alpha}, \overline{\alpha}]$, then the conditional optimal extrinsic reward function is*

$$P(x) = g^{-1}(x)x - E(n) + \int_{x}^{g(\alpha_n)} \chi\,\tilde{}\,\mathrm{d}g^{-1}(\chi) + \underline{x}(\alpha_n - \widehat{\alpha}). \tag{15}$$

The users' optimal effort level is

$$g(\alpha) = \begin{cases} \overline{x}, & \alpha \in [\underline{\alpha}, \widetilde{\alpha}], \\ \frac{\mu f(\alpha)}{\alpha f(\alpha)+F(\alpha)} - 1, & \alpha \in [\widetilde{\alpha}, \widehat{\alpha}], \\ \underline{x}, & \alpha \in [\widehat{\alpha}, \alpha_n]. \end{cases} \tag{16}$$

In particular, $g^{-1}(\overline{x}) = \widetilde{\alpha}$ and $g^{-1}(\underline{x}) = \widehat{\alpha}$.

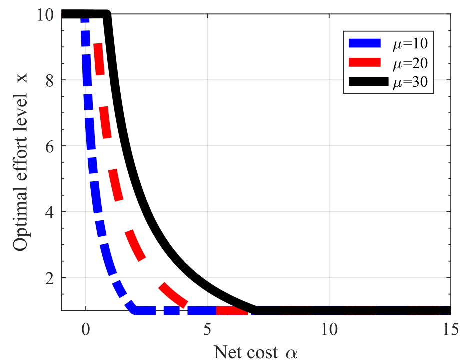


Fig. 3. The user's optimal effort level $g(\alpha)$. $x \in [1, 10], F(\cdot) \sim \mathrm{UNIF}(-1, 15)$.

Corollary 1. *The conditional optimal extrinsic reward function $P(\cdot)$ is non-negative and monotonically increasing.*

In Proposition 5, conditions $\widetilde{\alpha} > 0$ and $E(n) \leq \underline{x}\alpha_n$ are both reasonable. $\widetilde{\alpha}$ is the threshold net cost, below which a user will make the maximum effort $\overline{x}$. Users with negative net costs—and those with low positive net costs as well—will exert the maximum effort, with certain extrinsic rewards. Therefore, $\widetilde{\alpha}$ is assumed to be positive. $E(n) \leq \underline{x}\alpha_n$ indicates that network effects cannot fully cover the cost of the marginal user who makes the minimum effort; otherwise, the crowdsourcer will not provide any extrinsic rewards to them.

Note that the conditional optimal extrinsic reward functions, Equations (11), (13), and (15), are functions of users' effort level x, which is observable by the crowdsourcer, but not users' net cost α, which is private information. $P(\cdot)$ also depends on α_n, which is known by the crowdsourcer as the participation level n is set as the target by the crowdsourcer. Furthermore, the crowdsourcer is aware of users' response to the extrinsic reward function, and can use Proposition 5 to derive function $g(\cdot)$, which is indispensible in determining $P(\cdot)$. In the conditional optimal extrinsic reward function, the term $\alpha_n x$ or $g^{-1}(x)x$ can be regarded as the compensation for the users' cost; the term $-E(n)$ shows how network effects help curtail the crowdsourcer's payment to users; the rest of the terms are necessary to realize the targeted participation level. More specifically, it is ensured that $u_i > 0, \forall \alpha_i < \alpha_n, u_i < 0, \forall \alpha_i > \alpha_n$ and $\alpha_i = 0, \alpha_i = \alpha_n$. Interestingly, a user's optimal effort level is not affected by network effects, which are counteracted by the second term of extrinsic reward functions.

Corollary 2. ***Strict individual rationality.*** *With the extrinsic reward functions given by Proposition 5, every participant receives strictly positive utility, i.e., $u_i > 0, \forall \alpha_i \in [\underline{\alpha}, \alpha_n)$. In particular, the marginal user's utility is zero, i.e., $u_i = 0, \alpha_i = \alpha_n$.*

As the net cost increases, a user's optimal effort level declines, as shown in Figure 3. μ reflects the crowdsourcer's appreciation for users' contributions. If μ is higher, then the crowdsourcer is willing to elicit more user contributions with higher extrinsic rewards.

2.3.3 Optimal Participation Level. Proposition 5 gives the conditional optimal extrinsic reward function for a targeted participation level. By comparing the crowdsourcer's utility under each participation level with the conditional optimal extrinsic reward function, we can find the most lucrative participation level, and the corresponding global optimal extrinsic reward function.

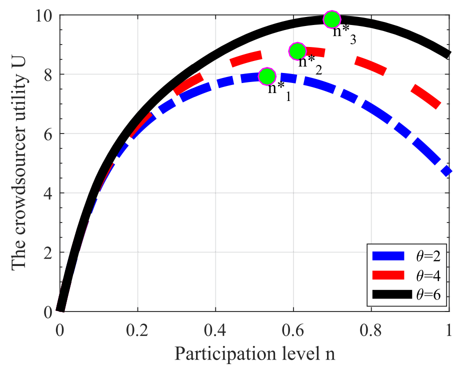


Fig. 4. The crowdsourcer's utility under different participation levels with conditional optimal extrinsic reward function. $\gamma = 1/2, x \in [1, 10], F(\cdot) \sim \text{UNIF}(-1, 15)$.

Proposition 6. ***Optimal Participation Level.*** *The optimal participation level is $n^* = \arg\max_n U$, in which*

$$U = \begin{cases} n\Big[\mu \ln(1+\overline{x}) - \overline{x}\alpha_n + E(n)\Big], & n \in [0, F(\widetilde{\alpha})), \\ U_A + \int_{\widetilde{\alpha}}^{\alpha_n} \Big[\mu \ln(1+g(\alpha)) - \\ (\alpha + \frac{F(\alpha)}{f(\alpha)})g(\alpha)\Big] \mathrm{d}F(\alpha) + E(n)n, & n \in [F(\widetilde{\alpha}), F(\widehat{\alpha})), \\ U_B + n\Big[\mu \ln(1+\underline{x}) + E(n) - \underline{x}\alpha_n\Big], & n \in [F(\widehat{\alpha}), 1], \end{cases} \tag{17}$$

in which $U_A = [\mu \ln(1+\overline{x}) - \overline{x}\widetilde{\alpha}]F(\widetilde{\alpha})$, $U_B = F(\widetilde{\alpha})[\mu \ln(1+\overline{x}) - \overline{x}\widetilde{\alpha}] - F(\widehat{\alpha})[\mu \ln(1+\underline{x}) - \underline{x}\widehat{\alpha}] + \int_{\widetilde{\alpha}}^{\widehat{\alpha}} [\mu \ln(1+x(\alpha)) - x(\alpha)\alpha - \frac{F(\alpha)}{f(\alpha)} g(\alpha)] \mathrm{d}F(\alpha)$.

The key idea of Proposition 6 is to achieve the participation level that maximizes the crowdsourcer's utility. Figure 4 illustrates the attainable utility under each participation level with the conditional optimal extrinsic reward function. The optimal participation level rests at the peak of each curve.

Figure 5 shows the optimal extrinsic reward function, the equilibrium participation level, and the crowdsourcer's utility under the flexible extrinsic reward mechanism. The flexible extrinsic reward mechanism remunerates users for different levels of contributions, as shown in Figure 5(a). It can be observed that the extrinsic reward function is concave, that is, a user's extrinsic reward increases with her effort level, but the marginal return decreases. Similar to Figure 2, network effects boost the equilibrium participation level and the crowdsourcer's utility, as shown in Figures 5(b) and 5(c).

2.3.4 Fixed vs. Flexible Extrinsic Reward Mechanisms. The flexible extrinsic reward mechanism is more efficient than the fixed extrinsic reward mechanism, as verified by Figure 6. Although the flexible mechanism requires more disbursement from the crowdsourcer than the fixed mechanism to provide the extrinsic rewards to incentivize users (Figure 6(a)), it induces a higher user contribution level in return. With the fixed mechanism, most participants will only provide a minimum level of effort; while with the flexible mechanism, participants are stimulated to work harder in exchange for higher extrinsic rewards. As a result, the crowdsourcer has a higher overall utility with the flexible mechanism, as shown in Figure 6(b). Interestingly, users also have a higher aggregate utility with the flexible mechanism, since the flexible mechanism gives a higher payment and motivates more users to participate. Both extrinsic rewards and intrinsic rewards induced by network effects are augmented. This suggests that the interests of users and the crowdsourcer are

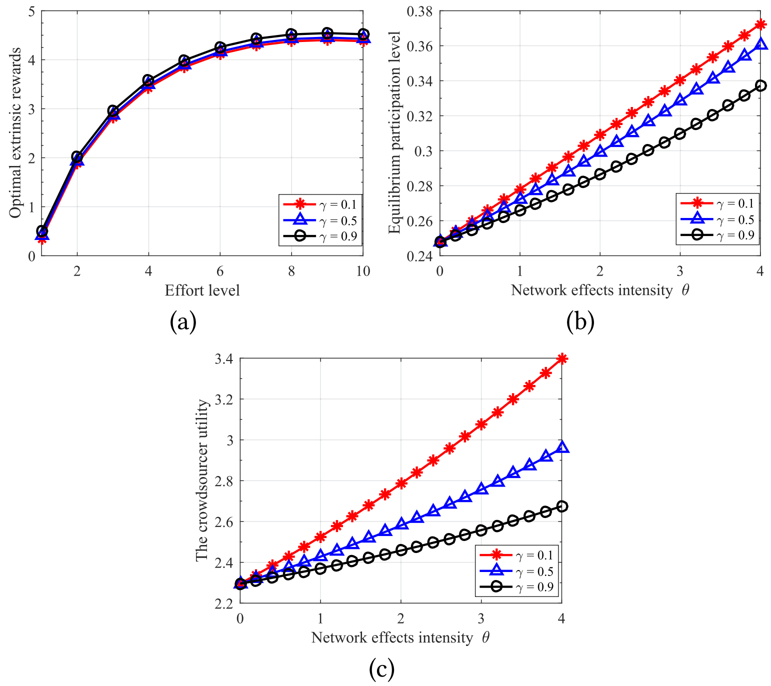


Fig. 5. Flexible extrinsic reward mechanism. $x \in [1, 10], F(\cdot) \sim \mathrm{UNIF}(-1, 15)$.

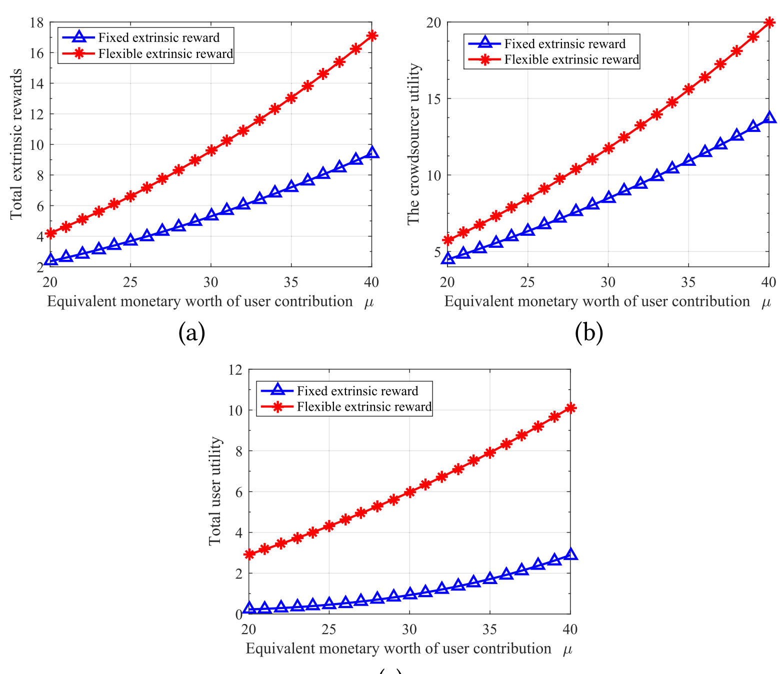


Fig. 6. Fixed vs. flexible extrinsic reward mechanisms $\theta = 1, \gamma = 1/2, F(\cdot) \sim \mathrm{UNIF}(-1, 20)$.

not necessarily in conflict with each other. Quite the contrary, a thriving crowdsourcing system with a higher participation level and a higher user contribution level will be valuable to both users and the crowdsourcer. Note that in Propositions 3 and 5, to compute the optimal fixed or flexible extrinsic rewards, the crowdsourcer needs to determine the equivalent monetary worth of users' contributions μ, and estimate the maximum user effort $\overline{x}$, the minimum user effort $\underline{x}$ and the cumulative distribution function of the user type $F(\alpha)$. The crowdsourcer can choose a proper μ to balance the monetary cost and the expected user contribution. From historical data on user contributions, the crowdsourcer can determine the maximum and minimum user effort $\overline{x}$ and $\underline{x}$. It is more difficult to obtain the cumulative distribution function of user types. The crowdsourcer may conduct measurement study or collect information through questionnaires to help derive $F(\alpha)$.

3 HETEROGENEOUS NETWORK EFFECTS

In this section, we analyze the scenario where the intrinsic rewards from network effects are heterogeneous for different users.

3.1 System Model

In Section 2, we assume that the network effect is $E(n) = \theta n^{\gamma}$, which is homogeneous for everyone, and only depends on the number of participants in the crowdsourcing system. However, different users gain different intrinsic rewards through network effects as they have different social relationships with others. Heterogeneous crowdsourcing has been well studied in existing works [16]. In this article, we focus on the influence of heterogeneous network effects on intrinsic reward. Social relationship plays a crucial role in both crowdsourcing and network effect model. In our settings, users are heterogeneous not only in terms of their effort level but also regarding their social relationship with other users. More specifically, the heterogeneous network effect enjoyed by users not only depends on her own effort level x_i but also the effort levels of other users, especially of those who have a significant influence on her, i.e., users with a large g_{ik}. A user who is more susceptible to other users' influence will benefit more from network effects. If user i inputs no effort, i.e., $x_i = 0$, then she will not be able to benefit from network effects. We assume that g_{ik} is public information, and the crowdsourcer can construct the social relationship matrix G, in which the entry of the ith row and the jth column is g_{ij}. Instead of a linear function, in this section, we adopt a convex quadratic function for the net cost as $\alpha_i x_i + \beta_i x_i^2$, in which α_i and β_i are positive parameters. This is reasonable, since the economic law of diminishing returns leads to increasing marginal costs. In this article, we assume that α_i and β_i are known by the crowdsourcer. If α_i and β_i are unknown, then we can easily leverage the Baysian game model, and use the distribution of α_i and β_i to compute their expected values.

User i's utility u_i is the sum of the intrinsic rewards from the network effects and the extrinsic rewards from the crowdsourcer, minus her net cost:

$$u_i = x_i \cdot \sum_{k=1}^{N} g_{ik} x_k + P(x_i) - \left(\alpha_i x_i + \beta_i x_i^2\right). \tag{18}$$

The crowdsourcer's utility is the aggregated contribution from all users minus the total extrinsic rewards[4]:

$$U = \sum_{i=1}^{N} (\mu x_i - P(x_i)). \tag{19}$$

[4]For tractability, we assume that the contribution from user i equals her effort level x_i, but not $\ln(1 + x_i)$ as in the case of homogeneous network effects in Section 2. A more complicated contribution function will be the future direction.

Here, we use the summation, i.e., $\sum$, rather than the integration, i.e., $\int$, because the parameters in users' net cost functions, i.e., α_i and β_i, are known, and we calculate users' true costs instead of their expected costs. For simplicity, we assume that a user's extrinsic reward is a linear function of her effort level, $P(x_i) = p_i x_i$. In fixed extrinsic reward mechanism, p_i is the same for all users, i.e., $p_i = p, \forall i$, while in flexible extrinsic reward mechanism, p_i is different for different users.

In the following analysis, we first analyze the flexible extrinsic reward mechanism, then study the fixed extrinsic reward mechanism. The former is simpler than the latter in the case of heterogeneous network effects, since participation level variation is easier to derive under the flexible mechanism.

3.2 Flexible Extrinsic Reward Mechanism

Flexible extrinsic reward mechanism fully differentiates different users according to their costs and social relationships with other users. It is an efficient way for the crowdsourcer to exploit the potential of each and every user.

3.2.1 Equilibrium User Effort Level. Being informed of her extrinsic reward p_i, user i strategically decides her effort level x_i. Since user i's utility function (18) depends on the effort levels of other users, her best effort level is also affected by the choice of other users.

Proposition 7. ***Optimal Effort Level.*** *The optimal effort level x_i^* of user i is a function of effort levels of other users $\boldsymbol{x}_{-i}$, the extrinsic reward p_i, and her cost parameters α_i, β_i:*

$$x_i^* = \max\left\{\frac{\sum_{k=1}^{N} g_{ik}x_k + p_i - \alpha_i}{2\beta_i}, 0\right\}. \tag{20}$$

Proof. The first derivative of user i's utility function Equation (18) is

$$\frac{\partial u_i}{\partial x_i} = \sum_{k=1}^{N} g_{ik}x_k + p_i - \alpha_i - 2\beta_i x_i.$$

The second derivative of user i's utility function Equation (18) is

$$\frac{\partial^2 u_i}{\partial x_i^2} = -2\beta_i < 0.$$

Thus, the utility function Equation (18) is maximized when $\frac{\partial u_i}{\partial x_i} = 0$, which yields the optimal effort level in Equation (20). □

A user's optimal effort level will change if other users alter their effort levels, but when every user adopts their effort levels according to Equation (20), the system will reach an equilibrium, i.e., no user is willing to unilaterally change her effort level. Define matrix $B \in \mathbb{R}^{n\times n}$, in which $B_{ij} = 2\beta_i$, if $i = j$; otherwise, $B_{ij} = 0$.

Proposition 8. ***Equilibrium Effort Level.*** *Let S denote the set of users with positive effort level, i.e., $x_i^* > 0$. If the condition $2\beta_i > \sum_{k=1}^{n} x_{ki}, \forall i$ is satisfied, given the crowdsourcer's extrinsic reward mechanism $\boldsymbol{p}$, then the equilibrium effort levels of users in S are*

$$\boldsymbol{x}_S = (B_S - G_S)^{-1}(\boldsymbol{p}_S - \boldsymbol{\alpha}_S), \tag{21}$$

in which $\boldsymbol{x}_S, \boldsymbol{\alpha}_S, \boldsymbol{p}_S$ are the (column) vectors of $x_i, a_i, p_i, i \in S$, respectively; B_S and G_S are submatrices of B and G, consisting of users in S. The equilibrium effort levels of users not in S are $x_i = 0, \forall i \notin S$.

Proof. For users in S, we have

$$2\beta_i x_i = \sum_{k=1}^{N} g_{ik} x_k + p_i - \alpha_i.$$

Thus, we have

$$\begin{aligned} B_S \boldsymbol{x}_S &= G_S \boldsymbol{x}_S + \boldsymbol{p}_S - \boldsymbol{\alpha}_S, \\ (B_S - G_S)\boldsymbol{x}_S &= \boldsymbol{p}_S - \boldsymbol{\alpha}_S. \end{aligned}$$

If the condition $2\beta_i > \sum_{k=1}^{N} g_{ik}, \forall i$ is satisfied, then $(B_S - G_S)$ is invertible. Hence, we have the equilibrium effort levels of users in S as Equation (21). □

According to Equation (21), the optimal effort level of a user increases with the extrinsic reward p_i, but decreases with her cost parameters α_i and β_i.

3.2.2 Optimal Flexible Extrinsic Rewards. Having derived users' equilibrium effort level, now we can turn to the crowdsourcer. Before we obtain the optimal flexible extrinsic reward mechanism, we have the following lemma.

Lemma 2. *If conditions $2\beta_i > \sum_{k=1}^{N} g_{ik}, \forall i$ and $\mu > \alpha_i, \forall i$ are satisfied, then the optimal extrinsic rewards $\boldsymbol{p}$ will induce every user to participate in the crowdsourcing system, i.e., $x_i^* > 0, \forall i$.*

Proof. We prove by contradiction. Assume that under the optimal extrinsic rewards $\boldsymbol{p}^*$, there exists user i whose equilibrium effort level is $x_i^* = 0$. We will construct another extrinsic rewards $\boldsymbol{p}'$ by increasing p_i and decreasing $p_j, \forall j \neq i$. Under $\boldsymbol{p}'$, user i will participate with a positive effort level, and other users will not change their effort levels, so that the utility of the crowdsourcer will increase. The newly constructed extrinsic rewards $\boldsymbol{p}'$ is

$$\begin{aligned} p_i' &= \alpha_i + \delta, \\ p_j' &= p_j^* - \frac{g_{ji}}{2\beta_i}\left(p_i' - \alpha_i + \sum_{k=1}^{N} g_{ik} x_k^*\right), \forall j \neq i, \end{aligned}$$

in which $0 < \delta < \mu - \alpha_i$. Therefore, the equilibrium users' effort levels are

$$\begin{aligned} x_i' &= \frac{p_i' - \alpha_i + \sum_{k=1}^{N} g_{ik} x_k'}{2\beta_i}, \\ x_j' &= \max\left\{\frac{p_j' - \alpha_j + \sum_{k=1}^{N} g_{jk} x_k'}{2\beta_j}, 0\right\}. \end{aligned}$$

We have $x_i' > 0$ as $p_i' > \alpha_i$. Now we will prove that all the other users' equilibrium effort levels are the same under $\boldsymbol{p}$ and $\boldsymbol{p}'$.

Look at a particular user j. Suppose that none of the other users changes her effort level, i.e., $x_k' = x_k^*, \forall k \neq i, k \neq j$, we have

$$\begin{aligned} x_j' &= \max\left\{\frac{p_j' - \alpha_j + \sum_{k=1}^{N} g_{jk} x_k'}{2\beta_j}, 0\right\} \\ &= \max\left\{\frac{p_j' - \alpha_j + g_{ji} x_i' + \sum_{k=1, k\neq i}^{N} g_{jk} x_k'}{2\beta_j}, 0\right\} \\ &= \max\left\{\frac{p_j^* - \alpha_j}{2\beta_j} - \frac{1}{2\beta_j}\frac{g_{ji}}{2\beta_i}\left(p_i' - \alpha_i + \sum_{k=1}^{N} g_{ik} x_k^*\right)\right. \end{aligned}$$

$$
\begin{aligned}
&+ \frac{g_{ji}}{2\beta_j}\frac{p_i' - \alpha_i + \sum_{k=1}^{N} g_{ik}x_k'}{2\beta_i} + \frac{\sum_{k=1,k\neq i}^{N} g_{jk}x_k'}{2\beta_j}, 0\Bigg\} \\
= \max &\Bigg\{\frac{p_j^* - \alpha_j + \sum_{k=1,k\neq i}^{N} g_{jk}x_k'}{2\beta_j} \\
&+ \frac{g_{ji}}{4\beta_j\beta_i}\sum_{k=1}^{n} g_{ik}(x_k' - x_k^*), 0\Bigg\} = x_j^*.
\end{aligned}
$$

This shows that the effort level of user i becomes positive while other users do not change their effort levels.

The crowdsourcer's utility becomes

$$U' = (\mu - p_i')x_i' + \sum_{j=1,j\neq i} (\mu - p_j')x_j^*.$$

We have $\mu > p_i'$ and $p_j' < p_j$, therefore, $U' > U^*$. This contradicts the assumption that extrinsic rewards $\boldsymbol{p}^*$ is optimal. Therefore, the optimal extrinsic rewards will ensure that every user participates with a positive effort level. □

Let $\mathbf{1}$ denote the (column) vector whose entries are all 1. We present the optimal flexible extrinsic reward mechanism as follows.

Proposition 9. ***Optimal Flexible Extrinsic Rewards.*** *If conditions $2\beta_i > \sum_{k=1}^{N} g_{ik}, \forall i$ and $\mu > \alpha_i, \forall i$ are satisfied, then the optimal flexible extrinsic rewards for the crowdsourcer is*

$$\boldsymbol{p} = \boldsymbol{\alpha} + (B - G)\left(B - \frac{G + G^T}{2}\right)^{-1} \frac{\mu\mathbf{1} - \boldsymbol{\alpha}}{2}. \tag{22}$$

A special case is, if G is a symmetric matrix, i.e., $g_{ik} = g_{ki}, \forall i, k$, then the optimal flexible extrinsic rewards for the crowdsourcer is

$$\boldsymbol{p} = \frac{\boldsymbol{\alpha} + \mu\mathbf{1}}{2}. \tag{23}$$

Proof. According to Lemma 2, every user inputs a positive effort level, so we have

$$(B - G)\boldsymbol{x}^* = \boldsymbol{p} - \boldsymbol{\alpha}. \tag{24}$$

More specifically, we have $x_i^* = (\sum_{k=1}^{N} g_{ik}x_k^* + p_i - \alpha_i)/(2\beta_i)$, which yields $p_i = 2\beta_i x_i^* + \alpha_i - \sum_{k=1}^{N} g_{ik}x_k^*$. Substitute p_i in Equation (3), we have

$$U = \sum_{i=1}^{N}\left[\mu x_i^* - \left(2\beta_i x_i^* + \alpha_i - \sum_{k=1}^{N} g_{ik}x_k^*\right)x_i^*\right].$$

To maximize U through $\boldsymbol{p}$ is equivalent to maximizing U through $\boldsymbol{x}^*$ induced by $\boldsymbol{p}$. Therefore, we have

$$\frac{\partial U}{\partial x_i^*} = \mu - \left(4\beta_i x_i^* + \alpha_i - \sum_{k=1}^{N} g_{ik}x_k^*\right) + \sum_{k=1}^{N} g_{ki}x_k^* = 0.$$

We have

$$
\begin{aligned}
\mu\mathbf{1} - \boldsymbol{\alpha} &= (2B - (G + G^T))\boldsymbol{x}^*, \\
\Rightarrow \boldsymbol{x}^* &= \left(B - \frac{G + G^T}{2}\right)^{-1} \frac{\mu\mathbf{1} - \boldsymbol{\alpha}}{2}.
\end{aligned}
$$

Substitute $\boldsymbol{x}^*$ in Equation (24) and we can get the optimal flexible extrinsic rewards as Equation (22). □

According to Proposition 9, if G is symmetric, the optimal extrisinc rewards are irrelavent of the network effects. The intuition is as follows. On the one hand, the crowdsourcer intends to subsidize those users who have a significant influence on other users through network effects, e.g., user i with large $g_{ki}, \forall k \neq i$. On the other hand, the crowdsourcer can cut extrinsic rewards to those users who receive considerable intrinsic rewards through network effects, e.g, user i with a large $g_{ki}, \forall k \neq i$. When G is symmetric, these two opposing effects cancel out each other. Proposition 9 also indicates that the crowdsourcer will provide higher extrinsic rewards to users with higher costs, i.e., larger α_i.

With the help of the network effects, the crowdsoucer can incentivize users with lower extrinsic rewards. We give an example to show that the optimal extrinsic rewards given by Equation (22) may be less than the costs of users. Assume there are three users in the crowdsourcing system, with $\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 3$, $\beta_1 = \beta_2 = \beta_3 = 20$, and $\mu = 100$. The social relationship among users are

$$G = \begin{pmatrix} 0 & 10 & 10 \\ 1 & 0 & 10 \\ 20 & 10 & 0 \end{pmatrix}.$$

According to Equation (22), we can calculate the optimal extrinsic rewards $\boldsymbol{p}^* = (54.03, 62.61, 38.61)$, and the equilibrium users effort levels $\boldsymbol{x}^* = (2.58, 2.27, 2.75)$. The crowdsoucer's utility is 371.88, and users' utilities are $u_1 = 133.01, u_2 = 102.71, u_3 = 150.82$. The extrinsic reward for user 3 is the fewest, yet user 3 inputs the highest effort level. This is because user 3 can receive a higher intrinsic reward from other users through network effects. We can check that the cost for user 3 is $1 * 2.75 + 20 * 2.75^2 = 154.00$, higher than the compensation $38.61 * 2.75 = 106.18$ from the crowdsourcer. This shows that the intrinsic rewards from the network effects help the crowdsourcer incentivize users with less expenditure.

3.3 Fixed Extrinsic Reward Mechanism

When the extrinsic reward is fixed, i.e., $p_i = p, \forall i$, the utility function of user i becomes

$$u_i = x_i \cdot \sum_{k=1}^{N} g_{ik} x_k + p x_i - \left(\alpha_i x_i + \beta_i x_i^2\right). \tag{25}$$

The crowdsourcer's utility function becomes

$$U = \sum_{i=1}^{N} (\mu - p) x_i. \tag{26}$$

3.3.1 Optimal User's Effort Level. Similar to Proposition 7, users' optimal effort levels are as follows.

Proposition 10. ***Optimal Effort Level.*** *The optimal effort level x_i^* of user i is a function of effort levels of other users $\mathbf{x}_{-i}$, the fixed extrinsic reward p, and her cost parameters α_i, β_i:*

$$x_i^* = \max\left\{\frac{\sum_{k=1}^{n} g_{ki} x_k + p - \alpha_i}{2\beta_i}, 0\right\}. \tag{27}$$

Lemma 3. *User i's optimal effort level x_i^* is an increasing function of the fixed extrinsic reward p. In particular, if $x_i^* > 0$, x_i^* is a strictly increasing function of the fixed extrinsic reward p.*

We ignore the proof of Proposition 10 as it is similar to the proof of Proposition 7. Lemma 3 is naturally true according to Proposition 10.

3.3.2 Optimal Fixed Extrinsic Reward. If all users are motivated by the fixed extrinsic reward p to participate in the crowdsourcing system, then we have

$$\boldsymbol{x}^* = (B - G)^{-1}(p\mathbf{1} - \boldsymbol{\alpha}).$$

Substitute $\boldsymbol{x}$ in the crowdsourcer's utility function Equation (26), we have

$$U = (\mu - p)\mathbf{1}^T(B - G)^{-1}(p\mathbf{1} - \boldsymbol{\alpha}).$$

Taking the first derivative, we can get the optimal fixed extrinsic reward as

$$p^* = \frac{\mathbf{1}^T(B - G)^{-1}(\boldsymbol{\alpha} + \mu\mathbf{1})}{2\mathbf{1}^T(B - G)^{-1}\mathbf{1}}. \tag{28}$$

Unfortunately, the fixed extrinsic reward in Equation (28) is optimal only if the crowdsourcer deliberately intends to reach a full participation level. It is possible that the crowdsourcer can gain a higher utility by involving only part of the users; in other words, the crowdsourcer may be better off with a certain targeted set of participants. Let $\mathcal{I} = \{1, 2, \ldots, N\}$ denote the entire set of users. If the extrinsic reward is high enough, then every user in $\mathcal{I}$ will participate with positive effort levels. As the extrinsic reward gradually decreases, more and more users will leave the crowdsourcing system.

Lemma 4. *Define*

$$\begin{aligned}
i_1 &= \arg\max_{i \in \mathcal{I}} \frac{\left((I - GB^{-1})^{-1}\boldsymbol{\alpha}\right)_i}{\left((I - GB^{-1})^{-1}\mathbf{1}\right)_i}, \\
p_1 &= \max_{i \in \mathcal{I}} \frac{\left((I - GB^{-1})^{-1}\boldsymbol{\alpha}\right)_i}{\left((I - GB^{-1})^{-1}\mathbf{1}\right)_i}, \\
S_1 &= \mathcal{I} \setminus \{i_1\}, \\
i_k &= \arg\max_{i \in S_k} \frac{\left((I - G_{S_k}B_{S_k}^{-1})^{-1}\boldsymbol{\alpha}_{S_k}\right)_i}{\left((I - G_{S_k}B_{S_k}^{-1})^{-1}\mathbf{1}\right)_i}, \\
p_k &= \max_{i \in S_k} \frac{\left((I - G_{S_k}B_{S_k}^{-1})^{-1}\boldsymbol{\alpha}_{S_k}\right)_i}{\left((I - G_{S_k}B_{S_k}^{-1})^{-1}\mathbf{1}\right)_i}, \\
S_{k+1} &= S_k \setminus \{i_k\},
\end{aligned}$$

in which I is the identical matrix, $(\cdot)_i$ is the ith entry of the vector. We have:

(1) $p_{k+1} \le p_k, \forall k$.
(2) If $p > p_1$, then every user will participate in the crowdsourcing system. If $p \in [p_{k+1}, p_k]$, then users in S_k will participate, while users in $\mathcal{I} \setminus S_k$ will quit.

In Lemma 4, p_1 is the minimum extrinsic reward to guarantee a full participation. As the crowdsoucer reduces the extrinsic reward from p_k to p_{k+1}, user i_k's effort level will become zero. In other words, a minimum extrinsic reward of p_k is required to keep user i_k to in the crowdsourcing system.

Proof. Assume that users in set S will choose to join the crowdsourcing system, we have

$$\boldsymbol{x}_S = (B_S - G_S)^{-1}(p\mathbf{1} - \boldsymbol{\alpha}_S) = B_S(I - G_S B_S^{-1})^{-1}(p\mathbf{1} - \boldsymbol{\alpha}_S).$$

For user i in S, we have

$$x_i = \frac{p\Big((I - G_S B_S^{-1})^{-1}\Big)_i - \Big((I - G_S B_S^{-1})^{-1}\boldsymbol{\alpha}_S\Big)_i}{2\beta_i}.$$

If the extrinsic reward $p > p_1$ and $S = \mathcal{I}$, then we can check that $x_i > 0, \forall i \in \mathcal{I}$. If the extrinsic reward drops to p_1, then user i_1 will stop making any effort. Furthermore, if the extrinsic reward changes from p_k to p_{k+1}, user i_k's effort level will change from positive to zero. Since user's effort level increases with the extrinsic reward according to Lemma 3, it must be true that $p_{k+1} < p_k$. The second claim is true due to the construction of the series of p_k. □

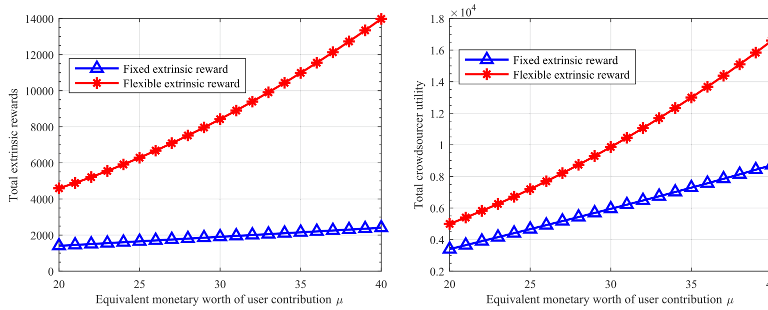
Lemma 4 inspires us to propose Algorithm 1 to find the optimal fixed extrinsic reward by iteratively decreasing the extrinsic rewards and shrinking the participating user set. In the kth iteration, we assume that the extrinsic reward falls into the region $[p_{k+1}, p_k]$. Being aware that users in S_k will have positive effort levels, we compute the conditional optimal extrinsic reward $p^*_{S_k}$ regarding only the users in S_k. If $p^*_{S_k}$ exceeds the bound of $[p_{k+1}, p_k]$, then we adopt p_{k+1} or p_k as the conditional optimal extrinsic reward for users in S_k. If the utility of the crowdsoucer is improved by $p^*_{S_k}$, then we update the optimal extrinsic reward as $p^*_{S_k}$. Such iterations continue until the targeted user set S_k becomes zero.

Note that in Proposition 9 and Algorithm 1, to obtain the optimal fixed or flexible extrinsic rewards under heterogeneous network effects, apart from the equivalent monetary worth of users'

ALGORITHM 1: The Optimal Fixed Extrinsic Reward

Input: Social relationship matrix G, cost parameters $\alpha_i, \beta_i, \forall i$, equivalent monetary worth of users' contributions μ.
Output: The optimal fixed extrinsic reward p^* and the maximum crowdsourcer's utility U^*.

Initialize $k = 0, U^* = 0, S_k = \mathcal{I}, p_0 = +\infty$.
Calculate p_1 according to Lemma 4.
while S_k is not empty **do**
  Compute $p^*_{S_k} = \frac{\mathbf{1}^T(B_{S_k} - G_{S_k})^{-1}(\boldsymbol{\alpha} + \mu\mathbf{1})}{2\mathbf{1}^T(B_{S_k} - G_{S_k})^{-1}\mathbf{1}}$.
  if $p^*_{S_k} > p_k$ **then**
    $p^*_{S_k} = p_k$.
  else
    if $p^*_{S_k} < p_{k+1}$ **then**
      $p^*_{S_k} = p_k$.
    end if
  end if
  $U = (\mu - p^*_{S_k})\mathbf{1}^T(B_{S_k} - G_{S_k})^{-1}(p^*_{S_k}\mathbf{1} - \boldsymbol{\alpha})$.
  if $U > U^*$ **then**
    $p^* = p^*_{S_k}, U = U^*$.
  end if
  $k = k + 1$.
  Update S_k, p_k according to Lemma 4.
end while



(a) (b)

Fig. 7. Fixed vs. flexible extrinsic reward mechanism: $n = 300, G \sim \text{UNIF}(0, 1)_{n\times n}; \boldsymbol{\alpha}, \boldsymbol{\beta} \sim \text{UNIF}(0, 10)_n$.

contributions μ, the crowdsourcer has to know the social relationship matrix G and the cost parameters α_i, β_i. The social relationship matrix can be constructed based on users' interactions, e.g., a user may *befriend* or *follow* another user. The strength of their relationship g_{ij} can be inferred by the frequency of their interactions, e.g., comment or reply. The cost parameters α_i and β_i can be attained in a similar way as under the homogeneous network effects.

We use the example in Section 3.2 to explain the process of Algorithm 1. In the first round, we can compute that $p_1 = 95.00$. The conditional optimal extrinsic reward that solicits a positive effort from all users is 51.00, which is out of the range $[p_1, +\inf)$, so we adjust the conditional optimal extrinsic reward as 95.00. The utility of the crowdsourcer can be obtained as 70.93. In the second round, we can compute that $p_2 = 5.00$. When the extrinsic reward is less than 95.00, user 2 will leave the crowdsourcing system. The conditional optimal extrinsic reward, considering only user 1 and 3, is 50.95, and the corresponding utility of the crowdsourcer is 189.00, higher than 70.93. Thus, we update the global optimal extrinsic reward as 50.95. In the third iteration, further decreasing the extrinsic reward below p_2 but above $p_3 = 1.00$, user 1 will be the only remaining user who inputs positive effort. In this case, the optimal extrinsic reward is 50.5, exceeding p_2. Therefore, we adjust the conditional optimal extrinsic reward as 5.00, which leads to a utility of 9.5 for the crowdsourcer. As a result, the crowdsourcer is able to glean a maximum utility of 189.00 with the optimal extrinsic reward is $p^* = 50.95$. Users' effort levels are $\boldsymbol{x} = (1.77, 0, 2.08)$, and their utilities are $u_1 = 62.64, u_2 = 0, u_3 = 86.84$.

Recall that in Section 3.2, the optimal flexible extrinsic rewards are $\boldsymbol{p}^* = (54.03, 62.61, 38.61)$; users' effort levels are $\boldsymbol{x} = (2.58, 2.27, 2.75)$; the crowdsourcer's utility is 371.88; the users' utilities are $u_1 = 133.01, u_2 = 102.71, u_3 = 150.82$. This shows that the flexible extrinsic reward mechanism is more effective in motivating users to contribute to the crowdsourcing system. With the fixed extrinsic reward mechanism, the crowdsourcer has to give up user 2 who requires a higher extrinsic reward. In comparison, with the flexible extrinsic reward mechanism, the crowdsourcer is able to differentiate extrinsic rewards to different users, and she chooses to pay more to user 2 who has a high cost but exerts a considerable influence on other users through network effects. It is also verified that the flexible extrinsic reward mechanism leads to higher utilities of both the crowdsourcer and the users than the fixed extrinsic reward mechanism.

We compare the crowdsourcer's utility and the extrinsic reward between fixed and flexible extrinsic reward schemes under heterogeneous network effects. As shown in Figure 7, similar to the homogeneous network effect scenario, the flexible extrinsic reward scheme pays a higher total extrinsic reward while bringing a higher utility to the crowdsourcer, since the users are motivated to contribute more efforts.

4 RELATED WORK

Extrinsic rewards in crowdsourcing. Existing works have used reverse auctions and Stackelberg games to model extrinsic rewards. With reverse auctions, the crowdsourcer selects users based on the bids, which reflect users' anticipated extrinsic rewards [8, 12, 25]. The complexity of reverse auctions is high, making them impractical in real-world implementations. With Stackelberg games, the crowdsourcer determines the optimal extrinsic rewards, while users compete for these rewards by making strategic decisions on their contribution levels [6, 22]. Different from these two multi-winner mechanisms, in References [15, 17], the authors proposed a winner-take-all mechanism, where a single best or designated user gets all the extrinsic rewards. In contrast, we focused on the interplay between extrinsic and intrinsic rewards in this article, with a focus on the impact of network effects.

Network effects. Network effects have been extensively discussed in telecommunication networks [5], the open-source software community [3], and social networks [13]. As the crowdsourcing systems connect a large number of participants, network effects can be observed [23]. Due to network effects, users obtain higher intrinsic rewards when the total number of participants increases, thus requiring less extrinsic rewards to compensate their costs. However, there is a lack of existing works that take advantage of network effects for more efficient extrinsic rewards design.

Empirical studies on intrinsic and extrinsic rewards. User behavior under the influence of extrinsic and intrinsic rewards have been explored by some empirical studies. Schweizer et al. [24] have used the feedback for crowdsourcing tasks as a potential intrinsic reward. Anawar et al. [2] have adopted self-determination theory to explain intrinsic motivations in a weight loss crowdsourcing system. Competitive extrinsic rewards are found to be more efficient than fixed extrinsic rewards in References [18, 20, 21]. However, intrinsic rewards incurred by network effects, as well as how intrinsic and extrinsic rewards interact with each other, have not been examined in existing empirical studies.

5 CONCLUSION

In this article, we propose a new framework for extrinsic reward design in crowdsourcing systems, which exploits network effects and intrinsic rewards. Instead of assuming a fixed participant population, we show how user participation levels evolve as a result of the interactions between extrinsic rewards and network effects. We first consider the scenario where every user experiences homogeneous network effects depending on the total number of participants, then extend to the scenario where the network effects are heterogeneous for users with different social relationships. For each scenario, we propose a *fixed* and a *flexible* extrinsic reward mechanism, designed to help a crowdsourcer to enlist more users and attain a higher payoff by considering network effects. In particular, the flexible mechanism is more efficient in incentivizing users to partipate and contribute to crowdsourcing systems, improving the utilities of both users and the crowdsourcer. Simulations and examples have verified that the proposed extrinsic reward mechanisms have outperformed existing ones that did not take network effects and the corresponding intrinsic rewards into consideration.

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