

Channel-relay price pair: towards arbitrating incentives in wireless *ad hoc* networks

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Summary

Cooperation in wireless *ad hoc* networks has twofold implications. First, each wireless node does not excessively and greedily inject traffic to the shared wireless channel. Second, intermediate nodes voluntarily relay traffic for upstream nodes towards the destination at the cost of its own private resource. Such an assumption supports almost all existing research when it comes to protocol design in *ad hoc* networks. We believe that without appropriate incentive mechanisms, the nodes are inherently selfish (unwilling to contribute its private resource to relay traffic) and greedy (unfairly sharing the wireless channel). In this paper, we present a *price pair* mechanism to arbitrate resource allocation and to provide incentives simultaneously such that cooperation is promoted and the desired global optimal network operating point is reached by convergence with a fully decentralized self-optimizing algorithm. Such desired network-wide global optimum is characterized with the concept of Nash bargaining solution (NBS), which not only provides the Pareto optimal point for the network, but is also consistent with the fairness axioms of game theory. We simulate the price pair mechanism and report encouraging results to support and validate our theoretical claims. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: price; game theory; *ad hoc* network; wireless network

1. Introduction

Nodes in wireless *ad hoc* networks not only share the wireless channel in the same local neighborhood, but also relay traffic so that destinations multiple hops away may be reached. In almost all previous work related to wireless *ad hoc* networks, the following two fundamental assumptions are made. First, nodes do not excessively inject traffic to the locally shared wireless channel. Second, intermediate nodes voluntarily relay traffic for upstream nodes towards the destination. In this paper, we believe that such assumptions may not hold in realistic scenarios, at least

not without appropriate incentive-based mechanisms. In fact, they behave in quite the contrary fashion: they are both *greedy* when it comes to sharing public resource (wireless channel) and *selfish* when it comes to contributing private resource (such as battery energy). In other words, the network may fail to function at all in realistic scenarios once neither assumption holds.

The only way to solve these problems is to design appropriate incentive mechanisms to not only encourage cooperative behavior of selfish nodes, but also curb unfair and excessive resource usage when sharing a common resource pool, such as a

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shared channel. Such designs of incentives should optimize towards a clearly specified objective, which is a desired optimal operating point of the wireless network. At such an optimal point, resources are shared fairly, and the levels of cooperation are adequate for all necessary data communications and network functions. This paper exactly targets this critical issue in multi-hop wireless networks.

Our original contributions are twofold. First, we clearly characterize the desired network-wide optimal operating point using a game theoretic framework, based on the concept of Nash bargaining solution (NBS). NBS naturally encapsulates two favorable properties: (1) Pareto efficiency in terms of resource usage; and (2) a set of fairness axioms with respect to resource allocations. Using this framework, the problem of finding the desired globally optimal operating point may be formulated as a non-linear optimization problem. Second, we propose a decentralized algorithm that uses a *price pair* mechanism to arbitrate incentives. With a pair of prices, localized self-optimization by individual nodes naturally converges to globally optimal network operating points. Within the price pair, the *channel price* regulates greedy usage of the shared wireless channel, while the *relay price* encourages traffic relaying. Effectively, our price pair mechanism transforms non-cooperative behavior in wireless *ad hoc* networks to a *cooperative game*, whose optimal operating points demonstrate more advantageous properties than the usual Nash equilibrium in typical non-cooperative environments.

The essence of our paper is to integrate the mechanisms that use pricing as signals to (1) fairly allocate resources; and (2) adequately incentivize cooperative behavior. Though there exists previous work towards either one of these objectives, we are not aware of existing work that integrates both prices into a coherent framework. Such integration becomes more complicated if we consider the unique channel contention characteristics in wireless *ad hoc* networks, where the traffic flows contend in multiple *contention cliques*. Considerations of such unique complications in *ad hoc* networks are beyond all of the existing work in the area of pricing or incentives.

The remainder of this paper is organized as follows. Section 2 presents some preliminaries before formal treatment of this topic. Section 3 defines the desired network operating points using the concept of NBS. We present the distributed algorithm in Section 4. We show simulation results in Section 5, present related

work in Section 6, and finally conclude the paper in Section 7.

2. Network Model

We consider a wireless *ad hoc* network which consists of a set of nodes $\mathcal{N} = \{1, 2, \dots, N\}$. In this network, only nodes that are within the transmission range of each other can communicate directly and form a *wireless link*. We model such a network as a bidirectional graph $\mathcal{G}_N = (\mathcal{N}, \mathcal{L})$, where $\mathcal{L} = \{1, 2, \dots, L\}$ is the set of wireless links.

In such a network, a wireless node $i \in \mathcal{N}$ may establish an end-to-end flow, or simply *flow*, f_i with rate x_i to another node. Flow f_i is assumed to be elastic: it requires a minimum rate of x_i^m and a maximum rate of x_i^M , that is, $x_i^m \leq x_i \leq x_i^M$. In general, f_i flows through multiple hops in the network, passing a set of wireless links. We use this set of wireless links to represent f_i , that is, $f_i \subset \mathcal{L}$. We denote the set of relaying nodes for flow f_i as $R(f_i)$, and the destination of f_i as $D(f_i)$. For simplicity of exposition, we further define $H(f_i) = R(f_i) \cup \{D(f_i), i\}$ as the set of nodes f_i traverses, and $K(f_i) = H(f_i) - \{i\}$. A single-hop data transmission along a particular wireless link is referred to as a *subflow* and is a part of a flow. Several subflows from different flows along the same wireless link form an *aggregated subflow*.

In such a network, nodes compete for two types of resources: shared wireless channel and individual nodes' relaying cost (such as energy). The availability of these resources constrains the solution space of resource allocations. We proceed to analyze the characteristics of both types of resources.

2.1. Shared Wireless Channel: Location-Dependent Contention

The shared-medium multi-hop nature of wireless *ad hoc* networks presents unique characteristics of *location-dependent contention* and *spatial reuse* of spectrum. Compared with wireline networks where flows contend only at the router with other simultaneous flows through the same router (contention in the time domain), the unique characteristics of multi-hop wireless networks show that flows also compete for shared channel bandwidth if they are within the transmission ranges of each other (contention in the spatial domain).

In particular, two subflows contend with each other if either the source or destination of one subflow is within the transmission range of the source or

destination of the other.[†] The locality of wireless transmissions implies that the degree of contention for the shared medium is location dependent. On the other hand, two subflows that are geographically far away have the potential to transmit simultaneously, reusing the wireless channel.

We now formulate the resource constraints that reflect the unique characteristics of wireless *ad hoc* networks. First, let us consider a set of mutually contending subflows. In this set, only one subflow can transmit at a time. Intuitively, the aggregated rate of all subflows in this set can not exceed the channel capacity. Formally, we consider a subflow contention graph. In this graph, each vertex corresponds to an aggregated subflow in the original network. Each edge in the graph denotes that two aggregated subflows which correspond to the two vertices, contend with each other. Formally, let $\mathcal{V}_C = \cup_{i \in \mathcal{N}} f_i \subseteq \mathcal{L}$ be the set of aggregated subflows in network \mathcal{G}_N , then a bidirectional graph $\mathcal{G}_C = (\mathcal{V}_C, \mathcal{E}_C)$ is a *subflow contention graph* of network \mathcal{G}_N .

In a graph, a complete subgraph is referred to as a *clique*. A *maximal clique* is defined as a clique that is not contained in any other cliques.[‡] In a subflow contention graph, the vertices in a maximal clique represent a maximal set of mutually contending subflows. Intuitively, each maximal clique in a subflow contention graph represents a ‘maximal distinct contention region,’ since at most one subflow in the clique can transmit at any time and adding any other subflows into this clique will introduce the possibility of simultaneous transmissions. For simplicity, we use the set of vertices in a clique to represent the clique, and denote it as q . Furthermore, we denote the set of all maximal cliques in a subflow contention graph as \mathcal{Q} .

Here, we illustrate the above concepts using an example. The network topology and the flows in the example are shown in Figure 1(a). The corresponding subflow contention graph is shown in Figure 1(b). In this example, there are three maximal cliques in the contention graph: $q_1 = \{\{1, 2\}, \{3, 2\}, \{3, 4\}, \{3, 6\}\}$, $q_2 = \{\{3, 2\}, \{3, 4\}, \{4, 5\}, \{3, 6\}\}$ and $q_3 = \{\{3, 2\}, \{3, 4\}, \{3, 6\}, \{6, 7\}\}$.

We proceed to consider the problem of allocating rates to wireless links. We claim that a *rate allocation*

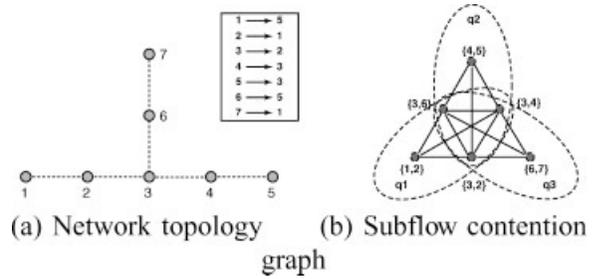


Fig. 1. Example of wireless *ad hoc* network and its subflow contention graph. (a) Network topology and (b) Subflow contention.

$y = (y_l, l \in \mathcal{L})$ is *feasible*, if there exists a collision-free transmission schedule that allocates y_l to l . We now formalize the condition implied by such a feasible rate allocation.

Lemma 1. *If a rate allocation $y = (y_l, l \in \mathcal{L})$ is feasible, then the following condition is satisfied:*

$$\forall q \in \mathcal{Q}, \sum_{l \in q} y_l \leq C \quad (1)$$

where C is the channel capacity.

Equation (1) gives an upper bound on the rate allocations to the wireless links. In practice, however, such a bound may not be tight, especially with carrier-sensing-multiple-access-based wireless networks (such as IEEE 802.11). In this case, we introduce C_q , the *achievable* channel capacity at a clique q . More formally, if $\sum_{l \in q} y_l \leq C_q$ then $y = (y_l, l \in \mathcal{L})$ is feasible. To this end, we observe that each maximal clique may be regarded as an independent channel resource unit with capacity C_q . It motivates the use of the *maximal clique* as a basic resource unit for pricing in wireless *ad hoc* networks, as compared to the notion of a *link* in wireline networks.

We now proceed to consider resource constraints on rate allocations among flows. To facilitate discussions, we define a clique-flow matrix $\mathbf{R} = \{R_{q_i}\}$, where $R_{q_i} = |q \cap f_i|$ represents the number of subflows that flow f has in the clique q . If we treat a maximal clique as an independent resource, then the clique-flow matrix \mathbf{R} represents the ‘resource usage pattern’ of each flow. Let the vector $\mathbf{C} = (C_q, q \in \mathcal{Q})$ be the vector of achievable channel capacities in each of the cliques. Constraints with respect to rate allocations to end-to-end flows in wireless *ad hoc* networks are presented in the following proposition.

[†]If we assume that the interference range is greater than the transmission range, the contention model can be straightforwardly extended.

[‡]Note that *maximal clique* has a different definition from *maximum clique* of a graph, which is the maximal clique with the largest number of vertices.

Proposition 1. In a wireless *ad hoc* network $\mathcal{G}_N = (\mathcal{N}, \mathcal{L})$, there exists a feasible rate allocation $\mathbf{x} = (x_i, i \in \mathcal{N})$, if and only if $\mathbf{R}\mathbf{x} \leq \mathbf{C}$.

Proof: It is obvious that $\mathbf{R}\mathbf{x} \leq \mathbf{C} \Leftrightarrow \forall q \in \mathcal{Q}, \sum_{i \in \mathcal{N}} R_{qi} x_i \leq C_q$. By the definition of \mathbf{R} , we have $\sum_{i \in \mathcal{N}} R_{qi} x_i = \sum_{l \in \mathcal{Q}} y_l$. The result follows naturally from Lemma 1 and its following discussions. ■

2.2. Costs of Relays

Relaying traffic for upstream nodes apparently incurs cost, since localized resources need to be consumed, such as energy, CPU cycle, and memory space. Without loss of generality, we use energy levels on each node as an example to characterize such costs of relays. Given a minimum expected lifetime in the network, each node j has a budget on its energy consumption rate, denoted as E_j . Here, we consider two types of energy consumption related to packet transmission: (1) e^r as the energy consumed for receiving a unit flow; (2) e^s as the energy consumed for transmitting a unit flow. Then the energy consumption at node j is $x_j e^s + \sum_{i:j \in R(f_i)} x_i (e^r + e^s) + \sum_{i:j=D(f_i)} x_i e^r$. As the energy consumption rate can not exceed the energy budget, we have the following relation:

$$x_j e^s + \sum_{i:j \in R(f_i)} x_i (e^r + e^s) + \sum_{i:j=D(f_i)} x_i e^r \leq E_j \quad (2)$$

We now proceed to define a $N \times N$ matrix \mathbf{B} as follows.

$$\mathbf{B}_{ji} = \begin{cases} c r e^s & \text{if } j = i \\ e^r + e^s & \text{if node } j \text{ forwards packets for flow } f_i, \\ & \text{that is, } j \in R(f_i) \\ e^r & \text{if node } j \text{ is the destination of flow } f_i, \\ & \text{that is, } j = D(f_i) \\ 0 & \text{otherwise} \end{cases}$$

\mathbf{B} specifies the relaying relation among nodes in the *ad hoc* network. To summarize, the local constraint on energy can be formalized as follows:

$$\mathbf{B} \cdot \mathbf{x} \leq \mathbf{E} \quad (4)$$

where $\mathbf{E} = (E_j, j \in \mathcal{N})$ is the energy consumption budget vector.

3. Problem Formulation

In this section, we characterize the desired network-wide optimal operating point using a game theoretic framework, based on the concept of NBS. NBS naturally encapsulates two favorable properties: (1) Pareto efficiency in terms of resource usage; and (2) a set of fairness axioms with respect to resource allocations. Using this framework, the problem of finding the desired globally optimal operating point may be formulated as a non-linear optimization problem. We show how such a global optimization problem may be decomposed into localized greedy optimization problems via a price pair.

3.1. Nash Bargaining Solution: a Game Theoretical Formulation

We present the basic concepts and results of NBS from game theory [1] and show how it can characterize and formulate the desired network operation point, towards which our price pair mechanism should converge.

The basic setting of the problem is as follows: the set of nodes \mathcal{N} in the wireless *ad hoc* network \mathcal{G}_N constitutes a set of players in the game. They compete for the use of a fixed amount of resources (wireless channel and costs of relays such as energy). The rate allocation $\mathbf{x} = (x_i, i \in \mathcal{N})$ is the utility vector of all players in the game. Let $S \subset \mathfrak{R}^N$ be the set of all feasible utility vectors. We assume that S is a non-empty convex closed and bounded set. Further, we denote the initial agreement point of the game as \mathbf{x}^* , which is the guaranteed utilities of players without any cooperation in order to enter the game.

In such a problem setting, a *bargaining problem* is any (S, \mathbf{x}^*) where $\{\mathbf{x} \in S | \mathbf{x} \geq \mathbf{x}^*\} \neq \emptyset$. In other words, a bargaining problem is actually a set of rate allocations \mathbf{x} that is acceptable to all players. Let $\mathfrak{B} = \{(S, \mathbf{x}^*)\}$ denote the set of all bargaining problems. It then follows that a *bargaining solution* is any function $\varphi : \mathfrak{B} \rightarrow \mathfrak{R}^N$, so that $\forall (S, \mathbf{x}^*) \in \mathfrak{B}, \varphi(S, \mathbf{x}^*) \in S$. A bargaining solution actually specifies finding a rate allocation within all acceptable allocations. A natural question about the bargaining solution is how such a function φ is to be defined. There are two reasonable properties desired for a bargaining solution: (1) efficient use of resources; and (2) fair allocation among all players. These two conditions are precisely encapsulated by the concept of NBS defined as follows.

Definition 1 (Nash Bargaining Solution). A bargaining solution $\varphi: \mathfrak{B} \rightarrow \mathfrak{R}^N$ is a NBS, if $\bar{\mathbf{x}} = \varphi(S, \mathbf{x}^*)$ satisfies the *Nash Axioms* A1–A6.

- A1 (individual rationality) $\bar{\mathbf{x}} \geq \mathbf{x}^*$;
 A2 (feasibility) $\bar{\mathbf{x}} \in S$;
 A3 (pareto optimality), If $\forall \mathbf{x} \in S, \mathbf{x} \geq \bar{\mathbf{x}}$, then $\mathbf{x} = \bar{\mathbf{x}}$;
 A4 (independence of irrelevant alternatives) If $\bar{\mathbf{x}} \in T \subset S$ and $\bar{\mathbf{x}} = \varphi(S, \mathbf{x}^*)$, then $\bar{\mathbf{x}} = \varphi(T, \mathbf{x}^*)$;
 A5 (independence of linear transformation) Let T be obtained from S by the linear transformation $\phi(\mathbf{x})$ with

$$\phi(\mathbf{x})_i = a_i x_i + b_i, a_i, b_i > 0, i = 1, 2, \dots, N$$

Then if $\varphi(S, \mathbf{x}^*) = \bar{\mathbf{x}}$, then $\varphi(T, \phi(\mathbf{x}^*)) = \phi(\bar{\mathbf{x}})$;

- A6 (symmetry) Suppose S is symmetric with respect to a subset $J \subseteq \{1, 2, \dots, N\}$ of indices, then φ is symmetry, which means that if $\mathbf{x} \in S$ and for $i, j \in J, x_i^* = x_j^*$, then $\bar{x}_i = \bar{x}_j$.

The above axioms encapsulate both the concept of Pareto optimality (A3) and the concept of fairness (A4–A6). Pareto optimality means that there is no other point which gives strict superior utility for all the players simultaneously. The definition of Pareto optimality reflects the condition of efficient use of resources, where there are no ‘idle’ resources in the network. The existing work in game theory [1] and its application in wireline communication networks [2] establish the following results for NBS.

Proposition 2. There exists a unique function φ defined on all bargaining problems (S, \mathbf{x}^*) that satisfies axioms A1–A6, that is, a unique NBS exists. Moreover, the unique solution $\bar{\mathbf{x}}$ is a unique vector that solves the following maximization problems:

$$N_1 : \max_{\mathbf{x}} \prod_{i \in \mathcal{N}} (\bar{x}_i - x_i^*) \quad (5)$$

Equivalently,

$$N_2 : \max_{\mathbf{x}} \sum_{i \in \mathcal{N}} \ln(\bar{x}_i - x_i^*) \quad (6)$$

3.2. Network Operating Points: Optimal and Fair Rate Allocations

We assume that each player involved in the game can be guaranteed with its minimum rate vector \mathbf{x}^m . Thus,

the minimum rate vector \mathbf{x}^m can be regarded as an initial agreement point of the game \mathbf{x}^* . From the discussions in Section 3A, it is clear that the NBS formulates the desired network operation point, which is fair to all nodes while efficient from the network’s point of view. Thus, the problem of finding the globally optimal resource allocation is transformed to solve the NBS of its corresponding game, which is the solution of the following non-linear optimization problem by Proposition 2.

$$P : \text{maximize} \quad \sum_{i \in \mathcal{N}} \ln(x_i - x_i^m) \quad (7)$$

$$\text{subject to} \quad \mathbf{A} \cdot \mathbf{x} \leq \mathbf{C} \quad (8)$$

$$\mathbf{B} \cdot \mathbf{x} \leq \mathbf{E} \quad (9)$$

$$\text{over} \quad \mathbf{x}^m \leq \mathbf{x} \leq \mathbf{x}^M \quad (10)$$

The objective function in Equation (7) of the optimization problem corresponds to the optimization problem whose solution is NBS. The constraints of the optimization problem Equations (8) and (9) are the constraints from the shared wireless channel and costs of relays, respectively, as discussed in Section 2.

Note that the objective function $\sum_{i \in \mathcal{N}} \ln(x_i - x_i^m)$ is strictly concave. In addition, the feasible region of the optimization problem is non-empty, convex, and compact. By non-linear optimization theory, there exists a maximizing value of argument \mathbf{x} for the above optimization problem. Let us consider the Lagrangian form of the optimization problem P :

$$\begin{aligned} L(\mathbf{x}, \boldsymbol{\mu}^\alpha, \boldsymbol{\mu}^\beta) \\ = \sum_{i \in \mathcal{N}} \ln(x_i - x_i^m) + \boldsymbol{\mu}^\alpha (\mathbf{C} - \mathbf{A}\mathbf{x}) + \boldsymbol{\mu}^\beta (\mathbf{E} - \mathbf{B}\mathbf{x}) \end{aligned} \quad (11)$$

where $\boldsymbol{\mu}^\alpha = (\mu_q^\alpha, q \in \mathcal{Q})$, $\boldsymbol{\mu}^\beta = (\mu_j^\beta, j \in \mathcal{N})$ are two vectors of Lagrange multipliers. The first-order Kuhn–Tucker conditions are necessary and sufficient for optimality of problem P . Thus, for $i \in \mathcal{N}$, the following conditions hold:

$$\frac{1}{x_i - x_i^m} - \sum_{q \in \mathcal{Q}} \mu_q^\alpha A_{qi} - \sum_{j \in \mathcal{N}} \mu_j^\beta B_{ji} = 0 \quad (12)$$

$$\boldsymbol{\mu}^\alpha (\mathbf{C} - \mathbf{A}\mathbf{x}) = 0; \boldsymbol{\mu}^\alpha \geq 0 \quad (13)$$

$$\boldsymbol{\mu}^\beta (\mathbf{E} - \mathbf{B}\mathbf{x}) = 0; \boldsymbol{\mu}^\beta \geq 0 \quad (14)$$

$$\mathbf{x}^m \leq \mathbf{x} \leq \mathbf{x}^M \quad (15)$$

In the Lagrangian form shown in Equation (11), the Lagrange multipliers μ_q^α can be regarded as the implied cost of unit flow accessing the channel at the maximal clique q . In other words, μ_q^α is the *shadow price* of clique q , called *channel price*. This price corresponds to the shared channel resource constraint. The Lagrange multipliers μ_j^β can be regarded as the implied relay cost of unit flow at node j . In other words, μ_j^β is the *shadow price* of relay at node j , called *relay price*. This pair of prices (μ^α, μ^β) will be used as incentives so that localized self-optimizing decision can implement the global optimum. In particular, the price μ^α will signal a ‘charge’ (contrary to incentives) to the shared channel usage and regulate the greedy behavior, while the price μ^β will provide incentives to traffic relays at intermediate nodes and regulate the selfish behavior of wireless nodes.

Let us denote

$$\lambda_i^\alpha = \sum_{q \in \mathcal{Q}} \mu_q^\alpha A_{qi} \quad (16)$$

$$\lambda_i^\beta = \sum_{j \in \mathcal{N}} \mu_j^\beta B_{ji} \quad (17)$$

Clearly,

$$\lambda_i^\alpha = \sum_{q: f_i \cap q \neq \emptyset} \mu_q^\alpha A_{qi} \quad (18)$$

$$= \sum_{l: l \in f_i} \sum_{q: l \in q} \mu_q^\alpha \quad (19)$$

$$\lambda_i^\beta = \sum_{j \in H(f_i)} \mu_j^\beta B_{ji} \quad (20)$$

$$= \mu_i^\beta e^s + \sum_{j \in R(f_i)} \mu_j^\beta (e^s + e^r) + \mu_{D(f_i)}^\beta e^r \quad (21)$$

Then λ_i^α and λ_i^β are the prices for node i , which is the source of flow f_i , for accessing shared channels and relay services, respectively. For channel usage, node i needs to pay for *all the maximal cliques* that it traverses. For each clique, the price to pay is the product of the number of wireless links that f_i traverses in this clique and the shadow price of this clique as in Equation (18). Alternatively, the price of

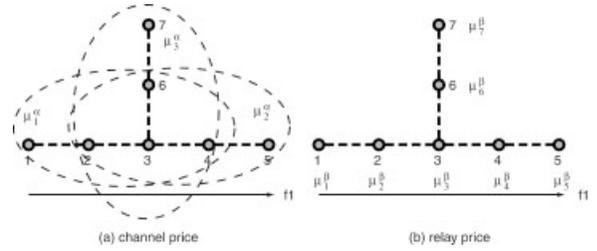


Fig. 2. An example of the price pair mechanism. (a) Channel price and (b) relay price.

flow f_i is the aggregated price of all its subflows. For each subflow, its price is *the aggregated price of the maximal cliques* that it belongs to as in Equation (19). Note that such a pricing policy for end-to-end flows is fundamentally different from the pricing models in wireline networks, where a flow’s price is the aggregation of the link prices which it traverses. Such difference is rooted at the nature of the shared medium—the interference and spatial reuse of wireless channel in an *ad hoc* network. For traffic relay services, node i needs to pay for the relay costs of all relaying nodes of f_i , including itself and the destination. Using flow f_1 as an example, the channel price model is illustrated in Figure 2(a) and the relay price model is illustrated in Figure 2(b). The channel price for f_1 is $\lambda_1^\alpha = 3\mu_1^\alpha + 3\mu_2^\alpha + 2\mu_3^\alpha$; and the relay price $\lambda_1^\beta = 2\mu_1^\beta + 3\mu_2^\beta + 3\mu_3^\beta + 3\mu_4^\beta + \mu_5^\beta$, where the energy consumed for receiving is given as $e^r = 1$, and for transmitting as $e^s = 2$.

To summarize, we have the following results with respect to the globally optimal network operating point.

Theorem 1. There exists a unique solution to problem P (i.e., unique NBS), which is characterized as follows: There exist two vectors $\mu^\alpha = (\mu_q^\alpha, q \in \mathcal{Q})$, $\mu^\beta = (\mu_j^\beta, j \in \mathcal{N})$ such that

$$x_i = \left[\frac{1}{\lambda_i^\alpha + \lambda_i^\beta} + x_i^m \right]_{x_i^m}^{x_i^M}, \text{ for } i \in \mathcal{N} \quad (22)$$

$$\mu^\alpha (C - Ax) = 0; \mu^\alpha \geq 0 \quad (23)$$

$$\mu^\beta (E - Bx) = 0; \mu^\beta \geq 0 \quad (24)$$

where $[z]_a^b = \max\{\min\{z, b\}, a\}$.

3.3. Local Strategies: Self-Optimizing Decisions

With the understanding of the global optimal point, we now study how this point can be achieved in a distributed manner by localized self-optimizing decisions at each individual node. The key to this goal is to use the pair of prices (μ^α, μ^β) as signals to coordinate the distributed decisions.

First, we study the conditions that the prices need to satisfy in order to incentivize local node decisions so that they could implement the global network optimum. Let us consider the following problems:

Channel($\mathbf{A}, \mathbf{C}; \mu^\alpha$):

$$\text{maximize } \sum_{i \in \mathcal{N}} \lambda_i^\alpha x_i \quad (25)$$

$$\text{subject to } \mathbf{A}\mathbf{x} \leq \mathbf{C} \quad (26)$$

$$\text{over } \mathbf{x}^m \leq \mathbf{x} \leq \mathbf{x}^M \quad (27)$$

where λ_i^α is a function of μ^α , as defined in Equation (16). Problem *Channel*($\mathbf{A}, \mathbf{C}; \mu^\alpha$) maximizes the total revenue of channel based on charging λ_i^α per unit of bandwidth to user i subject to the channel capacity constraint.

Relay($\mathbf{B}, \mathbf{E}; \mu^\beta$):

$$\text{maximize } \sum_{i \in \mathcal{N}} \lambda_i^\beta x_i \quad (28)$$

$$\text{subject to } \mathbf{B}\mathbf{x} \leq \mathbf{E} \quad (29)$$

$$\text{over } \mathbf{x}^m \leq \mathbf{x} \leq \mathbf{x}^M \quad (30)$$

where λ_i^β is a function of μ^β , as defined in Equation (17). Problem *Relay*($\mathbf{B}, \mathbf{E}; \mu^\beta$) maximizes the total relay revenue of all nodes subject to the relay cost constraint at each individual node.

Now let us consider a local self-optimizing decision at each node i which corresponds to the following problem:

Node_i($\lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta$):

$$\text{maximize } \ln(x_i - x_i^m) - \lambda_i^\alpha x_i - \lambda_i^\beta x_i + \mu_i^\beta E_i \quad (31)$$

$$\text{over } x_i^m \leq x_i \leq x_i^M \quad (32)$$

The relationship between localized node decision and the global optimal network operating point is then given as follows.

Theorem 2: There exist vectors μ^α, μ^β and \mathbf{x} such that

- (1) x_i is the unique solution to *Node_i*($\lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta$);
- (2) \mathbf{x} solves *Channel*($\mathbf{A}, \mathbf{C}; \mu^\alpha$);
- (3) \mathbf{x} solves *Relay*($\mathbf{B}, \mathbf{E}; \mu^\beta$);

Then \mathbf{x} also solves problem P .

The proof of this theorem is given in our technical report [3].

Let us denote

$$\Phi(x_i) = \ln(x_i - x_i^m) - \lambda_i^\alpha x_i - \lambda_i^\beta x_i + \mu_i^\beta E_i \quad (33)$$

Now we show that $\Phi(x_i)$ reflects the ‘net benefit’ of node i and problem *Node_i*($\lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta$) maximizes the node i ’s net benefit. This claim is based on the following two observations. First, node i does not pay its relay cost to itself from a local point of view, while the flow relay price λ_i^β , which is defined from a global point of view, contains the price of relay for itself. Thus from a *local* point of view, the cost of node i is:

$$\lambda_i^\alpha x_i + \sum_{j \in \mathcal{K}(f_i)} B_{ji} = (\lambda_i^\alpha + \lambda_i^\beta) x_i - \mu_i^\beta e^s x_i \quad (34)$$

And the revenue of node i is:

$$\sum_{j: i \in \mathcal{K}(f_j)} \mu_i^\beta B_{ij} x_j = \mu_i^\beta \sum_{j: i \in \mathcal{H}(f_j)} B_{ij} x_j - \mu_i^\beta e^s x_i \quad (35)$$

Second, from Theorem 2 we have

$$\mu_i^\beta = 0, \text{ if } \sum_{j: i \in \mathcal{H}(f_j)} B_{ij} x_j < E_i \quad (36)$$

$$\mu_i^\beta > 0, \text{ if } \sum_{j: i \in \mathcal{H}(f_j)} B_{ij} x_j = E_i \quad (37)$$

Thus it is clear that $\Phi(x_i)$ reflects the ‘net benefit’ of node i , which is the difference between utility, revenue 3 and cost.

4. Algorithm

Although the global problem P can be mathematically solvable in a centralized fashion, it is impractical for realistic operations in wireless *ad hoc* networks. In

this section, we present a distributed iterative algorithm for price calculation and resource arbitration. We show that the iterative algorithm converges to the global optimum, and maximizes the local net benefit at each node simultaneously.

4.1. Dual Problem

In order to achieve a distributed solution, we first look at the dual problem of P as follows.

$$D: \min_{\mu^\alpha, \mu^\beta \geq 0} D(\mu^\alpha, \mu^\beta) \quad (38)$$

where

$$\begin{aligned} D(\mu^\alpha, \mu^\beta) &= \max_{\mathbf{x}} L(\mathbf{x}, \mu^\alpha, \mu^\beta) \\ &= \max_{\mathbf{x}} \sum_{i \in \mathcal{N}} \underbrace{(\ln(x_i - x_i^m) - (\lambda_i^\alpha + \lambda_i^\beta)x_i + \mu_i^\beta E_i)}_{\Phi(x_i)} \\ &\quad + \sum_{q \in \mathcal{Q}} \mu_q^\alpha C_q \end{aligned}$$

Note that $\Phi(x_i)$ is node i 's 'net benefit.' By the separation nature of Lagrangian form, maximizing $L(\mathbf{x}, \mu^\alpha, \mu^\beta)$ can be decomposed into separately maximizing $\Phi(x_i)$ for each node $i \in \mathcal{N}$ (Section 3.4.2 in Reference [4]). We now have

$$D(\mu^\alpha, \mu^\beta) = \sum_{i \in \mathcal{N}} \max_{x_i^m \leq x_i \leq x_i^M} \{\Phi(x_i)\} + \sum_{q \in \mathcal{Q}} \mu_q^\alpha C_q \quad (39)$$

As $\Phi(\cdot)$ is strictly concave and twice continuously differentiable, a unique maximizer of $\Phi(x_i)$ exists when

$$\frac{d\Phi(x_i)}{dx_i} = \frac{1}{x_i - x_i^m} - (\lambda_i^\alpha + \lambda_i^\beta) = 0$$

We define the maximizer as follows:

$$x_i(\lambda_i^\alpha, \lambda_i^\beta) = \arg \max_{x_i^m \leq x_i \leq x_i^M} \{\Phi(x_i)\} \quad (40)$$

Note that $x_i(\lambda_i^\alpha, \lambda_i^\beta)$ is usually called demand function, which reflects the optimal rate for node i with channel price as λ_i^α and relay price as λ_i^β .

4.2. Distributed Algorithm

We solve the dual problem D using the gradient projection method [4]. In this method, μ^α and μ^β are

adjusted in the opposite direction to the gradient $\nabla D(\mu^\alpha, \mu^\beta)$:

$$\mu_q^\alpha(t+1) = [\mu_q^\alpha(t) - \gamma \frac{\partial D(\mu^\alpha(t), \mu^\beta(t))}{\partial \mu_q^\alpha}]_+ \quad (41)$$

$$\mu_j^\beta(t+1) = [\mu_j^\beta(t) - \gamma \frac{\partial D(\mu^\alpha(t), \mu^\beta(t))}{\partial \mu_j^\beta}]_+ \quad (42)$$

where γ is the stepsize. $D(\mu^\alpha, \mu^\beta)$ is continuously differentiable since $\ln(\cdot)$ is strictly concave [4]. Thus, it follows that

$$\frac{\partial D(\mu^\alpha, \mu^\beta)}{\partial \mu_q^\alpha} = C_q - \sum_{i: f_i \cap q \neq \emptyset} x_i(\lambda_i^\alpha, \lambda_i^\beta) A_{qi} \quad (43)$$

$$\frac{\partial D(\mu^\alpha, \mu^\beta)}{\partial \mu_j^\beta} = E_j - \sum_{i: j \in H(f_i)} x_i(\lambda_i^\alpha, \lambda_i^\beta) B_{ji} \quad (44)$$

Substituting Equation (43) into (41) and (44) into (42), we have

$$\begin{aligned} \mu_q^\alpha(t+1) &= \\ &[\mu_q^\alpha(t) + \gamma(\sum_{i: f_i \cap q \neq \emptyset} x_i(\lambda_i^\alpha(t), \lambda_i^\beta(t)) A_{qi} - C_q)]_+ \end{aligned} \quad (45)$$

$$\mu_j^\beta(t+1) = [\mu_j^\beta(t) + \gamma(\sum_{i: j \in H(f_i)} x_i(\lambda_i^\alpha(t), \lambda_i^\beta(t)) B_{ji} - E_j)]_+ \quad (46)$$

Equations (45) and (46) reflect the law of supply and demand. If the demand for bandwidth at clique q exceeds its supply C_q , the channel constraint is violated. Thus, the channel price μ_q^α is raised. Otherwise, μ_q^α is reduced. Similarly, in Equation (46), if the demand for energy at node j exceeds its budget E_j , the energy constraint is violated. Thus, the relay price μ_j^β is raised. Otherwise, μ_j^β is reduced.

We summarize our algorithm in Table I, where clique q and node i are deemed as entities capable of computing and communicating.

We now show the property of this distributed iterative algorithm. Let us define $Y(i) = \sum_q A_{qi} + \sum_j B_{ji}$, and $\bar{Y} = \max_{i \in \mathcal{N}} Y(i)$. Further, we define $U(q) = \sum_{i \in \mathcal{N}} A_{qi}$

Table I. Distributed algorithm.

Clique price update (by clique q): at time $t = 1, 2, \dots$

- 1 Receive rates $x_i(t)$ from flows f_i where $f_i \cap q \neq \emptyset$
- 2 Update price $\mu_q^\alpha(t+1) = [\mu_q^\alpha(t) + \gamma(\sum_{f_i \cap q \neq \emptyset} x_i(t)A_{qi} - C_q)]^+$
- 3 Send $\mu_q^\alpha(t+1)$ to flows f_i where $f_i \cap q \neq \emptyset$

Relay price update (by node j): at time $t = 1, 2, \dots$

- 1 Receive rates $x_i(t)$ from flows f_i where $j \in H(f_i)$
- 2 Update price $\mu_j^\beta(t+1) = [\mu_j^\beta(t) + \gamma(\sum_{ij \in H(f_i)} x_i(t)B_{ji} - E_j)]^+$
- 3 Send $\mu_j^\beta(t+1)$ to flows f_i where $j \in H(f_i)$

Rate update (by node i): at time $t = 1, 2, \dots$

- 1 Receive prices $\mu_q^\alpha(t)$ from q where $f_i \cap q \neq \emptyset$
- 2 Receive relay prices $\mu_j^\beta(t)$ from j where $j \in H(f_i)$
- 3 Calculate

$$\lambda_i^\alpha = \sum_{q: f_i \cap q \neq \emptyset} \mu_q^\alpha A_{qi}$$

$$\lambda_i^\beta = \sum_{j: j \in H(f_i)} \mu_j^\beta B_{ji}$$
- 5 Adjust rate $x_i(t+1) = x_i(\lambda_i^\alpha, \lambda_i^\beta)$
- 4 Send $x_i(t+1)$ to corresponding cliques.

and $\bar{U} = \max_{q \in \mathcal{Q}} U(q)$; $V(j) = \sum_{i \in \mathcal{N}} B_{ji}$ and $\bar{V} = \max_{j \in \mathcal{N}} V(j)$; $\bar{Z} = \max\{\bar{U}, \bar{V}\}$. Let $\kappa_i = (x_i^M - x_i^m)^2$ and $\bar{\kappa} = \max_{i \in \mathcal{N}} \kappa_i$.

Theorem 3 (*Global convergence and optimality*). Suppose $0 < \gamma < 2/\bar{\kappa}\bar{Y}\bar{Z}$. Starting from any initial rates $\mathbf{x}^m \leq \mathbf{x}(0) \leq \mathbf{x}^M$, and prices $\boldsymbol{\mu}^\alpha(0) \geq 0$ and $\boldsymbol{\mu}^\beta(0) \geq 0$, every accumulation point $(\mathbf{x}^*, \boldsymbol{\mu}^{\alpha*}, \boldsymbol{\mu}^{\beta*})$ of the sequence $(\mathbf{x}(t), \boldsymbol{\mu}^\alpha(t), \boldsymbol{\mu}^\beta(t))$ generated by the algorithm in Table I is primal-dual optimal.

The reader is referred to our technical report [3] for a detailed proof. Though there exists a unique maximizer \mathbf{x}^* to the problem P , there may be multiple dual optimal prices, since only the flow price is constrained at optimality according to $U'_f(x_f^*) = \lambda_i^{\alpha*} + \lambda_i^{\beta*}$. Theorem 3 does not guarantee convergence to a unique vector $(\mathbf{x}^*, \boldsymbol{\mu}^{\alpha*}, \boldsymbol{\mu}^{\beta*})$, though any convergent subsequence leads to the optimal rate allocation \mathbf{x}^* .

In the above iterative algorithm, a maximal clique is regarded as a network element that can carry out the functions of price calculation and notification. However, a maximal clique is only a concept defined based on subflow contention graph. To deploy the algorithm in an actual *ad hoc* network, the above tasks of a maximal clique need to be carried out by the nodes that constitute the clique in a distributed fashion. For the implementation details, readers are referred to our report [3].

5. Simulation Results

We present the simulation results of our price pair mechanism and the distributed algorithm on a simple network as shown in Figure 1. The reader is referred to our technical report [3] for extensive results and performance evaluation.

In the first experiment, the network parameters are as follows: channel capacity $C_q = 2$ Mbps, for $q = 1, 2, 3$; relay cost $E_j = 2$ for $j = 1, 2, 3, 5, 6, 7$ and $E_4 = 3$. The minimum and maximum rate requirement of flows are $\{x_i^m = 0$ Mbps and $x_i^M = 2$ Mbps, for all $i = 1, \dots, 7$. Step size $\gamma = 0.05$. It is obvious that the minimum rate requirement can be guaranteed. We show the convergence behavior of our iterative algorithm in this experiment. As shown in Figure 3, the algorithm converges to a global network equilibrium within about 800 iterations. At the equilibrium point, the optimal resource allocation and prices are listed in Table II.

In the second set of experiments, we show how both channel price and relay price are necessary to regulate and incentivize the network to operate at its optimal point. In particular, we simulate on the same network as in the first experiment with only one price. The results in comparison with the result based on price

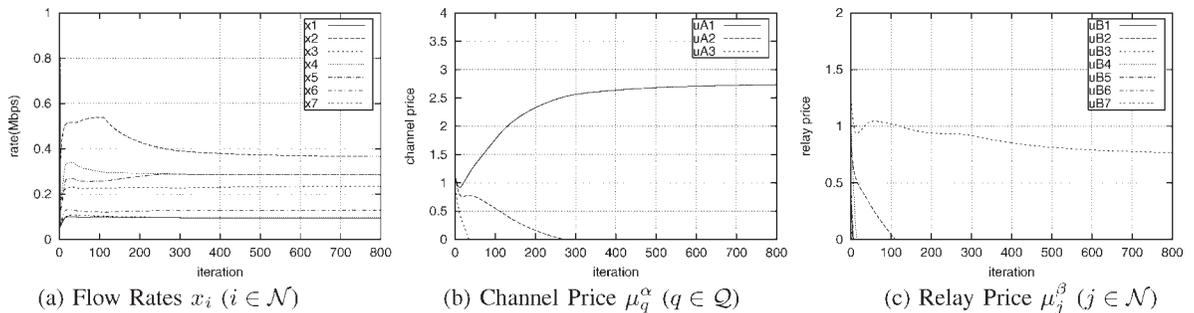


Fig. 3. Convergence of the algorithm on the network shown in Figure 1. (a) Flow rates $x_i (i \in \mathcal{N})$, (b) Channel price $\mu_q^\alpha (q \in \mathcal{Q})$ and (c) relay price $\mu_j^\beta (j \in \mathcal{N})$.

Table II. Equilibrium rate comparison.

Rate(Mbps)	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	$\sum_{i=1}^7 \ln(x_i)$
Price pair	0.095	0.364	0.235	0.286	0.286	0.129	0.096	-11.7
Channel price only	0.092	0.307	0.286	0.286	0.259	0.134	0.096	-11.8
Relay price only	0.081	0.540	0.137	0.314	0.314	0.105	0.081	-12.2

pair are shown in Table II. It is easy to see that the network does not operate at its desired point (i.e., NBS), when only one price (either channel price or relay price) is used. Above results show that in a typical network environment, using either only channel price to regulate the usage of shared wireless channel or only relay price to incentivize traffic relay for other nodes can *not* reach the adequate level of cooperation, where global network operates at its optimal point.

6. Related Work

The problem of optimal and fair resource allocation has been extensively studied in the context of wireline networks. Among these works, pricing has been shown to be an effective approach to achieve distributed solution for flow control [5–7] and service differentiation [8]. Simultaneously, game theory is applied to model resource sharing among multiple users, e.g., [2,9].

The main difference that distinguishes our work from the existing works is rooted in the unique characteristics of resource models of *ad hoc* network. First, the allocation mechanism needs to consider two types of resources, namely, the shared channel and the private resource such as energy. Second, due to the location-dependent contention and spatial reuse of the shared channel resource, the channel price is associated with a maximal clique in the subflow contention graph, rather than a wireline link. This presents a different pricing policy for end-to-end flows.

Incentives in wireless networks have stimulated much research interests. (e.g., in the context of *ad hoc* networks [10–12] and in wireless LAN [13]). In particular, the work in Reference [10] presents virtual credit-based mechanisms to stimulate cooperation in *ad hoc* networks, where virtual credits (so called nuglets) are awarded for packet forwarding. Some approaches [14,15] use a reputation-based me-

chanism where selfish or misbehaving nodes are identified, isolated, or punished. Our work distinguishes from the existing works in that, it does not only promote cooperation in packet forwarding, more importantly, it studies at what level of cooperation the network operates at its optimal point, and how to achieve such cooperation using pricing as incentives.

Non-cooperative game theory has been used to model the relaying behavior among nodes in *ad hoc* networks in References [11,12]. By designing appropriate game strategies and analyzing the Nash equilibrium of the corresponding relaying game, these works show the existence of a network operating point where node cooperation is promoted. In our work, Nash bargaining solution is used to characterize the global network operating point, which usually demonstrates more advantageous properties, such as Pareto optimality and fairness, than the usual Nash equilibrium in a non-cooperative game.

There are also previous works that address the issue of resource allocation [16] and use a price-based approach [17]. However, the *ad hoc* network models in these works do not consider the shared nature of the wireless channel, and thus their solutions are not able to capture the unique issues in wireless *ad hoc* networks. Moreover, the price-based distributed algorithm presented in [16] only converges to a network optimum when its utility function takes certain a special form, and such a utility function does not satisfy the fairness axiom.

7. Concluding Remarks

This paper presents a price pair mechanism that both regulates greedy behaviors and incentivizes selfish users in *ad hoc* networks. A pair of prices is the centerpiece of this mechanism: (1) the channel price that reflects the unique characteristics of location-dependent contention in *ad hoc* networks, and regulates the usage of shared wireless channel; (2) the relay price that gives incentives to reach the adequate

level of cooperation with respect to traffic relay. By using such a price pair as a signal, the decentralized self-optimizing decisions at each individual node converges to the global network optimal operation point.

References

- Owen G. *Game Theory*. Academic Press: New York, USA, 1995.
- Yaiche H, Mazumdar RR, Rosenberg C. A game theoretic framework for bandwidth allocation and pricing in broadband networks. *IEEE/ACM Transactions on Networking* 2000; **8**(5): 667–678.
- Xue Y, Li B, Nahrstedt K. Channel-relay price pair: towards arbitrating incentives in wireless ad hoc networks. Tech. Rep. UIUCDCS-R-2004-2476, UILU-ENG-2004-1779, University of Illinois at Urbana Champaign, 2004.
- Bertsekas D, Tsitsiklis J. *Parallel and Distributed Computation: Numerical Methods*. Athena Scientific Belmont MA, 1997. Prentice-Hall, 1989.
- Kelly FP, Maulloo AK, Tan DKH. Rate control in communication networks: shadow prices, proportional fairness and stability. *Journal of the Operational Research Society* 1998; **49**: 237–252.
- Low SH, Lapsley DE. Optimization flow control: basic algorithm and convergence. *IEEE/ACM Transactions on Networking* 1999; **7**(6): 861–874.
- Kunniyur S, Srikant R. End-to-end congestion control: utility functions, random losses and ecn marks. In *Proceedings of INFOCOM*, 2000.
- Cocchi R, Shenker S, Estrin D, Zhang L. Pricing in computer networks: motivation, formulation, and example. *IEEE/ACM Transactions on Networking* 1993; **1**(6): 614–627.
- Cao XR, Shen HX, Milito R, Wirth P. Internet pricing with a game theoretical approach: concepts and examples. *IEEE/ACM Transactions on Networking* 2002; **10**(2): 208–216.
- Buttayan L, Hubaux JP. Stimulating cooperation in self-organizing mobile ad hoc networks. *ACM/Kluwer Mobile Networks and Applications* 2003; **8**(5): 579–592.
- Srinivasan V, Nuggehalli P, Chiasserini CF, Rao RR. Cooperation in Wireless Ad Hoc Networks. In *Proceedings of IEEE INFOCOM*, 2003.
- Urpi A, Bonuccelli M, Giordano S. Modelinig cooperation in mobile ad hoc networks: a formal description of selfishness. In *Proceedings of Workshop modeling and optimization in mobile, ad hoc and wireless networks (WiOpt)*, 2003.
- Liao R, Wouhaybi R, Campbell A. Incentive engineering in wireless lan based access networks. In *Proceedings of 10th IEEE International Conference on Network Protocols (ICNP)*, November, 2002.
- Buchegger S, Le Boudec JY. Performance analysis of the confidant protocol (cooperation of nodes—fairness in dynamic ad-hoc networks. In *Proceedings of MobiHoc*, June 2002.
- Zhong S, Yang Y, Chen J. Sprite: a simple, cheat-proof, credit-based system for mobile ad hoc networks. In *Proceedings of IEEE INFOCOM*, 2003.
- Qiu Y, Marbach P. Bandwith allocation in ad-hoc networks: a price-based approach. In *Proceedings of INFOCOM*, 2003.
- Crowcroft J, Gibbens R, Kelly F, Ostring S. Modelling incentives for collaboration in mobile ad hoc networks. In *Proceedings of workshop modeling and optimization in mobile, ad hoc and wireless networks (WiOpt)*, 2003.

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