Scattered Random Network Coding for Efficient Transmission in Multi-hop Wireless Networks

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***Abstract*—Packet error rate dramatically increases when transmissions go over multiple hops in wireless networks, leading substantial throughput performance degradation. However, in the real wireless channels, the bit error probabilities vary across different bit positions in one modulation symbol, and the corruption of the packet is largely due to the incurred errors on those "bad" bit positions. To improve the throughput performance in multi-hop wireless networks, in this paper, we propose a novel *scattered* random network coding (referred to as *S-RNC*) scheme which further exploits the usefulness of random network coding and takes advantage of the error position diversity. In *S-RNC*, the random network coded blocks are classified into different groups and certain groups of blocks are selected as the *protected* blocks. The sender and relays always scatter the bits of these protected coded blocks on "good" bit positions (with low error probability) and the rest on "bad" bit positions (with high error probability). Rather than sharing the same error rate across all blocks in the conventional transmission scheme, the error probabilities of protected blocks in *S-RNC* decrease significantly even over *multiple hops*, which is helpful to achieve overall higher throughput, especially under poor channel conditions. Corroborating our intuition, our extensive simulation results show that *S-RNC* is able to improve the throughput performance substantially in multi-hop mode of wireless networks.**

***Index Terms*—Network Coding, Error Control, Multi-hop, Relay, Random Network Coding**

I. Introduction

It is common knowledge that errors are inherently present in unreliable wireless channels. Multi-hop wireless networks suffer from low throughput as errors expand over multiple hops. It is very crucial to design an efficient transmission and error control scheme to effectively maximize the achievable throughput in various transmission scenarios in multi-hop wireless networks even when unpredictable and time-varying errors exist.

With respect to the objective of maximizing the throughput, *network coding* has been originally proposed in information theory [1], [2] and has since emerged as one of the most promising information theoretic approaches to improve throughput performance. It has been successfully applied in wireless networks to opportunistically take advantage of multiple routes from the sender to the receiver in unicast flows [3], [4]. In order to provide more reliability in network coding, joint channel coding and network coding has been studied [5], [6], [7]. As a single error in one received network coding packet would typically render the entire transmission, it is important to eliminate erroneous data when performing decoding. It also applies when the erroneous network coding packet is combined with other received network coding packets to decode the transmitted message. In these works, channel coding and network coding is jointly designed to protect network coding packets from errors. In [8], MAC layer Random Network Coding has been introduced to avoid the overhead and provide resilience in the transmission. It is shown that random network coding helps to improve throughput performance substantially as compared with traditional hybrid automatic repeat request (HARQ). Rather than working on MAC layer, physical layer network coding was proposed in [9] by Zhang *et al.* to cancel the interference. Analog network coding [10] was later introduced to perform network coding in physical layer without the requirement of synchronization. [11] further exploits the benefits random network coding, and performs the network coding across the symbols within the packet in order to provide resilient and error control in fine granularity. Alamdar *et al.* proposed pre-coded transmission scheme using random network coding rather than frequency diversity, achieving significant performance improvement [12]. These works take advantage of the rateless property of random network coding: all data blocks (packet) are encoded as the random linear combination of the original packets and all independent coded blocks are equally useful [13], [14]; a receiver is analogous to holding a "bucket" to collect every fine "rain drop" (correct block) without dictating which block is from which source; once the "bucket" is full, the original data blocks can be correctly reconstructed. In this sense, random network coding can be considered as a rateless code.

When random network coding is applied to multi-hop wireless networks, the sender encodes the data and transmits the coded blocks to the relay which will perform forwarding to the end receiver. The block error rate increases significantly as the coded blocks travel hop by hop, leading data corruption with high probability when reaching the end receiver. Under such condition, the receiver's "bucket" is filled in much lower rate as all the erroneous blocks should be thrown away and can not be used to fill the "bucket." With our simulation results shown in Sec. III, with signal to noise ratio (SNR) 6, decoding error probability of approximately 0.7 for two-hop transmission degrades to approximately 0.99 for six-hop transmission. With increasing number of hops, the throughput performance

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downgrades dramatically even with random network coding applied.

However, due to the properties of modulation techniques, all the bits do not have the same bit error probability in one corrupted modulation symbol (one symbol contains a few bits, *e.g.*, 4 Quadrature Amplitude Modulation(QAM) contains 2 bits, 16QAM contains 4 bits, and 64QAM contains 6 bits). The bit error rates vary across different bit positions in one symbol of a certain modulation. Normally, errors only incur on the "bad" bit positions (with high error probability), which would eventually corrupt the whole symbol and further the whole data block. In such case, the whole data block is wasted due to a few errors on "bad" bit positions. If the data block size is larger, the more bits would be wasted. Once modulation is selected between the source and the destination, "good" and "bad" bit positions can be autonomously decided. As described in Sec. II-B, in case of 16QAM, when the Gray mapping rule is used, the 1st and the 3rd bit positions are "good" ones and the 2nd and the 4th bit positions are "bad" ones. Similarly, "good" and "bad" bit positions of 64QAM and 256QAM can be decided. However, in case of 4QAM and BPSK, bit position diversity can not be achieved due to symmetry of constellation points. The error rate diversity causes the inefficiency of multi-hop transmissions. *How to mitigate such negative effect and improve the overall throughput over multi-hop transmission* is the key challenge to the design of efficient transmission scheme in multi-hop wireless networks. *Can the diversity of bit error rate be turned into a positive factor? Is random network coding still helpful here?*

In this paper, we propose a novel scattered random network coding, referred to as *S-RNC*. *S-RNC* is built on top of the research findings in [8], [11] and [12] where random network coding is used as an error control scheme in MAC or physical layers and we showed the superiority of the proposed random network coding scheme to the conventional error control schemes such as HARQ. Different from the previous approaches of random network coding, *S-RNC* classifies the coded blocks into different groups and scatters the bits of corresponding groups' coded blocks into "good" or "bad" bit positions in order to take advantage of the diverse bit error rates. In this way, the groups of blocks whose bits are put into "good" positions are protected over multiple hops with lowered error rate. With such error separation, we can further exploit the usefulness of random network coding. Due to its favorable rateless property, the protected data eventually help to increase the overall packet delivery rate.

The intuition is shown in an illustrative example of Fig. 1. When *S-RNC* is applied, the base station puts the bits of block A to "good" bit positions (with low error probability) and the bits of block B to "bad" bit positions (with high error probability). By traveling via two hops (through Relay 1 and Relay 2), B is corrupted (the darkness indicates the error bits in the blocks). From the figure, we can see more and more bits in B are corrupted in multi-hop transmissions, since the bits of B are scattered on "bad" bit positions which would lead to errors with higher probability on each hop. On the other hand, A is protected and error free since the bits reside on "good" bit positions with low error rate on each hop.

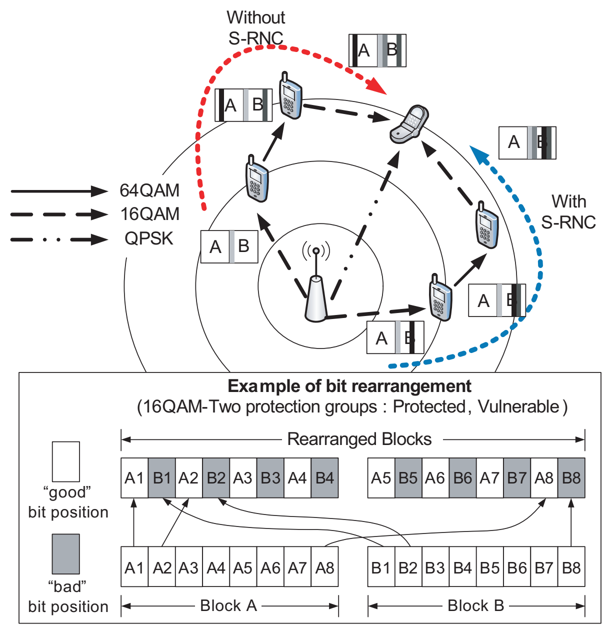

Fig. 1. Not all the bits are in error in one corrupted block. Different bit positions have different bit error rates under a certain modulation scheme. In the multi-hop transmission, we can lower the error rates on the blocks, the bits of which we scatter to the "good" positions. The darkness indicates the error bits in the blocks during the transmissions. In the example of bit rearrangement for blocks, each of which consists of 8 bits, under 16QAM modulation, the bits of block A are placed in "good" bit positions with lower bit error rates and the bits of block B are placed in "bad" bit positions with higher error rates.

Without such error separation, A and B may both be in error as they have the same high error rate. Thus, *S-RNC* protects certain blocks and prevents them from getting "dirty" when traveling over multiple hops, so that not all coded blocks are corrupted as contrasted with the conventional scheme. These protected coded blocks will make key contribution to the overall throughput by facilitating the receivers to get "clean" coded blocks faster. The intuition of block protection by bit rearrangement is also shown in Fig. 1.

The design objective of *S-RNC* is to realize all the potential benefits described above. To achieve such an objective, there are a number of challenges:

- ▷ How does *S-RNC* lower the error rates of protected blocks by rearranging the bits?
- ▷ How does *S-RNC* help to improve the overall throughput in multi-hop transmissions?
- ▷ How is *S-RNC* integrated with the modulation techniques adopted at the physical layer?

Our responses to these challenges constitute the flow of presentation in this paper. In Sec. II, we present the design of *S-RNC* in details, and the theoretical analysis is provided. We evaluate *S-RNC* in Sec. III. All results — coincide with our intuition — show that the significant gains can be obtained with the proposed scheme. Finally, we conclude the paper in Sec. IV.

II. Scattered Random Network Coding

S-RNC is designed specifically to explore the benefits provided by both network coding and error diversity in multi-hop wireless networks. The basic idea of *S-RNC* is to prevent all data from sharing the same error probability at the receiver side by protecting certain data against getting "dirty" over multiple hops. Protecting blocks from errors is the most important function of random network coding as the single error in one received block corrupts the entire received block [15]. Thus, *S-RNC* is able to facilitate the receivers to collect "clean" data more efficiently. These protected data will contribute to the overall throughput, as all the linearly independent coded blocks are equally innovative with random network coding. In this section, we present *S-RNC* in detail including network coding process, performance analysis, as well as the protocol.

A. Random Network Coding Process

Random network coding is the basic component in *S-RNC*. In the system, the transmitter divides input bit streams into segments (or referred to as generation in the literature) of fixed length. Each segment is further divided into certain number of *blocks*. Let n be the number of blocks in one segment, we denote n as the *batch size*, and let x_i ($i = 1, 2, \cdots, n$) be the blocks, and c_{ji} ($i = 1, 2, \cdots, n$) be the set of random coefficients generated in a given Galois field ($GF(2^m)$). A coded block, y_j, can then be produced as $y_j = \sum_{i=1}^{n} c_{ji} \cdot x_i$. Encoding operation can also be denoted in matrix form: $\mathbf{y} = \mathbf{C} \cdot \mathbf{x}$, where $y_j \in \mathbf{y}$, $x_i \in \mathbf{x}$, and $c_{ji} \in \mathbf{C}$. Each coded block is essentially a linear combination of all or a subset of the original data blocks. In this way, the encoder is able to generate a virtually unlimited number of coded blocks y_j ($j = 1, 2, \cdots$) using different sets of coefficients, and any n of independent coded blocks can be used to decode by inverting a matrix of coding coefficients. This is usually referred to as the *rateless* property. The design of *S-RNC* is based on the random network coding process described here.

Random network coding can be viewed as one of Maximum Rank Distance Code [15], [16]. However, Gabidulin Code [16], which is an important family of Maximum Rank Distance Codes, is not a rateless code. In order to perform efficient decoding of random network coding with any n correctly received coded blocks, we have to guarantee any n coefficient vectors, used to produce n coded blocks, are linearly independent with each other even when the number of coded blocks can be unlimited. To achieve such performance, we propose a coefficient code book matrix, defined as follows:

$$\mathbf{V}'(\mathbf{v}, k, n) = \begin{pmatrix} v_1 & v_1^2 & \dots & v_1^n \\ v_2 & v_2^2 & \dots & v_2^n \\ \vdots & \vdots & \vdots & \vdots \\ v_k & v_k^2 & \dots & v_k^n \end{pmatrix}$$

where $2^m - 2 \geq k \geq n$ when $GF(2^m)$ is used. The coefficient code book matrix is essentially the Vandermonde matrix without the first column (the column with all 1's). It can guarantee any n row vectors are linearly independent with each other and can be used as coefficients for encoding, if and only if the elements in its first column $\mathbf{v} = [v_1, v_2, ..., v_k]'$ are distinct elements of $GF(2^m)$ except 1. As we can randomly generate the first column and make the code book thereafter, essentially we can generate random code book for both encoding and decoding with random linear code approach. With such favorable property, it is very convenient and efficient to construct the coefficients matrix by selecting any row vectors from this coefficient code book matrix. Once k and m are decided and the first column of each row is selected the rest of columns can be automatically generated. If the first column elements are randomly selected, the selected elements need to be informed to the receivers which perform random network decoding. In order to reduce the overhead of transmitting the coefficient matrix, the matrix can be pre-generated following a certain rule and only the index can be transmitted. More discussion on the protocol overhead will be provided in Sec. II-D. When small k is used in symbol-level network coding, more accurate symbol level operation is possible but rateless property of random network coding is limited because the number of redundant coded blocks ($n-k$) are limited by the field size m [11]. Therefore, proper field size m needs to be carefully selected considering the tradeoff between the accuracy of random network coding operation and the freedom for the rateless property. If m is big (normally $m = 8$), virtually unlimited independent coded blocks can be produced.

B. *Is* S-RNC *Helpful in Multi-hop Transmissions?*

S-RNC is built on random network coding, and all the coded blocks are transmitted in modulation symbols at the physical layer. One symbol contains several bits. For example, there are 4 bits using 16QAM modulation, and 6 bits with 64QAM modulation. As we elaborated before, different bit positions may have different error probabilities. In case of 16QAM, when the Gray mapping rule is used, bit error probabilities on four bit positions (P_1,P_2,P_3 and P_4) are not equal [17]: $P_1 \approx P_3 < P_2 \approx P_4$. We denote P_1 and P_3 as $P_{16}(1)$ and P_2 and P_4 as $P_{16}(2)$. Similarly, the bit error probabilities of different bit positions under 64QAM are not equal either [17]: $P_1 \approx P_4 < P_2 \approx P_5 < P_3 \approx P_6$. We denote P_1 and P_4 as $P_{64}(1)$, P_2 and P_5 as $P_{64}(2)$, and P_3 and P_6 as $P_{64}(3)$. Table I lists bit error probabilities of 16QAM and 64QAM based on our simulation under AWGN channel condition.

To fully exploit the benefits provided by this diversity, we scatter the bits of coded blocks according to the bit error probabilities on different positions. Using the case of 16QAM as an example, the coded blocks can be divided into two groups: protected and vulnerable. For the transmission on each hop, the sender puts the bits of protected blocks to "good" bit positions and vulnerable bits to "bad" bit positions. Without loss of generality, we use the general case under 16QAM to rigorously show how *S-RNC* helps to lower the block error rate. Assume two coded blocks, each of which consists of s bits, are transmitted over h hops. If *S-RNC* is not used, the block error rates of the both blocks (denoted as P_A and P_B) after h-hop traveling are:

$$P_A = P_B = 1 - (1 - P_{16}(1))^{\frac{s}{2} \times h}(1 - P_{16}(2))^{\frac{s}{2} \times h} \quad (1)$$

When *S-RNC* is employed, the block error rates of the two

SNR	16 QAM				64 QAM					
	bit 1	bit 2	bit 3	bit 4	bit 1	bit 2	bit 3	bit 4	bit 5	bit 6
1	0.0283	0.0852	0.0266	0.0823	0.014	0.0281	0.0573	0.0135	0.0278	0.0574
2	0.0196	0.057	0.0193	0.0575	0.009	0.0191	0.0366	0.0097	0.0188	0.0387
3	0.0109	0.0333	0.0112	0.035	0.006	0.0117	0.0229	0.0056	0.0119	0.0216
4	0.0062	0.0184	0.0065	0.019	0.0028	0.0063	0.0123	0.0032	0.0065	0.0127
5	0.0031	0.009	0.0029	0.009	0.0015	0.003	0.0059	0.0015	0.003	0.006
6	0.0012	0.0034	0.0014	0.0036	0.00054	0.0013	0.0024	0.00064	0.0014	0.0023
7	0.00025	0.001	0.00034	0.0011	0.00018	0.0004	0.0007	0.00021	0.00029	0.00077

TABLE I
BIT ERROR PROBABILITY PER BIT POSITION FOR 16QAM AND 64QAM MODULATIONS

blocks (denoted as $P_A^{'}$ and $P_B^{'}$) will be:

$$P_A^{'} = 1 - (1 - P_{16}(1))^{s \times h} \quad (2)$$

$$P_B^{'} = 1 - (1 - P_{16}(2))^{s \times h} \quad (3)$$

Obviously, $P_A^{'} < P_A = P_B < P_B^{'}$, since $P_{16}(1) < P_{16}(2)$. It shows the error rate of block A is lowered and that of block B is increased after *S-RNC*. When the number of hops is large, block A with *S-RNC* is with much higher probability to be error free than transmission without S-RNC.

This idea is inspired by *constellation rearrangement*, which has been studied in terms of HARQ [18], [19], [20]. The idea of constellation rearrangement of HARQ is simple. In case of retransmission, if bits are transmitted mapped to the same bit positions of modulation symbols as the original transmission, bits on "bad" bit positions will be very likely to have errors again. Therefore, if bits transmitted on the "bad" positions in the first transmission mapped to the "good" positions in the retransmission by rearranging constellation, the performance can be enhanced. However, in order to implement constellation rearrangement, the sender and the receiver have to be fully aware of HARQ operation and perform rearrangement considering whether the transmission is retransmission or not. In *S-RNC*, rearranging bits of the random network coded blocks is able to prevent all blocks from getting "polluted" over multi-hop transmissions by protecting "clean" blocks against getting "dirty" while making "dirty" blocks "dirtier." As random network coding is applied, all the linearly independent coded blocks are equally innovative. By "collecting" more "clean" protected blocks, the receiver is able to decode and get useful data more efficiently. *Is S-RNC helpful to improve the overall throughput performance in multi-hop transmissions?* We use a simple analysis to show the benefits of *S-RNC* on throughput. According to Eq. (1) - (3), it is straightforward to obtain the expected block delivery rates with and without *S-RNC* (denoted as T_{S-RNC} and T respectively) in h-hop transmission as follows:

$$\begin{aligned} T_{S-RNC} &= \frac{1}{2}(1 - P_A^{'} + 1 - P_B^{'}) \\ &= \frac{1}{2}[(1 - P_{16}(1))^{s \times h} + (1 - P_{16}(2))^{s \times h}] \quad (4) \\ T &= 1 - P_A = 1 - P_B \\ &= (1 - P_{16}(1))^{\frac{s}{2} \times h}(1 - P_{16}(2))^{\frac{s}{2} \times h} \quad (5) \end{aligned}$$

Note that $(1 - P_{16}(1)), (1 - P_{16}(2)) > 0$. From the arithmetic/geometric mean inequality, it is obvious to obtain $T_{S-RNC} > T$, where $P_{16}(1) \neq P_{16}(2)$.

Therefore, by applying *S-RNC*, the throughput can be increased by collecting more "clean" blocks at the receiver. S-RNC's throughput enhancement solely comes from the favorable property of random network coding. Throughput enhancement of channel coding can not be achieved through bit position diversity. Because channel coding is able to correct some errors within a received block, it is not so important to protect some blocks. Average bit error rate, which can not be enhanced with bit rearrangement, is more important for channel coding. Also, spread errors across a packet is more desirable for channel coding than bursty errors. This is why channel coding employs bit interleaver.

C. Protocol Design

Now, we are ready to show the whole picture of *S-RNC* protocol. Fig. 2 is a block diagram showing how scattered random network coding works in a two hop transmission case. The sender first divides input data into segments and each segment is divided into a number of blocks. We denote the blocks as $\mathbf{x} = [x_1, x_2, \cdots, x_n]$. The random network code encoder generates random network coded blocks ($\mathbf{y} = [y_1, y_2, \cdots, y_n, \cdots]$) by performing random network encoding and appending Cyclic Redundancy Check (CRC) codes. Then, the bits of random network coded blocks are scattered to form scattered random network coded blocks ($\mathbf{y}' = [y_1', y_2', \cdots, y_n', \cdots]$) using *S-RNC* algorithms according to the used modulation scheme. After that, the data are modulated into symbols ($\mathbf{s} = [s_1, s_2, \cdots, s_F, \cdots]$) and the modulated symbols are transmitted to the relay. Upon receiving the data by the relay, it has to check whether the same modulation scheme would be used in the transmission on the next hop. If so, the relay just simply amplifies and forwards the received signals. Otherwise, the relay demodulates the detected symbols ($\mathbf{s^{d1}} = \left[s_1^{d1}, s_2^{d1}, \cdots, s_F^{d1}, \cdots\right]$) to get scattered random network coded blocks ($\mathbf{y}' = [y_1', y_2', \cdots, y_n', \cdots]$) and performs bit restoration, which is the reverse process as bit rearrangement, to recover the originally transmitted random network coded blocks ($\mathbf{y} = [y_1, y_2, \cdots, y_n, \cdots]$) and then performs *S-RNC* algorithms according to the new modulation scheme which produces the newly rearranged scattered random network coded blocks ($\mathbf{y}'' = [y_1'', y_2'', \cdots, y_n'', \cdots]$). Thus, the protected coded blocks can be continuously protected in the following transmissions to the receiver. The receiver demodulates the received symbols ($\mathbf{s^{d2}} = \left[s_1^{d2}, s_2^{d2}, \cdots, s_F^{d2}, \cdots\right]$)

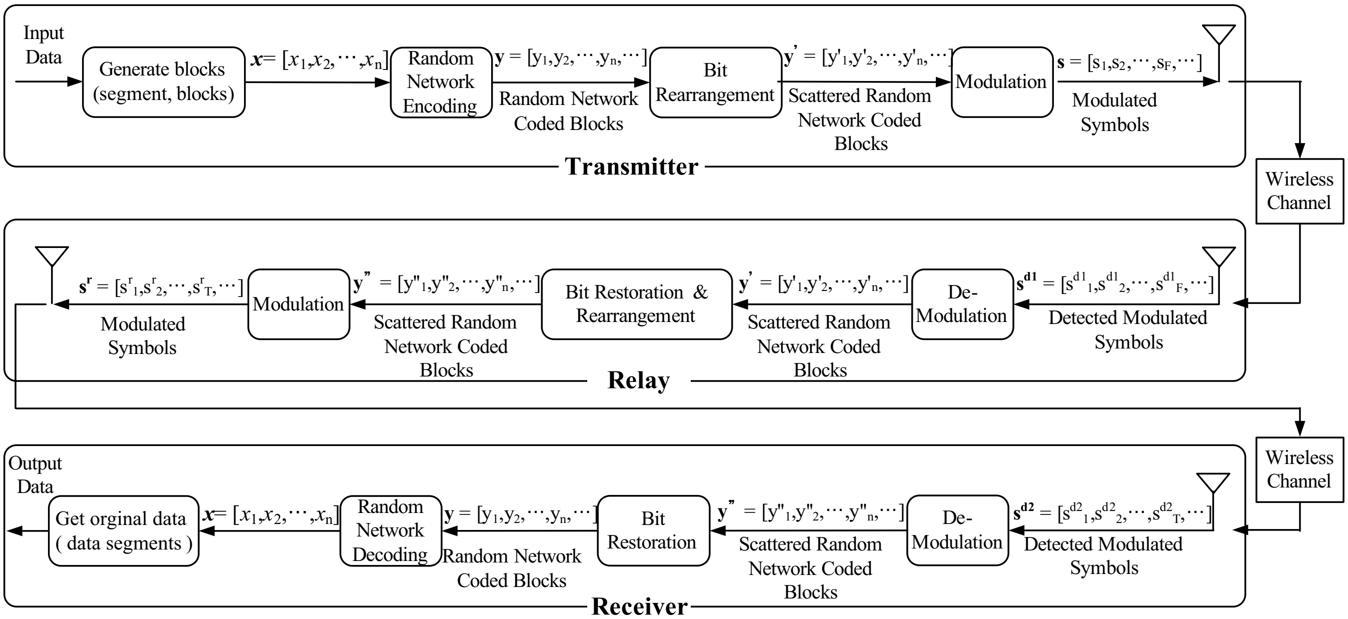

Fig. 2. Block diagram showing how scattered random network coding works.

and acquires scattered random network coded blocks ($\mathbf{y}'' = [y_1'', y_2'', \cdots, y_n'', \cdots]$). By performing bit restoration, random network coded blocks ($\mathbf{y} = [y_1, y_2, \cdots, y_n, \cdots]$) can be obtained. Finally, the original data ($\mathbf{x} = [x_1, x_2, \cdots, x_n]$) can be reconstructed by performing random network decoding with *clean blocks* where block errors can be detected using CRC.

D. Computation Complexity and Protocol Overhead

As neither base stations nor relay stations have constraints with respect to energy and computational power, we are only concerned with the computation overhead at mobile stations. Nowadays, the power of handsets is developing fast. Even a mobile device, nowadays, like cell phones, have much memory cache and strong computing ability. According to the results in [21], random network coding is almost “free” with the current processors. The coding speed could reach 1248 Mbps for 16 blocks of 32 KB each and 348 Mbps for 64 blocks of 32 KB each. Although it indeed adds computation overhead in some degree, it keeps the overhead within practical limits. The bit rearrangement and restoration processes in *S-RNC* can be performed simply using hardware at the physical layer and very fast without generating overhead in higher level.

In random network coding, random coefficients are normally embedded in the transmitted blocks as a packet header which would generate certain amount of overhead. In order to reduce the overhead of communicating random coefficients, the coefficient code book matrix, as we proposed in Sec. II-A, can be pre-generated and be kept at the sender, the receiver, and the relays for encoding and decoding. The sender then only transmits the index of the coefficients in the matrix that are used for encoding to the relays or the receiver, as a part of the session control information before starting to transmit actual data. The relay or receiver is able to obtain the coding coefficients according to the index and perform decoding. This scheme is able to dramatically mitigate the protocol overhead in the transmissions. *S-RNC* adopts this scheme and there is no header overhead totally. Overhead between the conventional random network coding and the proposed scheme can be easily compared with the following simple example. If Galois field ($GF(2^8)$) is used, n is 10 and k is 20, the coefficient matrix size is 20×10. Every packet has to carry 200 bytes overhead in the header for the random coefficients. In our proposed scheme with the given Galois field size, n and k, the coefficient code book matrix size of $2^8 - 2 = 254 \times 10$ can be obtained. Since the coefficient code book matrix can be autonomously generated once Galois field size, n and k are decided, the source can transmit the starting row number of the matrix which only requires one byte, 0.5% of the conventional random network coding overhead. Thereafter, because the source can use subsequent coefficients, the source just transmit the starting row numbers in the session control information when the session is established. Therefore, *S-RNC* has no header overhead and very low overhead for coefficients information delivery. Because n and k would be much large values in the normal setting, the overhead of the conventional method is substantially larger than the proposed scheme.

III. Performance Evaluation

Now, we evaluate the performance of *S-RNC*. The evaluation scenario is as follows: a large number of coded blocks are transmitted via multiple hops (two and six hops) using 16QAM modulation scheme in Additive White Gaussian Noise (AWGN) channel. In the evaluation, we set the block size to be 8 bits which is selected through thorough study in [11]. For *S-RNC*, we protect the odd numbered blocks by scattering the bits to “good” bit positions, and the even numbered blocks are scattered to “bad” positions.

We first measure the block error rate performance with and without *S-RNC*. The evaluation is performed via both

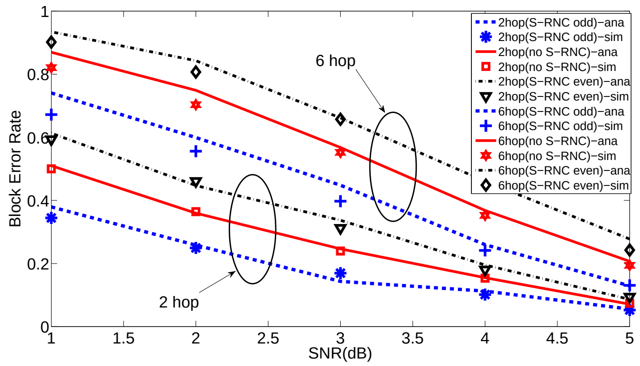

Fig. 3. Block error rates performance with and without S-RNC under 2-hop and 6-hop transmissions using 16QAM in AWGN channel. “no S-RNC,” “S-RNC odd” and “S-RNC even” represent, respectively, the performance without S-RNC, the performance of odd numbered blocks with S-RNC (protected), and the performance of even numbered blocks with S-RNC. “ana” and “sim” denote analysis and simulation respectively.

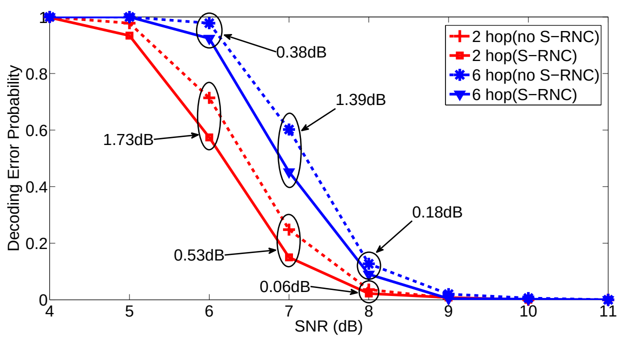

Fig. 4. Decoding performance at the receiver for 2-hop and 6-hop transmissions using 16QAM in AWGN channel. “S-RNC,” and “no S-RNC” stand for the schemes with S-RNC and without S-RNC respectively. The decoding performance represents the end-to-end throughput in the multi-hop transmissions.

numerical analysis and extensive simulation. For numerical analysis, we adopt the calculation in Eq. (4) - (5). For simulation implementation, we take advantage of the latest communication toolbox in MATLAB which is efficient to evaluate the performance of physical-layer protocols. The evaluation results are shown in Fig. 3, where we easily observe that block error rates of odd numbered blocks in *S-RNC* significantly decreases (approximately 50% in average) compared with the average block error rates in the scheme without *S-RNC*. The gains are more substantial under poor channel conditions. The results clearly show the improved performance on protected blocks over multiple hops.

We then perform simulations to evaluate *S-RNC* and examine whether it helps to “collect” more fine “rain drops” and hence to improve the overall throughput. In this simulation, we set packet size to be 512 bits as in [11] and k to be $2 \times n$. Fig. 4 shows the results which coincide with our intuition. The decoding error probability at the receiver significantly decrease which means the receiver has higher probability to “collect clean blocks” and thus achieve higher end-to-end throughput. The average gain can reach up to 0.46dB for two-hop transmission. This gain is even larger when the number

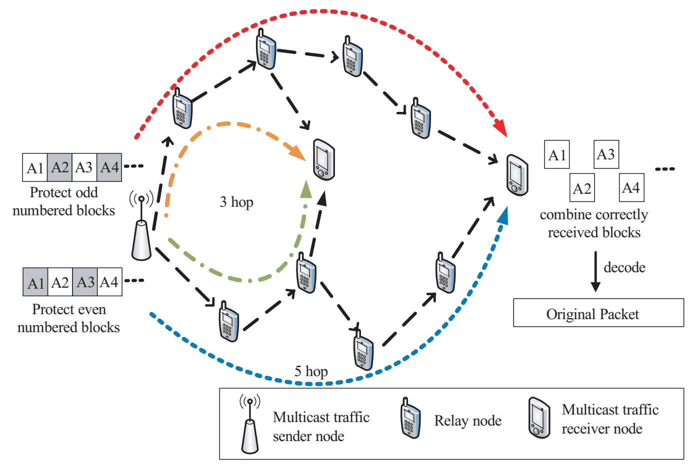

Fig. 5. Multicast scenario via multi-path over multi-hop. Example of 3-hop and 5-hop multicast traffic transmissions to two receiver nodes in multi-path multi-hop network topology.

of hops increases, reaching 1.24dB in average for six-hop transmission. All these results demonstrate *S-RNC* is able to improve the throughput performance significantly in multi-hop wireless networks.

Now, simulations are performed with realistic wireless channel condition of Rayleigh fading channel. Moving speed = 30km/h, path delay $= [0, 1, 2] \times 1e - 7$ and path gain $= [0, -3, -9]$ are used for Rayleigh fading simulations. Simulations are performed with two scenarios: 1. single-path multi-hop transmission, 2. multi-path multi-hop transmission with multicast traffic. Due to Rayleigh fading, it is very likely most of blocks have one or more bit errors without using *S-RNC*. This is confirmed with the single-path multi-hop transmission simulation in line network. However, if *S-RNC* is used bit errors due to Rayleigh fading only affects some of blocks, i.e., some blocks are protected with bit rearrangement. When *S-RNC* is utilized in the multi-path multi-hop networks with multicast traffic as shown in Fig. 5, system performance can be improved because different set of blocks can be protected over different paths which allows the receiver nodes to collect more number of clean blocks. In this scenario, multicast traffic receiver nodes receive multicast traffic via multiple paths over multiple hops. In this scenario, we just showed two multi-paths over 3-hop and 5-hop and only showed two receiver nodes for simplification reason. The simulation is performed with the scenario shown in Fig. 5, and performance is measured based on the average link quality (SNR) between two relay nodes because relay nodes are typically selected based on their radio signal quality from the previous node. The same previously explained realistic Rayleigh fading setting is used in this simulation. For fair comparison,in the multi-path multi-hop scenario, in the case of not using *S-RNC*, signal level multi-path combining is employed in the receiver. The simulation results for both scenarios are shown in Fig. 6. As we can see from the results, by utilizing *S-RNC* some blocks are protected over multiple hops in line network scenario and different set of blocks can be protected over different paths leading to high probability of successful decoding in multi-path multi-hop

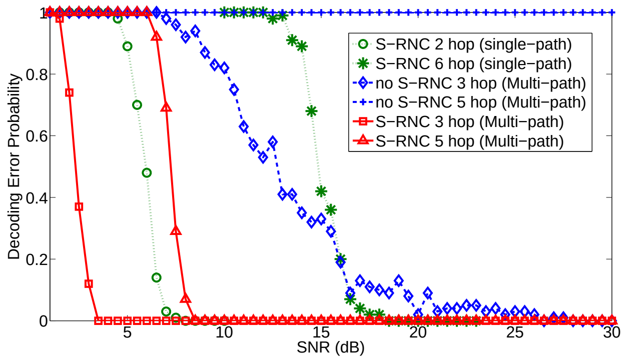

Fig. 6. Decoding performance at the receiver in Rayleigh fading channel. Scenario 1: 2-hop and 6-hop transmission using 16QAM in single-path line network topology. Scenario 2: 3-hop and 5-hop transmissions using 16QAM in multi-path multi-hop network topology with multicast traffic.

scenario. The implication of the multi-path multi-hop result is that if *S-RNC* is employed, the distance between relay nodes can be longer than the one without *S-RNC*.

IV. Concluding Remarks

In this paper, we have explored essential reasons on poor throughput performance in multi-hop wireless networks. A novel design of scattered random network coding (referred to as S-RNC) is proposed to achieve more efficient transmissions accordingly. The intuition is quite simple to narrate: taking advantage of the diverse bit error rates in the modulation symbols, *S-RNC* rearranges and scatters the bits of random network coded blocks and thus certain blocks can be effectively protected over multiple hops without sharing the same error probability as in the conventional scheme. With the rateless property of random network coding, the protected blocks eventually make key contribution to the overall end-to-end throughput in multi-hop transmissions. Via theoretical analysis and simulation evaluation, the proposed scheme is shown to be efficient and resilient to wireless channel errors, especially with increasing number of hops, thus improving the system performance substantially.

References

[1] R. Ahlswede, N. Cai, S. R. Li, and R. W. Yeung, "Network Information Flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204–1216, July 2000.

[2] R. Koetter and M. Medard, "An Algebraic Approach to Network Coding," *IEEE/ACM Transactions on Networking*, vol. 11, no. 5, pp. 782–795, October 2003.

[3] S. Chachulski, M. Jennings, S. Katti, and D. Katabi, "Trading Structure for Randomness in Wireless Opportunistic Routing," in *Proc. of ACM SIGCOMM*, 2007.

[4] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "XORs in The Air: Practical Wireless Network Coding," in *Proc. of ACM SIGCOMM*, 2006.

[5] R. Koetter and F. Kschischang, "Coding for Errors and Erasures in Random Network Coding," *Information Theory, IEEE Transactions on*, vol. 54, no. 8, pp. 3579 –3591, August 2008.

[6] L. Xiao, T. Fuja, J. Kliewer, and D. Costello, "A Network Coding Approach to Cooperative Diversity," *Information Theory, IEEE Transactions on*, vol. 53, no. 10, pp. 3714 –3722, October 2007.

[7] M. Riemensberger, Y. Sagduyu, M. Honig, and W. Utschick, "Training overhead for decoding random linear network codes in wireless networks," *Selected Areas in Communications, IEEE Journal on*, vol. 27, no. 5, pp. 729 –737, June 2009.

[8] J. Jin, B. Li, and T. Kong, "Is Random Network Coding Helpful in WiMAX?" in *Proc. of IEEE INFOCOM*, 2008.

[9] S. Zhang, S. C. Liew, and P. P. Lam, "Physical-Layer Network Coding," in *Proc. of ACM MobiCom*, 2006.

[10] S. Katti, D. Katabi, H. Balakrishnan, and M. Medard, "Symbol-Level Network Coding for Wireless Mesh Networks," in *Proc. of ACM SIGCOMM*, 2008.

[11] R. Y. Kim, J. Jin, and B. Li, "Drizzle: Cooperative Symbol-Level Network Coding in WiMAX," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 3, pp. 1415–1432, March 2010.

[12] A. Yazdi, S. Sorour, S. Valaee, and R. Kim, "Reducing Symbol Loss Probability in the Downlink of an OFDMA Based Wireless Network," in *Proc. of IEEE ICC*, May 2008, pp. 3485–3489.

[13] T. Ho, R. Koetter, M. Medard, D. Karger, and M. Effros, "The Benefits of Coding Over Routing in a Randomized Setting," in *Proc. of International Symposium on Information Theory (ISIT)*, 2003.

[14] T. Ho, M. Medard, R. Koetter, D. Karger, M. Effros, J. Shi, and B. Leong, "A random linear network coding approach to multicast," *Information Theory, IEEE Transactions on*, vol. 52, no. 10, pp. 4413–4430, Oct. 2006.

[15] D. Silva, F. Kschischang, and R. Koetter, "A Rank-Metric Approach to Error Control in Random Network Coding," in *IEEE Transactions on Information Theory*, vol. 54, 2008, pp. 3951–3967.

[16] E. Gabidulin, "Theory of codes with maximum rank distance," in *Probl. Inform. Transm*, vol. 21, no. 1, 1985, pp. 1–12.

[17] M. P. Fitz and J. P. Seymour, "On the bit error probability of QAM modulation ," in *International Journal of Wireless Information Networks*, 1994.

[18] C. Wengerter, A. von Elbwart, E. Seidel, G. Velev, and M. Schmitt, "Advanced hybrid arq technique employing a signal constellation rearrangement," in *Vehicular Technology Conference, 2002. Proceedings. VTC 2002-Fall. 2002 IEEE 56th*, vol. 4, September 2002, pp. 2002 – 2006 vol.4.

[19] M.-l. Jia, J.-m. Kuang, and Z.-w. He, "Enhanced HARQ Employing LDPC Coded Constellation Rearrangement with 64QAM," in *International Conference on Communication Technology*, 2006.

[20] Z. Zhao, L. Zhou, X. Zheng, and S.-E. Park, "Enhanced constellation rearrangement for harq with symbol combining," in *Mobile WiMAX Symposium, 2009. MWS '09. IEEE*, July 2009, pp. 11 –15.

[21] H. Shojania and B. Li, "Parallelized Progressive Network Coding With Hardware Acceleration," in *Proc. of 5th IEEE International Workshop on Quality of Service (IWQoS)*, 2007.