Distributed Topology Control in Wireless Sensor Networks with Asymmetric Links

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Abstract— **Topology control with per-node transmission power adjustment in wireless sensor networks has been shown to be effective with respect to prolonging network lifetime via power conservation and increasing network capacity via better spatial bandwidth reuse. In this work, we consider the problem of topology control in a network of heterogeneous wireless devices with different maximum transmission ranges, where asymmetric wireless links are not uncommon. In such an environment, we present a distributed topology control algorithm to calculate the per-node minimum transmission power, so that (1) reachability between any two nodes is guaranteed to be the same as in the initial topology; and (2) nodal transmission power is minimized to cover the least number of surrounding nodes. Analysis and simulation results demonstrate the correctness and effectiveness of our proposed algorithm.**

I. INTRODUCTION

Wireless sensor networks are formed by a collection of *power-conscious* wireless-capable sensors without the support of pre-existing infrastructure, possibly by unplanned deployment. Topology control via per-node transmission power adjustment has been shown to be effective in extending network lifetime and increasing network capacity (due to better spatial reuse of spectrum). The flip side of the coin is, with a reduced transmission range on each node, basic *reachability* from one node to another may be jeopardized. This problem is further exacerbated when we consider a network of heterogeneous wireless devices with *different maximum transmission ranges*, where **asymmetric** (or *uni-directional*) wireless links are not uncommon in the topology.

There exists considerable previous work addressing the topology control problem of minimizing nodal transmission power, with guarantees of network connectivity. For example, Wattenhofer *et al.* [1] proposed a fully distributed algorithm that only relies on directional information between nodes. Ramanathan *et al.* [2] presented a centralized topology control algorithm, along with a distributed heuristic. It has not discussed, however, guarantees on connectivity. Unlike the above deterministic guarantee of connectivity, Santi *et al.* [3] analyzed the connectivity of a sensor ad hoc network using a probabilistic approach in order to find out the minimum transmission power to be used at all nodes. The lower and upper bound on the probability of network connectivity are derived for certain transmission range assignments. Lloyd *et al.* [4] continued research towards this direction, with sound theoretical analysis on the properties of generic topology

control protocols in minimizing the maximum power adopted and the total energy consumed in the network. Rodoplu *et al.* [5] presented an topology control algorithm that is most similar to our proposal, requiring location information and working on vicinity topologies on each node in a distributed fashion. With the wealth of results related to topology control, *none* of the previous work has extensive discussions on the problem introduced by asymmetric (uni-directional) wireless links, and proposed algorithms tailored to this specific scenario.

When the existence of asymmetric links is not assumed in order to simplify the problem to tractable theoretical models, the following two issues are unavoidably introduced. First, if all links in the original topology are symmetric, it is impossible to assume the use of different transmission ranges among nodes. Second, if asymmetric links are allowed to exist in the finalized topology, the derived minimum-power topology may become more power-efficient since transmission ranges may be further reduced.

By placing asymmetric wireless links in the scope and spotlight of our work, we design a distributed topology control algorithm that enjoys the following favorable properties: First, the algorithm converges rapidly. For stationary sensor networks, the minimum power topology is finalized in a single pass. Second, the algorithm is not complex computationally, while still effective to guarantee the bi-directional multi-hop reachability between nodes in the network. Third, since information exchange between nodes is limited to the local neighborhood, the algorithm scales well to large networks.

The remainder of the paper is organized as follows. Sec. II describes our system model. Our distributed topology control algorithm is presented and analyzed in Sec. III and IV. In Sec. V, we show the correctness and effectiveness of our algorithm with simulation results. We conclude the paper and summarize its highlights in Sec. VI.

II. MODEL

In this work, we consider a wireless sensor network as a network of heterogeneous sensors, referred to as *nodes*. All nodes are arbitrarily deployed in a two-dimensional plane. Each node is equipped with an omni-directional antenna with adjustable transmission power. Since nodes are heterogeneous, they have different maximum transmission powers and radio ranges. For node i, we use P_i to denote its transmission power, $P_i^{\max}$ as its maximum transmission power (or, alternatively,

full power), and P_{ij} as the transmission power required for node i to reach node j. Under the assumption that the transmission medium is symmetric (and that asymmetric links are only due to the different ranges), we have $P_{ij} = P_{ji}$. Since $P_i^{\max} \neq P_j^{\max}$ for $i \neq j$, in the situation where $P_i^{\max} \geq P_{ij} > P_j^{\max}$, there exists an asymmetric link $\overrightarrow{L_{ij}}$ in the network topology since $P_{ji} > P_j^{\max}$ (impossible for j to reach i with its full power). Our work focuses on such asymmetric links.

Due to the existence of asymmetric links, the topology where each node transmits with its maximum transmission power is naturally a directed graph, referred to as the *maximum topology* $\overrightarrow{G} = (V, \overrightarrow{L})$. $\overrightarrow{G}$ can be either strongly connected, weakly connected, or disconnected. In a *strongly connected* $\overrightarrow{G}$, there is a directed, possibly multi-hop, path from any source to any destination. In a *weakly connected* $\overrightarrow{G}$, there exists pairs of nodes that only one of them can reach the other via multiple hops. Finally, in a *disconnected* $\overrightarrow{G}$, their exist pairs of nodes that can not reach each other.

The objective of our distributed topology control algorithm is to derive a *minimum-power topology* $\overrightarrow{G'}$ that is strongly connected, guaranteeing multi-hop reachability from any source to any destination in the directed graph. We assume that the algorithm begins with a strongly connected maximum topology. The topology control algorithm is assumed to start with a *strongly connected* $\overrightarrow{G}$. The need of such an assumption may be easily derived by contradiction.

When a node, such as node i, sends a message in the network, it broadcasts the message at a specific power level, in the range of $(0, P_i^{\max}]$. We use the path loss model commonly adopted by previous work [5], [6], where the power of the received signal is found to have a distance dependence of $1/d^n$, where d is the propagation distance and exponent n ranges from 2 to 5 depending on the environment. Despite this simplifying assumption, our algorithm works correctly with any path loss models as long as a node knows the path loss models of its neighboring nodes, which is achievable via local notifications without violating the distributed nature of the algorithm. We further assume that location information (x_i, y_i) is available to node i, for all nodes in the network. However, node i is not aware of the locations of other nodes in the network.

Finally, our solution is based on the existence of a MAC layer protocol [7] or sub-routing layer [8] that are aware of asymmetric links to ensure the network protocols function correctly with the presence of asymmetric wireless links.

III. ALGORITHM

Our topology control algorithm starts with the strongly connected maximum topology $\overrightarrow{G}$ of a wireless sensor network, and generates its minimum-power topology $\overrightarrow{G'}$ with a guarantee of the same bi-directional (and possibly multi-hop) reachability between any node-pairs. It is a fully distributed algorithm since each node runs the algorithm based on its local information, and possibly at different times. No synchronization is required among nodes in the network. After every node in $\overrightarrow{G}$ finishes running the algorithm, the network topology converges to $\overrightarrow{G'}$. Without loss of generality, we focus on an arbitrary node, i, and present the algorithm in three phases.

A. Phase 1: Establishing the vicinity topology

The skeleton is described as follows. Node i broadcasts a message, referred to as the *initialization request* (IRQ) message, using its maximum transmission power $P_i^{\max}$. The set of nodes that receive the IRQ message are referred to as the *vicinity nodes* of node i, denoted as V_i. The IRQ message includes the location of $i, (x_i, y_i)$, as well as $P_i^{\max}$. Upon receiving such an IRQ message, each node j in V_i replies to node i with an *initialization reply* (IRP) message, with its location (x_j, y_j) and $P_j^{\max}$.

In order for nodes in V_i to decide the transmission powers for sending the IRP messages, we discuss the following two cases.

(1) For a node $j \in V_i$, if $P_j^{\max} \geq P_{ij}$, j can reach node i via the single-hop link $\overrightarrow{L_{ji}}$.

(2) If $P_j^{\max} < P_{ij}$, j must find a multi-hop path to reach i. There are at least three solutions. (a) j uses $P_j^{\max}$ to broadcast its IRP message with a special bit toggled to signal that the IRP may need to be relayed. When any other nodes receive such an IRP not addressed to themselves, they assist with relaying the message by re-broadcasting with their maximum transmission power. (b) j can send the IRP message via network layer packet routing protocols to i. Due to the previous assumption of a strongly connected $\overrightarrow{G}$, there exists a directed multi-hop path for node j to reach node i. A better approach in this case is to have j piggy-back the IRP message to i when it sends data packets to node i. (c) Node j takes advantage of the services provided by the sub-routing layer [8] to pass the IRP message back to i.

Having the knowledge of the locations and maximum transmission powers for itself and all its vicinity nodes, and under the assumption that the path loss models of all vicinity nodes are coherent, node i may derive the existence of the *vicinity edges*. For any two nodes $j, k \in V_i$, link $\overrightarrow{L_{jk}}$ is defined as one of i's vicinity edges, if $P_j^{\max} \geq P_{jk}$. Consequently, node i constructs its local *vicinity topology* that includes all its vicinity nodes, itself and the discovered *vicinity edges*. If node i's vicinity topology is denoted as $\overrightarrow{G_i}$, and the collection of its vicinity edges is denoted as $\overrightarrow{E_i}$, we obtain a weighted, directed graph with source vertex i and weight function $w : \overrightarrow{E_i} \to \mathbf{R}$:

$$\overrightarrow{G_i} = (V_i, \overrightarrow{E_i})$$

where the weight for each directed edge, $w(u_i, u_j)$, is the power required to reach u_j from u_i on the edge $u_i \to u_j$, equivalent to $P_{u_i u_j}$.

B. Phase 2: Deriving the minimum-power vicinity tree

With the knowledge of the weighted, directed topology $\overrightarrow{G_i}$, the weight, W_l, of a directed path $l = u_0 \to u_1 \to \ldots \to u_k$ from node u_0 to u_k is the sum of edge weights along the path, *i.e.*, $W_l = \sum_{m=1}^{k} w(u_{m-1}, u_m)$. The minimum power for node i to reach j is $\min(W_p)$ for all available paths p
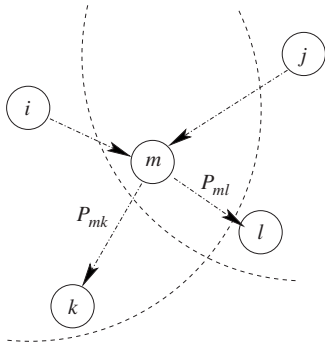
Fig. 1. An example of convergence on a node's transmission range

To be executed at each node $i \in V$:
    Find $\overrightarrow{G_i}$ with full power $P_i^{\max}$
    $\overrightarrow{G_{is}} = \text{ShortestPath}(\overrightarrow{G_i})$
    for (each link $\overrightarrow{E_{jk}} \in \overrightarrow{G_{is}}$)
        Node i sends P_{jk} to node j

To be executed at each node $j \in V_i$:
    Extract P_{jk} from PR message from node i
    $P_j = \max(P_{jk}, P_j)$

from i to j. In essence, we attempt to find the shortest path in $\overrightarrow{G_i}$ from i to j. In this case, node i may execute a single-source shortest-paths algorithm, such as the Bellman-Ford or Dijkstra's algorithms (since edge weights are nonnegative), to derive the *minimum-power vicinity tree* $\overrightarrow{G_{is}} = (V_{is}, \overrightarrow{E_{is}})$. In fact, $\overrightarrow{G_{is}}$ is a typical shortest-paths tree from i to all other nodes in V_i, with the following additional properties:

- *Property 1.* Since there does not exist unreachable nodes in the in $\overrightarrow{G_i}$, we have $V_{is} = V_i$, and $\overrightarrow{E_{is}} \subseteq \overrightarrow{E_i}$.
- *Property 2.* The derivation of $\overrightarrow{G_{is}}$ depends solely on the edge weights, which does not assume a specific propagation model. However, with different path loss models, $\overrightarrow{G_{is}}$ may be different.

C. Phase 3: Propagation of transmission powers

In this phase, node i needs to calculate the transmission power needed for itself and each vicinity node in V_i, to ensure that all its minimum-power paths exist in the final minimum-power network topology. Specifically, for node i itself and each node in set V_i, the transmission power is assigned as the power required to reach the furthest one-hop downstream nodes in node i's minimum-power vicinity tree $\overrightarrow{G_{is}}$. Node i first adopts the minimum power assigned to itself, and then sends the minimum power required for each vicinity node with a explicit *power request* (PR) *message*.

Upon receiving the PR message, a vicinity node j compares the power requirement from i with its current power setting. If i requires a stronger transmission power at node j, node j increases its power accordingly. Otherwise, it discards the PR message. Note that its existing setting is assigned by itself or any other nodes that have executed the algorithm earlier than node i and propagated the PR message.

For example, in Fig. 1, we observe that node m is a vicinity node of both i and j, *i.e.*, $m \in \overrightarrow{V_{is}}$ and $m \in \overrightarrow{V_{js}}$. Given that $P_{mk} > P_{ml}$, if node i runs the algorithm first, P_m is set as P_{mk} by node i. When node j executes the algorithm at a later time, P_m is required to be P_{ml}. Although the current power setting on node m is $P_{mk} > P_{ml}$, node m can not reduce its transmission power to P_{ml} since it should not violate the reachability requirement in node i's minimum-power tree. Finally, our proposed algorithm is summarized in Table I.

D. Further Optimizations

The resulting minimum-power topology can be enhanced through the application of expiration or discard schemes of the PR message without violating the connectivity requirement. Particularly, (a) all PR messages expire at a node upon its completion of our algorithm; (b) a node discards the PR message that asks it to reach a node already listed as one of its vicinity nodes.

In the previous discussion, a node, such as node j, receives the PR messages from other nodes for it to act as routers for them in order to reach the downstream nodes in their minimum-power trees. However, we further observe that (a) that specific downstream node is one of node j's vicinity nodes; (b) node j may be able to find a shorter route to reach that downstream node through relaying at other vicinity nodes. Based on observation (a), as long as node j can reach that downstream node through a single-hop or multi-hop path, the connectivity for other nodes is not affected; based on the observation (b), in case node j finds a shorter path to that downstream node compared with the direct wireless link, it may further reduce its transmission power. Therefore, node j can safely discard all the PR messages it has received upon the completion of shortest path algorithm on its vicinity topology without violating the connectivity requirement from other nodes.

An example of the scenario is illustrated in Fig. 2. In this scenario, node A finds that the most power-efficient path to reach node C is $\overrightarrow{L_{AB}} \to \overrightarrow{L_{BC}}$ due to the fact that $P_{AB} + P_{BC} < P_{AC}$. As a result, node A sends a PR message to node B to have $P_B \geq P_{BC}$ such that node B can relay traffic from node A to node C. However, when node B executes the shortest path algorithm over its vicinity topology, it finds that rather than reaching node C directly, it is more power-efficient to reach node C via node D. This shorter path is unknown to node A as node D is not node A's vicinity node. Hence, node B sets its transmission power as P_{BD} rather than P_{BC} as indicated in the PR message from node A to achieve higher power efficiency. Eventually, node A is connected to node C through $\overrightarrow{L_{AB}} \to \overrightarrow{L_{BD}} \to \overrightarrow{L_{DC}}$.

Such expiration of PR messages does not imply that PR messages can be removed from our algorithm completely.

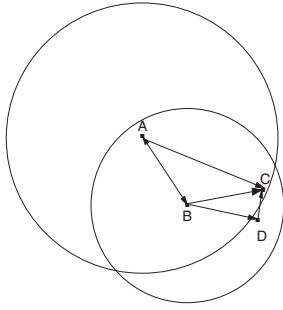Fig. 2. A scenario of further optimized nodal transmission range

Fig. 3. Two ways for node i to reach node j.

The time interval between two subsequent executions of our algorithm is expected to be long due to the stationary nature of the network. A new node may join the network during the time interval. The existence of the PR messages makes the algorithm adaptive to such topological changes.

IV. PROPERTIES AND ANALYSIS

We show that our proposed algorithm converges to a minimum-power network topology with the same reachability compared with the maximum topology, and is scalable to large-scale sensor networks.

A. Scalability

Our algorithm is a fully distributed algorithm to be executed on each node in the network. Since every node in the network can run the algorithm independently based on its local information, despite the size of the network, the execution of our algorithm is limited to its vicinity topology only. Moreover, the message exchange is restricted to its vicinity topology as well. Thus, our algorithm is scalable to networks composed of a large number of nodes.

Furthermore, due to the asynchronous execution at each node, our algorithm reduces the control overhead for synchronization when networks get larger, so that it is scalable to larger networks.

Finally, when the network becomes denser with more nodes, our algorithm can adjust each node's transmission power at each node to ensure that it is the minimum to guarantee the network connectivity. The average node degree and consequently the network contention level are consistent, despite the node density of the maximum topology. Therefore, our algorithm is scalable to the large and dense networks in terms of average node degree and the congestion levels in the network.

B. Convergence of the algorithm

In a sensor network, our algorithm is executed at each node periodically and independently. The duration of the period is a uniform system parameter. It depends on the nature of the sensor network, particularly, how dynamic the network topology is. The period tends to be shorter for a network with more frequent topological changes.

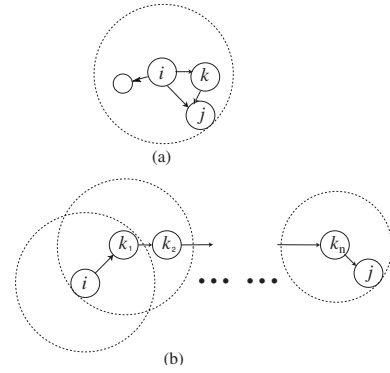As the algorithm is fully distributed, a node assigns the transmission powers with its local information only. The power assignment does not rely on other nodes' decisions about their powers. Therefore, the network converges to the final minimum-power topology once every node completes the execution of our algorithm.

Since the nodes may start running our algorithm at any time, the convergence time for our algorithm varies. We investigate the upper bound of the convergence time. For the worst case, given that the interval between two subsequent executions is T and the earliest node starts the execution at time 0, the latest node may initiate the execution at an instant just before time T during the iteration. The network converges in a duration of T. Therefore, the convergence time of our algorithm is in the range of $(0, T]$.

C. Guarantee of the network connectivity

Having shown the convergence property, the resulting minimum-power topology, $\overrightarrow{G'} = (V', \overrightarrow{E'})$, also demonstrates the following properties.

Theorem 1. The minimum-power topology guarantees the same reachability between any two nodes compared with the maximum topology, *i.e.*, it is strongly connected since the maximum topology is strongly connected.

Proof: Without loss of generality, we randomly take two nodes, node i and node j as an example. Since the maximum topology is strongly connected, there exists a directed path for node i to reach node j. We need to show that node i is still able to reach node j in the final minimum-power topology.

In the maximum topology, node i can reach node j in two ways: (a) node j is one of node i's vicinity nodes; (b) node i can reach node j via relaying at intermediate nodes such as a list of nodes $k_1, k_2, \ldots, k_n$. Both cases are shown in Fig. 3.

Case (a): Node $j \in V_i$. Based on the definition of vicinity nodes, link $\overrightarrow{L_{ij}}$ exists in the maximum topology. After running the shortest path algorithm, although $\overrightarrow{L_{ij}}$ may no longer be present in node i's minimum-power vicinity tree (due to the existence of a more power-efficient path for node i to reach node j, e.g., via $\overrightarrow{L_{ik}}$ and $\overrightarrow{L_{kj}}$), the reachability from node i to node j still exists. Since Phase 3 of the algorithm is designed to ensure every path in node i's minimum-power vicinity tree is valid, node i can definitely reach node j in the minimum-power topology of the network.

Case (b): Node i reaches node j via relaying at intermediate nodes, such as nodes $k_1, k_2, \ldots, k_n$. Thus, we have $k_1 \in V_i$, $k_2 \in V_{k_1}, \ldots, j \in V_{k_n}$. Subject to the results of case (a), in the minimum-power topology, node i can reach node k_1, node k_1 can reach node k_2, which continues until node k_n eventually reaches node j. As a result, node i can reach node j in the minimum-power topology of the network. □

D. Approximation of minimum power

Theorem 2. The transmission power at each node in the minimum-power topology approximates the minimum to guarantee the network connectivity.

Proof: According to our proposed algorithm, in the resulting topology, for any node i, $P_i = P_{ij}$, where $P_{ij} = \max\{P_{ik} | k \in$ {node i's one-hop downstream nodes in $\overrightarrow{G_{is}}$}}.

We first assume that for $P_i < P_{ij}$, the network can still be strongly connected and meet the minimum-power requirement. In this case, $\overrightarrow{L_{ij}}$ no longer exists in node i's minimum-power vicinity tree. Since node j is a one-hop downstream node of node i in node i's minimum-power vicinity tree, the elimination of link $\overrightarrow{L_{ij}}$ may imply that (1) node i is still able to reach node j, and consequently downstream nodes of node j in node i's minimum-power vicinity tree, via multi-hop paths which are not as power-efficient as the direct link $\overrightarrow{L_{ij}}$; or (2) node i is isolated from both node j and some (if not all) downstream nodes of node j in node i's minimum-power vicinity tree. Either of the two consequences leads to the failure to achieve a minimum-power topology with the guarantee of the network connectivity.

It is observed that although link $\overrightarrow{L_{ij}}$ is the most power-efficient path for node i to reach node j in its vicinity topology, there may exist a shorter path through a node which is not node i's vicinity node. For example, Fig. 4 illustrates that there may be a more power-efficient path for node i to reach node j through node k and node m. Since node m is not node i's vicinity node, $\overrightarrow{L_{km}}$ is not present in node i's vicinity topology. Node i has no knowledge of the existence of such a path based on its local information only. The adoption of a sub-optimal power P_{ij} is caused by the lack of global information. In a dense network, the probability of such cases is low, as multi-hop connections are common in node i's minimum-power vicinity tree. Hence, our distributed approach approximates the minimum-power topology that can be established by a centralized solution.

To generalize, the transmission power of each node in the resulting topology from our algorithm approximates the minimum-power requirement to guarantee the network connectivity. □

V. SIMULATION

In order to show the correctness and effectiveness of our algorithm, we carry out experiments to measure the reachability, power efficiency and scalability of the final topology. We consider networks with both directed and undirected maximum topologies. There are n nodes uniformly distributed in a network area of 100 meters by 100 meters, where n is in
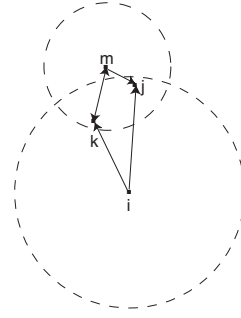

Fig. 4. An example of the existence of shortest path in node i's vicinity topology

the range of $[2, 50]$. The path loss model adopted is $1/d^2$. The maximum transmission power of every node is 50 meters to form a network with undirected maximum topology. The maximum transmission power of every node is randomly distributed in the range of $[40m, 60m]$ to form a maximum topology with directed maximum topology.

A. Reachability

In order to measure if the reachability between any two nodes are the same after the execution of our algorithm, we introduce a parameter, *reachability failure ratio*, which is denoted as γ. Reachability failure ratio is defined as the ratio of the count of reachability failures between any two nodes in the network over n^2.

In the maximum topology, since the network is strongly connected, every node can reach other nodes through multi-hop connection. The reachability failure ratio $\gamma = 0$. In a topology that every node can only reach itself, reachability failure ratio hits the maximum, which is $(n-1)/n$.

In networks with undirected maximum topology, the experimental results show that as long as $\gamma = 0$ is true for the maximum topology, $\gamma = 0$ always holds in the final topology when the number of nodes in the network varies from 2 to 50. The experiments demonstrate the same result in networks with directed maximum topology. We can conclude that the final minimum-power topology generated by our topology control algorithm guarantees the reachability between any two nodes in the network.

B. Power efficiency

Power efficiency is defined as the ratio of the total saved transmission power over the total maximum transmission power at all nodes and denoted as η. In the worst case that each node still uses its maximum transmission power after the execution of the topology control algorithm, the power efficiency is the minimum value, $\eta = 0$. The higher the power efficiency, the more power saved on transmitting in the network. Theoretically, the upper bound of η is 1. However, practically, η can never be 1 as the transmission power can not be 0 in order to ensure the network connectivity.

From the experimental results, when the network is sparse with fewer than 10 nodes, the average power efficiency is 0.5,
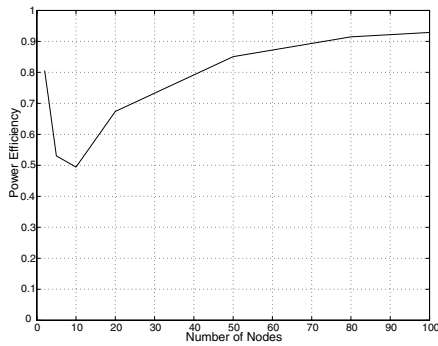
Fig. 5. Power efficiency in the networks with undirected maximum topology
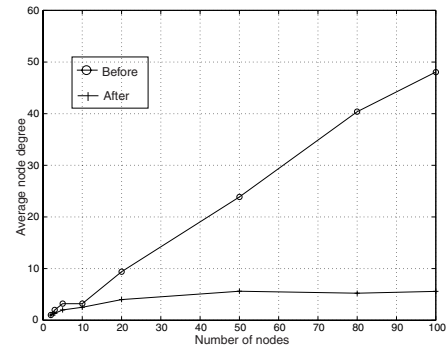

Fig. 7. Average node degree in networks with undirected maximum topology
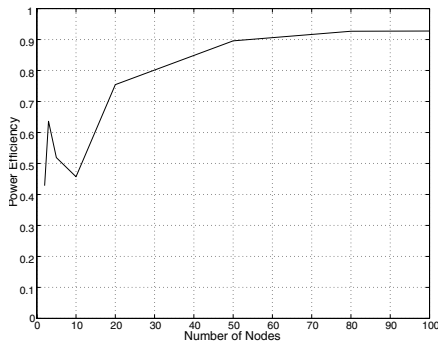

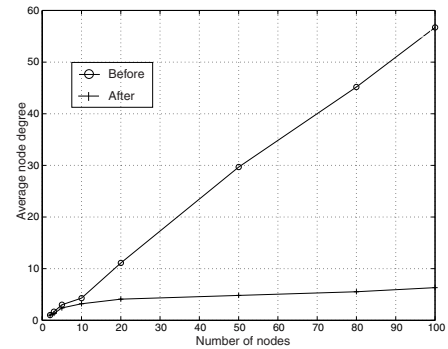Fig. 6. Power efficiency in the networks with directed maximum topology

Fig. 8. Average node degree in networks with directed maximum topology

which means that on average, a node can save 50% of the transmission power. Power efficiency is low in order to ensure the connectivity among the sparsely deployed nodes.

When the network becomes denser with more nodes, the achieved power efficiency increases to $80\% - 90\%$. This is because with more nodes in a certain area, the nodes are closer to each other. Hence, the nodes need lower transmission power to ensure the reachability between them.

C. Scalability

To evaluate the influence of our algorithm on the larger networks, we consider average node degree in the networks with both undirected and directed maximum topologies before and after the execution of our algorithm. The results are shown in Fig. 7 and 8 respectively.

In both Fig. 7 and 8, in the maximum topology, when the number of nodes increases, a node has more neighbors and the average node degree increases consequently. In comparison, with our algorithm, despite the number of nodes in the network, by adjusting the transmission power at each node, the average node degree remains constant. Furthermore, we observe that this achievement is done through localized message exchange and individual independent decision. Therefore, our algorithm is scalable to larger networks.

VI. Concluding Remarks

In this work, we propose a simple yet efficient distributed topology control algorithm. Through analysis and simulation,

we prove that our algorithm provides a solution to the topology control problem in a network of heterogeneous wireless devices with different maximum transmission ranges. The resulting minimum-power topology is shown to guarantee that (a) reachability between any two nodes is guaranteed to be the same as the maximum topology; and (b) nodal transmission range is minimized to cover the least number of surrounding nodes.

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