Many-to-Many Matching for Combinatorial Spectrum Trading

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***Abstract*—Dynamic spectrum access (DAS) is an efficient way to redistribute spare channels among users. Conventionally, dynamic spectrum access is conducted through (double) spectrum auction, where a third-party auctioneer collects bids from buyers and sellers, and determines the spectrum allocation. Rather than placing bids only on individual channels, combinatorial spectrum auction allows buyers to express their valuations for different combinations of channels. However, auction mechanisms are generally vulnerable to the collusion between the auctioneer and buyers or sellers. Furthermore, to find the optimal allocation in combinatorial auction is usually NP-hard. In this paper, we propose to leverage a many-to-many matching framework to realize combinatorial spectrum trading. Unlike traditional many-to-many matching problem, spectrum matching is more challenging, because spectrum allocation is interference-limited rather than quota-limited. To deal with this problem, we propose a novel matching algorithm, which takes buyers' interference relationship into consideration. We theoretically prove that the matching result is individual rational, strong pairwise stable and is a subgame-perfect Nash equilibrium of the corresponding spectrum bargaining game. Simulation results show that the proposed algorithm can converge to a stable matching within a few iterations.**

***Index Terms*—Dynamic spectrum access, spectrum trading, spectrum matching**

I. Introduction

Spectrum is an indispensible yet limited resource for wireless communication. With the rapid development of wireless technologies, service providers are facing a critical spectrum crunch thanks to the burgeoning growth of wireless applications and services. The acquisition of a spectrum license is one of the most difficult and costly procedures for any new wireless service and its market entry. In order to solve this dilemma, most of the countries have specific departments to regulate spectrum usage, e.g., Federal Communications Commission in the U.S. [1] and Radio Administration Bureau (RAB) in China [2]. Traditional spectrum allocation or auction is usually conducted on a long-term basis over large geographical regions. However, this will limit buyers' participation because of the large amount of capital required.

There is increasing evidence that spectrum resources are not being efficiently utilized [3]. Dynamic spectrum access (DAS), therefore, is proposed to cater to the increasing spectrum demand [4]. A wireless service provider can sell spare spectrum or buy additional spectrum according her needs. Spectrum auction is a common way to realize DAS. In a (double) spectrum auction, a third-party auctioneer collects bids from buyers and sellers, and then allocates spectrum in a centralized manner. Combinatorial spectrum auction, a special auction format, allows buyers to bid on combinations of channels, rather than individual channels, which enables buyers to express their preferences for different combinations. For example, continuous channels are easier to manage than non-continuous channels, and buyers may be willing to pay a higher price for continuous channels. Unfortunately, finding the optimal allocation in conventional combinatorial auction is usually NP-hard. With spectrum heterogeneity and spatial reusability, the combinatorial auction will become even more complicated [5].

In this paper, we leverage matching as an alternative framework for dynamic spectrum access. More specifically, to realize a combinatorial auction alike spectrum market, we propose to use many-to-many matching, where a buyer can purchase multiple channels, and a seller's channel can be assigned to multiple non-interfering buyers. Instead of maximizing social welfare, the aim of matching is to achieve a stable status. The stability concept is attractive because it keeps an equilibrium status for both sellers and buyers. Besides, the matching process can be free from a third-party auctioneer, avoiding potential collusions between the auctioneer and the buyers or sellers. In fact, stable matching has been widely applied to computer science, such as resource management in the cloud [6].

Matching has been widely studied in the economics and mathematics communities. In the pioneer work of Gale and Shapley [7], *deferred-acceptance* algorithms are proposed to reach a stable matching for the marriage problem (one-to-one matching) and the college admission problem (many-to-one matching). Compared with one-to-one matching and many-to-one matching, many-to-many matching is much more complicated, and it is more difficult to reach a stable matching result. Based on the deferred-acceptance algorithm, in [8], the authors proposed a competitive-adjustment process for labor markets with perfect information. In [9], an iterative T-algorithm is proposed, which can realize many nice properties, such as pairwise stability, setwise stability and core stability.

In this paper, we propose a novel many-to-many spectrum matching framework for combinatorial spectrum trading. Buyers can freely express their preferences for different combinations of channels, and the same channel can be reused by multiple non-interfering buyers. To address spectrum heterogeneity, different interference graphs are constructed for different channels to determine spectrum reuse [10]. We propose an algorithm, which can reach a stable matching result, and also improve spectrum utilization through spectrum reuse. We make the following key contributions:

- We propose a many-to-many spectrum matching framework to realize combinatorial spectrum trading for dynamic spectrum access.
- We propose a new matching algorithm to address spectrum heterogeneity and spectrum reuse. We prove that the matching result is individual rational, strong pairwise stable, and is a subgame-perfect Nash equilibrium of the corresponding spectrum bargaining game.
- We conduct extensive simulations to evaluate the performance of the proposed many-to-many spectrum matching framework. It is shown that it takes only a few iterations for the proposed algorithm to reach a stable matching result.

The rest of the paper is organized as follows. We describe the system model in details in Section II. In Section III, we present the many-to-many spectrum matching framework and matching algorithm. Simulation results are shown in Section IV. We briefly review the related work in Section V, and finally summarize our work in Section VI.

II. System Model

Assume there is a set of sellers $\mathcal{M} = \{1, 2, ..., m\}$ and a set of buyers $\mathcal{N} = \{1, 2, ..., n\}$ in the market. Each seller owns one channel, which can be matched to multiple non-interfering buyers. Buyer j has a basic price offer for all channels $B_j = (b_{1,j}, b_{2,j}, ..., b_{m,j})$, in which $b_{i,j}$ is buyer j's valuation for a single channel i. For a bundle of channels $\mathcal{A}$, buyer j may be willing to pay more than the sum $\sum_{i\in\mathcal{A}} b_{i,j}$, for example, two continuous channels may bring more benefit to a buyer. We will express such complementariness of channels in buyers' preference profiles.

The key feature of spectrum allocation is interference-restricted spacial reuse. To characterize interference heterogeneity of different channels, we construct a series of interference graphs $\{G^i = (V, E^i)\}_{i=1}^m$, in which each node $v \in V$ represents a buyer, and each edge $e^i \in E^i$ connects a pair of interfering buyers on channel i. Let $e^i_{j,j'} \in \{0, 1\}$ represent the interference status between buyers j and j' regarding channel i.

The preference profile $\succ_i$ of seller i is a complete, reflexive, and transitive binary relation on all sets of buyers $2^{\mathcal{N}}$. Due to the interference contraint, a seller prefers the empty set $\emptyset$ to any buyer set that contains interfering buyers. For two buyer sets that are both interference-free (not any two buyers in the set interfere with each other), the seller prefers the one with a higher aggregate basic offer price. Let $\mathcal{S}, \mathcal{S}' \in 2^{\mathcal{N}}$ denote buyer sets, we have:

- If $\mathcal{S}$ is interference-free, $\mathcal{S} \succ_i \emptyset$; if $\mathcal{S}$ is not interference free, $\emptyset \succ_i \mathcal{S}$.
- If $\mathcal{S}$ is interference-free, but $\mathcal{S}'$ is not, $\mathcal{S} \succ_i \mathcal{S}'$, vice versa.
- If both of $\mathcal{S}$ and $\mathcal{S}'$ are interference-free, $\mathcal{S} \succ_i \mathcal{S}' \iff \sum_{j\in S} b_{i,j} > \sum_{j'\in S'} b_{i,j'}$.
- If neither of $\mathcal{S}$ and $\mathcal{S}'$ is interference-free, the seller randomly decides the preference relation between $\mathcal{S}$ and $\mathcal{S}'$.

The preference profile $\succ_j$ of buyer j is a complete, reflexive, and transitive binary relation on all sets of channels (sellers) $2^{\mathcal{M}}$. The preference profile on all sets of channels instead of individual channels are quite expressive to cater to buyers' requirements. First, buyers are able to express their preference for certain bundles of channels. For example, buyer j may have $\{s_1, s_2\} \succ_j \{s_1, s_3\}$, which means that she prefers the continuous channels $\{s_1, s_2\}$ to non-continuous channels $\{s_1, s_3\}$. Second, buyers can easily comply with their budget constraints by an preferring empty set to large (therefore expensive) channel bundles. For example, buyer j may have $\emptyset \succ_j \{s_1, s_2, s_3\}$ since the aggregate basic offer price of $\{s_1, s_2, s_3\}$ exceeds her budget. Our proposed many-to-many spectrum matching algorithm works with general preference profiles without any restrictions.

III. Spectrum Matching

A. Preliminaries

We formally define many-to-many spectrum matching as follows.

Definition 1. *(Many-to-Many Spectrum Matching). Given the set of sellers $\mathcal{M}$ and the set of buyers $\mathcal{N}$, a many-to-many spectrum matching is a mapping μ from the set $\mathcal{M} \bigcup \mathcal{N}$ into the set of all subsets of $\mathcal{M} \bigcup \mathcal{N}$ (i.e., $2^{\mathcal{M} \bigcup \mathcal{N}}$), such that*

- *For every seller $i \in \mathcal{M}, \mu(i) \subseteq 2^{\mathcal{N}}$;*
- *For every buyer $j \in \mathcal{N}, \mu(j) \subseteq 2^{\mathcal{M}}$;*
- *For every seller i and buyer j, $j \subseteq \mu(i)$ if and only if $i \subseteq \mu(j)$.*

We also define the pre-matching, which will be the intermediate result in our proposed matching algorithm.

Definition 2. *(Pre-matching). A pre-matching is a pair $\nu = (\nu_m, \nu_n)$, in which ν_m is a mapping from the seller set $\mathcal{M}$ into all subsets of buyers $2^{\mathcal{N}}$, and ν_n is a mapping from the buyer set $\mathcal{N}$ into all subsets of seller $2^{\mathcal{M}}$, that is,*

- *For every seller $i \in \mathcal{M}, \nu_m(i) \in 2^{\mathcal{N}}$;*
- *For every buyer $j \in \mathcal{N}, \nu_n(j) \in 2^{\mathcal{M}}$.*

Note that a pre-matching is a matching if ν is such that $\nu_m(i) = j$ if and only if $\nu_n(j) = i$ for all $i \in \mathcal{M}, j \in \mathcal{N}$.

Given a buyer j and a set of sellers $\mathcal{S}$, let $Ch(\mathcal{S}, \succ_j)$ denote buyers j's most preferred subset of $\mathcal{S}$ according to j's preference relation $\succ_j$. More specifically, $Ch(\mathcal{S}, \succ_j)$ is the unique subset $\mathcal{S}'$ of $\mathcal{S}$ such that $\mathcal{S}' \succ_j \mathcal{S}''$ for all $\mathcal{S}'' \subseteq \mathcal{S}, \mathcal{S}'' \neq \mathcal{S}'$. Similarly ,given seller i and a set of buyers $\mathcal{S}$, $Ch(S, \succ_i)$ is seller i's most preferred subset of $\mathcal{S}$ according to i's preference relation $\succ_i$.

B. Matching Algorithm

We propose a many-to-many matching algorithm to realize a stable and interference-free spectrum matching, as shown in Algorithm 1. The key component in the matching algorithm is the $T(\cdot)$ operation on the pre-matching ν. We iteratively perform the $T(\cdot)$ operation until we reached a fixed point ($T(\nu) = \nu$) or the number iterations reached the threshold. To define the $T(\cdot)$, we first introduce two sets. Given a pre-matching $\nu = (\nu_m, \nu_n)$, we have:

$$\begin{aligned} U(i,\nu) &= \{j \in \mathcal{N} : i \in Ch(\nu_n(j) \cup \{i\}, \succ_j)\} \\ V(j,\nu) &= \{i \in \mathcal{M} : j \in Ch(\nu_m(i) \cup \{j\}, \succ_i)\} \end{aligned} \tag{1}$$

The set $U(i,\nu)$ is the set of buyers who are willing to obtain seller i's channel, given their currently matched channels. Similarly, the set $V(j,\nu)$ is the set of sellers who are willing to sell their channels to buyer j, given their currently matched buyers (and their interference relationships).

Now, we can define the $T(\cdot)$ operation as:

$$T(\nu) = \begin{cases} Ch(U(i,\nu), \succ_i), \forall i \in \mathcal{M} \\ Ch(V(j,\nu), \succ_j), \forall j \in \mathcal{N} \end{cases} \tag{2}$$

The purpose of the $T(\cdot)$ operation on seller i is to find the optimal set of buyers among those, who are willing to purchase seller i's channel. Likewise, the purpose the $T(\cdot)$ operation on buyer j is to find the optimal set of sellers (channels) among those, who are willing to sell their channels to buyer j.

Though similar to the definition in [9], the $T(\cdot)$ operation is quite different for spectrum matching. The main reason is that spectrum matching exhibits buyer externality: the matching result of a buyer will affect those of other buyers. More specifically, if a buyer is matched to a seller, her interfering neighbors will be unwilling to be matched to the same seller, even if they have a high preference for that seller's channel. For the same reason, when choosing the optimal set of buyers, a seller not only consider their basic offer price, but also their interference relationship.

In (2), the operation $Ch(U(i,\nu), \succ_i)$ requires seller i to find the optimal set of buyers with the highest total basic offer price, given the set $U(i,\nu)$. This is equivalent to finding the maximum weighted independent set (MWIS) on the interference graph G^i regarding buyers in $U(i,\nu)$. However, it has been proved that the MWIS problem is NP-hard. A naive brute-force solution is to exhaustively search all possible subsets of $U(i,\nu)$, resulting in exponential running time. To address this problem, we adopt the greedy algorithm in [11]. The key idea of the greedy algorithm is to select the buyer with the maximum price/degree ratio, remove her and all her neighbors, and repeat this process until the graph becomes empty. The selected buyers at all iterations is the output independent set.

It is proved in [9] that the fixed point $\nu = T(\nu)$ is a matching, that is, $\nu_m(i) = j$ if and only if $\nu_n(j) = i$ for all $i \in \mathcal{M}$ and $j \in \mathcal{N}$. However, due to the complexity of spectrum matching, it is analytically difficult to prove that this property can still be preserved. Our simulation results in Section IV numerically show that the output of Algorithm 1 is indeed a matching, the properties of which we will analyze in Section III-C. We will work on theoretically proving this in our future work.

In Algorithm 1, the input pre-matching ν is not specified. The simplest way is to initiate the pre-matching as $\nu_m(i) = \emptyset, \nu_n(j) = \emptyset, \forall i \in \mathcal{M}, j \in \mathcal{N}$. An alternative way is to randomly assign one buyer to one channel at the start (assuming there are more buyers than sellers), i.e., $\nu_m(i) = j, \forall i \in \mathcal{M}; \nu_n(j) = i, \text{if } \exists i, \nu_m(i) = j, \text{otherwise}, \nu_n(j) = \emptyset$. Now we give a toy example to show how the proposed algorithm works.

Algorithm 1 Many-to-many Spectrum Matching

Input: Preference relation $\succ$; interference graphs $\{G^i\}_{i=1}^M$; interference-free pre-matching ν_0; iteration threshold Num.
Output: An interference-free (pre-)matching ν

```
iteration = 0.
ν = T(ν_0).
while ν ≠ ν_0 or iteration ≤ Num do
    ν_0 = ν.
    ν = T(ν_0).
    iteration = iteration + 1.
end while
```

Toy example. Suppose the set of sellers is $\mathcal{M} = \{i_1, i_2, i_3\}$ and the set of buyers is $\mathcal{N} = \{j_1, j_2, j_3, j_4\}$. The basic offer of buyer j_1 is $(1,3,5)$, of buyer j_2 is $(5,1,3)$, of buyer j_3 is $(1,5,3)$, of buyer j_4 is $(3,5,1)$. Therefore, we can construct the buyers' preference profiles as:

$$\begin{aligned} &\succ_{j_1}: \{i_2,i_3\} \succ_{j_1} \{i_1,i_3\} \succ_{j_1} \{i_1,i_2\} \succ_{j_1} i_3 \succ_{j_1} i_2 \succ_{j_1} i_1 \\ &\succ_{j_2}: \{i_1,i_3\} \succ_{j_2} \{i_1,i_2\} \succ_{j_2} \{i_2,i_3\} \succ_{j_2} i_1 \succ_{j_2} i_3 \succ_{j_2} i_2 \\ &\succ_{j_3}: \{i_2,i_3\} \succ_{j_3} \{i_1,i_2\} \succ_{j_3} \{i_1,i_3\} \succ_{j_3} i_2 \succ_{j_3} i_3 \succ_{j_3} i_1 \\ &\succ_{j_4}: \{i_1,i_2\} \succ_{j_4} \{i_2,i_3\} \succ_{j_4} \{i_1,i_3\} \succ_{j_4} i_2 \succ_{j_4} i_1 \succ_{j_4} i_3 \end{aligned} \tag{3}$$

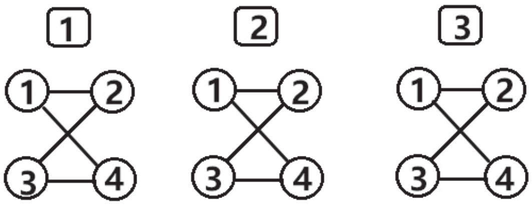
We assume that no buyer can afford the set $\{i_1, i_2, i_3\}$, so we omit this set, which is less preferred than the empty set due to budget constraint. The buyers' interference graph is shown in Fig. 1,so we can construct the sellers' preference profiles are:

$$\begin{aligned} &\succ_{i_1}: \{j_2,j_4\} \succ_{i_1} j_2 \succ_{i_1} j_4 \succ_{i_1} \{j_1,j_3\} \succ_{i_1} j_1 \succ_{i_1} j_3 \\ &\succ_{i_2}: \{j_1,j_3\} \succ_{i_2} \{j_2,j_4\} \succ_{i_2} j_3 \succ_{i_2} j_4 \succ_{i_2} j_1 \succ_{i_2} j_2 \\ &\succ_{i_3}: \{j_1,j_3\} \succ_{i_3} j_1 \succ_{i_3} \{j_2,j_4\} \succ_{i_3} j_2 \succ_{i_3} j_3 \succ_{i_3} j_4 \end{aligned} \tag{4}$$

We assume that the pre-matching is $\nu_m(i) = \nu_n(j) = \emptyset, \forall i, j$. In the first iteration, for seller i_1, $U(i_1, \nu) = \mathcal{N}$, and it can be easily found that the optimal buyer set is $\nu_m(i_1) = \{j_2, j_4\}$; for buyer j_1, $V(j_1, \nu) = \mathcal{M}$, so that after the $T(\cdot)$ operation, we have $\nu_n(j_1) = \{i_2, i_3\}$. The results after the first iteration is shown in the third row of Table I. Similarly, we can proceed through the second iteration, whose result can be checked as a fixed point and a matching.

TABLE I: A Toy Example

	$\nu_m(i_1)$	$\nu_m(i_2)$	$\nu_m(i_3)$	$\nu_n(j_1)$	$\nu_n(j_2)$	$\nu_n(j_3)$	$\nu_n(j_4)$
0	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
1	$\{j_2,j_4\}$	$\{j_1,j_3\}$	$\{j_1,j_3\}$	$\{i_2,i_3\}$	$\{i_1,i_3\}$	$\{i_2,i_3\}$	$\{i_1,i_2\}$
2	$\{j_2,j_4\}$	$\{j_1,j_3\}$	$\{j_1,j_3\}$	$\{i_2,i_3\}$	i_1	$\{i_2,i_3\}$	i_1
3	$\{j_2,j_4\}$	$\{j_1,j_3\}$	$\{j_1,j_3\}$	$\{i_2,i_3\}$	i_1	$\{i_2,i_3\}$	i_1



The interference graph.

Fig. 1: A Toy example.

C. Properties

In this section, we first prove that the matching result of the proposed Algorithm 1 is individual rational in general cases. Then, we show that with specific preference profiles, the matching result is strong pairwise stable, and is a subgame-perfect Nash equilibrium of the corresponding spectrum bargaining game.

Definition 3. *(Individual rational).*

A matching result is blocked by seller i if she prefers not to be matched to some of her currently matched buyers. In other words, $\exists S \subseteq \mu(i)$, $\big(\mu(i) \setminus S\big) \succ_i \mu(i)$.

A matching result is blocked by buyer j if she prefers not to be matched to some of her currently matched sellers. In other words, $\exists S \subseteq \mu(j)$, $\big(\mu(j) \setminus S\big) \succ_j \mu(j)$.

A matching result is individual rational if it is not blocked by any buyer or seller.

Proposition 1. *The matching result of the proposed Algorithm 1 is individual rational.*

Proof. let μ be the matching result. μ is a fixed point of the $T(\cdot)$ operation. So for any buyers $j \in \mathcal{N}$:

$$\begin{aligned} Ch(\mu_n(j), \succ_j) &= Ch(U(Ch(U(j,\mu), \succ_j)), \succ_j) \\ &= Ch(U(j,\mu), \succ_j) = \mu_n(j) \end{aligned} \tag{5}$$

$Ch(\mu_n(j), \succ_j) = \mu_n(j)$ indicates that there is no *block* for buyers. The proof for sellers is similar. Therefore, the matching result of the proposed Algorithm 1 is individual rational. □

Now, we consider a special constraint, *substitutability*, on buyers' preference profiles. First introduced in [8], substitutability is widely studied in the matching literature.

Definition 4. *(Substitutability). A buyer j's preference profile $\succ_j$ satisfies substitutability, if for any seller set S and $S' \subseteq S$, $i \in Ch(S \cup i, \succ_j) \Rightarrow i \in Ch(S' \cup i, \succ_j)$.*

An interpretation of substitutability is that, if buyer j wants to have channel i among an available channel set $S \cup i$, then she still wants channel i among a smaller channel set $S' \cup i$. Substitutable preference profile can be easily satisfied, as shown by the following example. There are three sellers/channels $\{i_1, i_2, i_3\}$, buyer j's basic offer is $(b_{i_i,j}, b_{i_2,j}, b_{i_3,j}) = (1, 2, 3)$. For any two-channel bundles of i and i', buyer j is willing to pay $1.1 * (b_{i,j} + b_{i',j})$. The three-channel bundle exceeds buyer j's budget constraint. Following this logic, buyer j's preference profile is:

$$\begin{aligned} &\{i_2, i_3\} \succ_j \{i_1, i_3\} \succ_j \{i_1, i_2\} \succ_j i_3 \\ &\quad \succ_j i_2 \succ_j i_1 \succ_j \emptyset \succ_j \{i_1, i_2, i_3\} \end{aligned} \tag{6}$$

It can be easily checked that this preference profile is substitutable. When all buyers' preference profiles satisfy substitutability, the matching results of the proposed Algorithm 1 is strong pairwise stable and is a subgame-perfect nash equilibrium of the corresponding spectrum bargaining game.

Definition 5. *(Strong pairwise stability).*

A matching result is blocked by a pair $(S, j) \in 2^{\mathcal{M}} \times \mathcal{N}$ in which $S \neq \emptyset$, if $S \cap \mu(j) = \emptyset, S \subseteq Ch(\mu(j) \cup S, \succ_j)$, and $j \in Ch(\mu(i) \cup j, \succ_i)$ for all $i \in S$.

A matching result is strong pairwise stable if it is not blocked by any pair of buyer and seller and individual rational.

Proposition 2. *The matching result of the proposed Algorithm 1 is strong pairwise stable.*

Proof. Assume there is a block pair (S, j) and $S \neq \emptyset$. Since $j \in Ch(\mu(i) \cup j, \succ_i), \forall i \in S$, by definition of $V(j,\mu)$, we have $S \subseteq V(j,\mu)$. Thus, for any subset $A \subseteq \mu(j)$, as $\mu(j) \subseteq V(j,\mu)$, we have $A \cup S \subseteq V(j,\mu)$. Due to individual rationality, $\mu(j) = Ch(V(j,\mu), \succ_j)$. So $\mu(j) \succeq_j Ch(A \cup S, \succ_j) \succeq_j A \cup S$, in which $\succeq_j$ means that the two sets may be the same. This contradicts the fact that $S \cap \mu(j) = \emptyset$ and $S \subseteq Ch(\mu(j) \cup S, \succ_j)$. Therefore, there can not be any blocks in the matching result μ. □

Now we regard the spectrum matching as a non-cooperative bargaining game. To begin with, every buyer j proposes a set of channels $\eta_j \subseteq \mathcal{M}$. After observing the proposals ,every seller i proposes a set of buyers $\xi_i \subseteq \mathcal{N}$. All buyers or sellers make the proposals simultaneously. A buyer and a seller will be matched if seller i proposes to buyer j and buyer j proposes to seller i. The strategy space for buyer j is $\eta_j \subseteq \mathcal{M}$, for seller i is $\xi_i(\eta) \subseteq \mathcal{N}$. Now we define *Subgame-Perfect Nash Equilibrium*(SPNE) for such a bargaining game.

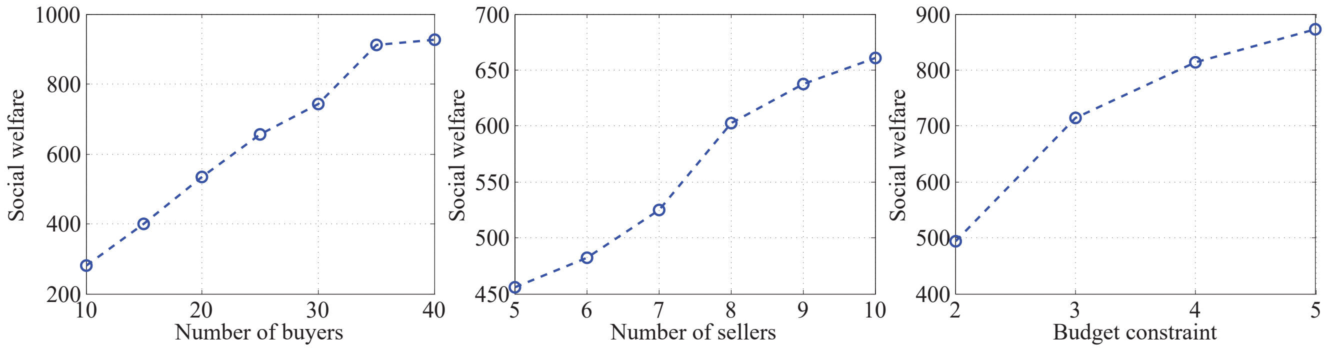
Definition 6. *(Subgame-Perfect Nash Equilibrium). Given the preference profiles $\succ$ of buyers and sellers, a strategy profile (η^*, ξ^*) is called a Subgame-Perfect Nash Equilibrium (SPNE), if $\forall i \in \mathcal{M}, j \in \mathcal{N}$,*

$$\begin{aligned} &\eta_j^* \cap \{i : j \in \xi_i^*(\eta^*)\} \succeq_i P \cap \{i : j \in \xi_i^*(P, \eta_{-j}^*)\}, \forall P \subseteq \mathcal{M} \\ &\xi_i^*(\eta) \cap \{j : i \in \eta_j^*\} \succeq_j S, \forall S \subseteq \{j : i \in \eta_j^*\} \end{aligned} \tag{7}$$

In other words, (η^*, ξ^*) is SPNE if η_j^* is optimal given other buyers' proposal η_{-j}^*, and $\xi_i^*(\eta^*)$ is an optimal proposal given all buyers' proposal η^*.

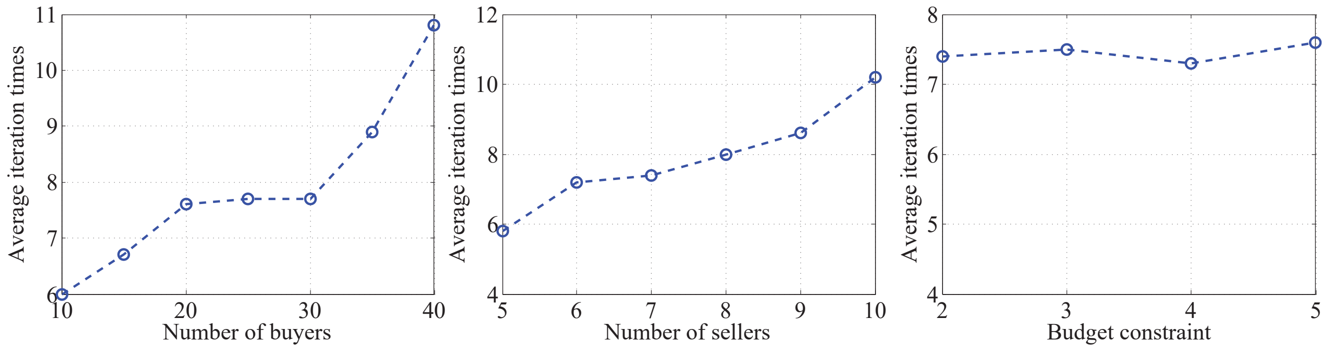
Proposition 3. *The matching result of the proposed Algorithm 1 is an SPNE of the spectrum bargaining game.*

Proof. Let μ be the matching result of the proposed Algorithm 1. Define (η^*, ξ^*) as $\eta_j^* = \mu(j)$ and $\xi_i^*(\eta^*) = Ch(\{j : i \in \eta_j^*\}, \succ_i)$. Let μ' be the outcome of the strategy (η^*, ξ^*) .Now we show that (η^*, ξ^*) is an SPNE and $\mu' = \mu$.



(a) (b) (c)

Fig. 2: Social welfare of the proposed many-to-many matching algorithm. (a) $M = 10$; (b) $N = 25$; (c) $M = 8, N = 30$.



(a) (b) (c)

Fig. 3: Iteration times of the proposed many-to-many matching algorithm. (a) $M = 10$; (b) $N = 25$; (c) $M = 8, N = 30$.

For any i and j, $\{j : i \in \eta_j^*\} \cup j = \mu(i) \cup j$, therefore,

$$\begin{aligned} &\{i : j \in Ch(\{j : i \in \eta_j^*\} \cup j, \succ_i)\} \\ &= \{i : j \in Ch(\mu(i) \cup j), \succ_i\} = U(i, \mu). \end{aligned} \tag{8}$$

So we have $\eta_j^* = \mu(j) = Ch(U(j, \mu), \succ_j)$, which means that η_j^* is optimal given η_{-j}^*. By definitions we know that $\xi_i^*(\eta')$ is optimal, given η^*. Thus (η^*, ξ^*) is an SPNE.

Since μ is a matching, we have $j \in \mu(i)$ if and only if $i \in \mu(j) = \eta_j^*$, so that $\{j : i \in \eta_j^*\} = \mu(i)$. According to individual rationality, we have $\xi_i^*(\eta^*) = Ch(\{j : i \in \eta_j^*\}, \succ_i) = Ch(\mu(i), \succ_i) = \mu(i)$. Therefore, $i \in \mu'(j)$ if and only if $i \in \eta_j^* = \mu(j)$, and $j \in \mu'(i)$ if and only if $j \in \xi_i^* = \mu(i)$. This proves that $\mu' = \mu$.

□

IV. Simulation

A. Simulation Settings

We assume that buyers are located in a 10×10 square. The transmission range of a channel is randomly chosen in the range $(0, 5]$. Based on buyers' locations and the transmission range of a channel, we can construct an interference graph of all channels. Preference profiles are generated as follows. We first assume that buyers' basic offer prices for individual channels are uniformly distributed in $(0, 10]$, based on which we can construct sellers' preference profiles according to Section II. Then, we assume that for a combination of channels S, buyer j is willing to pay $\alpha \sum_{i \in S} b_{i,j}$, in which $\alpha > 1$ is a gain factor. If the number of channels in the combination is higher, α is higher. But there is a threshold on the size of a combination, beyond which we assume that the buyer is not willing to purchase the combination due to budget constraints. Buyers' preference profiles are based on their willingness to pay for a combination of channels.

B. Performance of the Proposed Matching Algorithm

The influence of the number of buyers, the number of sellers and the budget contraint (the threshold on the size of channel combinations) on social welfare of the matching result is shown in Fig. 2. When the number of buyers increases, social welfare grows quickly at first, then slows down because more buyers compete for a limited number of channels, and the chance of obtaining channels becomes smaller. Social welfare also goes up with the number of sellers because more channels are available for the buyers to acquire. As buyers have higher budgets, social welfare improves, because buyers can attain more channels with higher budgets. We have checked that all fixed points of the simulation results are matching (instead of pre-matching), and the iteration times of the proposed many-to-many matching algorithm is shown in Fig. 3. We can see that the proposed algorithm can converge to the fixed-point within a few iterations. The iteration times are mainly affected by the number of buyers and the number of sellers. Changes in budget constraints do not have a significant impact on iteration times as the number of buyers and sellers stay the same.

V. Related Work

Matching-based resource allocation. In 1963, Gale and Shapley [7] first proposed the *deferred-acceptance* algorithm to reach stable matching results for marriage problem (one-to-one matching) and college admission problem (many-to-one matching). Existence of pairwise-stable matching in many-to-many matching has been studied by previous literature [12], [13]. In [9], the authors proposed a fixed-point based T-algorithm to realize strong pairwise-stable matching in many-to-many matching. The concept of fixed-point matching is also used in matching contexts by [14], [15] Matching has been widely applied in computer science. In [6], the authors proposed online and offline algorithms to match virtual machines and heterogeneous sized jobs in the cloud. In [16], the authors proposed to match Device-to-Device users to cellular users for resource sharing. In [17], the authors proposed to match secondary users to primary users for data relay. However, none of these matching framework can be applied to spectrum matching, which features interference constraint based spatial reuse.

Combinatorial auction. There are many previous works on combinatorial auction. Due to the intractibility of combinatorial auctions, a number of greedy algorithms have been proposed with bounded approximation ratio [18]–[20]. Combinatorial spectrum auction is studied in [21], [22]. However, the combinatorial auction has the disadvantage of intractibility and collusion, which can be addressed by the spectrum matching framework.

VI. Conclusion

In this paper, we propose the first many-to-many matching framework for combinatorial spectrum trading. Compared with combinatorial auction which is usually NP-hard, spectrum matching is easier to implement and is immune to collusion between the auctioneer and buyers or sellers. In many-to-many spectrum matching, buyers are given the freedom to express their preference for different combinations of channels. Spatial reuse is considered based on heterogeneous interference graphs, making spectrum matching different from all other traditional matching problems. We propose a matching algorithm, which generates an interference-free matching result. We theoretically prove that the matching result is individual rational, strong pairwise stable and is a subgame-perfect Nash equilibrium of the corresponding spectrum bargaining game. We conducted extensive simulations to evaluate the performance of the proposed many-to-many matching framework. It is shown that social welfare increases with the number of buyers or sellers, as well as buyers' budget constraints.

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