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Signed-PageRank: An Efficient Influence Maximization Framework for Signed Social Networks

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Abstract—Influence maximization in social networks is of great importance for marketing new products. Signed social networks with both positive (friends) and negative (foes) relationships pose new challenges and opportunities, since the influence of negative relationships can be leveraged to promote information propagation. In this paper, we study the problem of influence maximization for advertisement recommendation in signed social networks. We propose a new framework to characterize the information propagation process in signed social networks, which models the dynamics of individuals' beliefs and attitudes towards the advertisement based on recommendations from both positive and negative neighbours. To achieve influence maximization in signed social networks, we design a novel Signed-PageRank (SPR) algorithm, which selects the initial seed nodes by jointly considering their positive and negative connections with the rest of the network. Our extensive experimental results confirm that our proposed SPR algorithm can effectively and efficiently influence a broader range of individuals in the signed social networks than benchmark algorithms on both synthetic and real datasets.

Index Terms—Signed Social Networks, Influence Maximization, Information Propagation, Recommendation.

1 Introduction

Social networks consist of individuals who form emotional connections with each other, such as friends, enemies, relatives, neighbours, or collaborators. Such connections in social networks can be leveraged to conduct marketing activities by enterprises, e.g., advertisement recommendation. For example, a budget-limited company with a new product can provide incentives to a set of influential users who spread the advertisement as widely as possible in social networks, which may boost the propagation efficiency. Influence maximization in social networks is of major interests to both industry and academia. The goal is to find the k most influential users (or nodes), called the seed nodes, to initiate information propagation across the whole network. It is desirable to recruit minimal seed nodes to reduce costs but maximize information propagation to gain profits, e.g., for advertisement recommendation.

Popular social platforms, such as Facebook, Twitter, YouTube, LinkedIn and many others, have explicit connection information between users, e.g., friend lists. However, it is also important to differentiate positive and negative relationship between users, e.g., the fact that Alice *follows* Bob may mean that Alice likes or dislikes Bob and wants to be informed of Bob's update. Furthermore, users interact with each other through comments, which reveal implicitly positive and negative relationships between users. A social network with both weights and labels on each edge to indicate positive and negative relationships (e.g., friend and foe, trust and distrust) is referred to as a signed social network. Compared with unsigned social networks, signed social networks provide richer information on social relations that can be leveraged to enhance influence maximization. Existing research in [31], [36] have demonstrated that negative links are as effective as positive links, and can also significantly enhance recommendation process. The critical issue is how to quantify the influence of a neighbor in information propagation. Although existing works that did not differentiate signed and unsigned networks can deal with both positive and negative weights, we aim at better exploiting the characteristics of signed networks to refine the model of information propagation and strengthen the effect of influence maximization. In particular, we use the absolute value of the weight to quantify the degree of influence from a friend or a foe, i.e., the belief of a user is drawn more towards the belief of a friend with a tighter bond and is diverted more from the belief of a foe with a stronger hostility, and utilize the label to compute the proportion of friends and foes among all neighbors to further quantify the degree of positive and negative influences.

Ad recommendation in signed social networks can be viewed as an influence diffusion process through the weighted directed graph $G(V, E)$. As show in Figure 1, the seed nodes propagate an advertisement to their neighbours. There are two major challenges for influence maximization in signed social networks:

- ***How do we model information propagation in signed social networks?***

Since information propagation in social networks is similar to the spread of diseases, epidemic models are often used to

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characterize information propagation in social networks [16], [4], [9]. The Susceptible/Infectious/Removed (SIR) model [20] and the Independent Cascade (IC) model [2] are two traditional epidemic models that have been adapted for ad recommendation in unsigned social networks. The SIR model is used to predict the spread of a disease, where individuals switch among the infectious/susceptible/removed statuses (the infectious individuals are the ones who are infected and can be a new source of infection, the susceptible individuals are vulnerable to an epidemic, the removed individuals are the ones who are immune to the epidemic). The IC model is a sequential decision model in which individuals make their decisions based on the previous decisions made by others.

However, compared with unsigned social networks, information propagation in signed social networks is much more complex, since negative links may have an adverse influence on information propagation. The situation is more complicated if an individual receives ad recommendations from both friends and foes concurrently (called "parallel recommendation"). Therefore, to model information propagation in signed social networks, we need to deal with recommendations from negative links, especially the parallel recommendations.

In this paper, inspired by the SIR and the IC models, we propose a new framework to model information propagation for ad recommendation in signed social networks. More specifically, the information propagation process is characterized by dynamic changes of individuals' opinions for the advertisement (referred to as the *belief*). We have carefully designed belief update rules for signed social networks by incorporating influence from both positive and negative links to tackle the problem of parallel recommendations.

- ***How do we achieve influence maximization in signed social networks?***

The other challenge is influence maximization in signed social networks, i.e., how we would choose appropriate initial seed nodes for ad recommendation to maximize the number of individuals who eventually accept the advertisement. Influence maximization problem has been proved to be NP-hard [19], thus greedy algorithms and linear threshold models are often proposed to achieve sub-optimal results in linear time [16], [4]. Nevertheless, greedy algorithm and linear threshold models are neither efficient nor scalable. Some studies designed improved greedy algorithm and linear threshold models to reduce the operational time, but at the expense of degraded performance.

Most existing works on influence maximization focused on unsigned social networks, and there is a lack of works for signed social networks. Compared with unsigned social networks, the structure of relationships in signed social networks is more complicated, which poses great challenges for selecting seed nodes to maximize influence propagation. Without considering the sign of edges, in unsigned social networks, a user with a huge number of connections can be deemed as influential. However, in signed social networks, a well-connected user cannot be chosen for ad recommendation if most of her relationships are negative. Furthermore, during information propagation, the influence via positive edges may be strengthened while the influence via negative edges may be adverse, which makes the problem of influence maximization more complicated. There have been several existing works on influence maximization in signed networks. In [18], SRWR was proposed for personalized ranking in signed networks, where a signed random surfer changes her sign for walking by considering negative edges. In [25], PRIM problem was formulated to find the seed nodes with maximum positive influence or maximum negative influence in signed social networks, and the IC model was extended to the signed social networks as the Polarity-related Independent Cascade (IC-P) diffusion model. However, the SRWR algorithm assumes that the random walker goes back to the original seed node with a restart probability, which is not suitable for ad recommendation scenarios. The PRIM problem aims at either positive influence maximization or negative influence maximization, but did not address the problem of how to integrate both positive and negative influences to reach influence maximization.

To tackle the problems above, inspired by the efficient PageRank algorithm, we design a novel Signed-PageRank (SPR) algorithm to rank the importance of individuals for seed node selection. The traditional PageRank algorithm is used to sort the importance of web pages based on topological properties of web graphs [33]. The PageRank algorithm has been applied to study sign changes and seed node selection in signed social networks [5], [18]. In [5], the ranks of nodes are computed separately on the subgraph with only positive links and on that with only negative links. In [18], a personalized ranking approach is designed based on a walker who randomly moves and restarts in the signed network with certain restrictions. In comparison, our proposed SPR ranks the importance of nodes jointly considering the influence of both positive and negative links with computable influence factors.

In this paper, we make the following key contributions:

- We propose a new framework to characterize information propagation for ad recommendation in signed social networks. As far as we are concerned, previous works only study either positive influence or negative influence separately in signed social networks, while we make the first attempt to consider the scenario where an individual may receive recommendations from both friends and foes, and integrate both positive and negative influence for belief update of the individual.
- We design a novel Signed-PageRank algorithm for influence maximization in signed social networks. To the best of our knowledge, we are the first to extend the PageRank algorithm for influence maximization in signed social networks, which can effectively select the most influential seed nodes by jointly considering both positive and negative links.
- We evaluate our proposed algorithm with extensive experiments on both synthetic datasets and large-scale real datasets. The results verify that the proposed algorithm outperforms existing solutions in terms of both running time and influence maximization. This shows that the proposed algorithm can be readily applied to signed social networks to help broaden the spread of information and gain a high profit for entrepreneurs.

The rest of the paper is organized as follows. We describe our system model in Section 2. Our information propagation framework and the SPR algorithm for influence maximization in signed social networks are presented in Section 3. Section 4 demonstrates our experimental results on both synthetic and real datasets. Section 5 reviews related work, and Section 6 concludes the paper.

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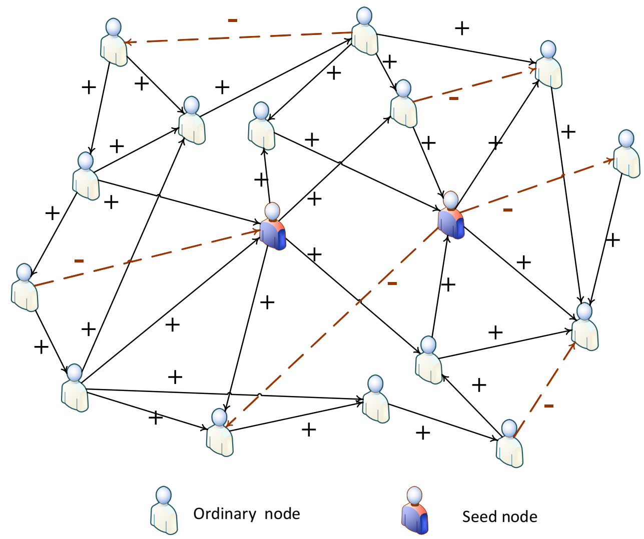

Fig. 1: Signed social network $G(V, E)$. E^+ and E^- are denoted as black solid arrows and red dotted arrows respectively. Individuals in red are seed nodes with an attitude of 1 and will recommend the advertisement to their neighbours.

2 System Model

We consider a weighed directed graph (digraph) $G(V, E)$ to model the individuals and their social relationships, where V is the set of individuals, E is the set of directed edges between individuals. There are N nodes/individuals, i.e., $|V| = N$. Each edge in E is associated with a weight $w_{i,j} \in [0, 1]$, which represents the probability of successful recommendation from individual v_i to individual v_j. The graph is asymmetrical, i.e., it is possible that $w_{i,j} \neq w_{j,i}$. Every edge is also assigned a unique label $l_{i,j}$ as $+$ (value 1) or $-$ (value -1), representing positive and negative relations, respectively [17][1], e.g., approval or disapproval of one another's comments on social platforms, trust or distrust one another's reviews about a certain product on social platforms. A positive directed edge indicates friendliness of one individual towards the other, while a negative link implies hostility of one individual towards the other. A graph G equipped with signs on each edge is called a signed graph.

Given a signed graph $G(V, E)$, let E^+ and E^- be the set of positive edges ($l_{i,j} = 1$) and negative edges ($l_{i,j} = -1$), respectively. We have $E^+ \cap E^- = \varnothing$, and $E^+ \cup E^- = E$. We use $G^+ = (V, E^+)$ and $G^- = (V, E^-)$ to denote the signed subgraph consisting of all positive edges E^+ and all negative edges E^-, respectively. We have $G = G^+ + G^-$. We assume that the underlying graph G is connected and the negative subgraph G^- is nonempty.

Every individual in the signed graph G has a subjective consciousness for an advertisement, called *belief*, which will evolve over time. The belief of individual v_i at time t is denoted as $x_{i,t} \in [0, 1]$. In the inception phase, the set $X_0 = \{x_{i,0} \mid v_i \in V\}$ is known. Note that $x_{i,t}$ is different from $w_{i,j}$, where $x_{i,t}$ is the belief of v_i in the advertisement, while $w_{i,j}$ is the level of influence of v_i on v_j, which will affect the changes of v_j's belief if v_j receives an ad recommendation from v_i. The belief $x_{i,t}$ of every node v_i, $v_i \in V$ will be updated over time via a probabilistic model based on ad propagations through interactions between individuals. An individual is more likely to increase her belief in the advertisement due to a recommendation from a friend, while a recommendation from a foe may have an adverse effect on her belief.

1. Apart from positive and negative relationships, the connections between users may be neutral. We will consider neutral connections with sign 0 in signed social networks in future networks.

An individual v_i is receptive to an ad recommendation within a fixed time window $T_i = [T_i^l, T_i^u]$, called a *recommendation cycle*, in which T_i^l and T_i^u are the start and the end time for receiving ad recommendation, respectively. In other words, individual v_i can only recommend advertisements to her neighbour v_j within T_j, and v_j's belief will be updated during T_j but will remain unchanged before T_j^l and after T_j^u.

Depending on their beliefs, each individual forms one of two opposite attitudes for an advertisement. We use $A_{i,t} \in \{0, 1\}$ to denote the attitude of individual v_i at time t, and $A_{i,t}$ follows the binomial distribution with probability of belief $x_{i,t}$. $A_{i,t} = 1$ indicates that v_i accepts and approves the advertisement, while $A_{i,t} = 0$ implies that v_i rejects the advertisement. The attitude of an individual determines whether she will propagate/recommend the advertisement or not. Individuals with attitude $A_{i,t} = 1$ are deemed as seed nodes, who will recommend the advertisement to their neighbours (both friends and foes) at time t. For example, v_1, v_2, and v_3 are mutual neighbors in the network. At time t, the belief of v_1 is $x_{1,t} = 0.8$, meaning that v_1 has a relatively strong belief in the ad; the beliefs of v_2 and v_3 are $x_{2,t} = x_{3,t} = 0.2$, showing their doubts in the ad. The actual attitudes of v_1, v_2, and v_3 are binomial distribution with probability of their beliefs, so that we assume that the attitude of v_1 turns out to be $A_{1,t} = 1$, and the attitudes of v_2 and v_3 turn out to be $x_{2,t} = x_{3,t} = 0$. Therefore, v_1 will recommend the ad to v_2 and v_3 since v_1 accepts the ad and helps propagate the ad. t is within the recommendation cycle of v_2, i.e., $T_2^l \leq t \leq T_2^u$, thus v_2 will update her belief at $t+1$ based on the sign of her relation with v_1 (a friend or foe) and her relations with other recommenders. The recommendation cycle of v_3 terminates at $t-1$, i.e., $T_3^l = t-1$, thus v_3 will not accept recommendations from anyone. In fact, v_3's attitude will stay as 0 until the end of the time horizon. After updating her belief, the attitude of v_2 is redrawn from the binomial distribution with probability of her belief. v_2 may or may not change their attitudes. v_1 will not change her belief or attitude once she has accepted the ad. The level of ad propagation in $G(V, E)$ at time t depends on the attitude set $A_t = \{A_{i,t} \mid v_i \in V\}$. As show in Figure 1, black solid arrows and red dotted arrows represent directed positive and negative relationships, respectively. Individuals in red have an attitude of 1 (indicating that they approve the advertisement) and will recommend the advertisement to their neighbours.

We summarize the key notations in Table 1.

3 Influence Maximization for Ad Recommendation in Signed Social Networks

In this section, we first present the viral marketing framework for information propagation in signed social networks based on the traditional IC model and SIR model, then we study the dynamic update for individuals' beliefs, addressing the problem of parallel relations that involve both positive and negative relationships in particular. Finally, based on the PageRank algorithm and dynamic belief updates, we propose a new Signed-PageRank algorithm to

TABLE 1: Key notations

Notation	Definition
$G(V,E)$	a weighted directed graph as the signed social network
$w_{i,j}$	the level of influence of v_i on v_j
W	the set of weights
E^+	the set of positive edges
E^-	the set of negative edges
G^+	the subgraph of friends
G^-	the subgraph of foes
N	the number of individuals
$l_{i,j}$	the label of the edge (positive/negative)
$\mathbf{L}$	the matrix of all labels
$x_{i,t}$	the belief of node v_i in the advertisement at time t
$p_{i,t}$	the recovery probability of node v_i at time t
X_0	the set of all beliefs at the inception phase
$A_{i,t}$	the attitude of node v_i at time t
A_t	the set of all attitudes at time t
T_i	the recommendation cycle of v_i
T	the set of all recommendation cycles
H_i^{out}	the set of out-degree neighbours of v_i
H_i^{in}	the set of in-degree neighbours of v_i
H_i^{in+}	the set of in-degree friends of v_i
H_i^{in-}	the set of in-degree foes of v_i
$\mathcal{S}_t$	the set of seed nodes with attitude 1 at time t
k	the number of initial seed nodes to be selected
α_i	positive embeddedness of v_i
β_i	negative embeddedness of v_i
$SPR_{i,\tau}$	SPR rank of v_i at iteration τ
d	the damping coefficient

achieve influence maximization for ad recommendation in signed social networks.

3.1 Information Propagation

Since ad propagation in social networks is similar to disease transmission in a population, we leverage viral marketing to characterize information propagation for ad recommendation [4], [16]. The Susceptible/Infectious/Removed (SIR) model and the Independent cascade (IC) model are two classical viral marketing models for unsigned networks, which we will extend to information propagation in signed social networks.

The traditional SIR model is designed for unsigned networks. For example, in Figure 1, according to the traditional SIR model, the individuals in red who have an attitude of 1, i.e., will recommend the advertisement to their neighbours as seed nodes, are in the infectious phase (contagious); the individuals in blue who have an attitude of 0 and will receive ad recommendation (within the recommendation cycle) are in the susceptible phase; the individuals who have an attitude of 0 and beyond the recommendation cycle are in the removed phase, i.e., these individuals will be removed from the social network since they will no longer receive ads from or recommend ads to anyone else.

In particular, at time t, individuals with an attitude of 1 will be the seed nodes who will propagate the advertisement to the rest of the network (infectious phase). The set of seed nodes at time t is denoted as $\mathcal{S}_t$. Let H_i^{out}/H_i^{in} denote the set of out-degree/in-degree neighbours of v_i, and H_i^{in+}/H_i^{in-} denote the set of in-degree friends/foes of v_i. At time t, if v_i has an attitude of 1 and her neighbour $v_j \in H_i^{out}$ has an attitude of 0, given that $t \in T_j$ (within the recommendation cycle of v_j), v_i will recommend the advertisement to v_j with a success probability depending on $w_{i,j}$. If v_j accepts the advertisement ($A_{j,t}$ becomes 1), she may revoke the acceptance with a probability of $p_{j,t}$ ($A_{j,t}$ recovers back to 0), where $p_{j,t} = 1 - x_{j,t}$. If v_j's attitude is 0 and her recommendation cycle ends, i.e., $t > T_j^u$, v_j is no longer susceptible and will be removed. The information propagation terminates when all individuals are either in the infectious phase or in the removed phase, and no individuals are in the susceptible phase. The IC model is similar to the SIR model, but it restricts that seed nodes can only recommend the advertisement once (successful or not), and will be removed in the next time slot.

However, we cannot directly apply the SIR model or the IC model to signed networks, since they do not consider the positive/negative relationships among individuals and their influence on ad recommendation. In particular, an individual in the susceptible phase may receive ad recommendations from both friends and foes simultaneously, making it difficult to determine her belief update under such contradictory influences. By addressing these problems, we extend the traditional SIR/IC model to characterize information propagation for ad recommendation in signed social networks. We give the formal definition of the information propagation process for signed social networks as follows.

Definition 1. Information Propagation for Signed Networks. Given a signed social network $G(V,E)$ and the parameters in Table 1, the information propagation for ad recommendation is:

- **Initiation**. At $t = 0$, all individuals hold the attitude of 0, and a set of individuals is selected by the proposed algorithm (explained in details in Section 3.3) as seed nodes to recommend the advertisement to their neighbours.
- **Propagation**. At $t > 0$, if the propagation process does not terminate, we have
  - *Infectious individuals*. Any individual v_i with attitude $A_{i,t} = 1$ is infectious, and will recommend the advertisement as a seed node to all her out-degree neighbours in H_i^{out}. Infectious individuals will not be influenced by ad recommendations from her neighbours, i.e., beliefs and attitudes of infectious individuals will remain unchanged.
  - *Susceptible individuals*. Any individual v_j with attitude $A_{i,t} = 0$ and within her recommendation cycle is susceptible. Susceptible individuals who receive ad recommendations from neighbours will update their beliefs according to certain rules (explained in details in Section 3.2). Note that if $t < T_j^l$, individual v_j is susceptible but will not receive ad recommendations from neighbours.
    - *Unaffected*. If v_j's attitude remains 0 after belief update, she is still in the susceptible phase.
    - *Recovered*. If v_j's attitude becomes 1 after belief update, she has a probability of p_j to recover (attitude switches back to 0), and is still in the susceptible phase.
    - *Infected*. If v_j's attitude becomes 1 after belief update and does not recover, she enters the infectious phase.
  - *Removed individuals*. Any individual v_j with attitude $A_{j,t} = 0$ and beyond her recommendation cycle, i.e., $t > T_j^u$, will be removed.

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Algorithm 1 InPro: Information Propagation Algorithm

Input: The signed social network $G(V, E)$, the number of initial seed nodes k, the set of beliefs X_t, the set of attitudes A_t, the matrix of labels $\mathbf{L}$, the set of recommendation cycles T, system time t.

1: **if** $t = 0$ **then**
2:     $\mathcal{S}_t = \mathsf{SPR}(G, X_0, \mathbf{L}, k)$.
3: **else**
4:     $\mathcal{S} = \{v_i | A_{i,t} = 1\}$.
5: **end if**
6: The set of susceptible individuals who will receive ad recommendation is $F_a = \{v_j | v_j \in V - \mathcal{S}, t \in [T_j^l, T_j^u]\}$.
7: The set of susceptible individuals who will not receive ad recommendation is $F_b = \{v_j | v_j \in V - \mathcal{S}, t < T_j^l\}$.
8: **if** F_a is empty and F_b is non-empty **then**
9:     $t = t + 1$.
10:     $\mathsf{InPro}(G, X_{t-1}, A_{t-1}, L, T, t)$.
11: **else if** F_a is non-empty **then**
12:     $t = t + 1$.
13:     $(X_t, A_t) = \mathsf{BUpdate}(G, X_{t-1}, A_{t-1}, L)$.
14:     **for all** $v_i \in (V - \mathcal{S})$ and $A_{i,t}$ becomes 1 **do**
15:         Recover $A_{i,t}$ from 1 to 0 with a probability of $p_{i,t}$.
16:     **end for**
17:     $\mathsf{InPro}(G, X_t, A_t, L, T, t)$.
18: **else**
19:     The information propagation terminates.
20: **end if**

- **Termination**. The information propagation terminates if all individuals are either infectious or removed.

According to Definition 1, we present the information propagation framework for ad recommendation in signed social networks in Algorithm 1, which iteratively updates the beliefs and attitudes of all individuals. At time $t = 0$, the set of seed nodes $\mathcal{S}_0$ are selected by Algorithm 3 for influence maximization, which will be explained in details in Section 3.3; at time $t > 0$ until termination, the set of seed nodes are the individuals whose attitude is 1. The BUpdate algorithm at line 13 is in Algorithm 2, which will be described in detail in Section 3.2.

3.2 Belief Update

When susceptible individuals receive ad recommendations, their beliefs will change. We assume that the beliefs of the individuals who have accepted the advertisement (the seed nodes) will no longer change. Previous works on information propagation for unsigned social networks design belief update rules without considering the impact of positive/negative relationships. The DeGroot's model [10] is proposed to characterize how a group of people reach a consensus, where the belief of an individual is updated as a convex combination of the beliefs of her neighbours. Inspired by the DeGroot's model, we design the belief update rules in signed social networks.

It is shown that people are more likely to trust their friends than their foes [3] [35]. This indicates that negative relations may have an adverse influence on information propagation, i.e., an ad recommendation from a foe may reduce the belief of an individual. We first consider a simple case where an individual receives ad recommendations from a set of friends or a set of foes. Leveraging the probability theory and the DeGroot's model, we design the following real-time belief update rules.

Proposition 1. Belief Update Rules. Suppose that individual v_i receives ad recommendations from her neighbours (either friends or foes) at time t. The belief of individual v_i is updated as:

- *Positive update.* If v_i receives ad recommendations from friends, i.e., $l_{j,i} = 1, \forall v_j \in H_i^{in} \cap \mathcal{S}_t$, v_i updates her belief as:

$$x_{i,t+1} = x_{i,t} + \alpha_i \cdot \sum_{v_j \in H_i^{in} \cap \mathcal{S}_t} w_{j,i} \cdot (x_{j,t} - x_{i,t}), \quad (1)$$

  where α_i is the positive embeddedness of individual v_i, and we have $0 \leqslant \alpha \leqslant 1$.

- *Negative update.* If v_i receives ad recommendations from foes, i.e., $l_{j,i} = -1, \forall v_j \in H_i^{in} \cap \mathcal{S}_t$, v_i updates her belief as:

$$x_{i,t+1} = x_{i,t} - \beta_i \cdot \sum_{v_j \in H_i^{in} \cap \mathcal{S}_t} w_{j,i} \cdot (x_{j,t} - x_{i,t}), \quad (2)$$

  where β_i is the negative embeddedness of individual v_i, and we have $0 \leqslant \beta \leqslant 1$.

To further quantify the degree of the influence of friends and foes, we introduce the positive and negative embeddedness of individual v_i based on the label of social relations. Obviously, the larger the proportion of friends, the greater the positive influence and the smaller the negative influence. Similarly, the larger the proportion of foes, the greater the negative influence and the smaller the positive influence. Thus, the positive and negative embeddedness of individual v_i is calculated as:

$$\alpha_i = \frac{|H_i^{in+}|}{|H_i^{in}|}, \beta_i = \frac{|H_i^{in-}|}{|H_i^{in}|}, \quad (3)$$

where $|H_i^{in}|$ is the number of in-degree neighbours of v_i, and $|H_i^{in+}|/|H_i^{in-}|$ is the number of positive/negative in-degree neighbours of v_i.

The positive update rule allows individuals to follow their friends' behaviors. The negative update rule estranges the beliefs of individuals from their foes. However, the rules above cannot cater to the scenario where individuals receive ad recommendations from both friends and foes.

Empirical studies show by experiments that concurrent recommendations from both friends and foes will exponentially increase the number of infected individuals and speed up ad spreading [32]. As shown in Figure 2, at time t, node u and node v in blue are susceptible individuals and other individuals in red are seed nodes. The red dotted arrows and black solid arrows stand for negative and positive relations, respectively. Individual v receives ad recommendations from two foes, while individual u receives ad recommendations from a foe and two friends. In this case, individual v can update her belief by Eq. (2), but individual u cannot. We refer to this scenario as *parallel recommendation*, which is defined as follows.

Definition 2. Parallel Recommendation. A parallel recommendation refers to the case where an individual simultaneously receives ad recommendations from her friends and foes.

To address the problem of parallel recommendation, we design the following belief update rules.

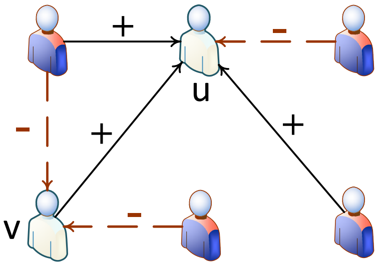

Fig. 2: Parallel recommendation. A signed social network with 4 seed nodes (in red) and 2 susceptible individuals (in blue), in which the red dotted arrows and black solid arrows denote negative and positive links, respectively.

Proposition 2. Belief Update Rules for Parallel Recommendations. Suppose that individual v_i receives ad recommendations her neighbours (both friends and foes) at time t. The belief of individual v_i is updated as:

$$x_{i,t+1} = x_{i,t} + \alpha_i \cdot \sum_{v_j \in H_i^{in+} \cap \mathcal{S}_t} w_{j,i} \cdot (x_{j,t} - x_{i,t}) - \beta_i \cdot \sum_{v_j \in H_i^{in-} \cap \mathcal{S}_t} w_{j,i} \cdot (x_{j,t} - x_{i,t}). \quad (4)$$

in which α_i and β_i are defined in Eq. (3).

Note that Proposition 2 is inclusive of Proposition 1, where Proposition 1 is a special case of Proposition 2.

Toy Example. Figure 3 illustrates four cases in belief updates with parallel recommendation at time t.

- In Figure 3(a), individual b has friends but no foes. Therefore, we can calculate that $\alpha = 1$ and $\beta = 0$. According to Eq. (4), the belief of b at time $t+1$ will be $x_{b,t+1} = 0.2 + 1 \cdot (0.3 \cdot (0.7 - 0.2) + 0.6 \cdot (0.5 - 0.2)) = 0.53$.
- In Figure 3(b), individual b has friends but not foes. Therefore, we can calculate that $\alpha = 1$ and $\beta = 0$. However, c's belief will decrease b's belief since c has a lower belief than b and b tries to follow her friend's opinion. The belief of b at time $t+1$ will be $x_{b,t+1} = 0.2 + 1 \cdot (0.3 \cdot (0.7 - 0.2) + 0.6 \cdot (0.1 - 0.2)) = 0.29$.
- In Figure 3(c) and Figure 3(d), individual b has one friend and one foe. Therefore, we can calculate that $\alpha = 0.5$ and $\beta = 0.5$. b will be adversely influenced by her foe c. In Figure 3(c), the belief of c is greater than b, thus the influence of c will decrease b's belief as $x_{b,t+1} = 0.2 + 0.5 \cdot 0.3 \cdot (0.7 - 0.2) - 0.5 \cdot 0.6 \cdot (0.5 - 0.2) = 0.185$. In contrast, in Figure 3(d), b's belief increases as $x_{b,t+1} = 0.2 + 0.5 \cdot 0.3 \cdot (0.7 - 0.2) - 0.5 \cdot 0.6 \cdot (0.1 - 0.2) = 0.305$ because her belief is higher than c.

At time t, user v_i will decide to accept or reject an ad recommendation based on her attitude $A_{i,t}$ towards the ad, which is usually affected by her belief $x_{i,t}$ in the ad. With a higher belief, the user is more likely to accept the ad with a higher probability. Therefore, we model the attitude of individual v_i at time t as the binomial distribution with probability $x_{i,t}$. In existing works, the voter model is usually used to decide the ad recommendation results [13], [28]. Individuals are assumed to definitely accept an ad if they receive recommendations with a total influence higher than a certain threshold in the voter model. The drawback of the voter model is a lack of uncertainty. There is still a small chance that an individual may not accept an ad even if the influence of recommenders is strong, and may indeed accept an ad even if the influence of recommenders is weak. In comparison, our model can well capture such uncertainty. The attitude of individual v_i will be updated as

$$A_{i,t} \sim B(1, x_{i,t}), \quad (5)$$

where $B(1, x_{i,t})$ denotes the binomial distribution with probability $x_{i,t}$ and 1 trial.

We summarize the belief update algorithm in Algorithm 2.

Algorithm 2 BUpdate: Belief Update Algorithm

Input: The signed social network $G(V, E)$, the set of beliefs X_t, the set of attitude A_t, the matrix of labels $\mathbf{L}$, the set of seed nodes $\mathcal{S}_t$, system time t.
Output: X_t, A_t.

for all $v_i \in V, j \in \mathcal{S}$ **do**
  Calculate positive embeddedness α_i and negative embeddedness β_i.
  $Pos = \sum_{v_j \in H_i^+ \cap \mathcal{S}_t} w_{j,i} \cdot (x_{j,t} - x_{i,t})$.
  $Neg = \sum_{v_j \in H_i^- \cap \mathcal{S}_t} w_{j,i} \cdot (x_{j,t} - x_{i,t})$.
  $x_{i,t+1} = x_{i,t} + \alpha_i \cdot Pos - \beta_i \cdot Neg$.
  Randomly generate $A_{i,t+1}$ following the binomial distribution $B(1, x_{i,t+1})$.
end for

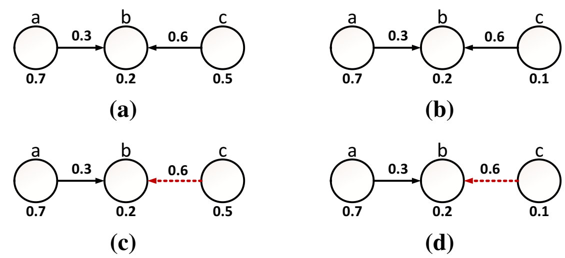

Fig. 3: An example of belief updates: The initial beliefs are given, and the black and red arrows stand for positive and negative links respectively.

3.3 Influence Maximization

Given the information propagation model in Section 3.1 and the belief update rules in Section 3.2, our aim is to achieve influence maximization by selecting initial seed nodes for ad recommendation such that as many nodes as possible will finally accept the ad. The key is to choose the most influential individuals to infiltrate other nodes and boost information propagation. PageRank algorithm is widely used to rank webpages according to their importance. Therefore, we adapt the PageRank algorithm to rank users according to their influence so that we can select the top-ranking users as initial seeds.

3.3.1 PageRank Algorithm

The PageRank algorithm assesses the importance of web pages based on topological properties of the web graph [33]. The rank of web pages will be iteratively updated via a random walker following directed edges of the graph. Let $PR_{i,\tau}$ denote the rank of the web page node v_i at iteration τ. $PR_{i,\tau}$ will be updated as:

$$PR_{i,\tau+1} = d \cdot \sum_{v_j \in H_i^{in}} \frac{PR_{j,\tau}}{|H_j^{out}|} + \frac{1-d}{N}, \forall v_i \in V, \quad (6)$$

where $d \in [0,1]$ is the damping coefficient to prevent page ranks from increasing indefinitely, H_i^{in} is the set of in-degree neighbours of node v_i, N is the number of nodes, and $|H_j^{out}|$ is the number of out-degree neighbours of node v_j.

We can define an out-degree adjacency matrix as:

$$\mathbf{F} = \begin{bmatrix} f_{1,1} & f_{1,2} & \cdots & f_{1,N} \\ f_{2,1} & \ddots & & \vdots \\ \vdots & & f_{i,j} & \\ f_{N,1} & \cdots & & f_{N,N} \end{bmatrix}, \tag{7}$$

where $f_{i,j} = 1/|H_j^{out}|$ if there is a directed edge from v_i to v_j; otherwise $f_{i,j} = 0$. Note that the sum of every row in matrix $\mathbf{F}$ equals 1. Matrix $\mathbf{F}$ can be regarded as a normalized fair allocation of weights. Combine Eqs. (6) and (7), we have:

$$\mathbf{PR}_{\tau+1} = d \cdot \mathbf{PR}_\tau \cdot \mathbf{F} + [(1-d)/N, \cdots, (1-d)/N]^T. \tag{8}$$

where $\mathbf{PR}_\tau = [PR_{1,\tau}, \cdots, PR_{N,\tau}]^T$. As Eq. (8) is convergent, the iteration process will terminate when $|PR_{i,\tau+1} - PR_{i,\tau}| < \varepsilon, \forall v_i \in V$, for a small ε. When $\tau = 0$, PR_0 must be normalized to conform $\forall v_i \in V, \sum_{i=1}^N PR_{i,0} = 1$.

The final convergence rank of all nodes can be regarded as their importance. Therefore, an individual with a higher rank is more suitable to be chosen as the seed node for ad recommendation.

Toy Example. Figure 4(a) illustrates a directed graph with $N = 5$. The traditional PageRank algorithm assumed that the initial belief of nodes are stochastic, thus we set the belief set $PR_0 = \{0.5, 0.7, 0.3, 0.8, 0.6\}$. Based on Eq. (6), the rank of node A can be calculated as:

$$PR_{A,\tau+1} = d \cdot \left[\frac{PR_{B,\tau}}{2} + \frac{PR_{C,\tau}}{2}\right] + \frac{1-d}{5}.$$

The out-degree adjacency matrix is:

$$\mathbf{F} = \begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 1/3 & 1/3 & 0 & 1/3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Suppose $Q = d \cdot \mathbf{F}$, $d = 0.85$ and $\varepsilon = 0.01$, we have

$$\mathbf{Q} = \begin{bmatrix} 0 & 0.2833 & 0.2833 & 0.2833 & 0 \\ 0.425 & 0 & 0 & 0.425 & 0 \\ 0.425 & 0 & 0 & 0 & 0.425 \\ 0 & 0.2833 & 0.2833 & 0 & 0.2833 \\ 0 & 0 & 0 & 0.85 & 0 \end{bmatrix}.$$

In the initial stage, we normalize PR_0 to meet the requirement $\sum_{i=1}^N PR_{i,0} = 1, \forall v_i \in V$, thus $PR_0 = \{0.1724, 0.2414, 0.1034, 0.2759, 0.2069\}$. According to Eq. (8), PR is updated as:

- $\tau = 1$: $PR_1 = \{0.1765, 0.157, 0.157, 0.3573, 0.1521\}$. None of nodes in V except A satisfy convergence conditions.
- $\tau = 2$: $PR_2 = \{0.1635, 0.1813, 0.1813, 0.2761, 0.198\}$. None of nodes in V satisfy convergence conditions.
- $\tau = 3$: $PR_3 = \{0.1841, 0.1545, 0.1545, 0.3216, 0.1852\}$. None of nodes in V satisfy convergence conditions.
- $\tau = 4$: $PR_4 = \{0.1614, 0.1733, 0.1733, 0.3053, 0.1868\}$. None of nodes in V except E satisfy convergence conditions.

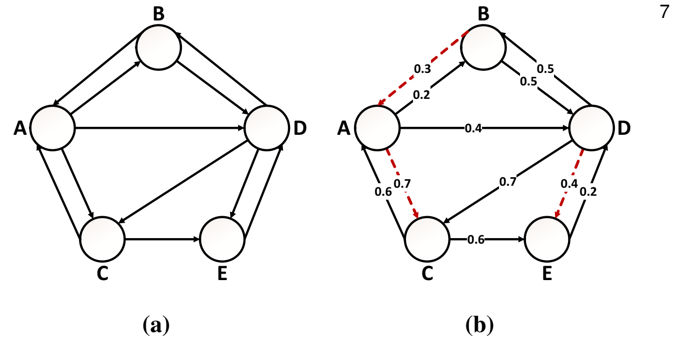

Fig. 4: Example of PageRank. (a) a directed graph without weight. (b) a signed weighted directed graph, the red dotted arrows are negative links.

- $\tau = 5$: $PR_5 = \{0.1773, 0.1622, 0.1622, 0.3081, 0.1901\}$. None of nodes in V except D and E satisfy convergence conditions.
- $\tau = 6$: $PR_5 = \{0.1679, 0.1675, 0.1675, 0.3108, 0.1862\}$. Every node in V satisfies convergence conditions.

After 6 rounds, the traditional PageRank algorithm converges, and the rank of nodes in $G(V, E)$ is $\{D, E, A, B, C\}$.

However, the traditional PageRank algorithm does not consider the labels of edges (positive/negative relations), which are of great importance to characterize the influence of friends or foes on individuals. In [5], an extended PageRank algorithm was proposed for signed networks, which focuses on the change of signs rather than influence maximization. In [5], an integrated PageRank algorithm was proposed to calculate the ranks in G^+ and G^-, respectively, but did not take into account local influence of signed social networks. Moreover, none of these works consider parallel recommendation and dynamic adaptation for belief updates.

3.3.2 *Signed-PageRank Algorithm*

Compared with greedy algorithms, PageRank algorithm works with matrix, which can greatly improve efficiency. Therefore, we propose a new algorithm, called Signed-PageRank (SPR), to rank nodes in the non-ascending order of their importance and choose top-ranking nodes as initial seed nodes for influence maximization in signed social networks.

Recall that in Eq. (4), the influence of individual v_j on the belief of individual v_i is $w_{j,i} \cdot (x_{j,t} - x_{i,t})$, and PR_{v_i} indicates network influence and status of v_i in PageRank, thus SPR should be calculated from a presenter standpoint, i.e., the influence for his neighbors. With a similar rationale, we update the rank of individual $\forall v_i \in V$ at iteration τ as:

$$SPR_{i,\tau+1} = \sum_{v_j \in H_i^{out}} (SPR_{i,\tau} - SPR_{j,\tau}) \cdot y_{i,j} + (1-d)/N, \tag{9}$$

where $y_{j,i} \in \mathbf{Y}$, and $\mathbf{Y}$ is the Signed-PageRank adjacency matrix with damping coefficient:

$$\mathbf{Y} = d \cdot (\widetilde{\mathbf{W}} * \mathbf{L}), \tag{10}$$

where $\widetilde{\mathbf{W}}{*}\mathbf{L}$ is the Hadamard product of $\widetilde{\mathbf{W}}$ and $\mathbf{L}$. $\widetilde{\mathbf{W}}$ is defined as:

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Algorithm 3 SPR: Signed PageRank Algorithm

Input: The signed social network $G(V, E)$, the initial set of beliefs X_0, the matrix of labels $\mathbf{L}$, the number of initial seed nodes k.
Output: The set of seed nodes $\mathcal{S}$.

1: $\tau = 0$.
2: Calculate the the normalized matrix $\widetilde{\mathbf{W}}$ = of W to make $\sum_{i=1}^{N} \widetilde{w}_{i,j} = 1, \forall v_j \in V$.
3: Calculate $\mathbf{Y}$ based on Eq. (10).
4: **for all** $v_i \in V$ **do**
5:     $SPR_{i,\tau} = x_{i,0}$.
6: **end for**
7: $Sort_\tau = [1, 2, ..., N]$.
8: $Sort_{\tau+1}$ = sort $SPR_{i,\tau}$ in a descending order.
9: **while** $\exists v_i \in V, Sort_{i,\tau+1} \neq Sort_{i,\tau}$ **do**
10:     $Sort_\tau = Sort_{\tau+1}$.
11:     **for all** $v_i \in V$ **do**
12:         $SPR_{i,\tau+1} = \sum_{j \in H_i^{out}} (SPR_{i,\tau} - SPR_{j,\tau}) \cdot y_{i,j} + (1-d)/N$.
13:     **end for**
14:     $Sort_{\tau+1}$ = sort $SPR_{i,\tau+1}$ in a descending order.
15:     $\tau = \tau + 1$.
16: **end while**
17: $\mathcal{S}$ = the first k individuals in $Sort_\tau$ as seed nodes.

$$\widetilde{\mathbf{W}} = \begin{bmatrix} \widetilde{w}_{1,1} & \cdots & \widetilde{w}_{1,N} \\ \vdots & \ddots & \vdots \\ \widetilde{w}_{N,1} & \cdots & \widetilde{w}_{N,N} \end{bmatrix},$$

where $\widetilde{\mathbf{W}}$ is the normalized matrix of weight $w_{i,j}$, i.e., $\forall v_j \in V$, $\sum_{j=1}^{N} \widetilde{w}_{i,j} = 1$. The initial rank of an individual is her belief, i.e., $SPR_{i,0} = x_{i,0}$.

The convergence condition $|PR_{i,\tau+1} - PR_{i,\tau}| < \varepsilon, \forall v_i \in V$ cannot be applied to $SPR_{i,\tau}$, since our extensive experiments show that the signed rank $SPR_{i,\tau}$ in Eq. (9) will go towards infinity as the number of iterations τ increases. However, our experiments also show that the sorted ranking order of individuals will converge. Therefore, we stipulate the termination condition of the proposed SPR algorithm as:

$$|Sort_{i,\tau+1} - Sort_{i,\tau}| = 0, \forall v_i \in V, \tag{11}$$

where $Sort_{i,\tau}$ is the sorted ranking order of individual v_i.

The proposed Signed-PageRank algorithm is presented in Algorithm 3. The computational complexity of Algorithm 3 is $O(n^2)$. In fact, line 11 ∼13 can be realized by basic matrix operations in MATLAB, which can effectively reduce the computational complexity to $O(n)$. Our experiments show that the final sorting order, which represents how important an individual is in the signed social networks, will not be affected by $SPR_{i,0}$. In other words, the proposed Signed-PageRank algorithm can generate a stable result given any initial values.

Toy Example. We will explain the entire ad recommendation process in a signed weighted social network as shown in Figure 4(b). Without loss of generality, we assume that individual v_i will accept the advertisement (the attitude becomes 1) and recommend it to her out-degree neighbours if her belief satisfies $x_{i,t} > 0.7$. Note that in real practice, the attitude $A_{i,t}$ will follow the binomial distribution $B(1, x_{i,t})$. We assume that $d = 0.85$, $k = 1$, the initial set of beliefs is $X_0 = \{0.5, 0.7, 0.3, 0.8, 0.6\}$, and the recommendation cycle is $T = \{[1, 3], [1, 2], [1, 6], [4, 8], [4, 5]\}$.

TABLE 2: Example - SPR update

Iteration	SPR update	sorted order
1	$-0.166, -0.28, -0.496, -0.326, -0.496$	A, B, D, C, E
2	$-0.589, -0.53, -0.67, -0.65, -0.798$	B, A, D, C, E
3	$-0.826, -0.779, -0.816, -0.758, -0.94$	D, B, C, A, E
4	$-0.917, -0.864, -0.912, -0.086, -0.968$	D, B, C, A, E

TABLE 3: Example - information propagation for ad recommendation

Time	Activity	Belief	Seeds
1	$D \to B, C$	$0.5, 0.75, 0.475, 0.8, 0.6$	B, D
2	$D \to C$; $B \to A$	$0.463, 0.75, 0.668, 0.8, 0.6$	B, D
3	$D \to C$; $B \to A$	$0.42, 0.75, 0.76, 0.8, 0.6$	B, C, D
4	$C, D \to E$	$0.42, 0.75, 0.76, 0.74, 0.61$	B, C, D
5	$C, D \to E$	$0.42, 0.75, 0.76, 0.74, 0.62$	B, C, D
6	/	$0.42, 0.75, 0.76, 0.74, 0.62$	B, C, D

Seed Node Selection. To begin with, we find the first k individuals with the highest influence based on the proposed Signed-PageRank algorithm. The label matrix $\mathbf{L}$ is:

$$\mathbf{L} = \begin{bmatrix} 0 & 1 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

The edge matrix W is:

$$W = \begin{bmatrix} 0 & 0.2 & 0.7 & 0.4 & 0 \\ 0.3 & 0 & 0 & 0.5 & 0 \\ 0.6 & 0 & 0 & 0 & 0.6 \\ 0 & 0.5 & 0.7 & 0 & 0.4 \\ 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}.$$

We can calculate the normalized matrix $\widetilde{\mathbf{W}}$ as:

$$\widetilde{\mathbf{W}} = \begin{bmatrix} 0 & 2/13 & 7/13 & 4/13 & 0 \\ 3/8 & 0 & 0 & 5/8 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 5/16 & 7/16 & 0 & 1/4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

According to Eq. (10), we can calculate the Signed-PageRank adjacency matrix $\mathbf{Y}$ as:

$$\mathbf{Y} = \begin{bmatrix} 0 & 0.1308 & -0.4577 & 0.2615 & 0 \\ -0.3187 & 0 & 0 & 0.5313 & 0 \\ 0.425 & 0 & 0 & 0 & 0.425 \\ 0 & 0.2656 & 0.3719 & 0 & -0.2125 \\ 0 & 0 & 0 & 0.85 & 0 \end{bmatrix}$$

Then, according to Eq. (9), we calculate SPR iteratively as shown in Table 2. For example, after initialization $SPR_0 = \{0.5, 0.7, 0.3, 0.8, 0.6\}$, we can compute $SPR_{A,1} = y_{1,2} \cdot (SPR_{A,0} - SPR_{B,0}) + y_{1,3} \cdot (SPR_{A,0} - SPR_{C,0}) + y_{1,4} \cdot (SPR_{A,0} - SPR_{D,0}) + (1-d)/5 = -0.166$.

Advertisement Recommendation. The selected seed nodes will initiate ad recommendation to neighbouring individuals, who will update their beliefs according to Eq. (4). In this way, the advertisement will spread in the signed social network. For simplicity, we ignore the recover process in this example.

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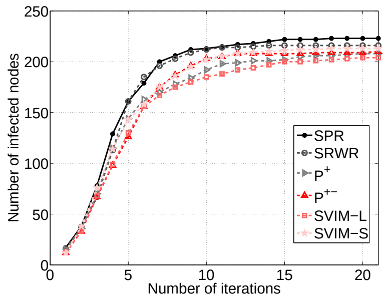
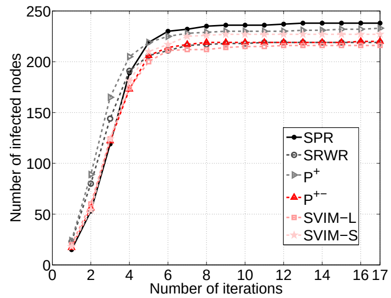
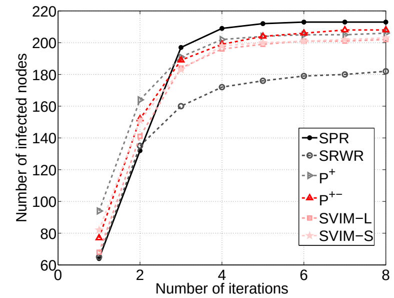


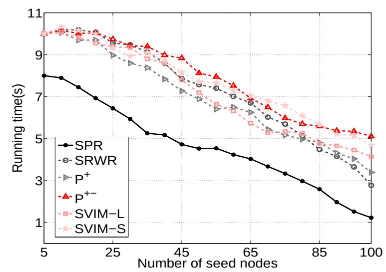
(a) $k = 5$



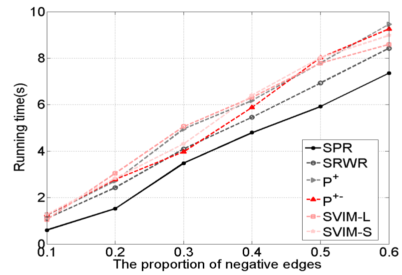
(b) $k = 20$



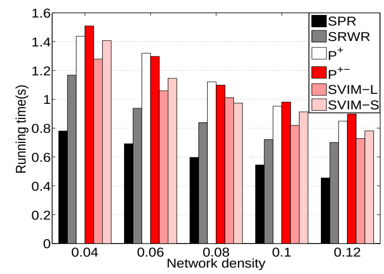
(c) $k = 50$



(d) $Dens = 0.04, P_{Neg} = 0.1$



(e) $k = 5, Dens = 0.04$



(f) $k = 5, P_{Neg} = 0.1$

Fig. 5: Performance comparison in synthetic signed social network with 300 nodes.

According to Eq. (3), the positive embeddedness of the five individuals can be calculated as $\alpha = \{0.5, 1, 0.5, 1, 0.5\}$, the negative embeddedness of the five individuals is $\beta = \{0.5, 0, 0.5, 0, 0.5\}$. As shown in Table 3, the information propagation for ad recommendation runs as follow:

- Time 1: $t = 1$, seed node D recommends the advertisement to neighbours B, C and E, and we have $t \in T_B = [1, 2]$, $t \in T_C = [1, 6]$, $t < T_E^l$. Then the beliefs of B and C are updated as:
$$x_{B,1} = x_{B,0} + \alpha_2 \cdot w_{4,2} \cdot (x_{D,0} - x_{B,0}) = 0.75,$$
$$x_{C,1} = x_{C,0} + \alpha_3 \cdot w_{4,3} \cdot (x_{D,0} - x_{B,0}) = 0.475.$$
Note that the beliefs of recommenders and of individuals who do not receive ad recommendations are unchanged. The belief set becomes $X_1 = \{0.5, 0.75, 0.475, 0.8, 0.6\}$. Since $x_{B,1} > 0.7$, the recommenders will be B and D.
- Time 2: $t = 2$, individual B recommends the advertisement to neighbour A, and individual D recommends the advertisement to neighbour C and E, and we have $t \in T_A = [1, 3]$, $t \in T_C = [1, 6]$, $t < T_E^l$. Then the beliefs of A and C are updated as:
$$x_{A,2} = x_{A,1} - \beta_1 \cdot w_{2,1} \cdot (x_{B,1} - x_{A,1}) = 0.463,$$
$$x_{C,2} = x_{C,1} + \alpha_4 \cdot w_{4,3} \cdot (x_{D,1} - x_{C,1}) = 0.668.$$
The belief set $X_2 = \{0.463, 0.75, 0.668, 0.8, 0.6\}$. the recommenders are unchanged.
- Time 3: $t = 3$, individual B recommends the advertisement to neighbour A, and individual D recommends the advertisement to neighbour C and E, and we have $t \in T_A = [1, 3]$, $t \in T_C = [1, 6]$, $t < T_E^l$. Then the belief of A and C is updated as:
$$x_{A,3} = x_{A,2} - \beta_1 \cdot w_{2,1} \cdot (x_{B,2} - x_{A,2}) = 0.42,$$
$$x_{C,3} = x_{C,2} + \alpha_4 \cdot w_{4,3} \cdot (x_{D,2} - x_{C,2}) = 0.76.$$
The belief set $X_3 = \{0.42, 0.75, 0.76, 0.8, 0.6\}$, Since $x_{C,3} > 0.7$, the recommenders will be B, $C$and D.
- Time 4: $t = 4$, individuals B, C recommends the advertisement to A, and individual C, D recommends the advertisement to neighbour E, we have $t > T_A^u$ and $t \in T_E = [4, 5]$, the belief of E is updated as
$$x_{E,4} = x_{E,3} + \alpha_5 \cdot w_{3,5} \cdot (x_{C,3} - x_{E,3}) - \beta_5 \cdot w_{4,5} \cdot (x_{C,3} - x_{E,3}) = 0.61.$$
The belief set $X_4 = \{0.42, 0.75, 0.76, 0.74, 0.61\}$, the recommenders are unchanged.
- Time 5: $t = 5$, individuals C and D recommend the advertisement to E simultaneously, we have $t \in T_E = [4, 5]$, the belief of E is updated as
$$x_{E,5} = x_{E,4} + \alpha_5 \cdot w_{3,5} \cdot (x_{C,4} - x_{E,4}) - \beta_5 \cdot w_{4,5} \cdot (x_{C,4} - x_{E,4}) = 0.62.$$
The belief set $X_5 = \{0.42, 0.75, 0.76, 0.74, 0.62\}$, the recommenders are unchanged.
- Time 6: $t = 6$, individuals C and D recommend the advertisement to E simultaneously, but we have $t > T_E^u$, thus the information propagation terminates.

After six rounds, the ad recommendation process stops and individuals B, C, D accept the advertisement, actually, individual E may accept this advertisement if his recommendation cycle delays.

4 EXPERIMENT

In this section, we evaluate the performance of our proposed framework with both synthetic datasets and a real-world dataset. We choose five benchmark algorithms for influence maximization of ad recommendations:

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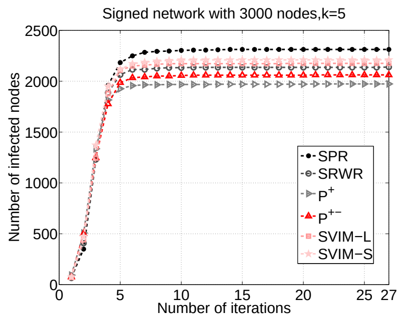
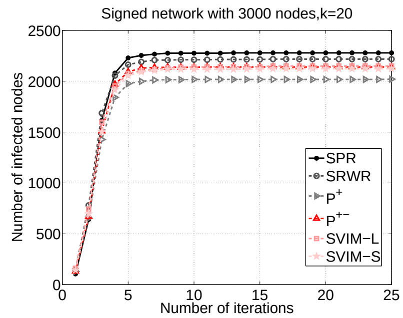
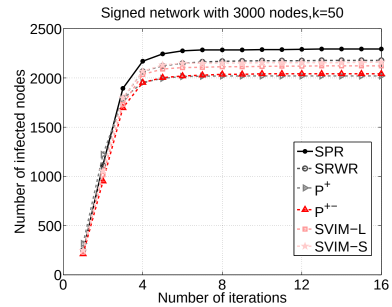


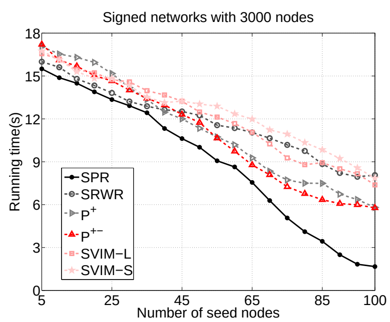
(a) $k = 5$



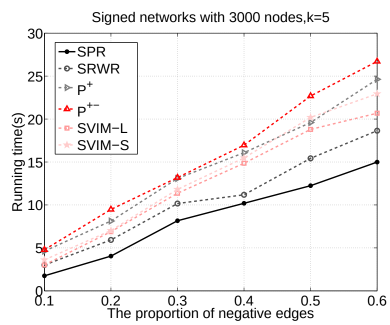
(b) $k = 20$



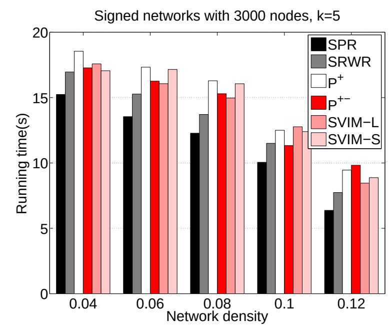
(c) $k = 50$



(d) $Dens = 0.04, P_{Neg} = 0.1$



(e) $k = 5, Dens = 0.04$



(f) $k = 5, P_{Neg} = 0.1$

Fig. 6: Performance comparison in synthetic signed social network with 3,000 nodes.

- P^+: choose the individuals with the highest weighted positive out-degree as seed nodes.
- P^{+-}: choose the individuals with the highest weighted out-degree as seed nodes.
- SRWR: the personalized ranking method in [18].
- SVIM-L: select initial seeds that maximize the long-term steady state influence coverage [27].
- SVIM-S: select initial seeds that maximize the short-term influence coverage [27].

The benchmark P^+ and P^{+-} are designed for unsigned social networks, ignoring the influence of positive/negative relationships on individuals. In SRWR, a random walker with a positive or negative sign moves in the signed networks. The walker will change her sign from positive to negative or vice versa when she encounters a negative edge, and will return to the start node with a certain probability. SVIM-L and SVIM-S are designed to find optimal solutions for influence maximization in both short-term and long-term cases in signed social networks.

4.1 Synthetic Datasets

We generate two synthetic datasets: 1) a social network with 300 individuals and 500 randomly-generated directed edges, 2) a social network with 3,000 individuals and 60,000 randomly-generated directed edges. The two generated social networks are sparse. The density of signed networks and proportion of negative links can be calculated as:

$$Dens = \frac{N_E}{N \cdot (N-1)}, P_{Neg} = \frac{N_{E^-}}{N_E},$$

where N_E and N_{E^-} are the total number of edges and the number of negative edges. The density of the two generated signed networks are 0.0056 and 0.0067, respectively, and the proportion of negative links is 0.04 for both networks.

Effectiveness. We run the experiments of information propagation for 1,000 times, and compare the average number of individuals who accept the advertisement (infected individuals with attitude 1). Figure 5(a)-(c) and Figure 6(a)-(c) show the number of infected individuals when the number of selected initial seed nodes is $k = 5$, $k = 20$, $k = 50$ respectively. We can see that the number of infected individuals by using the proposed SPR algorithm is 20% higher than the best benchmark algorithm, which confirms that the proposed SPR algorithm is more effective than benchmarks for broaden the range of advertisement propagation in signed social networks. We can observe that the number of infected individuals of SPR during early times may be lower than those of benchmark algorithms, but the number of infected individuals rises much faster than those of benchmark algorithms. The possible explanation is that SPR jointly consider positive and negative relationships in selecting initial seed nodes, which may not have many neighbours to recommend advertisement to during initial stages.

Efficiency. We show the number of iterations for SPR to convergence when k varies from 5 to 80. We run each experiment for 500 times and calculate the average required iterations for convergence. Figure 7 and Figure 8 show that SPR requires fewer iterations to converge in most cases than the benchmark algorithms. In particular, SPR converges much faster than benchmark algorithms when k varies from 17 to 30 in the signed social network with 300 nodes. Comparing Figure 7 and Figure 8, we find that the maximum number of iterations in the signed social network with 3,000 nodes is lower than that in the signed social network with 300 nodes, which is also true in Figure 5 and Figure 6. This is because the network density is 0.0056 in the signed social network with 300 nodes and is 0.0067 in the signed social network with 3,000 nodes. A higher network density implies that there are more neighbours for each individual.

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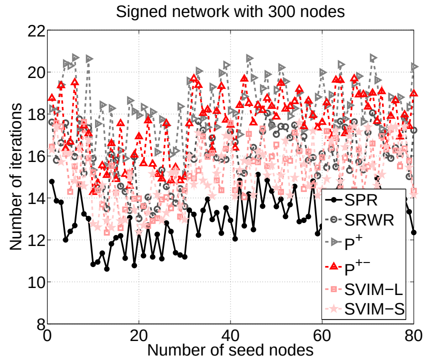

Fig. 7: Efficiency, synthetic signed networks with 300 nodes.

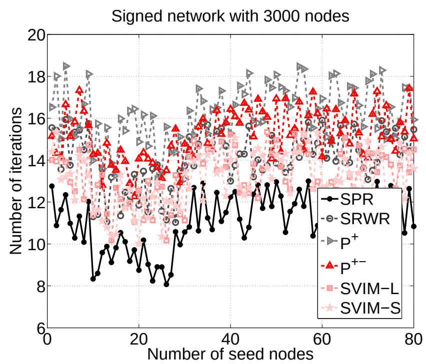

Fig. 8: Efficiency, synthetic signed networks with 3,000 nodes.

Since the belief of an individual will update more quickly if she has more neighbours, the probability of the individual accepting the advertisement will increase.

Running time. We show the duration between the start and the end of information propagation. We run each experiment for 500 times. The experimental results show that the propagation time of SPR is much shorter than that of the benchmarks. Figure 5(d) and Figure 6(d) explore the relationship between the propagation time and the number of seed nodes, given the density of the generated network as 0.04 and the proportion of negative links as 0.1. In SPR, the propagation takes less time as more seed nodes are chosen, because that advertisement will flood through the whole network faster with more seed nodes. As shown in Figure 5(e) and Figure 6(e), the propagation time grows with a higher proportion of negative edges, when the density of the generated network is 0.04 and the number of seed nodes is 5. Figure 5(f) and Figure 6(f) illustrate that propagation time decreases with network density.

4.2 Real Datasets

We evaluate the performance of SPR with two large online signed social network datasets Epinions and Slashdot [23]. Epinions is a consumer review site where individuals form positive or negative relationships with each other by agreeing or disagreeing with the product reviews they have written. Slashdot is a technology news website, where individuals can tag each other as friends or foes regarding the comments of the news.

Data processing. In Epinions datasets, the label $l_{i,j}$ is set as 1 if user i trusts user j, -1 if user i distrusts user j. For a product I_k reviewed by user j on Epinions, user i can rate how helpful the review of user j is from 1 to 6, i.e., $rate_{i,j,I_k} \in \{1, 2, 3, 4, 5, 6\}$.

TABLE 4: Statistics of real datasets

Dataset	Epinions	Slashdot
Number of nodes	131,828	82,140
Number of edges	841,372	549,202
Number of Positive Links	717,668	422,350
Number of Negative Links	123,704	123,322
Average clustering coefficient	0.1279	0.0588

The weight $w_{i,j}$ indicates the confidence of user i on user j, and we calculate $w_{i,j}$ based on the ratings of user i on user j.

$$w_{i,j} = \frac{\sum_{I_k=1}^{N_I} rate_{i,j,I_k}}{6 \cdot N_I},$$

where N_I is the number of all products.

The initial belief $x_{i,0}$ is the belief of user i in the advertisement in the inception phase. We calculate $x_{i,0}$ as the average ratings of user i for all her neighbor's reviews for all products (since we only consider one product, i.e., the advertisement, we average over all products on the Epinions).

$$x_{i,0} = \frac{\sum_{I_k=1}^{N_I} \sum_{j \in H_i^{out}} rate_{i,j,I_k}}{6 \cdot N_I \cdot |H_i^{out}|}.$$

The label matrix L, weight matrix W and belief sets X_0 for Slashdot dataset are constructed similar to those for Epinions. Since the datasets of Epinions and Slashdot are one-day snapshots without temporal evolution, the recommendation cycle $T_i = [T_i^l, T_i^u]$ for all users are randomly generated. T_i^l is randomly drawn in the range $[0, 50]$, and T_i^u is generated as the sum of T_i^l and a random time slot in $[0, 10]$.

Considering the network size in Epinions and Slashdot are too huge and redundant, we remove the isolated nodes and retain the first 20,000 nodes as the signed social networks. The main attributes of these two datasets are listed in Table 4.

Effectiveness. We run each experiment for 500 times and compare the average number of infected individuals. Figure 9(a)-(c)and Figure 10(a)-(c) show that SPR outperforms benchmark algorithms in both Epinions and Slashdot. The number of infected individuals of SPR is 8.4% higher than the best benchmark algorithm in Epinions and 8.8% higher than the best benchmark algorithm in Slashdot. Comparing Figure 9(a)-(c), we can find that if we increase the number of seed nodes, the final numbers of infected individuals are almost the same, but the number of rounds for ad propagation to terminate decreases slightly. In fact, most of ad recommendations are accomplished within 5 rounds thanks to our designed efficient information propagation framework.

Running time. We run each experiment for 500 times and compare the average running time. Figure 11 and Figure 12 illustrate that the running time shrinks as there are more initial seed nodes, similar to the trend in synthetic datasets. SPR reduces the running time compared with benchmark algorithms, which verifies the efficiency of SPR in information propagation.

The experimental results have verified that SPR outperforms benchmark algorithms in both the synthetic and the real datasets. In conclusion, the proposed SPR algorithm can effectively and efficiently select initial seed nodes for information maximization of ad recommendation in signed social networks.

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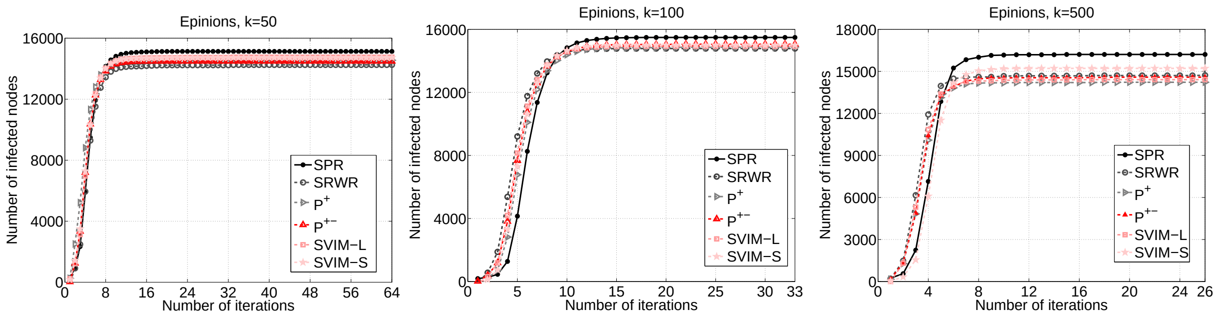
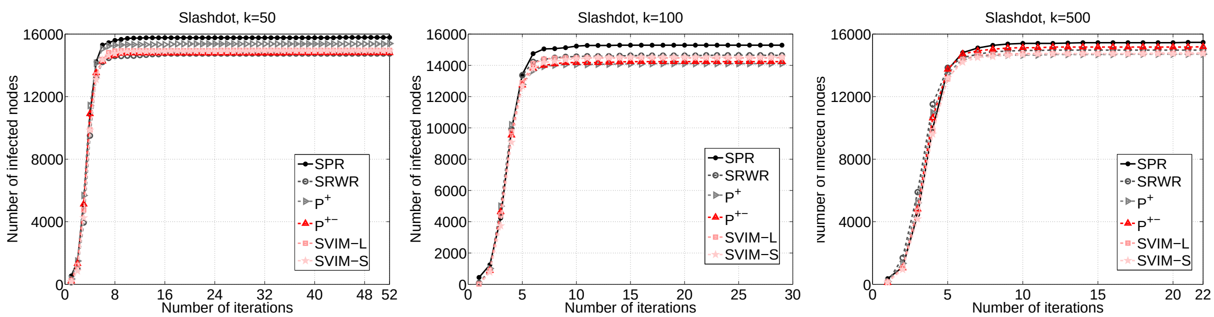


(a) $k = 50$ **(b)** $k = 100$ **(c)** $k = 500$

Fig. 9: Performance comparison, Epinions with 20000 nodes.



(a) $k = 50$ **(b)** $k = 100$ **(c)** $k = 500$

Fig. 10: Performance comparison, Slashdot with 20000 nodes.

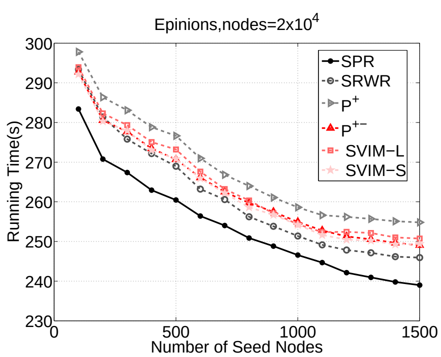


Fig. 11: Propagation time in Epinions.

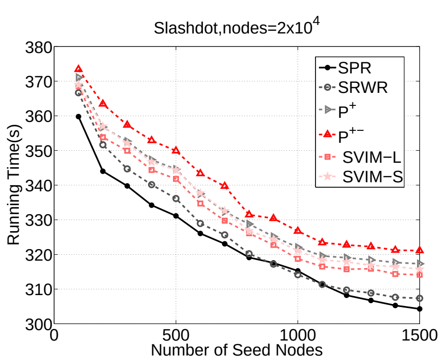


Fig. 12: Propagation time in Slashdot.

5 Related Work

5.1 Influence Maximization

The objective of influence maximization is to maximize the influence coverage in social networks with the minimum time and the minimum number of seed nodes. In [29], Li et al. presented a comprehensive survey of existing works on influence maximization and discussed future research directions. To achieve influence maximization, we need to find a certain number of most influential individuals (seed nodes), who will spread the information widely in social networks based on a specific propagation model. In [11], Domingos et al. first proposed the influence maximization model in social networks and formulate it as a Markov random field, then they designed a heuristic algorithm to achieve influence maximization. In [19], Kempe et al. extended the Independent Cascade (IC) model and Linear Threshold (LT) model for influence maximization, and proved that the optimization problem of selecting seed nodes is NP-hard. In [34], Shen et al. proposed a linear threshold-based diffusion model for signed social networks, which considered negative relationships between individuals for influence maximization. The natural greedy strategy was adopted to solve the problem of influence maximization, but the greedy algorithm is not scalable due to long operational time and complex calculations. In [7], [21], [6], [15] and [12], the researchers focused on designing efficient greedy algorithms and scalable heuristics with reduced running time but the performance of the algorithms is degraded. In [30], Liu et al. built a cascade diffusion-based model to distinguish positive influence spreading from negative influence spreading, and proposed a greedy algorithm to maximize the spreading of positive influence. In [13] and [37], the voter model was applied to characterize basic features of influence maximization. The voter model is a naive probabilistic model in which each node adopts randomly the opinion or the attitude of their neighbours. In epidemiology, the disease spread is similar to information propagation in social networks, thus a numbers of studies extended the traditional SIR and SIS epidemic model to study influence maximization [8], [20]. However, all these works do not distinguish different influence of ad recommendations from friends and foes, especially parallel recommendation.

5.2 Belief Dynamics

Belief dynamics has long been studied for social networks. In [14], Ghaderi et al. focused on the formation of beliefs about a specific topic in social networks. They assumed that each individual has an initial belief for a topic, then their beliefs will change based on the initial belief and the beliefs of their neighbours. In [1], according to both Bayesian and non-Bayesian models, Acemoglu et al. discussed the formation of beliefs on the structure of social relationships, and provided a mathematical model to combine the belief dynamics and the distribution of prior beliefs. Unfortunately, the Bayesian method needs the prior knowledge, which is hard to acquire. The DeGroot model described in [17] is a classical non-Bayesian model of opinions dynamics using a local update approach, which drives the belief of nodes closer to their friends. In [26], Li et al. provided the LT-IO model (Linear Threshold model with Instant Opinions) for influence maximization by considering the real-time attitudes of individuals. However, all these works study belief dynamics in unsigned social networks but not signed social networks with both positive and negative relationships. Based on the traditional DeGroot model and probability theory, we make the first attempt to study real-time belief dynamics for influence maximization in signed social network.

5.3 Signed Social Networks

In recent years, more attention has been paid to signed social networks that consist both positive and negative links [5], [35], [28], [22], [27], [24]. In [35], Tang et al. proposed RecSSN for recommendation in signed social networks, which captured the local and global information, and they demonstrated that users are more likely to be similar to their friends than foes. In [28] and [27], Li et al. extended the classical voter model to signed social networks, and analyzed the long-term and short-term dynamics for influence coverage. In [22], Kunegis et al. designed link prediction algorithms, which focused on measuring the local balance for graph drawing and clustering. In [24], an approach based on simulated annealing (SA) for influence maximization is proposed, but the performance of the algorithm is highly dependent on the initial values of parameters. In [5], the traditional PageRank algorithm is extended for signed social networks, which guarantees global convergence but ignores the influence of negative links. Most existing works on influence maximization in signed social networks consider either positive influence or negative influence. As far as we are concerned, we are the first to integrate positive and negative influences to address the problem of belief update in the case of parallel recommendation. Our extensive experiments have confirmed that the proposed Signed-PageRank algorithm is more effective in selecting seed nodes for influence maximization than existing algorithms.

6 Conclusion

In this paper, we have investigated the problem of influence maximization for ad recommendation in signed social networks. We have proposed a new framework to better describe the process of information propagation in signed social networks and designed belief update rules considering influence from both positive and negative relationships. To realize influence maximization, we have proposed a novel Signed-PageRank algorithm, which jointly takes account of the influence of positive and negative links when selecting initial seed nodes to boost ad recommendation. Experimental results demonstrate that the proposed Signed-PageRank algorithm outperforms the benchmark algorithms, improving the number of individuals who accept the advertisement by 20% on synthetic datasets, and by 8.4% and 8.8% in two real datasets.

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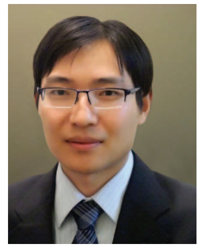

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