Cooperative Multicast Scheduling with Random Network Coding in WiMAX

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***Abstract*—The Multicast and Broadcast Service (MBS) in WiMAX has emerged as the next-generation wireless infrastructure to broadcast data or digital video. Multicast scheduling protocols play a critical role in achieving efficient multicast transmissions in MBS. However, the current state-of-the-art protocols, based on the shared-channel single-hop transmission model, do not exploit any potential advantages provided by the channel and cooperative diversity in multicast sessions, even while WiMAX OFDMA provides such convenience. The inefficient multicast transmission leads to the under-utilization of scarce wireless bandwidth.**

In this paper, we revisit the multicast scheduling problem, but with a new perspective in the specific case of MBS in WiMAX, considering the use of multiple ODFMA channels, multiple hops, and multiple paths simultaneously. Participating users in the multicast session are dynamically enabled as relays and concurrently communicate with others to supply more data. During the transmission, random network coding is adopted, which helps to significantly reduce the overhead. We design practical scheduling protocols by jointly studying the problems of channel and power allocation on relays, which are very critical for efficient cooperative communication. Protocols that are theoretically and practically feasible are provided to optimize multicast rates and to efficiently allocate resources in the network. Finally, with simulation studies, we evaluate our proposed protocols to highlight the effectiveness of cooperative communication and random network coding in multicast scheduling with respect to improving performance.

I. Introduction

The Multicast and Broadcast Service (MBS) in multi-channel wireless networks (*e.g.* IEEE 802.16 WiMAX [1]) has emerged as the next-generation wireless infrastructure to broadcast data or digital video. With the current mandate of MBS, the Base Station broadcasts or multicasts data in the downlink using robust modulation and coding schemes to provide reliable transmissions for all the users, as individual feedback (such as ARQ and HARQ) is not supported in MBS. However, such a dependence on using the most robust modulation and coding schemes to counter the most adverse channel quality among all users leads to the under-utilization of scarce wireless bandwidth: users with good channel conditions would not enjoy flow rates that are commensurate with their conditions, as the "least common denominator" is used to cater to users with poor channel conditions.

How to properly select a multicast rate? Multicast scheduling protocols play a critical role in achieving efficient multicast transmissions in multi-channel wireless networks. The main difficulty of multicast scheduling is caused by the diverse channel conditions of users in the multicast session. Research attention in most previous work [2], [3] is mainly focused on alleviating such a negative impact by formulating the multicast in a single-hop shared-channel communication model, and maximizing the total throughput in multicast users.

For a given multicast session, different downlink users experience different channel conditions, and the same channel experiences different gains on different transmission links, especially when user mobility is considered. This diversity may become a positive factor in multicast sessions, if we exploit its potential advantages by allowing users to cooperatively contribute to each other as relays. Such *cooperative communication* has been shown to improve throughput of multiple unicast sessions by simultaneously exploring the broadcast nature of a shared wireless channel and the cooperation among multiple users [4], but not fully explored and employed in multicast scheduling yet, especially in WiMAX MBS.

The adoption of Orthogonal Frequency Division Multiple Access (OFDMA) in WiMAX makes the use of multiple orthogonal sub-channels realistic, which allows for the additional convenience of supporting concurrent transmissions via different sub-channels without interference. However, multicast protocols that are currently proposed in MBS are primitive in nature, as they fail to embrace this unique advantages of WiMAX, and to take advantage of both channel and cooperative diversity to improve the multicast performance.

In this paper, we revisit the traditional multicast scheduling problem, but with a new perspective of considering multiple hops, multiple paths, and multiple channels at the same time, rather than the system models with a single shared channel. We seek to design protocols to dynamically assign multicast users as relays, and ask them to cooperatively transmit data to other peers. The basic idea, explained intuitively, is that users with good channel conditions can forward the received data to the remaining users who need help. In this case, the Base Station may use a much higher rate to multicast data to all users, leading to more efficient use of bandwidth.

The bad news, however, is that it is challenging to schedule transmissions in a cooperative fashion. Relays do not have sufficient knowledge on which packets their neighbors need. Blindly "pushing" packets that are not needed to other peers will incur a substantial degree of overhead. To address this challenge, we propose to take advantage of the favorable rate-

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less properties of *random network coding*, which has emerged as one of the most promising information theoretic approaches to improve throughput, especially in wireless networks [5]. With random network coding, all packets are encoded with random linear codes, and all coded data blocks could be considered equally useful and innovative. With the data fully mixed, relays can freely "push" innovative blocks to their downlink multicast members. Without dictating which packet is from which source, receivers only need to "hold" a "bucket" and collect a sufficient amount of data from their upstream nodes. With the help of random network coding, the overhead can be substantially mitigated in cooperative communication. The Base Station only needs to multicast coded blocks in a rateless fashion, until all users are able to reconstruct the original data by receiving a sufficient number of linearly independent coded blocks, regardless of their channel conditions.

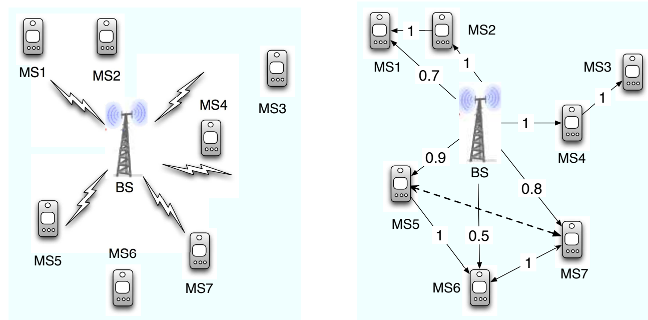
The salient highlight of our contributions in this paper is a multicast scheduling framework that exploits potential benefits made possible by multiple orthogonal sub-channels, cooperative communication, and random network coding, all in the realistic context of MBS in WiMAX. Such a system model has not been previously considered in the literature to our knowledge. The framework is formulated as a set of optimization problems, by jointly considering relay assignment, channel allocation and power control, which are very critical for efficient cooperative communication. Both theoretical and practical solutions are provided, and then evaluated in extensive simulations. Corroborating our intuition, our protocols are able to improve multicast throughput substantially.

The remainder of the paper is organized as follows. In Sec. II, we intuitively show the advantages of our multicast scheduling protocols with illustrative examples. In Sec. III, we review related work on multicast scheduling in wireless networks. From Sec. IV to Sec. VI, we present the design of our multicast scheduling protocols and conduct extensive simulation for evaluation. In Sec. VII, we analyze the overhead for our proposed protocols. Finally, we conclude our paper in Sec. VIII.

II. Illustrative Examples

We now use illustrative examples in WiMAX networks to show the potential benefits made possible by applying multiple channels, cooperative communication and random network coding in multicast scheduling. In the network, the Base Station (BS) multicasts data and Mobile Stations (MSs) collect the data in the downlink. According to the conventional multicast scheduling as shown in Fig. 1(a), the BS has to multicast data using robust modulation and coding schemes to ensure the reliable transmissions to all MSs. In the example, we assume the multicast rate is 5 packets per second. Thus, the total throughput at MS1 and MS2 is 10 packets per second. However, this reliability under-utilizes the wireless bandwidth, as the MSs in good channel conditions (MS2 in this example) get data in a conservative low rate.

To effectively use the wireless spectrum, the BS may use higher rate for multicasting. As shown in Fig. 1(b), the BS



(a) Conventional multicast scheme (b) Cooperative multicast scheme

Fig. 1. Illustrative examples to show the advantages of cooperative multicast scheduling with random network coding in WiMAX. The number on each link in (b) indicates the packet delivery rate from the BS to the MS.

multicasts data with 10 packets per second. Under this rate, MS2 still reliably receives all the packets due to its good channel condition, while MS1 only receives 70% as it is farther from the BS. Taking advantage of *relays* which are enabled in WiMAX, we ask MS2 who is close to MS1 to cooperatively transmit data to it through a separate sub-channel, aiming to compensate its loss. Via different paths, MS1 receives data simultaneously from both BS and MS2 and is able to collect 10 packets per second. Benefited from this cooperative communication, the total throughput on MS1 and MS2 **dramatically increases** to 20 packets per second. To get more gains, the BS even can use higher multicast rate. Although none of MSs is able to correctly receive all the data reliably (In the example, MS5, MS6 and MS7 get data directly from BS by 90%, 50% and 80% respectively), they could contribute to each other to achieve reliable transmissions with higher throughput, as shown in Fig. 1(b).

To fully exploit the potential benefits, we apply random network coding on the transmissions. All the enclosed packets are encoded and issued by the BS and equally innovative if they are linearly independent. The MS is able to produce new coded packets by encoding all the innovative ones it correctly receives and push them to the downlink MSs. With the help of random network coding, the overhead can be substantially reduced. Moreover, the transmission reliability can be maintained as the transmissions are performed in a rateless fashion until all users correctly receive sufficient number of coded packets.

The design objective of multicast scheduling with cooperative communication and random network coding in WiMAX is to realize all the potential benefits described in these intuitive examples. To achieve such an objective, there are a number of practical **challenges**:

- ▷ How to dynamically assign multicast users as relays and apply random network coding in cooperative communication to tightly fit in the design of WiMAX MBS?
- ▷ How to optimally allocate sub-channels for cooperative communication to obtain maximum benefits on multicast performance even with limited amount of bandwidth?
- ▷ How to efficiently allocate power for cooperative communication when the energy on relays is highly constrained?

Our responses to these challenges constitute the flow of presentation in this paper.

III. Related Work

Multicast scheduling has been extensively studied in the literature. The CDMA2000 1xEV-DO networks [6] adopt a simple multicast scheduling scheme that takes the most robust modulation and coding scheme to transmit data. As we have previously elaborated, such a scheme under-utilizes wireless resources. As a potential remedial solution, in [2], multicast members are divided into two groups with different levels of channel qualities. The sender transmits the same copy of each packet to two groups in two different time slots using different rates which best fit the channel quality in each group. It has been shown to improve the throughput performance. However, it is too conservative, especially when the number of users in poor channel conditions is very small. The sender still has to consume more time for multicasting the data to them. In [3], Kozat has investigated the optimal multicast rate by focusing each transmission onto a proper subset of multicast users, rather than trying to serve all the users at each channel use. It still works on the single-hop shared-channel scenario, and does not exploit the cooperative diversity in the broadcasting channels. In our simulations, we will evaluate it against our protocol with cooperative communication to show some of the advantages of our protocol in WiMAX networks.

In [7], Hou *et al.* attempted to utilize relays to help the users with poor channel conditions, and the proposed protocol is based on a two-phrase scheduling. It still suffers the same problems in [2] and does not exploit the channel and cooperative diversity in multicast channels. Our work differs from it in a number of important aspects. First, our proposed protocols rely on concurrent cooperative transmissions among multicast users via orthogonal OFDMA sub-channels and hence work in a substantially different system. Second, we propose to apply random network coding to effectively perform cooperative communication. Third, we design our protocols by solving optimization problems formulated to maximize the throughput performance. Finally, we specifically study the resource allocation problems in cooperative multicast scheduling, which are critical in practical systems.

IV. Multicast Scheduling with Cooperative Communication and Random Network Coding

We concentrate on the multicast scheduling in the time-slotted WiMAX MBS, where the Base Station (BS) serves as the multicast sender and keeps on broadcasting a big file, and the Mobile Stations (MSs) (also referred to as nodes) are the participating users in multicast sessions. Throughout the paper, we assume quasi-stationary channel conditions: any node's channel condition remains the same during a given time slot, and it varies independently from one time slot to another. The channel quality information on each link can be effectively estimated [8] and fully captured by the BS through Channel State Information (CSI) messages exchanged between the BS and each MS periodically in WiMAX [1]. The objective is to find the optimal multicast rate, as well as the most efficient cooperative communication schedule, to maximize the aggregate throughput on all users under a proportional fairness criteria which is able to strike a good balance between utilization and fairness and its robustness with respect to changes in topology and power constraints [9]. We perform scheduling at each time slot, thus the overall performance will be optimized in the long term [9].

A. Optimizing Multicast Scheduling

The objective of the multicast scheduling can be stated as,

$$\max_{R(t)} \quad \sum_{i\in\zeta} \frac{U_i(t)}{\overline{r}_i(t)} \tag{1}$$

where $R(t)$ denotes the multicast rate at time slot t. If modulation and coding scheme m (index) is used, $R(t) = R_m(t)$, where $m \in \{1, 2, \cdots, 6\}$, as there are mainly six schemes according to IEEE 802.16 standard [1]. $\overline{r}_i(t)$ denotes the average throughput at node i over time horizon $[1, t]$, which is kept track at each node and reported to the BS. ζ is the set of nodes in MBS, and the total number is G.

$U_i(t)$ is the throughput on node i at time slot t in Eq. (1), taking account for the transmissions both from BS directly, and from cooperative communication. At the starting point, we assume there is a channel pool with sufficient number of sub-channels, and each link can be assigned one sub-channel for cooperative communication if there exists such opportunities. We will study more complicated and realistic cases on channel and power allocation for cooperative communication in the following sections. All nodes work in the full-duplex mode and are equipped with multiple radios which support concurrent communication with multiple nodes in both downlink and uplink via separate sub-channels. Random network coding is applied in the transmissions, with which all the packets are considered to be fully random and linearly independent with high probability. Thus, we calculate $U_i(t)$ by,

$$U_i(t) = S_{m,i}(t)R_m(t) + \sum_{g\in\zeta} R_{gi}(t) \tag{2}$$

$S_{m,i}(t) \in [0, 1]$ is the packet deliver rate from the BS to node i when modulation and coding scheme m is used at time slot t. Exact closed-form packet deliver rate under coded modulations is not available, and we calculate it by using an accurate approximation for packet error rate in [10]. Note we specifically denote the multicast rate at time slot t as $R_m(t)$ ($m \in \{1, \cdots, 6\}$) to indicate that $S_{m,i}(t)$ depends on the multicast rate selection. $R_{gi}(t)$ ($\forall g, i \in \zeta$) is the maximum transmission rate that can be achieved on the link from node g to node i under certain channel conditions. It is subject to the following constraints:

$$0 \le R_{gi}(t) \le C_{gi}(t) \tag{3}$$

$$R_{gi}(t) \le \max\{0, \frac{B_g(t) - B_i(t)}{T}\} \tag{4}$$

(3) shows that the cooperative transmission rate is bounded by the capacity on the link (denoted as $C_{gi}(t)$). At the same

time, this rate is limited by the amount of innovative data that node g is able to contribute to node i. As random network coding is employed, a packet is innovative (or referred to as useful or new) if it is linearly independent from the other packets from the same segment. Checking for independence can be done using simple Gaussian Elimination. As we assume the packets are fully random and linearly independent with high probability, we can use (4) to describes this constraint, where $B_g(t)$ denotes the amount of innovative data buffered at node g at time slot t, and $B_i(t)$ indicates the same information at node i. T is the duration of one time slot. It is easy to get from this constraint: $R_{gi}(t) = 0, \text{if } g = i$.

Now we can see from Eq. (2) that $S_{m,i}(t)R_m(t)$ represents the throughput from BS, and $\sum_{g\in\zeta} R_{gi}(t)$ describes the cooperative throughput. The total throughput $U_i(t)$ is also constrained as the total data that each node receives can not exceed the amount the BS is able to provide,

$$U_i(t) \leq \sum_{h=1}^{t} R(h) - \frac{B_i(t)}{T} \quad \Rightarrow$$

$$\sum_{g\in\zeta} R_{gi}(t) \leq \sum_{h=1}^{t-1} R(h) - \frac{B_i(t)}{T} + (1 - S_{m,i}(t))R_m(t) \quad (5)$$

Overall, the multicast scheduling can be formulated as the optimization problem with the objective of (1), subject to (2) - (5). As there are six modulation and coding schemes, we can solve it using exhaustive search for all six possible schemes to get the optimal solution with **constant** time complexity.

B. Protocol Design

We design the multicast scheduling protocol based on the optimization above and by applying random network coding in the transmission. The BS holds all the original data, and separates the data into segments. A data segment (also referred to as a *generation* or a *group* in the literature) is further divided into n *blocks* with a fixed size. We can easily compute the number of blocks in one segment if the segment size is pre-determined. The BS randomly chooses a set of coding coefficients c_{ji} ($i = 1, 2, \cdots, n$) in a given Galois field. A coded block y_j can then be produced as $y_j = \sum_{i=1}^{n} c_{ji} \cdot x_i$. Each coded block is a linear combination of all or a subset of the original data blocks. In this way, the sender is able to generate a virtually unlimited number of coded blocks y_j ($j = 1, 2, \cdots$) using different sets of coefficients, and any n of these coded blocks can be used to decode by inverting a matrix of coding coefficients. This is usually referred to as the *rateless* property.

The BS multicasts the coded blocks in a rateless fashion, using the rate determined by solving the optimization problem we formulated above at each time slot. When a node receives a packet (coded block), it checks whether it contains new information, and ignores non-innovative packets. When performing cooperative communication, the node produces new coded blocks by creating random linear combinations of the coded blocks it has correctly received from the same segment and transmits them to its neighbors (the nodes within the sender's transmission range). Note the recoded blocks are still the linear combination of the original data blocks.

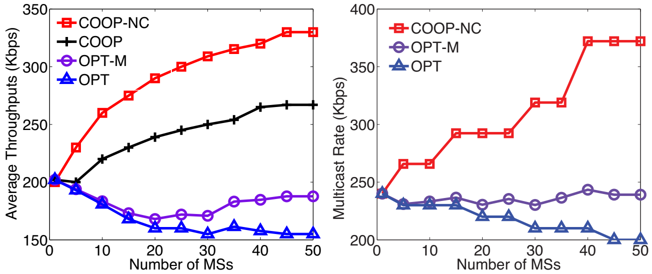
All the nodes collect the data and perform *progressive decoding* [11], with which the node is able to recover the entire original segment immediately after n coded blocks have been received for a segment, and sends the ACKnowledgement (ACK) back to the BS. When the BS receives the ACKs from all the nodes, it first multicasts a message to inform all nodes that the transmission for current segment is finished, and then starts to proceed the next segment. Upon receiving such message, all nodes flush the buffer and reset the time slot index $t = 0$, and also start the cooperative transmission for the next segment instead of the transmissions for the current segment.

C. Are Cooperative Communication and Random Network Coding Helpful?

We now resort to extensive simulations to evaluate the usefulness of cooperative communication and random network coding. To be realistic, the simulations are performed by emulating WiMAX MBS with typical parameters according to IEEE 802.16 standard [1] and WiMAX system evaluation methodology released by WiMAX forum. The evaluation is performed under the following scenario. The BS multicasts a large file to all MSs. To provide realistic time-varying channel conditions, each MS is allowed to move randomly in the service area of the BS, and its initial location is randomly chosen in the service region. We apply multi-path Rayleigh fading in the transmission, since the MS keeps on moving.

To evaluate the performance, we compare four multicast scheduling protocols: cooperative multicast scheduling with random network coding (denoted as "COOP-NC"), cooperative multicast scheduling *without* random network coding (denoted as "COOP"), optimal multicast scheduling (denoted as "OPT"), and optimal multicast scheduling with cooperative bandwidth (denoted as "OPT-M"). "COOP-NC" is performed under the design described in this section. "COOP" also follows this design, but without random network coding. MSs just randomly send the data in the buffer to their neighbors. "OPT" is the optimal scheduling protocol *without* applying cooperative communication and random network coding. We adopt the protocol in [3] and have simulated it to the best of our knowledge according to all the available details presented in the paper. "OPT-M" is also based on "OPT", but the BS uses more bandwidth by applying all the sub-channels assigned for cooperative communication in "COOP-NC" in multicasting. Thus, "COOP-NC" and "OPT-M" consume the same amount of bandwidth, with which the comparison is more fair.

Fig. 2(a) shows the performance on average throughput over time (1000-second simulation) of all protocols as a function of increasing number of MSs active in MBS. We observe from the results that "COOP-NC" performs best. Compared with "OPT", a 72% gain is achieved. For more fair comparison, "COOP-NC" shows its advantages by outperforming "OPT-M" with a margin of 58%. Such a throughput advantage should be considered substantial by any standard. It coincides



(a) Throughput vs. Number of MSs (b) Multicast rate vs. Number of MSs

Fig. 2. Throughput performance of four multicast scheduling protocols in a realistic WiMAX MBS scenario. Cooperative multicast scheduling with random network coding is able to achieve substantial throughput improvement by effectively utilizing the scarce wireless bandwidth.

with our intuition that *multicast scheduling with cooperative communication and random network coding naturally fits in the design of WiMAX MBS and is able to achieve significant throughput improvement due to its effective use of wireless spectrum*. Specifically, we examine the usefulness of random network coding. Evident from the results, "COOP-NC" outperforms "COOP" by 20% as random network coding effectively reduces the overhead. Another interesting result we get is the margin that "COOP-NC" and "COOP" outperform "OPT-M" and "OPT" becomes more substantial with increasing number of MSs. This observation indicates *more MSs create higher degree of cooperation which is able to benefit more on throughput performance*.

To further explore the advantages of cooperative communication and random network coding in multicast scheduling, we examine the performance on average multicast rate at the BS with the results shown in Fig. 2(b). When the number of MSs increases, the BS gradually uses higher multicast rates to transmit data when cooperative communication and random network coding are applied, which exactly shows the multicast bandwidth at the BS is more efficiently utilized. This result verifies and confirms — from a different aspect — the advantages of our protocol in WiMAX MBS.

V. Cooperative Multicast Scheduling with Channel Allocation

In practical systems like WiMAX, the OFDM channels are scarce resources and the number of channels to support cooperative communication is limited. Thus, it is very critical to efficiently allocate the channels for cooperative communication in the scheduling. Moreover, there are potential channel diversity gains in the networks, as sub-channel experiencing gain could vary from one link to another, allowing for the cooperate links to be assigned their best channels. In this section, we study the optimal multicast scheduling with constrained bandwidth resources, exploiting all the benefits provided by multi-user, multi-channel and cooperative diversity.

A. Optimizing Performance with Limited Bandwidth

Under limited resources, the scheduling turns to be a joint optimization problem (denoted as *COOP-CA-NC*), whose objective is to find not only the optimal multicast rate R_m but also efficient centralized channel allocation scheme to maximize overall throughput under the fairness criteria. To study it, we set a binary function $K_{gi}^{(n)} \in \{0,1\}$ to capture the assignment of sub-channel n to the cooperative transmission link from node g to node i, where $n \in \chi$ and χ denotes the set of sub-channels that are available for cooperative communication. The set of feasible assignments is denoted as K. To avoid interference in the cooperative communication, we set one sub-channel only can be assigned to one link,

$$\sum_{g\in\zeta}\sum_{i\in\zeta} K_{gi}^{(n)} \leq 1 \quad \forall n \in \chi \tag{6}$$

By considering the channel allocation, the throughput on each user (Eq. (2)) should be updated as follows,

$$U_i = S_{m,i}R_m + \sum_{g\in\zeta}\sum_{n\in\chi} K_{gi}^{(n)} R_{gi}^{(n)} \tag{7}$$

where $R_{gi}^{(n)}$ is the maximum rate that can be achieved when sub-channel n is assigned to the link from node g to node i.

Now we are ready to state the optimization objective as,

$$\max_{R,K} \quad \sum_{i\in\zeta}\frac{U_i}{\overline{r}_i} = \sum_{i\in\zeta}\frac{S_{m,i}R_m}{\overline{r}_i} + \sum_{i,g\in\zeta}\sum_{n\in\chi} K_{gi}^{(n)}\frac{R_{gi}^{(n)}}{\overline{r}_i} \tag{8}$$

As studied in the previous section, we use exhaustive search to get the optimal multicast rate. When we fix R_m in the search each time, $S_{m,i}$ can be determined. $\overline{r}_i$ is pre-determined since it is the average throughput before time slot t. Thus, the joint optimization problem can be decomposed, and the scheduling is reduced to the channel allocation problem for each search as stated in the following (denoted as *CA-NC*),

$$\max_{K} \quad \sum_{i,g\in\zeta}\sum_{n\in\chi} K_{gi}^{(n)}\omega_{gi}^{(n)} \tag{9}$$

where

$$\omega_{gi}^{(n)} = \frac{R_{gi}^{(n)}}{\overline{r}_i} \tag{10}$$

subject to (6), and following constraints (updating (3) - (5)):

$$0 \leq \omega_{gi}^{(n)} \leq \frac{C_{gi}^{(n)}}{\overline{r}_i} \qquad \forall g,i \in \zeta, n \in \chi \tag{11}$$

$$\sum_{n\in\chi} K_{gi}^{(n)}\omega_{gi}^{(n)} \leq \max\{0, \frac{B_g - B_i}{T\overline{r}_i}\} \qquad \forall g,i \in \zeta \tag{12}$$

$$\sum_{g\in\zeta}\sum_{n\in\chi} K_{gi}^{(n)}\omega_{gi}^{(n)} \leq \sum_{h=1}^{t-1}\frac{R(h)}{\overline{r}_i} - \frac{B_i}{T\overline{r}_i} + \frac{(1-S_{m,i})R_m}{\overline{r}_i} \tag{13}$$

Overall, we can get the optimal solution of joint optimization problem by exhaustive search and solving channel allocation problem. The procedure is stated in **Algorithm 1**.

However, the main problem of **Algorithm 1** is the difficulty of solving channel allocation problem *CA-NC*. It is a mixture integer program (MIP) which is NP hard in general. We formulate it to a maximum weighted bipartite matching (WBM) problem which is **equivalent** to the original problem and can be solved optimally with **polynomial** time complexity.

Construct a bipartite graph $A = (\Phi \times \chi, E)$. The vertices in Φ denote all the possible cooperative links (*e.g.* $(1,2)$ indicates the transmission link from node 1 to node 2. Note it is different from $(2,1)$ which represents the transmission link from node 2 to node 1). The set of sub-channels for cooperative transmissions is denoted by the vertex set χ.

Algorithm 1 Multicast scheduling with channel allocation

Set $Q = 0$.
for $m = 1$ to 6 **do**
  Set $Q_{MR} = \sum_{i \in \zeta} \frac{S_{m,i} R_m}{\overline{r}_i}$.
  Solve *CA-NC*. The optimal objective value is denoted as Q_{CA} and the optimal channel allocation is K_{CA}.
  if $Q_{MR} + Q_{CA} > Q$ **then**
    $Q = Q_{MR} + Q_{CA}$.
    $R_{OPT} = R_m$.
    $K_{OPT} = K_{CA}$.
  end if
end for
R_{OPT} is the optimal multicast rate and K_{OPT} is the optimal scheme for channel allocation.

The edge set E corresponds to $|\Phi| \times |\chi|$ edges connecting all possible pairs. The weight of each edge carries $\omega_{gi}^{(n)}$ as we defined in Eq. (10), which represents the maximum cooperative transmission rate that can be achieved if sub-channel n is assigned to link (g,i) subject to the proportional fairness criteria. In WBM, we initially set $\omega_{gi}^{(n)} = \frac{1}{\overline{r}_i} \min\{R_{gi}^{(n)}, \max\{0, \frac{B_g - Bi}{T}\}\}$. We exclude all links from Φ whenever $\omega_{gi}^{(n)} = 0$. $|\Phi|$ may be not equal to $|\chi|$. Thus we patch void vertices to χ or Φ to make $|\Phi| = |\chi|$. If a edge connects any void node, its weight is also set to be zero.

Given the above graphical setup, channel allocation problem can be solved by solving a WBM problem. The intuition is shown in Fig. 3. If vertex (g,i) in Φ and vertex n in χ are matched, we assign sub-channel n to link (g,i) and set $K_{gi}^{(n)} = 1$. The WBM problem can be solved using existing network flow algorithms such as the cost scaling algorithm [12].

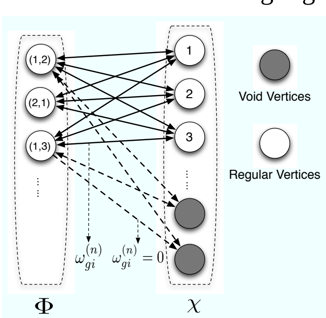


Fig. 3. Solving the channel allocation problem using maximum weighted bipartite matching algorithm.

Solving the WBM problem above may violate a few constraints. First, we consider constraint (12). The violation may happen when more than one sub-channels are assigned to one cooperative link, and the link capacity via multiple sub-channels may be over large. To solve this problem, we assign sub-channels by performing WBM in rounds. In each round, we remove the sub-channels that are already assigned in the previous round from set χ. Particularly, we update the weight on each edge by considering the constraint (12). Then, we solve the WBM problem in a new round.

Another constraint may be violated is (13). To solve this problem, we check whether the throughput of cooperative communication on each node i exceeds the upper limit at each round. If so, we favor the cooperative links with highest rates where efficient transmissions can be achieved. We assign sub-channels to those links and release the sub-channels assigned to other links. It is easy to find the solution by a simple search. After that, we have to omit all the links from Φ which cooperatively contribute to node i, since the maximum throughput on this node has already been reached. We can not assign any more sub-channels to these links in the following rounds. Overall, the approach is summarized in **Algorithm 2**.

Algorithm 2 Channel allocation algorithm using maximum weighted bipartite matching

Initiate $K_{gi}^{(n)} = 0, \forall (g,i) \in \Phi, \forall n \in \chi$.
Define $B_{gi} := \max\{0, \frac{B_g - B_i}{T\overline{r}_i}\}, \forall (g,i) \in \Phi$.
repeat
  Construct the bipartite graph, and patch the void nodes to make $|\Phi| = |\chi|$.
  Solve the WBM problem, and get the solutions as $K_{gi}^{(n)}$.
  for each $i \in \zeta$ **do**
    if (13) is violated **then**
      Define $T_g := \sum_{n \in \chi} K_{gi}^{(n)} \omega_{gi}^{(n)}, \forall g \in \zeta$.
      Define $D_1, D_2, \cdots, D_G$ to be the sorted array of T_g $(\forall g \in \zeta)$ in descending order.
      for $v = 1$ to G **do**
        if $\sum_{g' \leq v} D_{g'} \geq \sum_{h=1}^{t-1} \frac{R(h)}{\overline{r}_i} - \frac{B_i}{T\overline{r}_i} - \frac{(1-S_{m,i})R_m}{\overline{r}_i}$ **then**
          Define $\xi := \{g | \exists g' > v, \text{st. } T_g = D_{g'}\}$.
          Release channel assignment on $(g,i), \forall g \in \xi$.
          Exclude links (g,i) $\forall g \in \zeta$ from Φ .
          break
        end if
      end for
    end if
  end for
  for each $(g,i) \in \Phi$ **do**
    $B_{gi} = B_{gi} - \sum_{n \in \chi} K_{gi}^{(n)} \omega_{gi}^{(n)}$.
    for each $n \in \chi$ **do**
      $\omega_{gi}^{(n)} = \min\{\frac{R_{gi}^{(n)}}{\overline{r}_i}, B_{gi}\}$.
    end for
  end for
  Exclude the assigned channels in χ.
until All channels or all links are excluded

B. Channel Allocation with Channel Reuse

To fully utilize the available resources, we further exploit the advantages provided by the spatial reuse in the cooperative communication. It is straightforward that two links which do not include each other in the interference region could use the same sub-channels for communication without interference. The interference information in the network can be collected in a distributed fashion. If two nodes could correctly overhear the frequently exchanged handshake messages with each other (the transmission power is assumed to be equal for all nodes), we mark out that they are within each other's interference zone. An "interference table" I is defined as follows,

$$I_{ki} = \begin{cases} 1 & \text{If node } i \text{ is in interference zone of node } k \\ 0 & \text{Otherwise} \end{cases}$$

where $k, i \in \zeta$. Nodes will periodically update this table and send interference information to the BS. To prevent collision, channel reuse is not allowed in the interference zone. Thus, the channel assignment should follow the following constraints,

$$\sum_{i \in \zeta, i \neq k} I_{ki} \sum_{g \in \zeta} K_{gi}^{(n)} \leq 1 \quad \forall n \in \chi, \forall k \in \zeta \tag{14}$$

$$\sum_{g \in \zeta} K_{gi}^{(n)} \leq 1 \quad \forall n \in \chi, \forall i \in \zeta \tag{15}$$

Thus, the channel allocation problem should be updated with the consideration of channel reuse, which can be stated as (denoted as *CA-NC-reuse*),

$$\max_{K} \quad \sum_{i \in \zeta} a_i \tag{16}$$

$$\text{where} \quad a_i = \sum_{n \in \chi} \sum_{g \in \zeta} K_{gi}^{(n)} \omega_{gi}^{(n)} \tag{17}$$

subject to (11) - (15)

It is also a MIP, and we use the randomized rounding procedure (**Algorithm 3**) to solve it with **polynomial** time complexity.

Algorithm 3 Randomized rounding algorithm for channel allocation with channel reuse

1. Solve its relaxation (convex) with $K_{gi}^{(n)}$ being relaxed to [0,1]. Let the optimal fractional solutions be $K_{gi}^{*(n)}$ ($\forall g, i \in \zeta, \forall n \in \chi$).
2. Initiate $\hat{K}_{gi}^{(n)} = 0$ ($\forall g, i \in \zeta, \forall n \in \chi$).
3. **for** each $g, i \in \zeta, n \in \chi$ **do**
4.   Round $\hat{K}_{gi}^{(n)} = 1$ with probability $K_{gi}^{*(n)}$.
5.   **if** $\hat{K}_{gi}^{(n)} = 1$ **then**
6.   **if** (12) or (13) or (14) is violated **then**
7.   Set $\hat{K}_{gi}^{(n)} = 0$.
8.   **else**
9.   Set $\hat{K}_{ji}^{(n)} = 0$, $\forall j \neq g$.
10.   **end if**
11.   **end if**
12. **end for**
13. The optimal rounding solutions are $\hat{K}_{gi}^{(n)}$.

As designed in **Algorithm 3**, the rounding procedure ensures that all constraints are satisfied. We note (12), (13) and (14) are satisfied with high probability in practice, since the rate of cooperative communication is relatively much lower than the multicast rate and the transmission range of MSs is relatively much smaller than the serving area of the BS due to the power and bandwidth constraints. Thus, we can ignore them in the rounding procedure (line 6 in Algorithm 3). Now, we give the approximation factor for this randomized rounding algorithm under this assumption.

Lemma 1: $f_{1j}Y_1 + f_{2j}(1 - Y_1)Y_2 + \cdots + f_{lj}(\prod_{i=1}^{l-1}(1 - Y_i))Y_l \geq (1 - (1 - \frac{1}{l})^l) \sum_{1 \leq i \leq l} f_{ij} Y_i$ whenever $Y_i \geq 0$ for all i and $\sum_i Y_i \leq 1$ and $f_{1j} \geq f_{2j} \geq \cdots \geq f_{lj} \geq 0$.

Proof: refer to [13]. ■

Theorem 1: Algorithm 3 provides an approximation guarantee of at least $(1 - (1 - \frac{1}{G})^G)$, where G is the number of multicast users.

Proof: Without loss of generality, for each $i \in \zeta, n \in \chi$, assume that the sorted users are $1, 2, \cdots, G$ with $\omega_{1i}^{(n)} \geq \omega_{2i}^{(n)} \geq \cdots \geq \omega_{Gi}^{(n)} \geq 0$. The probability that sub-channel n is assigned to the link (u, i) in randomized rounding algorithm is $\prod_{j=1}^{u-1}(1 - K_{ji}^{*(n)}) K_{ui}^{*(n)}$, $\forall u \in \zeta$. Thus, the expected throughput contribution on node i to the objective function (16) can be stated as,

$$\sum_{u=1}^{G} (\prod_{j=1}^{u-1} (1 - K_{ji}^{*(n)}) K_{ui}^{*(n)}) \omega_{ui}^{(n)}$$

Using Lemma 1, we have,

$$\begin{aligned} \sum_{n \in \chi} \sum_{u=1}^{G} (\prod_{j=1}^{u-1} (1 - K_{ji}^{*(n)}) K_{ui}^{*(n)}) \omega_{ui}^{(n)} &\geq \\ \sum_{n \in \chi} (1 - (1 - \frac{1}{G})^G) \sum_{j \in \zeta} K_{ji}^{*(n)} \omega_{ji}^{(n)} &= \\ (1 - (1 - \frac{1}{G})^G) \sum_{n \in \chi} \sum_{j \in \zeta} K_{ji}^{*(n)} \omega_{ji}^{(n)} &= \\ (1 - (1 - \frac{1}{G})^G) a_i^* \qquad \forall i \in \zeta \end{aligned}$$

a_i^* is the throughput contribution of node i to the objective function (16) in the optimal fractional solution. Thus, we have the expected contribution of node i to the objective function in the rounding solution $E[\hat{a}_i]$ as:

$$E[\hat{a}_i] \geq (1 - (1 - \frac{1}{G})^G) a_i^*$$

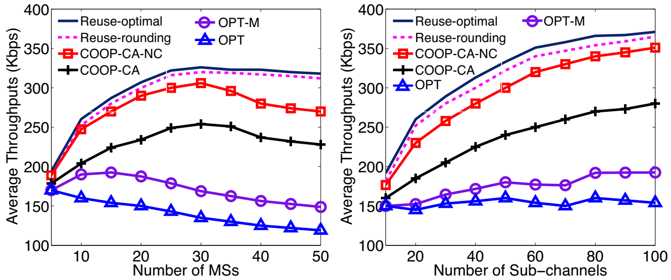
Thus, we have,

$$\sum_{i \in \zeta} E[\hat{a}_i] \geq (1 - (1 - \frac{1}{G})^G) \sum_{i \in \zeta} a_i^*$$

■

C. How efficient are the channels allocated?

To study the impact of the channel allocation and identify the performance gains offered by cooperative communication and random network coding with limited bandwidth resources, we perform a set of simulations under the same scenario in the previous section. Fig. 4(a) shows the average throughput over time (1000 seconds) as the function of increasing number of active MSs when the number of sub-channels is limited as 100. "COOP-CA-NC" which performs the multicast scheduling protocol with channel allocation as we designed in



(a) Throughput vs. No. of MSs (b) Throughput vs. No. of channels

Fig. 4. The performance of cooperative multicast scheduling with random network coding when the number of cooperative sub-channels is limited. The protocols with and without channel reuse algorithm are both evaluated.

this section beats the same protocol without random network coding ("COOP-CA") by 19%, and outperforms "OPT-M" and "OPT," by delivering 65% and 94% improvement respectively. It demonstrates: *by efficiently allocating sub-channels, cooperative communication with random network coding is helpful to achieve significant throughput improvement with very limited amount of bandwidth resources*. Further, we perform the simulations with fixed number of MSs, but with increasing number of sub-channels. Shown in Fig. 4(b), "COOP-CA-NC" outperforms others by a substantial margin. This improvement becomes more salient as the number of sub-channels increases. The intuition is: *more bandwidth resources for cooperative communication will benefit more on multicast performance*.

To evaluate the performance gains provided by channel reuse, we specifically conduct simulations by performing multicast scheduling with the design of channel reuse. From Fig. 4, we observe that multicast scheduling with channel reuse under randomized rounding algorithm (denoted as "Reuse-rounding") performs close to the optimum (denoted as "Reuse-optimal") within 95%. Moreover, "Reuse-rounding" further improves the throughput by 8% in average compared with "COOP-CA-NC" which already provides very satisfactory performance as we evaluated above. These results highlight the benefits achieved by our proposed protocols.

VI. Cooperative Multicast Scheduling with Power Allocation

One of the most critical problems in the practical systems is that the MS is very energy-constrained. Thus, the cooperative communication may not be fully performed with limited power on relays. In this section, we study the multicast scheduling from a different aspect, aiming to maximize the throughput by effectively allocating power on relays.

A. Maximizing Throughput with Limited Power

Let $S_{gi}^{(n)}$ denote the power that node g transmits data to node i if channel n is assigned on this link. S denotes the set of feasible power allocation schemes. As we note, the power for cooperative communication on each node is limited,

$$\sum_{n\in\chi}\sum_{i\in\zeta} S_{gi}^{(n)} \leq P_g \quad \forall g \in \zeta \tag{18}$$

where P_g is the power limit on each node.

Under the power constraint, we update constraint (11):

$$0 \leq \omega_{gi}^{(n)} \leq C_{gi}^{(n)} = BW/\overline{r}_i \cdot \log_2(1 + S_{gi}^{(n)}/\sigma_{gi}^{(n)}) \tag{19}$$

where BW denotes the channel bandwidth (the sub-channels are with equal bandwidth) and $\sigma_{gi}^{(n)}$ is the noise on the link.

Instead of only considering channel allocation in multicast scheduling as we designed in the previous section (*CA-NC* and *CA-NC-reuse*), we aim to optimize the performance by jointly accounting for both channel and power allocation. We state this new problem (denoted as *CA-PA*) as follows,

$$\begin{aligned} &\max_{K,S} \quad \textstyle\sum_{g,i\in\zeta}\sum_{n\in\chi} K_{gi}^{(n)}\omega_{gi}^{(n)} \\ &\text{subject to (12) - (15), (18) and (19)} \end{aligned} \tag{20}$$

We take dual problem by introducing a set of dual variables $\lambda_g \geq 0, g \in \zeta$. Thus, the objective (20) can be rewritten as,

$$\begin{aligned} &\max_{K,S} \quad \textstyle\sum_{g,i\in\zeta}\sum_{n\in\chi} K_{gi}^{(n)}\omega_{gi}^{(n)} \\ &+\textstyle\sum_{g\in\zeta}\lambda_g(P_g - \sum_{n\in\chi}\sum_{i\in\zeta} S_{gi}^{(n)}) \\ &\text{subject to (12) - (15), and (19)} \end{aligned} \tag{21}$$

As proved in [14], the original optimization problem (20) can be solved by solving its dual (21) with nearly zero duality gap when G is sufficiently large. We use the dual update method to solve the problem as shown in **Algorithm 4**.

Algorithm 4 Dual update method to solve joint channel and power allocation problem

Initialize λ (vector of the dual variables).
repeat
  Solve *CA-PA* with fixed λ.
  Update λ using the ellipsoid method [14].
until λ is converged.

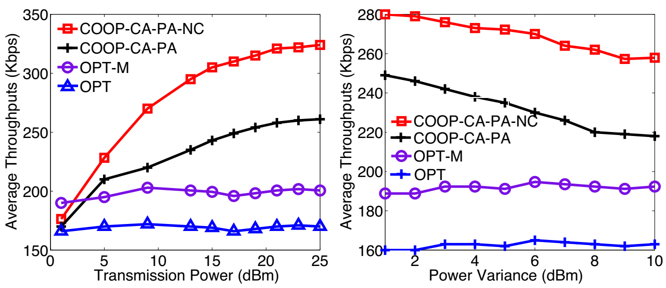
The hard part is to solve *CA-PA* even under fixed λ which is obviously nonconvex (MIP). Here, we adopt a heuristic approach with **polynomial** time complexity as given in **Algorithm 5**. This algorithm gives a good solution and λ always can be converged in various set-ups we tested.

Algorithm 5 Heuristic algorithm to solve joint channel and power allocation problem under fixed λ

Step 1: For the fixed λ, solve its relaxation (convex) with $K_{gi}^{(n)}$ being relaxed to [0,1]. Let the optimal channel allocation solutions be $K_{gi}^{*(n)}$ $(\forall g, i \in \zeta, \forall n \in \chi)$.
Step 2: Round $\hat{K}_{gi}^{(n)} = 1$ with probability $K_{gi}^{*(n)}$ $(\forall g, i \in \zeta, \forall n \in \chi)$. If $\hat{K}_{gi}^{(n)} = 1$, check whether all constraints are satisfied. Set $\hat{K}_{gi}^{(n)} = 0$ if not.
Step 3: Solve the convex optimization problem with fixed $\hat{K}_{gi}^{(n)}$, by taking $S_{gi}^{(n)}$ $(\forall g, i \in \zeta, \forall n \in \chi)$ as the variables. Let the optimal power allocation solutions be $S_{gi}^{*(n)}$.

B. What's the Impact of Power?

Finally, we evaluate the performance of our protocol with power allocation. The simulations are performed under increasing power limit at MSs, and Fig. 5(a) shows the average throughput across time and 50 MSs with same power limit. "COOP-CA-PA-NC" represents our cooperative multicast scheduling with random network coding, and especially applies both channel and power allocation algorithms as we designed in this section. It is not a surprise that "COOP-CA-PA-NC" outperforms all other protocols ("COOP-CA-PA" is the protocol with the same design as "COOP-CA-PA-NC" but without random network coding) with substantial gains. *By efficient power allocation, cooperative communication with random network coding could be well performed and achieve significant performance improvement, even with highly limited power on relays*. We observe from the results that the throughput increases dramatically as the transmission power rises up, which shows *more power the MSs use for cooperative communication could achieve more gains*.



(a) Throughput vs. Power Limit (b) Throughput vs. Standard Variance of Power Limit

Fig. 5. The performance of multicast scheduling with our power allocation algorithm in a power-constrained MBS.

Another set of simulations specifically study the impact of power on multicasting. We examine the throughput under increasing standard variances of power used for cooperative communication across different MSs. Fig. 5(b) shows that the throughput decreases as the variance increases. We can intuitively conclude from this observation: *maximum throughput performance gains can be obtained if each node performs cooperative communication by equally using its maximum power.* In our future work, we may study how to motivate MSs to make contributions to the networks for multicasting.

VII. Overhead Analysis

In closing, we study the protocol overhead. As BS has no power and computation constraints, we are only concerned with the computation overhead at MSs. Nowadays, even a mobile device like a cell phone has sufficient memory cache and strong computing ability. According to [11], random network coding is almost "free" with the current processors. Verified by our simulations, our protocols have an average running time of less than 5 ms (over Intel Core Duo machine running at 1.83 GHz and a memory of 2 GB), and are therefore suitable for typical WiMAX with scheduling durations of 5–10 ms. With respect to the communication overhead, the protocols require MSs to report the channel quality information (normally 5 bits per message) to the BS. This communication can be performed over the fast feedback channel in WiMAX and this channel state reporting is originally required in WiMAX standards [1]. Overall, our proposed protocols generate little communication overhead within practical limits.

VIII. Concluding Remarks

In this paper, we have studied — from a new perspective — the multicast scheduling problem. Previous work in the literature — almost without an exception — has solved the problem based on a shared-channel single-hop transmission model, which ignores the advantages provided by both channel and cooperative diversity in WiMAX where multiple channels are used. In contrast, we consider multicast scheduling with multi-hop multi-path transmissions over multiple OFDMA channels to fully exploit the advantages provided by cooperative communication and random network coding. The intuition is quite simple to narrate: cooperative communication with random network coding could favor the users with good channel conditions to enjoy high multicast flow rates from the source and cooperatively help others with poor channel conditions simultaneously with little overhead. We design multicast scheduling protocols which are tightly integrated with the design of WiMAX MBS, and study the critical problems of channel and power allocation for cooperative communication. Theoretical and practical solutions based on optimization are provided and further evaluated in extensive simulations. The highlight of this paper is our conclusion: multicast performance can be significantly improved by applying cooperative communication and random network coding with effective use of wireless spectrum.

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