APPENDIX

Proof of Theorem 3: The proof is based on the Chernoff Bound. For $v_i \in [0,1]$ independent random variables, let $S = \sum_i v_i, \nu = E[\sum_i v_i]$, then

$$P[S \leq (1-\delta)\nu] \leq e^{-\frac{\delta^2 \nu}{2}}.$$

Let $\Phi_i^1 = \sum_{c_1} \tilde{x}_i^{c_1} w_{i,j(i)}^{c_1}, \Phi_i^2 = \sum_{c_2} \tilde{y}_i^{c_2} w_{j(i),P}^{c_2}, \Phi_j^2 = \sum_{c_2} \tilde{y}_j^{c_2} w_j^{c_2}$. All $\Phi_i^1, \Phi_i^2, \Phi_j^2$ are sums of R.V's $\in [0,1]$. With $E[\Phi_i^2] = \hat{a}_i, E[\Phi_j^2] = \hat{b}_j$, we have the following

$$P[\Phi_i^2 \geq (1-\delta)\hat{a}_i] \geq 1 - e^{-\frac{\delta^2 \hat{a}_i}{2}}, P[\Phi_j^2 \geq (1-\delta)\hat{b}_j] \geq 1 - e^{-\frac{\delta^2 \hat{b}_j}{2}}.$$

Note that the aggregate marginal utility (flow) in the first time slot is obtained from the flow in the rounded solution in the second slot. Assume $\sum_{c_2} \tilde{y}_i^{c_2} w_{j(i),P}^{c_2} \geq (1-\delta)\hat{a}_i$. Since $E[\Phi_i^1 | \Phi_i^2] = \tilde{a}_i = \Phi_i^2/(1-\delta) \geq \hat{a}_i$, it can be shown that

$$P[\Phi_i^1 \geq (1-\delta)\hat{a}_i | \Phi_i^2] \geq 1 - e^{-\frac{\delta^2 \hat{a}_i}{2}}.$$

Now to ensure a net flow of at least $(1-\delta)\hat{a}_i$ at every PU and $(1-\delta)\hat{b}_j$ at every SU with high probability, we need:

$$P[\Phi_i^1, \Phi_i^2 \geq (1-\delta)\hat{a}_i] \geq \left(1 - e^{-\frac{\delta^2 \hat{a}_i}{2}}\right)^2 = \left(1 - \frac{1}{K^R}\right)^2,$$
$$P[\Phi_j^2 \geq (1-\delta)\hat{b}_j] \geq \left(1 - e^{-\frac{\delta^2 \hat{b}_j}{2}}\right)^2 = \left(1 - \frac{1}{K^R}\right)^2.$$

This results in $\delta = \sqrt{\frac{2\ln(K^R)}{\hat{a}_i}} = \sqrt{\frac{2\ln(K^R)}{\hat{b}_j}}$. Then the approximation bound B is given by

$$B = 1 - \sqrt{2\ln(K^R)} \frac{\sum_i \sqrt{\hat{a}_i} + \sum_j \sqrt{\hat{b}_j}}{\sum_i \hat{a}_i + \sum_j \hat{b}_j} \tag{1}$$

$\frac{\sum_i \sqrt{\hat{a}_i} + \sum_j \sqrt{\hat{b}_j}}{\sum_i \hat{a}_i + \sum_j \hat{b}_j}$ is maximized when $\hat{a}_i = \hat{b}_j = \hat{a} = \frac{\sum_i \hat{a}_i + \sum_j \hat{b}_j}{2N_S}$. By the normalized weight assumption $\sum_i \hat{a}_i + \sum_j \hat{b}_j \leq K^R$. If $c = \frac{\max_i \{w_{i,j(i)}^{c_1}, w_{j(i),P}^{c_2}\}}{\min_{i,j} \{w_{i,j(i)}^{c_1}, w_{j(i),P}^{c_2}\}} \geq 1$, then $\sum_i \hat{a}_i + \sum_j \hat{b}_j \geq \frac{K^R}{c}$. Substituting into (1), we have the following with high probability

$$B \geq 1 - \sqrt{\frac{4cN_S}{K^R} \ln(K^R)}.$$

$\square$