Achievable Throughput of Multiple Access Spectrum Systems Based on Cognitive Relay

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ABSTRACT

Cognitive relays form a special cooperation relationship among users in cognitive radio networks, and help increase the transmission rates of both primary users and secondary users. However, we observe that in certain scenarios, the use of relays may deteriorate the performance. In this paper, we propose a novel scheme in the MAC layer, called *CodeAssist*, by using network coding. It renders every relayed packet useful and recoverable as long as a sufficient number of coded packets are received by the intended terminal. *CodeAssist* is designed to apply network coding in every relay buffer, and moreover, leads to a lower bound of relayed packets for each secondary user. We also show numerical results for further demonstrations of the improved performance with *CodeAssist*.

Categories and Subject Descriptors

C.2.4 [**Computer-Communication Networks**]: Distributed Systems—*Distributed Applications*; C.4 [**Performance of Systems**]: Design Studies

General Terms

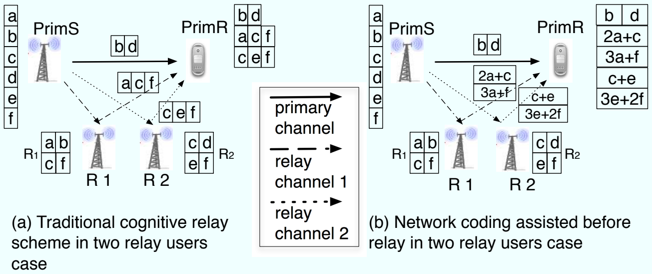
Algorithms, Design, Performance

1. INTRODUCTION

The inefficiency of using wireless spectrum has become a significant challenge in wireless communication [3]. The cognitive radio technology provides the capability of utilizing licensed spectral resources by dynamically accessing spectrum holes, which mitigates the problem. The cognitive principle states that, in a certain spectrum, the presence of the secondary transmission activity should be "transparent" to the primary user [16], which implies that secondary users (who have cognitive abilities) are free to use the spectrum as long as the transmission of primary users (users who pay to use the spectrum) do not. There exists prior work in the literature on maximizing the transmission throughput of the sub-channel for secondary users with consideration of interference [6]. Hence, several cooperation strategies are motivated in cognitive radio networks.

One cooperation strategy, referred to as *cognitive relaying*, is the primary focus in this paper. It refers to an approach of employing secondary users as relay nodes to maximize the throughput of both primary and secondary transmissions. A cognitive radio network with multiple transmission channels can be treated as a multiple-input multiple-output system aiming at efficiently utilizing the spectrum resource, which was introduced by Simeone *et al.* [16]. The related work on using cognitive relays focused on the maximization of throughput with two secondary cognitive relays, by minimizing the interference among relay channels in the PHY layer.

We argue that opportunities to further improve the performance of the cognitive radio system may not be limited to the physical layer. In this paper, we identify the following challenge in a cognitive relay system. Consider the example scenario in Fig. 1. Except the primary communication channel between the primary sender (PS) and the primary receiver (PR), there are also two relay channels, separately supported by two secondary users $R1$ and $R2$. The challenge is that these relays need to communicate with each other to discover the packet overlap in their respective buffers, so that redundant transmissions can be mitigated or avoided.



(a) Traditional cognitive relay scheme in two relay users case

(b) Network coding assisted before relay in two relay users case

Figure 1: The motivating scenario for network coding in a cognitive relay system to avoid redundant transmissions.

As illustrated in Fig. 1(a), packet c and f are simultaneously received by two secondary nodes. They are delivered twice in the traditional relay without any coordination. Due to synchrony in cognitive sensing, such a phenomenon occurs with high probability and decreases the effective throughput. To make matters worse,

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the messaging overhead of inter-relay coordination will be multiplied when the number of relay users increases. This renders such inter-relay coordination a solution far less ideal, even completely infeasible.

Is there a remedial strategy to reduce or eliminate the need for inter-relay coordination and the associated messaging overhead? We believe that random network coding can be used at both relays, as showed in Fig. 1(b). When random network coding is applied before relaying, each relay only needs to deliver a sufficient number of linearly independent packets. There is no need for inter-relay coordination.

In this paper, we present *CodeAssist*, a new algorithm design based on our intuitive insights in Fig. 1. It is developed in the scenario with one primary node and several homogeneous secondary nodes. As an efficient approach, network coding is combined with traditional relay schemes in buffering to eliminate the overlapping in relay buffers. Redundant packet deliveries are reduced, or even eliminated, since all delivered packets are linearly independent under network coding. In addition, low computational complexity is guaranteed by the scheme, and the necessary amount of buffering space is reduced. Meanwhile, the throughput approaches its theoretical upper bound.

The remainder of this paper is structured as follows. After presenting related work in Sec. 2, we first present the single-primary multi-secondary cognitive radio model in Sec. 3, and study its no-relay transmission performance. In Sec. 4, an additional relay scheme is explored. In Sec. 5, *CodeAssist* is introduced as a practical solution, and its feasibility and effectiveness are validated by numerical results. Finally, Sec. 6 concludes the paper.

2. RELATED WORK

The cognitive relay scheme is based on the detective capability of cognitive users to collaborate in the same spectrum [10]. With cognitive radio [3], it is applicable for users to detect the "white space" and hop among spectrums. To maximize the throughput, its application is broadened from a single relay to multiple relays [7]. Although existing works in the literature have focused on the interference among secondary users, the achievable rate region was given as a result of a multiple relay system consisted with one primary and multiple secondary users [1, 17]. Devroye *et al.* and Gambini *et al.* [6, 8, 9] proposed schemes that have mitigated not only interference, but also the detective error in the relay system. In [12, 17], Sridharan *et al.* and Jovicic *et al.* have focused on cooperative decoding of multiple channel information. These works have the same motivation that the performance for cognitive communication should be improved. They all pay more attention to potential challenges in the PHY layer, rather than the MAC layer.

However, redundant relaying in the MAC layer after synchronous sensing has also decreased the effective throughput. This potential problem was often solved by coordination in previous work. In Simeone *et al.*'s first work on cognitive relays [16], it proposed that the MAC protocols should be redesigned, following the PHY layer. It applied ACK (Acknowledgement) exchanging for every packet to solve the potential redundancy problem across multiple relays. Zhang *et al.* [18] gave a heuristic system model, which considered both the multiple relay scenario and the influence of MAC layer cooperation. In their model, utilizing the AP (Access Point) as a central control station can solve any possible problems in the MAC layer. However, we note that such a coordination strategy causes excessive overhead. In this paper, redundant relaying is avoided without coordination with the help of network coding, which has originally been proposed in information theory [2, 11]. It is inferred that random network coding can decrease the redundancy, caused by multi-channel transmission [14, 13]. In [4], a new network coding protocol has been presented and implemented to address redundancy in opportunistic routing in multi-hop 802.11-based networks. The system model assumes that all wireless links share the same wireless channel. In contrast, *CodeAssist* focuses on a cognitive relay setting, where multiple cognitive secondary users serve as relays for packets from the primary user. We believe that network coding has not been applied in the cognitive relay setting in the literature.

3. PRELIMINARY MODELING

We illustrate a cognitive relay network in Fig. 2. The network contains one primary licensed user, which exists as the master of the spectral resource in the network, and N secondary unlicensed users, which generally have cognitive equipment for the detection of idle spectrum.

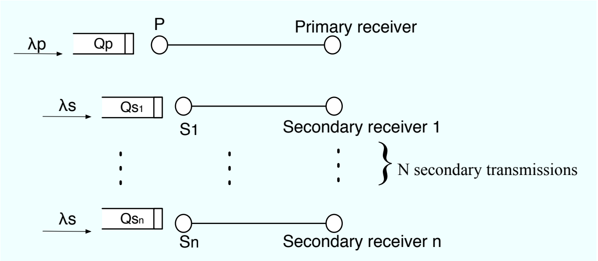


Figure 2: A basic model containing one primary transmission and N secondary transmissions.

In the MAC layer, the system is denoted as $\Omega(P, S, C)$, where P is the primary user, S is the set of secondary users $\{s_1, s_2, \ldots, s_n\}$, and C is the set of channels in the spectrum.

Each channel in C is equipped with a buffer, in which all receiving packets are stored in queue Q_i (i equals to p or s to distinguish the queue of the primary from that of the secondary). The packet arrival processes in each buffer are independent and i.i.d. Bernoulli processes, as defined with a mean of λ_p for the primary transmission and λ_s for every secondary transmission.

To guarantee the operation of the cognitive system, there is a constraint:

$$\lim_{t\to\infty} P[Q_p(t) = 0] \neq 0$$

for the primary transmission, in which $P[Q_p(t) = 0]$ means idle time-slots for secondary transmissions.

In the physical layer, we assume channels in C are all Rayleigh block-fading channels. Given channel parameters $(\beta_p, \beta_s, \lambda_p)$ and the power gain of each channel γ_i (i equals to p or s), $\gamma|h(t)|^2P < \beta$ is the guarantee against transmission failure in channels, where $h(t)$ is a zero-mean unit-variance stationary process and P is the transmission power and equals to 1.

We only pay attention to the transmission error occurred in every physical channel, whose probability is given by the preliminary model

$$P_e = P[\gamma|h(t)|^2P < \beta] = 1 - \exp(-\frac{\beta}{\gamma}). \tag{1}$$

For the sake of fair competition for the idle spectrum among secondary users, we apply TDMA as the spectrum access strategy and round robin as the spare spectral resource allocation scheme. Hence, all secondary users take turns to utilize the idle spectrum. For the simplicity of theoretical analysis, we ignore all the interference among transmissions and also assume all feedback from the primary receiver are correctly perceived in the primary transmission.

4. COGNITIVE RELAY AND PROBLEM FORMULATION

With the definition of a traditional cognitive system in Sec. 3, it is clear that each packet could be received correctly with the average of probability $1 - P_e$ without relay. The maximal throughput of the primary transmission is written as

$$\mu_p^{\max} = 1 - P_{e,p} = P[\gamma_p |h(t)|^2 P > \beta_p] = \exp(-\frac{\beta_p}{\gamma_p P_p}).$$

Little's theorem [15] indicates that the behavior of sub-channel transmissions is dependent among one other. Therefore, we have spare bandwidth for secondary users $P[Q_p(t) = 0] = 1 - \lambda_p/\mu_p^{\max}$. From Eq. (1), we obtain the maximum available bandwidth that secondary users have in the traditional case:

$$\mu_s^{\max} = \exp(-\frac{\beta_s}{\gamma_s P_s})(\frac{\mu_p^{\max} - \lambda_p}{\mu_p^{\max}}). \tag{2}$$

Furthermore, we obtain the basic constraint for the input rate of the primary user according to the Loynes' theorem. The queue service process is stable when the average incoming rate is less than the average outgoing rate:

$$\lambda_p < \mu_p < \exp(-\frac{\beta_p}{\gamma_p P_p}).$$

4.1 Multiple relay system: performance analysis

Cognitive nodes are commonly equipped with a strong cognitive ability, which promises more received packets in the sensing process than the primary receiver. According to the feature, the promising approach of relaying packets by secondary users instead of retransmitting from the primary user can be applied in cognitive radio networks. A relay scheme introduced in [16] has been proved helpful in a single relay cognitive system, with just one primary user and one secondary user. In this paper, we extend it to the case of multiple cognitive relays, and present its analysis and simulations.

We add N relay channels to the channel set C to the model in Sec. 3. These channels connect every secondary user with both the primary user and the primary receiver. The traditional relay is implemented as the following: After the sensing process, secondary users forward the sensed packets to the primary receiver. Two separate processes, the sensing process and the relay process, are simultaneously implemented on every secondary user. With the acknowledgement of the primary receiver, only packets lost in the primary transmission are forwarded by secondary users. The relay model is illustrated by Fig. 3. There are N relay channels assisted by N secondary users, respectively. In the relay subsystem, γ_{ps} is the sensing process gain and γ_{sp} is the relaying process gain. In addition, N cognitive queues are equipped for the relay and their average incoming rate of each queue is

$$\lambda_{ps} = \frac{\lambda_p}{\overline{\mu_p}^{\max}} P_{e,p}(1 - P_{e,ps}). \tag{3}$$

In Eq. 3, $\overline{\mu_p}^{\max}$ is the new upper bound of the primary transmission after the system update. In the relay case, the primary transmission gains the entire capacity of the main channel and N relay channels. We obtain the maximum as

$$\overline{\mu_p}^{\max} = 1 - P'_{e,p} = 1 - P_{e,p} P_{e,s}^N.$$

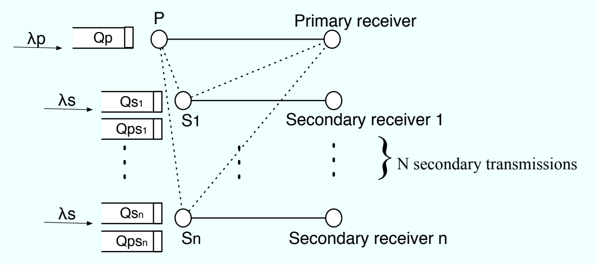


Figure 3: A modified system introduced with relay scheme.

Assuming that every packet has the probability ε to leave cognitive queues, the upper bound of the departure rate is given by

$$\begin{aligned} \mu_{ps}^{\max}(\varepsilon) &= \frac{1}{N} P[Q_p(t) = 0](1 - P_{e,sp})\varepsilon \\ &= \frac{1 - \lambda/\overline{\mu_p}^{\max}}{N} \cdot \exp(-\frac{\beta_p}{\gamma_{sp}})\varepsilon, \end{aligned} \tag{4}$$

and the maximal output of each secondary transmission with the variable ε

$$\mu_{s_i} = \frac{1}{N} P[Q_p(t) = 0](1 - \varepsilon)(1 - P_{e,s}). \tag{5}$$

The relay process is stationary when

$$\lambda_{ps} < \mu_{ps}^{\max},$$

which means

$$\varepsilon > N \cdot \frac{\lambda_p}{\overline{\mu_p}^{\max} - \lambda_p} \cdot (1 - \exp(-\beta_p/\gamma_p)) \frac{\exp(-\beta_p/\gamma_{ps})}{\exp(-\beta_p/\gamma_{sp})}. \tag{6}$$

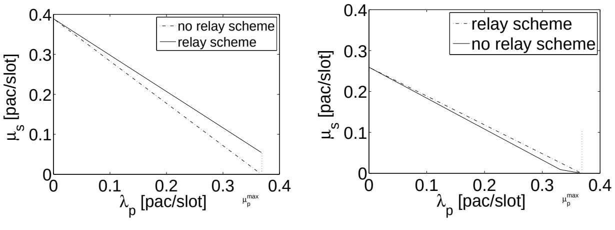
When ε reaches its minimal value, the system is in the sub-stable state. Therefore we obtain the maximal throughput of each secondary transmission with the mean

$$\begin{aligned} \mu_{s_i}^{\max} &= \frac{1}{N} P[Q_p(t) = 0](1 - \varepsilon_{\min})(1 - P_{e,s}) \\ &= \frac{1}{N} \frac{\overline{\mu_p}^{\max} - \lambda_p}{\overline{\mu_p}^{\max}} \exp(-\frac{\beta_s}{\gamma_s}) \end{aligned}$$

$$(1 - N \cdot \frac{\lambda_p}{\overline{\mu_p}^{\max} - \lambda_p} \cdot (1 - \exp(-\frac{\beta_p}{\gamma_p})) \frac{\exp(-\frac{\beta_p}{\gamma_{ps}})}{\exp(-\frac{\beta_p}{\gamma_{sp}})}). \tag{7}$$

4.2 Numerical results for the multiple relay system

The performance of system is evaluated by the maximum throughput of the individual secondary transmission. The parameters of the simulation are selected as $\gamma_p = 4\text{dB}, \gamma_s = \gamma_{sp} = \gamma_{ps} = 10\text{dB}$, $\beta_p = \beta_s = 4\text{dB}$.



(a) (b)

Figure 4: The maximum throughput μ_s versus λ_p. (a) results with two relay users; and (b) results with 3 relay users.

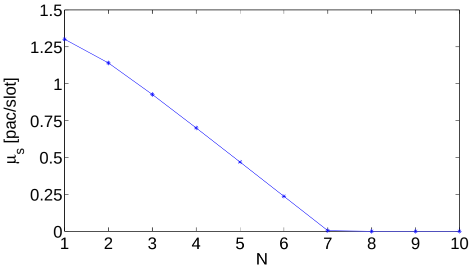


Figure 5: The ratio of the maximum throughput with normal relay transmission over that without relay transmission, with respect to the number of secondary users sharing the spectrum.

As it is illustrated in Fig. 4, we observe that the performance of the cognitive relay system is improved with one or two relay nodes active, but not when the number of relay nodes is larger than 3. When there are two cognitive relay nodes, the simulation result shows the line of the relay scheme is higher than that of the original scheme. However, when adding one more relay node, as illustrated in Fig. 1 as a toy example, the cognitive relay scheme shows an inferior level of performance when there are more than two relay nodes, as compared to the no relay case. In the next section, we are able to analyze this phenomenon by formulating it as a coupon collection problem.

We have observed that, though the number of packets sensed by secondary users increases, the number of packets sensed by individual secondary users as an independent process never changes. Moreover, those secondary users have no knowledge about what packets others have received. As we have stated, the packet-collecting processes are i.i.d. Therefore, the set of packets simultaneously sensed by different secondary users overlaps and are delivered redundantly. The transmission of the relays causes redundancy for the entire system. It also degraded the performance of the throughput when more than 3 secondary users participate.

One potential way to avoid the redundancy is to acknowledge every packet globally in the system. However, we notice that it involves delicate mechanisms such as waiting and feedback, which will introduce additional overhead by exchanging acknowledgement. The window scaling strategy, which usually compromises between the throughput and the overhead, cannot work satisfactorily with multiple cognitive relays without acknowledgements. Even if there are acknowledgements, centralized coordination for relay transmissions is still required. Otherwise, the system may operate with a low efficiency. In what follows, we present *CodeAssist*, that takes advantage of network coding to avoid inter-relay coordination.

5. CODEASSIST: ALGORITHM DESIGN

We mentioned in Sec. 4 that, the redundancy across multiple relays and the resulting need for coordination has been a major challenge towards the use of multiple cognitive relays in the system. Without a doubt, the redundancy can be avoided when every users know the local buffer of other users. However, overhead might be increased excessively, accompanied by a large amount of local information exchanges and a substantial degree of computational complexity. As an efficient solution, we introduce the *CodeAssist* algorithm based on network coding.

5.1 How effective is network coding?

In random network coding theory [5], a data segment (referred to as a generation in the original paper) is divided into n packets, each of which has a fixed number of bytes. When the segment is to be transmitted, the sender randomly chooses a set of coding coefficients in the Galois field $GF(2^8)$, and implements linear combination on these packets. Random network coding compresses multiple packets with randomly generated coefficients into one packet. In the theory of network coding, the receiver decodes coded packets and recovers original ones after receiving a sufficient number of packets.

Our cognitive relay algorithm with network coding, called *CodeAssist*, consists of three steps: (1) primary transmissions accompanied with the sensing by secondary users; (2) cognitive relay by secondary users with network coded packets; and (3) retransmission of missing packets, the need for which is mitigated by network coding. In *CodeAssist*, partial coding is applied instead of a global coding strategy in the entire transmission system. In step (2) and (3), only missing packets in the last step are coded and delivered. We deploy partial network coding in the relay buffering and retransmission scheme, for the sake of reducing complexity of the entire system.

Table 1: *CodeAssist*: cognitive relay based on network coding.

1. Build a buffer with a storage of m coded packets for each sensing queue.
2. Clear the buffer. Initialize the value of the data payload Y_a to 0 and randomly generate M coding coefficients $C_1^a, \ldots, C_M^a$ for the coding packet a $(a = 1, \ldots, m)$.
3. Receive M (larger than m) original packets $X_1, \ldots, X_M$. When receiving a new packet $X_b (1 \leq b \leq M)$, multiply the data payload of X_b with the matching coefficient $C_b^a (a = 1, \ldots, m)$ of every coding packet and add to the data payload Y_a. The packet a in the buffer would be stored with the linear combination $Y_a = \sum 1, M X_i \dot{C}_i^a$. It is applied to all packets in the buffer.
4. Transmit coded packets.
5. Check if it is the end of the buffer:
   a. If so, go to Step 2;
   b. Otherwise, repeat Step 4.

A heuristic approach is proposed with random network coding applied to buffers of secondary users. The algorithm is presented in Table 1, which conserves the amount of buffer space needed.

The algorithm is implemented when new packets arrive in every buffer. The ratio of the coding field size to the mapping field size in each buffer for the scheme will be discussed later in this paper. The relationship between the original coding field size M and the mapping field size m can affect the amount of coded packets in each buffer, and guarantee that the primary receiver can recover all the original packets.

5.2 How many coded packets are needed?

As mentioned, with random network coding, redundant relays can be avoided even without coordination. The only problem is to find out how many coded packets are needed for the relay.

Let R be the set of requested packets by the primary receiver, $\{R_{s_i}\}$ the subsets of cognitive packets in secondary users $s_i (i = 1, \cdots, N)$, and $R_{\Sigma s}$ the collection of all the subsets R_{s_i}, which means $R' = \bigcup_{i=1}^{N} R_{s_i}$. Each relay channel receives packets from the primary channel with the probability given by

$$1 - P_{e,ps} = \exp(-\frac{\beta_p}{\gamma_{ps}}).$$

The number of cognitive packets sensed in a certain time span in a single relay channel is given by $M = |R| \times 1 - P_{e,ps}$, which is the coding field size.

With the collaboration of N channels, the receiving probability by the primary receiver is

$$P_{sense} = 1 - P_{e,ps}^N = 1 - (1 - \exp(-\frac{\beta_p}{\gamma_{ps}}))^N. \qquad (8)$$

and the number of overall sensed packets is given by $|R_{\Sigma s}| = |R| \times P_{sense}$. According to network coding theory, to recover x independent original packets, it requires at least x independent coded packets. Hence, each relay buffer should store coded packets with at least $1/N$ as much as the number of packets in $R_{\Sigma s}$.

We have the following proposition inferred from the analysis:

PROPOSITION 1. *Assume M original packets are received by each individual secondary user. In the multiple cognitive relay system, it requires at least m coded packets for the relay. The ratio of M to m is given by*

$$\frac{m}{M} = \frac{1}{N} \cdot \frac{1 - P_{e,ps}^N}{1 - P_{e,ps}}. \qquad (9)$$

PROOF. Given any packet $r \in R$, it is assumed to be contained by n different subsets with the probability $C_n^N (1 - P_{e,ps})^n P_{e,ps}$. Given that packet r is covered by n different subsets, each subset only needs to relay $1/n$ part of the coded packets. Therefore the cognitive entropy of the individual packet by each secondary user is given by

$$E_{pac} = \sum \frac{1}{n} C_n^N (1 - P_{e,ps})^n P_{e,ps} = \frac{1}{N} \cdot \frac{1 - P_{e,ps}^N}{1 - P_{e,ps}}. \qquad (10)$$

Hence, the entropy that secondary users separately derive from M packets is

$$E_{ch} = M \times E_{pac}.$$

To guarantee all sensed information to be relayed, m should not be less than E_{ch}. Referring to Eq. (8) we conclude the expression

$$\begin{cases} \frac{|R_{s_i}|}{|R|} & = 1 - P_{e,ps} \\ \frac{|R_{\Sigma s}|}{|R|} & = P_{sense} \end{cases}$$

The subset $R_{s_i}(i = 1, \cdots, N)$ is compressed by network coding, with its coding coefficient vectors $\{h_{s_i,j}(j = 1, \cdots, m)\}$. $H_{s_i} = (h_{s_i,1}^T, \cdots, h_{s_i,m}^T)$ is the coding coefficient matrix, in which any two vectors are independent. H is the $|R| \times |R|$ matrix, which is the combination of $H_{s_i} = (h_{s_i,1}^T, \cdots, h_{s_i,k}^T)(i = (1, \cdots, N))$. Considering validation of the decoding process, the coding coefficient matrix H should be a full rank matrix.

To calculate the row rank of H, we initialize the system with just one secondary user, and define its sensing buffer with the packet subset $R_{s_1} = (r_1, \cdots, r_n)(n = (1 - P_{e,ps})\|R\|)$ and coding coefficients vectors $\{h_{s_1,j} = (e_1^{j,1}, \cdots, e_1^{j,n})(j = 1, \cdots, m)\}$. The encoding output is given by

$$\begin{aligned} B_{s_1} &= R_{s_1} \cdot H_{s_1} \\ &= (r_1, \cdots, r_n) \cdot \begin{bmatrix} e_1^{1,1} & \cdots & e_1^{m,1} \\ \vdots & \ddots & \vdots \\ e_1^{1,n} & \cdots & e_1^{m,n} \end{bmatrix}. \end{aligned} \qquad (11)$$

By adding another user in the relay system, we extend the original coding field with R_{s_2}. The coding matrix is also extended to

$$\begin{bmatrix} H_{s_1} & 0 \\ 0 & H_{s_2} \end{bmatrix} = \begin{bmatrix} e_1^{1,1} & \cdots & e_1^{m,1} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & 0 & \cdots & 0 \\ e_1^{1,j} & \cdots & e_1^{m,j} & e_2^{1,1} & \cdots & e_2^{m,1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ e_1^{1,n} & \cdots & e_1^{m,n} & e_2^{1,n-j+1} & \cdots & e_2^{m,n-j+1} \\ 0 & \cdots & 0 & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & e_2^{1,n} & \cdots & e_2^{m,n} \end{bmatrix}.$$

Any row vector in $(H_{s_1}0)^T$ is linearly independent with any other row vectors in $(0H_{s_2})^T$ with high probability. Therefore, $((H_{s_1}0)^T, (0H_{s_2})^T)$ is a full rank matrix with $rank_l = 2k$. With one additional sensing channel, the rank of the coding coefficient matrix is increased by k. Thus, the overall rank of H is obtained as

$$Rank_l = m \times N = \frac{1 - P_{e,ps}^n}{1 - P_{e,ps}} \cdot M.$$

Based on the definition of matrix H and its randomly generated components, the row rank of the system matrix H could be calculated as

$$\begin{aligned} Rank_r &= \frac{\sum_{k=0}^{N-1} (1 - P_{e,ps}) P_{e,ps}^k}{1 - P_{e,ps}} \cdot M \\ &= \frac{1 - Pe,ps^N}{1 - Pe,ps} \cdot M. \end{aligned}$$

With $Rank_l = Rank_r = |R|$, we conclude that H is a full rank matrix. Hence, all relay packets can be decoded, under the proposed scheme. ☐

5.3 Numerical Results of CodeAssist

With Proposition 1, *CodeAssist* can be applied to the multiple cognitive relay system. With *CodeAssist*, the ratio of the input rate to the output rate of each channel relaying packets is $\frac{\lambda}{\mu} = \frac{m}{M}$, which means

$$\lambda_{ps} \cdot \frac{m}{M} < \mu_{ps}^{\max}.$$

Hence, Eq. (6) is modified as

$$\frac{\lambda_p}{\overline{\mu_p}^{\max}} P_{e,p}(1 - P_{e,ps}) \frac{1}{N} \cdot \frac{1 - P_{e,ps}^N}{1 - P_{e,ps}} < \frac{1 - \frac{\lambda}{\overline{\mu_p}^{\max}}}{N} \cdot \exp(-\frac{\beta_p}{\gamma_{sp}})\varepsilon, \qquad (12)$$

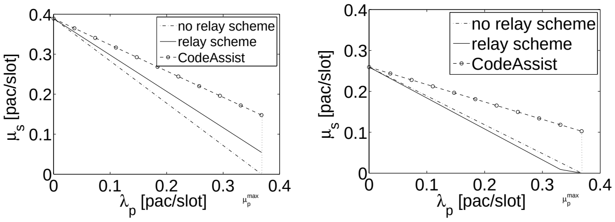
in which the minimal value of ε is now

$$\varepsilon > N \cdot \frac{1 - P_{e,ps}}{1 - P_{e,ps}^N} \frac{\lambda_p}{\overline{\mu_p}^{\max}} \cdot (1 - \exp(-\beta_p/\gamma_p)) \frac{\exp(-\beta_p/\gamma_{ps})}{\exp(-\beta_p/\gamma_{sp})}. \qquad (13)$$

We have simulated *CodeAssist* with the same parameters of the system in Sec. 4, and our results are shown in Fig. 6.

The improvement of the system performance based on network coding is clearly illustrated with Fig. 6. The relay transmission based on network coding performs the best in our comparison study, and network coding is more effective in a multiple cognitive relay system with three relays in *CodeAssist*, over the regular relay scheme without network coding.

In Fig. 7, it is also indicated that when the number of secondary users is large enough, the increasing contribution of the relay system compared to the primary transmission is non-significant. In general, we can determine the optimal number of secondary users to share and relay in a certain spectrum in advance, although *CodeAssist* still performs best in such a system, compared with existing approaches.



(a) (b)

Figure 6: The maximum throughput μ_s versus λ_p when using network coding. (a) results with two relay users; and (b) results with 3 relay users.

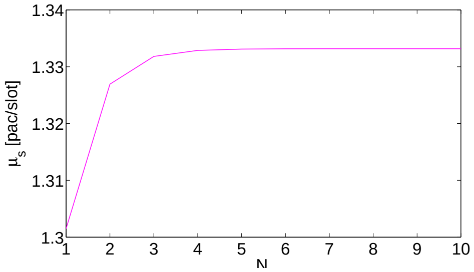


Figure 7: The Y axis denotes the ratio of maximum throughput of relay transmission based on *CodeAssist* over the no relay transmission, and the X axis denotes the number of secondary users sharing the spectrum.

6. CONCLUSION

In this paper, we studied a multiple cognitive radio channel system with cognitive relays. Theoretically, the upper bound of achievable rate is increased with the number of relay nodes in a multiple relay system. However, the system is not able to practically achieve such an expectation, due to redundancies across multiple relays. We proposed the *CodeAssist* algorithm that uses network coding to mitigate such redundancies. It helps decrease or even eliminate excessive redundancy across relays. Moreover, we have shown that our scheme can be practically implemented in cognitive radio systems with low computational costs. We show numerical results with simulations to verify the efficiency of cognitive relaying with network coding.

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