Network Coding Aware Dynamic Subcarrier Assignment in OFDMA Wireless Networks

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***Abstract*—Taking advantage of the frequency diversity and multiuser diversity in OFDMA based wireless networks, dynamic subcarrier assignment mechanisms have shown to be able to achieve much higher downlink capacity than static assignment. A rich literature exists that proposes MAC and physical layer schemes aiming at exploiting the diversity gain with low implementation complexity. In this paper, we propose a cross layer approach that explores the joint advantage of network coding and dynamic subcarrier assignment. Our algorithm improves the bandwidth efficiency of OFDMA downlink by encoding frames of the mobile stations that exchange information. We highlight a tradeoff between diversity gain and the network coding advantage, which is critical to the network performance. To explore the tradeoff, we formulate the coding aware dynamic assignment scheme as a mixed integer program, and design a polynomial time heuristic that can be used in practical systems. Based on a network flow formulation and a penalty scheme, our heuristic well approximates the performance of an optimal algorithm, in terms of both throughput and fairness.**

I. INTRODUCTION

The emerging generation of wireless standards such as 802.16 [1] have identified OFDMA (Orthogonal Frequency Division Multiple Access) as a promising technology enabling broadband wireless access. In OFDMA systems, the prescribed frequency band is divided into hundreds of orthogonal subbands called *subcarriers*. The base station (BS) assigns disjunctive sets of subcarriers to mobile stations (MS) which multiplex the available downlink capacity. In the original 802.16 PHY specification, subcarriers are either statically or randomly allocated to the MSs, oblivious of their diverse channel conditions. In reality, however, the fading profiles vary across the whole frequency band, and even the same subcarrier experiences independent attenuation when assigned to MSs in different locations. Such multiuser diversity have motivated **d**ynamic **s**ubcarrier **a**ssignment (DSA) mechanisms, which deliberately match each downlink to the set of subcarriers supporting higher throughput. It has been observed that an optimal DSA algorithm can achieve up to twice higher downlink throughput compared with static assignment schemes [2]. A large body of work has also focused on suboptimal algorithms aiming at achieving similar performance at lower implementation complexity [2].

In this paper, we add a new dimension to the literature of DSA, proposing a cross layer approach towards **c**oding **a**ware **d**ynamic **s**ubcarrier **a**ssignment (CADSA). Taking advantage of network coding, our CADSA algorithm combines the downlink frames heading towards different MSs, and transmits them through the same set of subcarriers, thereby significantly improving the bandwidth efficiency of OFDMA systems. As an intuitive justification, consider the scenario in Fig. 1, where two MSs are exchanging information with each other via the BS, creating an opportunity for network coding (henceforth referred to as *coding opportunity*). Traditional assignment algorithms will allocate disjoint set of subcarriers to the downlinks. In contrast, the CADSA algorithm XORs the two uplink frames and multicasts the combined frame via the two downlinks. The corresponding MSs receive the same frame, but can decode different information by XORing the combined frame with one that is known *a priori*. For instance, through the operation $B \oplus (A \oplus B)$, MS$_1$ directly obtains frame A, which originated from MS$_2$.

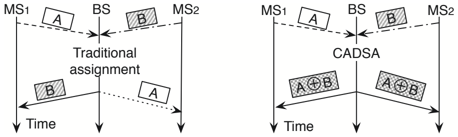

Fig. 1. The motivating scenario for coding aware subcarrier assignment in OFDMA systems. Different line patterns denote disjoint sets of subcarriers.

In an ideal case where all downlinks have coding opportunities and the subcarriers have uniform channel gains for all MSs, CADSA can save half of the subcarriers, achieving a two-fold increase in capacity, compared with traditional assignment algorithms. However, the benefits of network coding diminish in case of high multiuser diversity, when sharing the same subcarrier may result in underutilized bandwidth. For instance, if MS$_1$ in Fig. 1 is farther to the BS than MS$_2$ and has much lower channel gain, the capacity of both downlinks is bounded by the achievable rate of the downlink to MS$_1$. In such cases, it may be more preferable to assign subcarriers separately, in order to maintain bandwidth efficiency.

To quantify the benefits of network coding in practical fading environment, we formulate an optimization framework that provides upper bounds on the performance of CADSA. In view of its high complexity, we propose a suboptimal heuristic that achieves similar downlink capacity and fairness. Compared with traditional assignment schemes, our CADSA algorithm achieves much higher downlink rates, especially when the downlinks experience uniformly high SNR. In addition, we introduce a scheduling and coding mechanism to CADSA such that no additional overhead is induced compared with existing dynamic mechanisms for subcarrier assignment.

This paper proceeds as follows. In Sec. II, we review existing work on subcarrier assignment algorithms and network coding protocols in wireless networks. In Sec. III, we introduce the network models, as well as the scheduling and

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coding algorithm for CADSA. We continue to formulate the optimization framework and describe the heuristic algorithm in Sec. IV, and then evaluate their performance in Sec. V. Finally, Sec. VI concludes the paper.

II. RELATED WORK

Dynamic mechanisms for resource allocation in OFDMA downlink have been extensively investigated in literature. Existing algorithms are centered around two optimization frameworks: maximizing the sum capacity subject to power and fairness constraints, or minimizing the power budget subject to per-link rate and fairness constraints. Both optimization problems are essentially mixed-integer programs proven to be NP-hard [2]. Instead of tracking the optimal solution with exponential complexity, many suboptimal algorithms have been proposed. These algorithms generally involve two aspects: the subcarrier assignment and the power allocation. Subcarrier assignment schemes generally match each downlink with a set of subcarriers with high channel gains (see, *e.g.*, [3], [4]). Power allocation algorithms adaptively assigns transmission power to each subcarrier, which adjusts its modulation type according to the SNR at the receiver side. Most of the above algorithms reside in the MAC and PHY layers, without taking advantage of the network level paradigms such as the scenario in Fig. 1. A comprehensive survey of dynamic mechanisms in OFDMA networks has been presented in [2], to which we refer the readers for more details.

XOR network coding has already been implemented in 802.11 wireless mesh networks to improve the unicast throughput [5]. The basic idea is to locally search for coding opportunities, and XOR packets heading towards different next-hops, based on prior knowledge of whether they can be decoded. The seminal work has been followed by many other analysis and protocols. For example, [6] studied the joint design of network coding and routing, and quantified its optimal performance. This line of research has mostly focused on the 802.11 models. The coding advantage in a multichannel system like OFDMA has not been exploited.

III. SYSTEM MODELS

In this section, we introduce the underlying network models for CADSA. In addition, we describe the coding and scheduling algorithm used by CADSA.

A. Network Models

We consider a 802.16 cellular network where the base station can serve as an intermediate relay for MSs located in the same cell. Packets are transmitted from one MS to the BS through the uplink, and then switched to another MS via the downlink. We refer to such an end-to-end network flow as a *session*. When multiple sessions co-exist, it becomes critical to allocate subcarriers to the uplink and downlink of each session, in order to maximize the total network throughput while maintaining fairness. Such single-cell switching network models can be seen as a decomposition of multi-hop multi-cell OFDMA networks, such as 802.16j based wireless mesh network and its extensions.

We model the wireless fading environment by large scale path-loss and shadowing, along with small scale Rayleigh fading effects. Due to the multiuser diversity, the achievable rate of a subcarrier depends not only on its fading profile, but also on which link it is assigned to, and how much power it has been allocated by the BS. It has been observed that dynamic power allocation schemes achieve marginal performance gain, especially with small attenuation spread among different MSs [2], [3]. Therefore, we only focus on the CADSA with equal power allocation, *i.e.*, all subcarriers equally share the power budget, and perform AMC according to the received SNR.

B. Frame Scheduling and Network Coding Algorithm for CADSA

We assume the system is operating at TDD mode, *i.e.*, the uplinks and downlinks are activated alternately. In both uplink and downlink phase, the entire set of subcarriers are allocated to all sessions. As in most existing work [2], however, we only focus on the downlink subcarrier allocation. Specifically, before each downlink phase, the BS performs subcarrier assignment and XOR network coding simultaneously using the CADSA algorithm. The input to the CADSA algorithm includes the identity of each frame's destination MS and the channel gain of each subcarrier. The destination identity can be found in the network layer header field of each frame. The subcarrier's channel gain on the downlink is estimated at the MS using the training sequence in OFDMA systems [7], and then signaled to the BS via the uplink. To reduce the overhead, the feedback information only contains the best modulation type that a subcarrier can achieve given the current SNR.

Given the above information, the BS first searches for potential coding opportunities between each pair of frames heading towards different MSs. A coding opportunity exists for frames A and B if $D_A = S_B$ and $D_B = S_A$, where S_K and D_K denote frame K's source and destination, respectively. In this case, the CADSA encodes A and B into one frame, allowing the two downlinks BS $\rightarrow D_B$ and BS $\rightarrow D_A$ to share the same set of subcarriers. At the receiver side, the mobile station D_A extracts frame A with the operation $B \oplus (B \oplus A)$. Similar decoding algorithm applies for D_B.

For successful decoding, each receiver must determine the identities of the encoded sessions. Such information is implicit in CADSA. Since exactly two sessions (if any) can be encoded, the pairs of sessions that share the same downlink subcarriers are exactly the encoded pairs. The subcarrier assignment information can be found in the signaling field (DL-MAP and UL-MAP [1]) in each downlink frame. In addition, the receiver needs to determine the identity of the key frame that can decode the encoded frame. Assuming there is no backlogged frames at the BS (which is reasonable for QoS guaranteed OFDMA systems like 802.16), then the key is just the latest frame that the receiver sent out. With the above measure, the CADSA frame becomes self-contained — it introduces no additional overhead compared with the general DSA without network coding.

Admittedly, the dynamic subcarrier allocation (whether

CADSA or general DSA) introduces non-negligible overhead compared with static assignment, which is caused by the uplink feedback information indicating the modulation type. Fortunately, the overhead can be significantly reduced by coarse-grained adaptations (see, *e.g.*, [7]). Such overhead reduction techniques apply to our CADSA algorithm as well.

IV. Subcarrier Assignment Algorithms

A. The CADSA algorithms

1) The optimization framework: Denote ζ and Ω as the set of subcarriers and sessions, respectively. Let ϕ be the set of coding opportunities. Each element in ϕ is a vector (s,t), indicating that frames from session s and t satisfy the network coding condition, and thus can be combined into one frame. To avoid repeated count, we dictate $s<t$ for all $(s,t)\in\phi$. In addition, we define function $R(c,m)$ as the achievable rate of subcarrier c when assigned to mobile station m. Given the feedback about modulation type, it can be obtained by $R=\frac{b_m c_r}{T_s}$, where b_m is the number of bits in a modulated symbol; T_s and c_r are the symbol period and error control coding rate, respectively.

Our main objective is to assign an appropriate set of subcarriers to the downlink of each session, such that the total downlink capacity (*i.e.*, aggregate downlink throughput) of the switching network is maximized while no session is starved. Denote the throughput of session s as λ_s, then the objective function can be expressed as $\max\ \min_s \lambda_s$, or equivalently:

$$\max \lambda, \quad \text{subject to: } \lambda \le \lambda_s, \forall s\in\Omega \tag{1}$$

The downlink traffic of each session s consists of two classes: b_{st}, which is the amount contributed by subcarriers transmitting XORed frames for session s and t, $\forall(s,t)\in\phi$; and u_s, which is the amount of uncoded traffic carried by subcarriers uniquely assigned to session s. Therefore, we have:

$$\lambda_s = \textstyle\sum_t b_{st} + u_s, \forall s\in\Omega, (s,t)\in\phi$$

$$\text{and: } \lambda_s = \textstyle\sum_t b_{ts} + u_s, \forall s\in\Omega, (t,s)\in\phi \tag{2}$$

If two downlinks share one subcarrier, then the subcarrier's rate must conform to the one with lower achievable rate. Denote x_{cs} as a 0-1 variable indicating whether subcarrier c is assigned to the downlink of session s. Then, $\forall(s,t)\in\phi$,

$$b_{st} = \textstyle\sum_{c\in\zeta} \min(R(c,D_s),R(c,D_t))\cdot x_{cs}\cdot x_{ct} \tag{3}$$

where D_s is the destination MS for session s. Let $y^c_{st}\in\{0,1\}$ and $y^c_{st}=x_{cs}x_{ct}$, then the above nonlinear constraint is equivalent to the following linear constraints: $\forall(s,t)\in\phi$,

$$b_{st} = \textstyle\sum_{c\in\zeta} \min(R(c,D_s),R(c,D_t))\cdot y^c_{st}, \tag{4}$$

$$y^c_{st} \le x_{cs}, \text{and } y^c_{st}\le x_{ct}, \forall c\in\zeta \tag{5}$$

Furthermore, the amount of uncoded traffic can be obtained by subtracting the coded traffic from the total rate allocated to each session, *i.e.*, $\forall s\in\Omega$,

$$u_s = \sum_{c\in\zeta} R(c,D_s)x_{cs} - \sum_{c\in\zeta}\sum_t R(c,D_s)y^c_{st}, \forall(s,t)\in\phi, \text{and:}$$

$$u_s = \sum_{c\in\zeta} R(c,D_s)x_{cs} - \sum_{c\in\zeta}\sum_t R(c,D_s)y^c_{ts}, \forall(t,s)\in\phi \tag{6}$$

Finally, except for those carrying coded traffic, one subcarrier can only be allocated to at most one session. Therefore, we have the following constraint:

$$\sum_{s\in\Omega} x_{cs} - \sum_{(s,t)\in\phi} y^c_{st} \le 1, \forall c\in\zeta$$

$$\text{and: } \sum_{s\in\Omega} x_{cs} - \sum_{(t,s)\in\phi} y^c_{ts} \le 1, \forall c\in\zeta \tag{7}$$

Consequently, the CADSA optimization becomes a mixed-integer linear program, with the objective and constraints (1), (2), (4), (5), (6) and (7).

2) A Heuristic CADSA algorithm: The above CADSA mixed-integer program is NP-hard in general. Hence we propose a polynomial time heuristic algorithm that can be applied to the base station of real OFDMA cellular networks. Our basic idea is to assign subcarriers to each session in a round based manner. In each round, we employ an *assignment algorithm* to maximize the downlink capacity, and a *penalty algorithm* to ensure fairness.

In the *assignment algorithm*, we group the sessions into those with coding opportunities, and those requiring a unique set of subcarriers. For ease of exposition, we first formulate a graphical model for the assignment mechanism for the former group (graph A in Fig. 2). This graph contains three sets of nodes: the set of sessions Ω, the coding opportunities ϕ and the subcarriers ζ. A link assumes zero weight unless it is from ϕ to ζ, where the weight equals to the achievable rate when the link is matched to a specific coding opportunity. For instance, the weight of $P_1 \to C_1$ equals to $\min(R(C_1,D_{S_1}),R(C_1,D_{S_2}))$. All links have unity capacity except those from ϕ to ζ which can route two units of flows. Links from the virtual source S to each session impose the constraint that a session can only choose one coding opportunity (and correspondingly one subcarrier) in each round, while links from ζ to the virtual sink T ensure that a subcarrier can be assigned to at most one pair of sessions in ϕ.

Given the above graphical setup, the objective of the heuristic CADSA is equivalent to pushing the maximum units of flows from S to T, and choosing the paths in such a way that maximizes the total link weights. Observing that each session can have at most one coding opportunity, and only links from ϕ to ζ have non-zero weights, we can transform this problem into a max-weight max-flow problem on graph B (Fig. 2), where we patch void nodes (whose adjacent links have zero weights) to either ϕ or ζ such that $|\phi|=|\zeta|$. As a result, the original problem becomes weighted bipartite matching (WBM), which can be easily solved using existing network flow algorithms such as the cost scaling algorithm [8]. Once a subcarrier is occupied after the WBM procedure, it will be permanently removed from ζ. The algorithm terminates when no more subcarriers can be assigned in a round. The assignment problem for those sessions without coding opportunities can be solved in a similar manner, except that the set ϕ is eliminated.

In the *penalty algorithm*, we aim at providing a fair share of bandwidth for each session. Specifically, we enforce the following penalty condition for $\forall s\in\Omega$: $T_s - \frac{1}{|\Omega|}\sum_{r\in\Omega} T_r > R_{min}$, where T_s is the downlink throughput of session s in the current round. R_{min} is the achievable rate of a subcarrier when using the modulation type with the lowest rate. Sessions satisfying the penalty condition are gaining advantages over

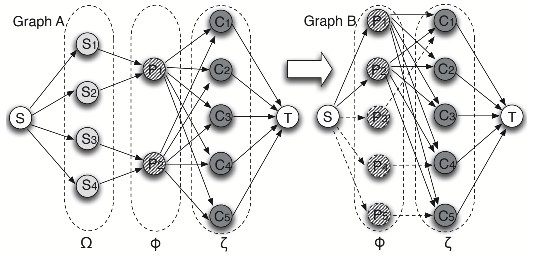

Fig. 2. The graphical model for the CADSA problem: sessions with coding opportunities. Dotted nodes are void nodes whose adjacent links have zero weights. Not all links from ϕ to ζ are shown.

the average by approximately one subcarrier, and will be prohibited from the next-round's assignment. Correspondingly, some nodes in Ω in the above graphs will be removed temporarily.

The computational load of the above CADSA heuristic is dominated by the the WBM algorithm that has polynomial complexity. Since we call the WBM algorithm for at most $\lceil \frac{|\zeta|}{|\Omega|} \rceil$ rounds, the overall complexity is still polynomial. Such an algorithm is well suited for implementation in the base station of real OFDMA systems.

B. The General DSA Algorithms

As a benchmark, we inspect the general DSA algorithm, *i.e.*, the dynamic subcarrier assignment algorithm without network coding. Such schemes have been extensively explored in the literature. Here we consider the optimization based solution with equal power allocation (see, *e.g.*, [3], [4]), as well as the corresponding suboptimal approximations [2]–[4] (henceforth referred to as *DSA heuristic*), which selects one subcarrier with the highest channel gain for each session iteratively, until no more subcarriers can be assigned. We provide more extensive evaluation of it together with the CADSA heuristic in the following section.

V. PERFORMANCE EVALUATION

In this section, we investigate the performance of the heuristic CADSA in comparison with the optimal solution, as well as the non-coding schemes.

A. Experiment Setup

The key of our experiment settings is to derive the achievable data rate of a subcarrier when it is allocated to an arbitrary MS. This requires computing the corresponding SNR value, and mapping the SNR to an achievable rate. To generate realistic results, we adopt empirical parameters to model the wireless fading environment, and configure the OFDMA system according to the 802.16 specification [1].

First, the channel impairment due to large scale fading is modeled by the log-normal model [9] with path-loss exponent 2.4 and shadowing-loss standard deviation 5.4dB. We assume that the shadowing loss varies on the time scale of 0.1 second.

The small scale fading effects are caused by movement of the MS in multipath environment, and modeled by the Rayleigh fading process. The inherent frequency selective property is characterized by an exponential power delay profile with delay spread 15 μs. The time selective nature is captured by the doppler spread, which depends on the MS's speed (throughout the simulation, the MSs are moving at pedestrian speed 2m/s, according to the random waypoint model with pause period 0.01s). The combined complex gain is generated using an improved Jakes-like method [9], which models the frequency correlation between adjacent subcarriers and the time correlation for each subcarrier.

Without loss of generality, we choose the following set of configurations from the 802.16d wirelessMAN-OFDMA specifications [1]. The system bandwidth is 7 MHz, centered around the 5 GHz frequency, and equally shared by all subcarriers. The maximum number of data subcarriers is 1536; subcarrier spacing is $3\frac{29}{32}$ kHz; symbol period T_s is $264\mu s$; downlink frame length T_f is 2 ms. Available modulation schemes include QPSK$\frac{1}{2}$ (error control coding rate), QPSK$\frac{3}{4}$, 16QAM$\frac{1}{2}$, 16QAM$\frac{3}{4}$, 64QAM$\frac{1}{2}$, and 64QAM$\frac{3}{4}$. The corresponding SNR thresholds are 6.0dB, 8.5dB, 11.5dB, 15dB, 19dB and 21dB [1]. When computing SNR, the BS transmission power, noise temperature and noise figure are 1W, 290K and 7dB, respectively. Both the BS and the MSs use omnidirectional single-antenna transceivers.

B. Experiment Results

We compare three subcarrier allocation schemes: the coding aware dynamic subcarrier assignment (CADSA) algorithm, dynamic subcarrier assignment without network coding (DSA), and the randomized subcarrier allocation mechanism (referred to as RAND). Similar to the scheme in 802.16, the RAND algorithm randomly allocates an equal number of subcarriers to each downlink, and chooses the modulation for each subcarrier according to its SNR value. Since the optimal solution for CADSA and DSA cannot be obtained for large scale scenarios using optimization software, we evaluate their LP-relaxations instead. The resulting linear-programming solutions impose upper bounds on the original mixed-integer programs.

1) Throughput comparison: We focus on the scenario where 8 mobile MSs are uniformly located in a circular cell with 0.6 km radius. We randomly start 20 pairwise sessions with constant bit rate traffic, assuming that the downlink of each session always has data to transmit. To limit the computation time of the linear programs, we only use 256 consecutively located data subcarriers of the entire frequency band. We compute the downlink capacity, (*i.e.*, the aggregate downlink throughput of all sessions) over one second.

As shown in Fig. 3, the performance gain of CADSA over DSA keeps consistently around 75%. The downlink capacity of the heuristic CADSA approximates the optimum well. Both CADSA and DSA outperform RAND by a significant margin. Notably, the throughput of the heuristic DSA can approach or even exceed the optimal values. This is at the cost of fairness, *i.e.*, there can be a certain gap between the max and min throughput of all sessions when running the heuristics. To quantify the difference in fairness, we compute the Jain's fairness index [10] for all the above schemes. Denote the throughput of session i as W_i, then the fairness index is

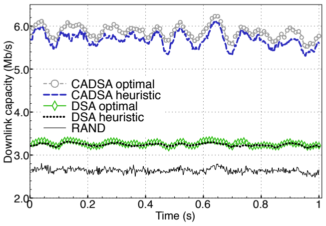

Fig. 3. The total downlink capacity as a function of time.

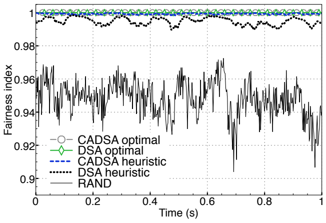

Fig. 4. The fairness index of each scheme as a function of time.

$F = \frac{(\sum_{i=1}^{|\Omega|} W_i)^2}{|\Omega| \sum_{i=1}^{|\Omega|} W_i^2}$. From Fig. 4, we see that the optimal LP solutions tend to achieve full fairness (*i.e.*, $F = 1$). The intuition behind is that the optimal algorithm can reduce the difference in throughput by switching subcarriers from high-throughput sessions to low-throughput sessions. In contrast, the heuristic DSA and RAND tend to deviate from the optimal fairness index. Remarkably, the fairness of the heuristic CADSA is quite close to the optimum, owning to its penalty mechanism.

Note that in these experiments, we assume that the sessions are paired so that each session is interested in exchanging information with another one, thus a coding opportunity exists for each session. In practice, not all sessions may have coding opportunities, and therefore the gains of network coding also depend on the fraction of sessions that can be encoded.

2) Influence of multiuser diversity: Generally, multiuser diversity (or attenuation spread) is reduced when we decrease the cell radius, since the MSs' difference in distances to the base station is reduced. In Fig. 5 and Fig. 6, we explore the influence of multiuser diversity on time-averaged downlink capacity and fairness. As we increase the cell radius, the average channel condition deteriorates, resulting in lower downlink capacity. Meanwhile, the attenuation spread becomes larger, making it harder for the heuristic DSA and RAND to ensure fairness. With the penalty mechanism, however, the heuristic CADSA keeps near-optimal fairness and yet much higher capacity, even under severe channel conditions.

In general, the dynamic subcarrier assignment algorithms outperform RAND in the scenarios with larger multiuser diversity [7], *i.e.*, the channel gains of different MSs vary substantially. However, to exploit the network coding advantage, it is preferable to encode the downlinks with similar channel gains, and assign the same subcarriers to them. Otherwise the downlink with a worse channel condition will undermine the shared downlink rate. Apparently, there is a trade-off between the diversity gain and network coding gain. We illustrate the trade-off in Fig. 7, where the diversity gain is reflected by the performance gain of the optimal DSA over RAND, and the coding gain is reflected by the performance gain of the optimal CADSA over DSA. We adopt the *minimum throughput* of all sessions as the performance metric, which is essentially the optimization objective of DSA and CADSA. We observe that with small attenuation spread, the coding gain approaches the 100% bound. When the MSs experience considerably different channel conditions, the coding gain diminishes. In contrast, the diversity gain increases with the attenuation spread. Balancing a trade-off between both schemes, the CADSA mechanism achieves significant performance improvement over the RAND.

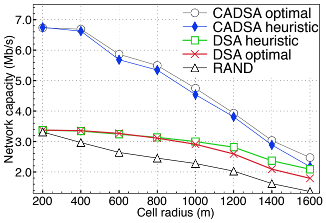

Fig. 5. Influence of multiuser diversity on the downlink capacity.

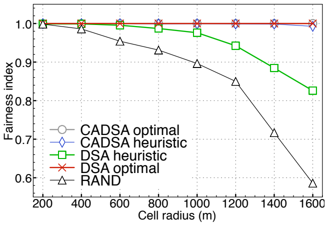

Fig. 6. Influence of multiuser diversity on fairness.

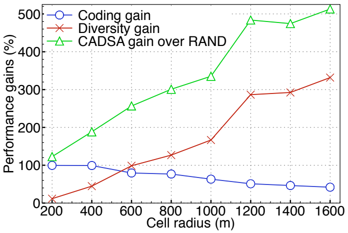

Fig. 7. Influence of attenuation spread on performance gains.

VI. CONCLUSION

In this paper, we proposed CADSA, a cross layer protocol that integrates network coding and dynamic subcarrier assignment for OFDMA systems. We formulated the optimal coding aware subcarrier assignment scheme, and approximated it with a suboptimal heuristic. Our simulations in a realistic fading environment and under 802.16 settings have demonstrated the advantages of the CADSA in efficiently utilizing available subcarriers. In addition, we identified an important tradeoff between the coding advantage and the diversity gain, which may need further exploration from an information theoretic perspective.

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